

# Logistic Regression Analytically Solves the 3D Navier Stokes Equations

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## Abstract

The velocity  $\mathbf{u} = 2\nu P(1-P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$  and pressure  $p$ , where  $p, \mathbf{u} \in C^\infty(\mathbb{R}^3 \times [0, \infty))$ ,  $P = \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}}$  have been verified and validated in commercial softwares that has implemented Vector Calculus.

First, fluid incompressibility is an essential condition for eliminating the term nonlinear

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = (\nabla \cdot \mathbf{u}) \begin{pmatrix} 2\nu P(1-P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ and in parallel simplify the viscosity term, } \nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}),$$

giving a manageable form of the 3DNS equations,  $\frac{\nabla p}{\rho_0} = -\frac{\partial \mathbf{u}}{\partial t} - \nu \nabla \times (\nabla \times \mathbf{u})$ .

Where,  $p - p_0 = -2P^2 \nu \rho_0 (k + 2\nu\mu_1^2 - 4\nu\mu_2^2 + 2\nu\mu_3^2 - 4P\nu\mu_1^2 + 8P\nu\mu_2^2 - 4P\nu\mu_3^2)$

Second, we obtain the explicit form of the pressure  $p$ , of the vorticity  $\mathbf{w} = \nabla \times \mathbf{u}$  and calculates the integral of the energy in all space, showing that it is limited.

Third, there is a set of polynomials, which also solve the 3DNS equations, with the fundamental polynomial being  $P(1-P)$ , which represent the probability density function for logistic. Finally, it is shown that logistic regression complements the 3DNS solution.

**Keywords:** 3D navier stokes, logistic regression, pythagorean theorem

## 1. Introduction

The 3D Navier Stokes equations are a set of nonlinear equations in partial derivatives, which describe the dynamics of Newtonian fluids and are the result of applying two physical principles of fluid mechanics: conservation of momentum and mass.

Its origin is in the contributions of mathematics and physics provided by two scientists, a French and an Irish; the French engineer and physicist expert in Mechanics of Continuous Media, Claude Louis Navier (1785-1836) and the mathematical physicist expert in Vector Calculus, George Gabriel Stokes of Irish nationality (1819-1903).

The question of whether smooth solutions exist for all time (global existence) or for arbitrarily large initial data is still an open problem. The existence or non-existence of global solutions to the Navier-Stokes equations in three dimensions is one of the unsolved problems in mathematics, known as the Navier-Stokes existence and smoothness problem, (Fefferman, 2017).

There are advances in the formal study of the NS3D equations, as well as their main properties in two dimensions, (Leray, 1934).

The Navier-Stokes equations are highly nonlinear partial differential equations that describe the motion of viscous fluids. They involve the velocity and pressure fields and include a nonlinear convection term and a viscous diffusion term. One of the challenging aspects of studying the Navier-Stokes equations is the presence of possible singularities or blow-up phenomena in the solutions, (Caffarelli et al., 1982).

Theoretical and experimental applications of the Navier Stokes 3D equations are essentially in fluid mechanics and statistics, (Landau & Lifshitz, 1987); (Girault & Raviart, 1986); (Huang, 1987). The 3DNS equations are employed in studying the interaction between fluids and solid structures. This includes the behavior of fluids in pipes, channels, and pipelines, as well as the response of structures subjected to fluid forces, such as bridges, dams, and offshore structures. The 3DNS equations form the foundation of fluid dynamics, providing a mathematical framework to study the motion and

behavior of fluids. They are used to analyze a wide range of fluid flows, including laminar and turbulent flows, boundary layers, jets, vortices, and more.

The atomic nucleus behaves as an incompressible fluid that also complies with 3DNS, which has allowed us to find some useful parameters in the advancement of nuclear physics and its applications, (Polyanin et al., (2002); (Kulish, 2002); (Auerbach & Yeverechyahu, 1975).

The predictive, explanatory and deductive power of mathematics is evident in the solution of NS3D equations, especially in applications in engineering, fluids, nuclear physics and climate. (Oganessian & Utyonkov, 2015); (Wong, 2004).

On the other hand, the use of artificial intelligence (AI) in three-dimensional Navier-Stokes equations has become an active field of research in fluid simulation and computational fluid dynamics (CFD). Artificial intelligence-assisted fluid dynamics (AI-aided CFD), focus on reducing the dimensionality of the problem, speeding up computations, and facilitating its uses in medicine and engineering. (Pohl et al., 2010); (Geant 4, 2016).

Also, in mechanical engineering, neural networks can be used to learn more compact representations of velocity and pressure fields, thus speeding up computation time.

Some practical applications of the 3DNS equations to Newtonian petroleum fluids have been developed to study the dynamics of catalysis in petroleum refining, giving successful results as shown below. (Calderon et al., 2021); (Jiménez et al., 2020); (Jiménez et al., 2019)

Modern approaches in the analysis and solution of the 3DNS equations have appeared, creating new perspectives that will allow to deepen the study of black holes and chaotic systems that appear in the evolution of hurricanes and waterspouts that negatively impact worldwide, which may be produced by climate change and the greenhouse effect. The most relevant researches are: (Fré & Trigiante, 2023); (Singh, 2023).

Finally, direct numerical simulations (DNS) are a computationally intensive approach in which the Navier-Stokes equations are solved fully explicitly, without making any assumptions or approximations. These simulations probabilists and determinists are based on very fine grids and require high computational power, with results close to those obtained by an analytical solution. (Riley et al., 1999, p. 157); (Bertozzi & Majda, 2002, p. 13).

To date (2023) there is not a properly accepted and generalized analytical solution for 3DNS, which is recognized by the international scientific community, in this sense the general analytical solution proposed in this article should be easy, powerful and with graphical implications of fluid dynamics with turbulence and vortices involved.

This new proposed analytical solution method meets the requirements of the 3DNS problem, being these: 1.- the handling of the nonlinearity of the 3DNS partial differential equations, 2.- the efficient management of the compressibility equation which is aged both in the nonlinear part and in the viscosity term with Laplacian, and 3.- the capture of the nonlinearity through the Navier Stokes polynomials. Moreover, these polynomials depend on a logistic probability function that integrates the methodological richness of the analysis and the probability theory, 4.- The vortices are managed in a simple way with the rotationals of the velocity vector field, whose components depend on the logistic probability function, and 5.- finally, by eliminating the nonlinear part of the 3DNS equations, we only have to apply the Taylor Theorem in three dimensions in order to find the pressure gradient and the pressure. With which the problem is completely solved.

The proposed method was not chosen, it simply chose us and could be identified when it effectively solved the 3DNS problem.

**2. Model**

Fluid velocity is defined as  $\mathbf{u} = 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$ , where  $P(x, y, z, t)$  is the logistic probability function which We called

strategic variable  $P(x, y, z, t) = \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}}$ . The term  $P(x, y, z, t)$  is defined in  $((x, y, z) \in \mathbb{R}^3, t \geq 0)$ , where constants  $k > 0, \mu_1 > 0, \mu_2 > 0, \mu_3 > 0$  and  $P(x, y, x, t)$  is a main function to build the general solution of the Navier-Stokes 3D equation, which has to satisfy the conditions (1) and (2), allowing us to analyze the dynamics of an incompressible fluid. (Pohl et al., 2010); (Geant 4, 2016); (Riley et al., 1999).

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \nu \nabla^2 \mathbf{u} - \frac{\nabla p}{\rho_0} \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \tag{1}$$

We will use an equation analogous to concentration equation of Physical Chemistry  $C = C_0 e^{kt}$ , where  $k$ , is velocity coefficient,  $p_0$  is the initial pressure of our fluid,  $\eta$  the dynamic viscosity and  $C_0$  the initial concentration of energetic fluid molecules.

It is evident that, in equilibrium state we can write  $\mu_1x + \mu_2y + \mu_3z - kt = 0$ , however, the Navier-Stokes equation precisely measures the behavior of the fluids out of equilibrium, so that:  $\mu_1x + \mu_2y + \mu_3z - kt \neq 0$ .

With,  $\mathbf{u} \in \mathbb{R}^3$  a known velocity vector,  $\rho_0$  constant density of fluid,  $\eta$  dynamic viscosity,  $\nu$  cinematic viscosity, and pressure gradient  $\nabla p$  in  $((x, y, z) \in \mathbb{R}^3, t \geq 0)$ .

Velocity and pressure are depending of  $(x, y, z)$  and  $t$ . We will write the condition of incompressibility.

$$\nabla \cdot \mathbf{u} = 0 \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \tag{2}$$

The initial conditions of fluid movement  $\mathbf{u}^0(x, y, z)$ , are determined for  $t = 0$ . Where speed  $\mathbf{u}^0$  must be  $C^\infty$  divergence-free vector.

$$\mathbf{u}(x, y, z, 0) = \mathbf{u}^0(x, y, z) \quad ((x, y, z) \in \mathbb{R}^3) \tag{3}$$

For physically reasonable solutions, we make sure  $\mathbf{u}(x, y, z, t)$  does not grow large as  $x \rightarrow \pm\infty, y \rightarrow \pm\infty, z \rightarrow \pm\infty$ . We will restrict attention to initial conditions  $\mathbf{u}^0$  that satisfy.

$$|\partial_x^\alpha \mathbf{u}^0| \leq C_{\alpha K} (1 + r)^{-K} \quad \text{on } \mathbb{R}^3 \text{ for any } \alpha \text{ and } K \tag{4}$$

The Clay Institute accepts a physically reasonable solution of (1), (2) and (3), only if it satisfies:

$$p, \mathbf{u} \in C^\infty(\mathbb{R}^3 \times [0, \infty)) \tag{5}$$

and the finite energy condition named bounded energy (Riley et al., 1999); (Bertozzi & Majda, 2002).

$$\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz \leq C \quad \text{for all } t \geq 0 \tag{6}$$

The function  $P$  permit us to build the general solution of the Navier Stokes equations, which permit us to satisfy conditions (1),(2),(3),(5) and (6). An introductory form of the use of Gaussian probability in the pressure and velocity field can be seen in (Bertozzi & Majda, 2002, p. 194).

### 2.1 Proof of Equations (1), (2), (3), (5), (6).

In order to construct a function that complies with the 3D Navier Stokes equations, we must verify that it is continuous, that is, infinitely differentiable, and that at infinity it is attenuated to the value of zero. This requirement fulfills the function  $P(x, y, z, t)$  which is also extremely flexible to handle linear parameters  $\mu_1x + \mu_2y + \mu_3z - kt$ , representing relevant physical realities such as the absorption of energy as the fluid moves  $\mu_1, \mu_2, \mu_3$  and the parameters of evolution over time,  $k$ .

$$P(x, y, z, t) = \frac{1}{1 + e^{\mu_1x + \mu_2y + \mu_3z - kt}} \tag{7}$$

It is extremely useful to explicitly present the forms that are recurrent in the development of this research, which are:  $P = \frac{1}{1 + e^{\mu_1x + \mu_2y + \mu_3z - kt}}$ ,  $(1 - P) = \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1}$ ,  $P(1 - P) = \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2}$  and  $\tanh((x\mu_1 + y\mu_2 + z\mu_3 - kt) / 2) = \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) - 1}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)} = 1 - 2P$ .

Logistic regression is used to find the parameters and constants  $\mu_1, \mu_2, \mu_3, k, \nu$  of the 3DNS solution, which depends on  $x, y, z, t$ , as follows:  $\ln\left(\frac{1-P}{P}\right) = \mu_1x + \mu_2y + \mu_3z - kt$ , where  $\mu_1^2 - \mu_2^2 + \mu_3^2 = 0$ .

### 2.2 Fluid Velocity and Acceleration

The velocity of the fluid is represented by the vector field  $\mathbf{u}(x, y, z, t)$ , as follows:

$$\mathbf{u} = \begin{pmatrix} 2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_1 \\ -2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_2 \\ 2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_3 \end{pmatrix} = 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \tag{8}$$

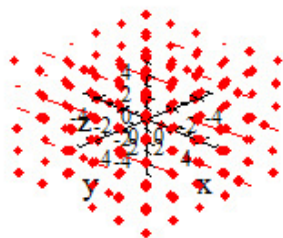


Figure 1. This figure is the graphic of the vector field in  $t=0$

For the initial time  $t = 0$  we can plot the velocity field of the fluid  $\mathbf{u}(x, y, z, t = 0) = \mathbf{u}^0$ . For this graphical representation we will maintain the fundamental condition,  $\mu_1^2 - \mu_2^2 + \mu_3^2 = 0$ ,  $\mu_1 = 4, \mu_2 = 5, \mu_3 = 3$ .

$$\mathbf{u}^0 = 2\nu \begin{pmatrix} 4 - \frac{\exp(4x+5y+3z)}{(\exp(4x+5y+3z)+1)^2} \\ -5 \frac{\exp(4x+5y+3z)}{(\exp(4x+5y+3z)+1)^2} \\ 3 - \frac{\exp(4x+5y+3z)}{(\exp(4x+5y+3z)+1)^2} \end{pmatrix}$$

### 2.3 Fluid Acceleration $\frac{\partial \mathbf{u}}{\partial t}$

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} &= 2\nu \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \frac{\partial}{\partial t} \left( \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \right) \\ \frac{\partial \mathbf{u}}{\partial t} &= \begin{pmatrix} 2k\nu\mu_1 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) - 1}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)} \\ -2k\nu\mu_2 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) - 1}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)} \\ 2k\nu\mu_3 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) - 1}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)} \end{pmatrix} \\ \frac{\partial \mathbf{u}}{\partial t} &= 2k\nu P(1 - P)(1 - 2P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \end{aligned} \tag{9}$$

### 2.4 Fluid Viscosity Term $\nu \nabla^2 \mathbf{u}$

Using the vector Laplacian we have,

$$\nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}) = (\nabla^2 u_x, \nabla^2 u_y, \nabla^2 u_z).$$

In the case of incompressible fluids  $\nabla \cdot \mathbf{u} = 0$ , giving us a greater simplification to the vector laplacian

$$\nu \nabla^2 \mathbf{u} = -\nu \nabla \times (\nabla \times \mathbf{u})$$

That is, the rotational of  $-\nu \nabla \times (\nabla \times \mathbf{u})$ , has the following form, *Annex of fundamental derivatives*:

$$\nu \nabla^2 \mathbf{u} = -\nu \nabla \times (\nabla \times \mathbf{u}) = -4\nu^2 P(1 - P)(6P^2 - 6P + 1) \begin{pmatrix} -\mu_1 \mu_2^2 \\ \mu_2 (\mu_1^2 + \mu_3^2) \\ -\mu_2^2 \mu_3 \end{pmatrix} \tag{10}$$

2.5 Equation of Incompressible Fluids,  $\nabla \cdot \mathbf{u}$

For this equation we directly apply the explicit form of the velocity field  $\mathbf{u}$ , as in: *Annex of fundamental derivatives*.

$$\nabla \cdot \mathbf{u} = -2\nu P(1 - P)(1 - 2P)(\mu_1^2 - \mu_2^2 + \mu_3^2) \tag{11}$$

Similarly, the incompressibility equation holds for  $\nabla \cdot \mathbf{u}^0 = \mathbf{0}$ .

2.6 Nonlinear Term,  $(\mathbf{u} \cdot \nabla) \mathbf{u}$

Recall the form of the velocity field vector given by equation (8) and taking the respective derivatives, see in: *Annex of fundamental derivatives*,

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = -4\nu^2 (P(1 - P))^2 (1 - 2P)(\mu_1^2 - \mu_2^2 + \mu_3^2) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} = (\nabla \cdot \mathbf{u}) \begin{pmatrix} 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \end{pmatrix}$$

The equation of fluid incompressibility is clearly embedded and present in the equation representing the nonlinear term.

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = (\nabla \cdot \mathbf{u}) \begin{pmatrix} 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{12}$$

2.7 Pressure and Pressure Gradient Equation  $\nabla p$ .

$$\frac{\nabla p}{\rho_0} = -\frac{\partial \mathbf{u}}{\partial t} - \nu \nabla \times (\nabla \times \mathbf{u}) \tag{13}$$

$$\frac{\nabla p}{\rho_0} = \left[ -\begin{pmatrix} 2\mu_1 k \nu (1 - 2P) \\ -2\mu_2 k \nu (1 - 2P) \\ 2\mu_3 k \nu (1 - 2P) \end{pmatrix} + \begin{pmatrix} 4\mu_1 \mu_2^2 \nu^2 (6P^2 - 6P + 1) \\ -4\mu_2 (\mu_1^2 + \mu_3^2) \nu^2 (6P^2 - 6P + 1) \\ 4\mu_2^2 \mu_3 \nu^2 (6P^2 - 6P + 1) \end{pmatrix} \right] P(1 - P)$$

Let's simplify it to make resolution easier.

$$\nabla p = \left[ \begin{matrix} 2\rho_0 \nu \mu_1 (12\nu P^2 \mu_2^2 - 12\nu P \mu_2^2 + 2kP + 2\nu \mu_2^2 - k) \\ -2\rho_0 \nu \mu_2 (12\nu P^2 \mu_1^2 + 12\nu P^2 \mu_3^2 - 12\nu P \mu_1^2 - 12\nu P \mu_3^2 + 2kP + 2\nu \mu_1^2 + 2\nu \mu_3^2 - k) \\ 2\rho_0 \nu \mu_3 (12\nu P^2 \mu_2^2 - 12\nu P \mu_2^2 + 2kP + 2\nu \mu_2^2 - k) \end{matrix} \right] P(1 - P) \tag{14}$$

We need to simply solve equation (14), to have the definitive form of the pressure.

2.8 Solving the Pressure Equation (13), (14)  $((x, y, z) \in \mathbb{R}^3, t \geq 0)$

Starting from equation (13), it is necessary to find the explicit form of the pressure, to meet this objective we will use the chain rule of the total differential of the pressure  $p$  as a function of the strategic variable  $P$ , (Riley et al., 1999, p. 157); (Bertozzi & Majda, 2002, p. 13)

$$\frac{dp}{dP} = \frac{\partial p}{\partial x} \frac{dx}{dP} + \frac{\partial p}{\partial y} \frac{dy}{dP} + \frac{\partial p}{\partial z} \frac{dz}{dP} = \frac{\partial p}{\partial x} \frac{dx}{dP} + \frac{\partial p}{\partial y} \frac{dy}{dP} + \frac{\partial p}{\partial z} \frac{dz}{dP} \tag{15}$$

The components of the pressure gradient can be obtained from equation (14), while the components  $\frac{dP}{dx} = -\mu_1 P(1 - P)$ ,  $\frac{dP}{dy} = -\mu_2 P(1 - P)$ ,  $\frac{dP}{dz} = -\mu_3 P(1 - P)$  of the equation of the strategic variable  $P = \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}}$ , with these results we

find each one of the components  $\frac{\partial p}{\partial x}$ ,  $\frac{\partial p}{\partial y}$ ,  $\frac{\partial p}{\partial z}$  of the differential equation  $\frac{dp}{dP}$ .

$$\frac{\partial p}{\partial x} = \frac{2\rho_0 \nu \mu_1 (12\nu P^2 \mu_2^2 - 12\nu P \mu_2^2 + 2kP + 2\nu \mu_2^2 - k) P(1 - P)}{-\mu_1 P(1 - P)} \tag{16}$$

$$\frac{\partial p}{\partial y} = - \frac{2\rho_0\nu\mu_2(12\nu P^2\mu_1^2 + 12\nu P^2\mu_3^2 - 12\nu P\mu_1^2 - 12\nu P\mu_3^2 + 2kP + 2\nu\mu_1^2 + 2\nu\mu_3^2 - k)P(1-P)}{-\mu_2 P(1-P)}, \tag{17}$$

$$\frac{\partial p}{\partial z} = \frac{2\rho_0\nu\mu_3(12\nu P^2\mu_2^2 - 12\nu P\mu_2^2 + 2kP + 2\nu\mu_2^2 - k)P(1-P)}{-\mu_3 P(1-P)}. \tag{18}$$

Making the respective replacements and the condition  $P(1-P) \neq 0$ , in equation (15) we arrive at the solution of differential equation (15).

$$\int_{p_0}^p dp = p - p_0 = \int_{P_0}^P \frac{\partial p}{\partial x} dP + \int_{P_0}^P \frac{\partial p}{\partial y} dP + \int_{P_0}^P \frac{\partial p}{\partial z} dP \tag{19}$$

$$p = p_0 - 2\nu\rho_0(P - P_0)(4\nu P^2\mu_2^2 + 4\nu P\mu_2^2 P_0 - 6\nu P\mu_2^2 + kP + 4\nu\mu_2^2 P_0^2 - 6\nu\mu_2^2 P_0 + 2\nu\mu_2^2 + kP_0 - k)$$

The function representing the pressure of fluid  $p$ , depends cubically on the strategic variable  $P$ , and is continuous. Without losing the generality we will graph the pressure for  $P_0 = 0$ .

$$p - p_0 = -2P^2\nu\rho_0(k + 2\nu\mu_1^2 - 4\nu\mu_2^2 + 2\nu\mu_3^2 - 4P\nu\mu_1^2 + 8P\nu\mu_2^2 - 4P\nu\mu_3^2)$$

2.9 Velocity Integral  $\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz \leq C$ .

Squaring the equation (8)  $\mathbf{u} = 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$ , We can easily find it.

$$\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz = 4\nu^2(\mu_1^2 + \mu_2^2 + \mu_3^2) \int_0^1 P^2(1 - P)^2 dP = \frac{4}{30}\nu^2(\mu_1^2 + \mu_2^2 + \mu_3^2) < C \quad \text{for all } t \geq 0 \quad (\text{bounded energy}).$$

### 3. Results

The results will be presented from three perspectives: graphic, theoretical and repeatability.

The graphs that we will present aim to see the behavior of the pressure and velocity of the fluid when time or space tend to infinity.

Repeatability in science refers to the ability of an experiment to give the same result over and over again and to be reproduced or replicated by others. This is important to draw accurate and reliable conclusions from experiments or simulation.

3.1 Asymptotic Convergence: If Space or Time Tends to Infinity Then  $P(1 - P) \rightarrow 0$

That is, with  $P = 0, P = 1$  the equations of Navier Stokes are fully fulfilled, especially when time and space tend to infinity:

$$\lim_{t \rightarrow \infty} P = \lim_{t \rightarrow \infty} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = 1$$

$$\lim_{x \rightarrow \infty} P = \lim_{x \rightarrow \infty} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = 0$$

$$\lim_{y \rightarrow \infty} P = \lim_{y \rightarrow \infty} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = 0$$

$$\lim_{z \rightarrow \infty} P = \lim_{z \rightarrow \infty} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = 0$$

**Proposition 1.** The function  $P(1 - P)$ , of the vector field  $\mathbf{u}$  is strictly decreasing in space  $(x, y, z)$  when  $P < 1/2$  and strictly increasing when  $P > 1/2$ .

A relevant contribution of this modeling is the introduction of the generalized variable  $Z = \mu_1 x + \mu_2 y + \mu_3 z - kt$ , which allows analysis in two dimensions  $(P, Z)$  without losing generality in the model in  $(x, y, z, t)$ .

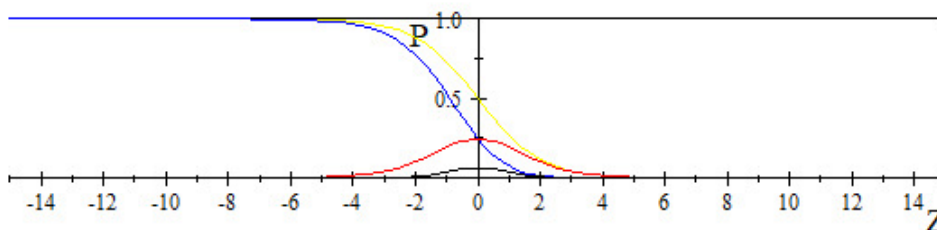


Figure 2. Explicit graph of the evolution of the functions P[yellow], P<sup>2</sup>[blue], P(1-P)[red], P<sup>2</sup>(1-P)<sup>2</sup>[black] you can see the advance of the probability in time and space, where the solitary wave is located at the origin

Graphically we visualize that the functions  $P, P^2$  are continuous, since they are strictly decreasing:

$$\frac{\partial}{\partial Z} P = -P(1 - P) < 0, \frac{\partial}{\partial Z} P^2 = -2P^2(1 - P) < 0.$$

Let be the vector field  $\mathbf{u} = 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$ , We can generalize and find the derivative with respect to  $Z \frac{\partial}{\partial Z} P(1 - P) = -P(1 - P)(1 - 2P)$  and we will verify that the function  $P(1 - P)$  is strictly decreasing in space  $(x, y, z)$  when  $P < 1/2$  and strictly increasing when  $P > 1/2$ . Therefore the components of the vector field are infinitely differentiable with respect to the generalized variable,  $Z$ .

$$P = \frac{1}{1 + e^{4x+5y+3z}}$$

Plot the generalized function  $Z = x\mu_1 + y\mu_2 + z\mu_3 - kt, ((x, y, z) \in \mathbb{R}^3, t \geq 0)$

$$P = \frac{1}{1+e^Z}, P^2 = \frac{1}{(1+e^Z)^2}, P(1 - P) = \frac{e^Z}{(1+e^Z)^2}, P^2(1 - P)^2 = \frac{e^{2Z}}{(1+e^Z)^4}$$

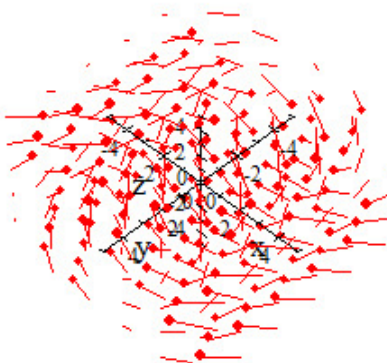


Figure 3. Fulfillment of Pygoras' theorem in the parameters, which means the fundamental condition of the Navier Stokes 3D equations

$$P = \frac{1}{1+e^{4x+5y+3z}}, \mu_1^2 - \mu_2^2 + \mu_3^2 = 0$$

### 3.2 Continuity of Functions

$$P = \frac{1}{1+e^{Z+5}}, P^2 = \frac{1}{(1+e^{Z+5})^2}, P(1 - P) = \frac{e^{Z+5}}{(1+e^{Z+5})^2}, P^2(1 - P)^2 = \frac{e^{2Z+10}}{(1+e^{Z+5})^4}$$

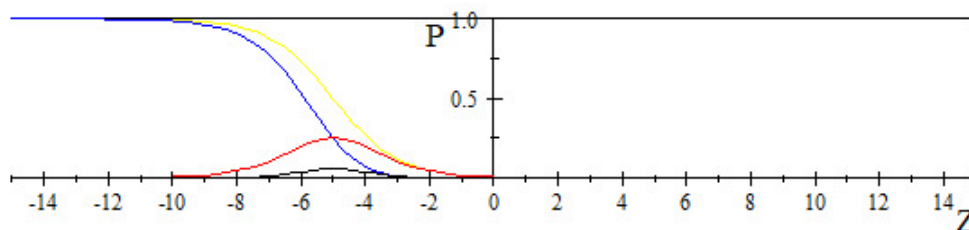


Figure 4. Explicit graph of the evolution of the functions P[yellow],P2[blue],P(1-P)[red],P2(1-P)2[black] you can see the advance of the probability in time and space, where the solitary wave is displaced

### 3.3 Navier Stokes Polynomials (NSP(n))

$$u_x = 2\nu\mu_1 P(1 - P)$$

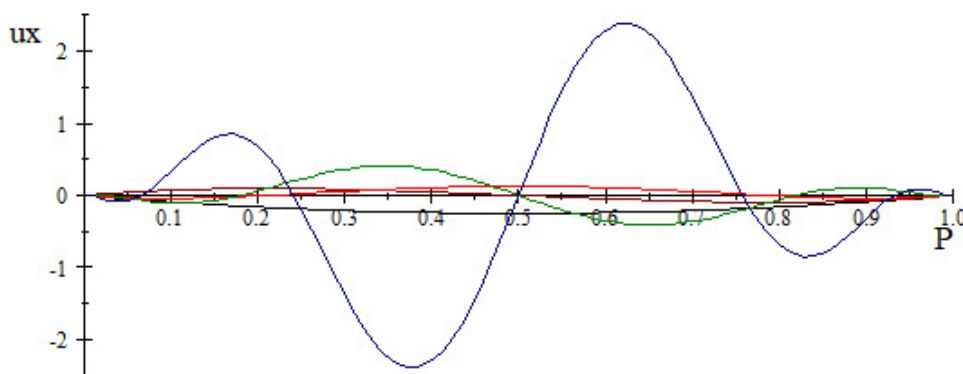


Figure 5. Navier Stokes polynomials, which generalize the solution of the 3DNS equations.  $\alpha=0,1,2,5,7$

### 3.4 Particular Cases NSP(n=0,1,2,3,4,5,6,7)

$$NSP(0) = P(1 - P), NSP(1) = 2P^3 - 3P^2 + P,$$

$$NSP(2) = 6P^4 - 12P^3 + 7P^2 - P, NSP(3) = 24P^5 - 60P^3 + 50P^2 - 15P^2 + P,$$

$$NSP(4) = 120P^6 - 360P^5 + 390P^4 - 180P^3 + 31P^2 - P,$$

$$NSP(5) = 720P^7 - 2520P^6 + 3360P^5 - 2100P^4 + 602P^3 - 63P^2 + P,$$

$$NSP(6) = 5040P^8 - 20160P^7 + 31920P^6 - 25200P^5 + 10206P^4 - 1932P^3 + 127P^2 - P,$$

$$NSP(7) = 40320P^9 - 181440P^8 + 332640P^7 - 317520P^6 + 166824P^5 - 46620P^4 + 6050P^3 - 255P^2 + P$$

$$\frac{\partial}{\partial x} u_x = \frac{\partial}{\partial x} (2\nu\mu_1 P(1 - P)) = -\mu_1 P(1 - P) \frac{\partial}{\partial P} (2\nu\mu_1 P(1 - P)) = -2\nu\mu_1^2 (2P - 1)(P - 1) P = -2\nu\mu_1^2 (2P^3 - 3P^2 + P)$$

$$\frac{\partial^2}{\partial x^2} u_x = \frac{\partial}{\partial x} (-2\nu\mu_1^2 (2P^3 - 3P^2 + P)) = -\mu_1 P(1 - P) \frac{\partial}{\partial P} (-2\nu\mu_1^2 (2P^3 - 3P^2 + P)) = +2\nu\mu_1^3 (P - 1) P (6P^2 - 6P + 1)$$

$$= 2\nu\mu_1^3 (6P^4 - 12P^3 + 7P^2 - P)$$

$$\frac{\partial^3}{\partial x^3} u_x = \frac{\partial}{\partial x} (2\nu\mu_1^3 (6P^4 - 12P^3 + 7P^2 - P)) = -\mu_1 P(1 - P) \frac{\partial}{\partial P} (2\nu\mu_1^3 (6P^4 - 12P^3 + 7P^2 - P))$$

$$= -2\nu\mu_1^4 (24P^5 - 60P^3 + 50P^2 - 15P^2 + P)$$

$$\frac{\partial^4}{\partial x^4} u_x = \frac{\partial}{\partial x} (-2\nu\mu_1^4 (24P^5 - 60P^4 + 50P^3 - 15P^2 + P)) =$$

$$2\nu\mu_1^5 (120P^6 - 360P^5 + 390P^4 - 180P^3 + 31P^2 - P)$$

$$\frac{\partial^5}{\partial x^5} u_x = \frac{\partial}{\partial x} (2\nu\mu_1^5 (120P^6 - 360P^5 + 390P^4 - 180P^3 + 31P^2 - P))$$

$$= -2\nu\mu_1^6 (720P^7 - 2520P^6 + 3360P^5 - 2100P^4 + 602P^3 - 63P^2 + P)$$

$$\frac{\partial^6}{\partial x^6} u_x = \frac{\partial}{\partial x} (-2\nu\mu_1^6 (720P^7 - 2520P^6 + 3360P^5 - 2100P^4 + 602P^3 - 63P^2 + P))$$

$$= 2\nu\mu_1^7 (5040P^8 - 20160P^7 + 31920P^6 - 25200P^5 + 10206P^4 - 1932P^3 + 127P^2 - P)$$



$$\frac{\partial^7}{\partial x^7} u_x = \frac{\partial}{\partial x} \left( 2\nu\mu_1^7 (5040P^8 - 20160P^7 + 31920P^6 - 25200P^5 + 10206P^4 - 1932P^3 + 127P^2 - P) \right) \\ = -2\nu\mu_1^8 (40320P^9 - 181440P^8 + 332640P^7 - 317520P^6 + 166824P^5 - 46620P^4 + 6050P^3 - 255P^2 + P)$$

### 3.5 Conclusions

Here, We present a new method in the resolution of the equations of Navier Stokes 3D, which consists of eliminating the nonlinear term and simplifying the term of the Laplacian, in this way it becomes totally manageable to obtain the pressure. An easy form of the velocity vector field that depends on the logistic probability law in a polynomial way is proposed, giving rise to a set of polynomials that also comply with 3DNS. Logistic regression is used to obtain the parameters and constants of the 3DNS solution, which depends on  $x, y, z, t$ .

1. This new approach is an analytical, graphical, easy and manageable representation of the dynamics of viscous fluids, where new elements appear in obtaining explicitly, the vector field of velocities and pressure: 1.-The introduction of the Navier Stokes polynomials that are also solutions of 3DNS, which depend on the logistic probability function. Moreover, these terms absorb the nonlinear information of the 3DNS differential equations. 2.- The propagation of the fluid front resembles a solitary wave which is visible and easy to plot with the Navier Stokes polynomials which in their simplest form represent the cumulative probability. The pressure has a direct dependence on the Navier Stokes polynomials.

2. We have integrated the geometrical part of the 3DNS equations with the probabilistic part of every physical phenomenon that depends on its moments, i.e. mean, standard deviation, skewness, kurtosis and entropy.

3. The logistic function and the logistic probability density are closely related to Chaos, so this method would be very helpful in problems of evolution and interaction of populations and infections. Evolutionary game theory and dynamics would have a great contribution by introducing this powerful method.

4. The method was not selected by us, it was the result of studying the power and vector field propagation graphs of the fluid by means of logistic regression in problems of Quantitative and Behavioral Finance (Quantum Games) with Energy Economics (Petroleum Catalysis and Hydrogen), finally combined with the viscous fluids of Chemical Engineering.

5. Logistic regression is used to find the parameters and constants  $\mu_1, \mu_2, \mu_3, k, v$  of the 3DNS solution, which depends on  $x, y,$

$z, t$ . Here the Pythagorean Theorem is fulfilled with the parameters  $\mu_1, \mu_2, \mu_3$ .

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### Authors contributions

Dr. Edward Henry Jimenez was responsible for study design, mathematical development and revising. All authors read and approved the final manuscript.

### Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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**Appendix A**

**Annex of fundamental derivatives:**  $\nabla \cdot \mathbf{u} = 0, \nu \nabla^2 \mathbf{u}$  and  $(\mathbf{u} \cdot \nabla) \mathbf{u} = 0$

When the fundamental equations are simplified we arrive at a manageable and simple form that allows us to find the scalar field of fluid pressure.  $\frac{\nabla p}{\rho_0} = -\frac{\partial \mathbf{u}}{\partial t} - \nu \nabla \times (\nabla \times \mathbf{u})$

**Derivatives of logistic density function**  $P = \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}}$

$$P = \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}}, Z = \mu_1 x + \mu_2 y + \mu_3 z - kt$$

$$\mathbf{u} = \begin{pmatrix} 2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_1 \\ -2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_2 \\ 2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_3 \end{pmatrix} = 2\nu P (1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$$

$$P = \frac{1}{1 + e^Z}, P (1 - P)$$

$$\frac{\partial}{\partial Z} \frac{1}{1 + e^Z} = -\frac{e^Z}{(e^Z + 1)^2} = -P (1 - P), \frac{\partial Z}{\partial x} = \mu_1$$

$$\begin{aligned} \frac{\partial P}{\partial x} &= \frac{\partial P}{\partial Z} \frac{\partial Z}{\partial x} = -\mu_1 P(1 - P) \\ \frac{\partial P}{\partial x} &= \frac{\partial}{\partial x} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = -\mu_1 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} = -\mu_1 P(1 - P) \\ \frac{\partial P}{\partial y} &= \frac{\partial}{\partial y} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = -\mu_2 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} = -\mu_2 P(1 - P) \\ \frac{\partial P}{\partial z} &= \frac{\partial}{\partial z} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = -\mu_3 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} = -\mu_3 P(1 - P) \\ \frac{\partial P}{\partial t} &= \frac{\partial}{\partial t} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = k \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} = kP(1 - P) \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial x} P(1 - P) &= \frac{\partial}{\partial x} P - 2P \frac{\partial}{\partial x} P = (1 - 2P) \frac{\partial}{\partial x} P = -\mu_1 P(1 - P)(1 - 2P) \\ \frac{\partial}{\partial y} P(1 - P) &= \frac{\partial}{\partial y} P - 2P \frac{\partial}{\partial y} P = (1 - 2P) \frac{\partial}{\partial y} P = -\mu_2 P(1 - P)(1 - 2P) \\ \frac{\partial}{\partial z} P(1 - P) &= \frac{\partial}{\partial z} P - 2P \frac{\partial}{\partial z} P = (1 - 2P) \frac{\partial}{\partial z} P = -\mu_3 P(1 - P)(1 - 2P) \\ \frac{\partial}{\partial t} P(1 - P) &= \frac{\partial}{\partial t} P - 2P \frac{\partial}{\partial t} P = (1 - 2P) \frac{\partial}{\partial t} P = kP(1 - P)(1 - 2P) \end{aligned}$$

**Equation condition of incompressible fluid.**

$$\nabla \cdot \mathbf{u} = \nabla \cdot \left( 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \right) = \mu_1 2\nu \frac{\partial}{\partial x} (P(1 - P)) - \mu_2 2\nu \frac{\partial}{\partial y} (P(1 - P)) + \mu_3 2\nu \frac{\partial}{\partial z} (P(1 - P))$$

$$\nabla \cdot \mathbf{u} = 2\nu \left( -\mu_1^2 + \mu_2^2 - \mu_3^2 \right) P(1 - P)(1 - 2P) = -2\nu \left( \mu_1^2 - \mu_2^2 + \mu_3^2 \right) P(1 - P)(1 - 2P)$$

**Nonlinear term equation:**  $(\mathbf{u} \cdot \nabla) \mathbf{u} = (\nabla \cdot \mathbf{u}) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = \left( 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \cdot \nabla \right) \mathbf{u} = 2\nu (P(1 - P)) \left( \mu_1 \frac{\partial}{\partial x} - \mu_2 \frac{\partial}{\partial y} + \mu_3 \frac{\partial}{\partial z} \right) 2\nu P(1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = 4\nu^2 (P(1 - P)) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \left( \mu_1 \frac{\partial}{\partial x} P(1 - P) - \mu_2 \frac{\partial}{\partial y} P(1 - P) + \mu_3 \frac{\partial}{\partial z} P(1 - P) \right)$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = 4\nu^2 (P(1 - P)) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \left[ -\mu_1^2 P(1 - P)(1 - 2P) + \mu_2^2 P(1 - P)(1 - 2P) + \mu_3^2 P(1 - P)(1 - 2P) \right]$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = -4\nu^2 (P(1 - P))^2 (1 - 2P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} = (\nabla \cdot \mathbf{u}) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

**Calculation of the rotational of u**

$$\begin{aligned} & -\nu \operatorname{curl} \left( \operatorname{curl} \begin{pmatrix} 2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_1 \\ -2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_2 \\ 2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \mu_3 \end{pmatrix} \right) \\ & - \left( \begin{aligned} & -4\nu^2 \mu_1 \mu_2^2 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \left( \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} - 4 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} + \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \right) \\ & 4\nu^2 \mu_2 (\mu_1^2 + \mu_3^2) \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \left( \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} - 4 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} + \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \right) \\ & -4\nu^2 \mu_2^2 \mu_3 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \left( \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} - 4 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} + \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \right) \end{aligned} \right) \\ & \nu \nabla^2 \mathbf{u} = -\nu \nabla \times (\nabla \times \mathbf{u}) = -4\nu^2 \begin{pmatrix} -\mu_1 \mu_2^2 \\ \mu_2 (\mu_1^2 + \mu_3^2) \\ -\mu_2^2 \mu_3 \end{pmatrix} P(1 - P) \left( (1 - P)^2 - 4P(1 - P) + P^2 \right) \end{aligned}$$

$$\nu \nabla^2 \mathbf{u} = -\nu \nabla \times (\nabla \times \mathbf{u}) = -4\nu^2 P(1-P)(6P^2 - 6P + 1) \begin{pmatrix} -\mu_1 \mu_2^2 \\ \mu_2 (\mu_1^2 + \mu_3^2) \\ -\mu_2^2 \mu_3 \end{pmatrix}$$

**Pressure integral terms,**  $\int_{P_0}^P dp = p - p_0$

$$\int_{P_0}^P dp = p - p_0 = \int_{P_0}^P \frac{\partial p}{\partial x} dP + \int_{P_0}^P \frac{\partial p}{\partial y} dP + \int_{P_0}^P \frac{\partial p}{\partial z} dP$$

Taking into consideration that each of the integrals of (19) are large we will do it one by one to finally join the partial results.

$$\int_{P_0}^P \frac{\partial p}{\partial x} dP = \int_{P_0}^P \frac{2\rho_0 \nu \mu_1 (12\nu P^2 \mu_2^2 - 12\nu P \mu_2^2 + 2kP + 2\nu \mu_2^2 - k)}{-\mu_1} dP = -2\nu \rho_0 (P - P_0) (4\nu P^2 \mu_2^2 + 4\nu P \mu_2^2 P_0 - 6\nu P \mu_2^2 + kP + 4\nu \mu_2^2 P_0^2 - 6\nu \mu_2^2 P_0 + 2\nu \mu_2^2 + kP_0 - k)$$

$$\int_{P_0}^P \frac{\partial p}{\partial y} dP = \int_{P_0}^P \frac{-2\rho_0 \nu \mu_2 (12\nu P^2 \mu_1^2 + 12\nu P^2 \mu_3^2 - 12\nu P \mu_1^2 - 12\nu P \mu_3^2 + 2kP + 2\nu \mu_1^2 + 2\nu \mu_3^2 - k)}{-\mu_2} dP = 2\nu \rho_0 (P - P_0) (4\nu P^2 \mu_1^2 + 4\nu P^2 \mu_3^2 + 4\nu P \mu_1^2 P_0 - 6\nu P \mu_1^2 + 4\nu P \mu_3^2 P_0 - 6\nu P \mu_3^2) + \dots$$

$$\dots + (kP + 4\nu \mu_1^2 P_0^2 - 6\nu \mu_1^2 P_0 + 2\nu \mu_1^2 + 4\nu \mu_3^2 P_0^2 - 6\nu \mu_3^2 P_0 + 2\nu \mu_3^2 + kP_0 - k)$$

$$\int_{P_0}^P \frac{\partial p}{\partial z} dP = \int_{P_0}^P \frac{2\rho_0 \nu \mu_3 (12\nu P^2 \mu_2^2 - 12\nu P \mu_2^2 + 2kP + 2\nu \mu_2^2 - k)}{-\mu_3} dP = -2\nu \rho_0 (P - P_0) (4\nu P^2 \mu_2^2 + 4\nu P \mu_2^2 P_0 - 6\nu P \mu_2^2 + kP + 4\nu \mu_2^2 P_0^2 - 6\nu \mu_2^2 P_0 + 2\nu \mu_2^2 + kP_0 - k)$$

Without losing generality we will evaluate for  $P_0 = 0$ .

$$p - p_0 = -4\nu \rho_0 P (4\nu P^2 \mu_2^2 - 6\nu P \mu_2^2 + kP + 2\nu \mu_2^2 - k) + 2\nu \rho_0 P (4\nu P^2 \mu_1^2 + 4\nu P^2 \mu_3^2 - 6\nu P \mu_1^2 - 6\nu P \mu_3^2 + kP + 2\nu \mu_1^2 + 2\nu \mu_3^2 - k)$$

$$p - p_0 = -2P(P - 1) \nu \rho_0 (k + 2\nu \mu_1^2 - 4\nu \mu_2^2 + 2\nu \mu_3^2 - 4P\nu \mu_1^2 + 8P\nu \mu_2^2 - 4P\nu \mu_3^2)$$

**Derivatives of Navier Stokes polynomial**  $P = \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}}$

$$P = \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}}, Z = \mu_1 x + \mu_2 y + \mu_3 z - kt$$

$$\mathbf{u} = \begin{pmatrix} 2\nu \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)^2} \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)} \mu_1 \\ -2\nu \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)^2} \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)} \mu_2 \\ 2\nu \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)^2} \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)} \mu_3 \end{pmatrix} = 2\nu P(1 - P)(1 - 2P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$$

$$P = \frac{1}{1 + e^Z}, P(1 - P)$$

$$\frac{\partial}{\partial Z} \frac{1}{1 + e^Z} = -\frac{e^Z}{(e^Z + 1)^2} = -P(1 - P), \frac{\partial Z}{\partial x} = \mu_1$$

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial Z} \frac{\partial Z}{\partial x} = -\mu_1 P(1 - P)$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = -\mu_1 \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)^2} = -\mu_1 P(1 - P) = \mu_1 P(P - 1)$$

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = -\mu_2 \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)^2} = -\mu_2 P(1 - P) = \mu_2 P(P - 1)$$

$$\frac{\partial P}{\partial z} = \frac{\partial}{\partial z} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = -\mu_3 \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)^2} = -\mu_3 P(1 - P) = \mu_3 P(P - 1)$$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial t} \frac{1}{1 + e^{\mu_1 x + \mu_2 y + \mu_3 z - kt}} = k \frac{\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt)}{(\exp(\mu_1 x + \mu_2 y + \mu_3 z - kt) + 1)^2} = kP(1 - P) = -kP(P - 1)$$

$$\frac{\partial}{\partial x} (2P^3 - 3P^2 + P) = (6P^2 - 6P + 1) \frac{\partial}{\partial x} P = \mu_1 P(P - 1)(6P^2 - 6P + 1)$$

$$\frac{\partial}{\partial y} (2P^3 - 3P^2 + P) = (6P^2 - 6P + 1) \frac{\partial}{\partial y} P = \mu_2 P(P - 1)(6P^2 - 6P + 1)$$

$$\frac{\partial}{\partial z} (2P^3 - 3P^2 + P) = (6P^2 - 6P + 1) \frac{\partial}{\partial z} P = \mu_3 P (P - 1) (6P^2 - 6P + 1)$$

$$\frac{\partial}{\partial r} (2P^3 - 3P^2 + P) = (6P^2 - 6P + 1) \frac{\partial}{\partial r} P = -kP (P - 1) (6P^2 - 6P + 1)$$

**Equation condition of incompressible fluid.**

$$\nabla \cdot \mathbf{u} = \nabla \cdot \left( 2\nu (2P^3 - 3P^2 + P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \right) = \mu_1 2\nu \frac{\partial}{\partial x} (2P^3 - 3P^2 + P) - \mu_2 2\nu \frac{\partial}{\partial y} (2P^3 - 3P^2 + P) + \mu_3 2\nu \frac{\partial}{\partial z} (2P^3 - 3P^2 + P)$$

$$\nabla \cdot \mathbf{u} = 2\nu (-\mu_1^2 + \mu_2^2 - \mu_3^2) (6P^2 - 6P + 1) P (P - 1) = -2\nu (\mu_1^2 - \mu_2^2 + \mu_3^2) (6P^2 - 6P + 1) P (P - 1) = 0$$

**Nonlinear term equation:  $(\mathbf{u} \cdot \nabla) \mathbf{u} = (\nabla \cdot \mathbf{u}) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$**

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = \left( 2\nu P (1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \right) \cdot \nabla \mathbf{u} = 2\nu (P (1 - P)) \left( \mu_1 \frac{\partial}{\partial x} - \mu_2 \frac{\partial}{\partial y} + \mu_3 \frac{\partial}{\partial z} \right) 2\nu P (1 - P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = 4\nu^2 (P (1 - P)) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \left( \mu_1 \frac{\partial}{\partial x} P (1 - P) - \mu_2 \frac{\partial}{\partial y} P (1 - P) + \mu_3 \frac{\partial}{\partial z} P (1 - P) \right)$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = 4\nu^2 (P (1 - P)) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \left[ -\mu_1^2 P (1 - P) (1 - 2P) + \mu_2^2 P (1 - P) (1 - 2P) + \mu_3^2 P (1 - P) (1 - 2P) \right]$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = -4\nu^2 (P (1 - P))^2 (1 - 2P) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} = (\nabla \cdot \mathbf{u}) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

**Calculation of the rotational velocity vector field  $\mathbf{u}$**

$$-\nu \text{curl} \left( \text{curl} \begin{pmatrix} 2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)} \mu_1 \\ -2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)} \mu_2 \\ 2\nu \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)} \mu_3 \end{pmatrix} \right) =$$

$$\left( \begin{aligned} & \nu \left( 16\nu\mu_1\mu_2^2 \frac{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^2}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^3} - 60\nu\mu_1\mu_2^2 \frac{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^3}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^4} + 48\nu\mu_1\mu_2^2 \frac{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^4}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^5} \right) \\ & - \nu \left( \frac{16\nu\mu_1^2\mu_2(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^2}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^3} + \frac{16\nu\mu_2\mu_3^2(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^2}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^3} - \frac{60\nu\mu_1^2\mu_2(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^3}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^4} - \dots \right) \\ & \nu \left( 16\nu\mu_2^2\mu_3 \frac{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^2}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^3} - 60\nu\mu_2^2\mu_3 \frac{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^3}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^4} + 48\nu\mu_2^2\mu_3 \frac{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt))^4}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^5} \right) \end{aligned} \right) =$$

$$\left( \begin{aligned} & 4\nu^2\mu_1\mu_2^2 \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^5} (\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt) - 7 \exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 4) \\ & - 4\nu^2\mu_2 (\mu_1^2 + \mu_3^2) \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^5} (\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt) - 7 \exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 4) \\ & 4\nu^2\mu_2^2\mu_3 \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^5} (\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt) - 7 \exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 4) \end{aligned} \right) =$$

$$\left( \begin{aligned} & 4\nu^2\mu_1\mu_2^2 \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^3} \left( \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} - 7 \frac{\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)} + 4 \frac{1}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^2} \right) \\ & - 4\nu^2\mu_2 (\mu_1^2 + \mu_3^2) \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^5} (\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt) - 7 \exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 4) \\ & 4\nu^2\mu_2^2\mu_3 \frac{\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt)}{(\exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 1)^5} (\exp(2x\mu_1 + 2y\mu_2 + 2z\mu_3 - 2kt) - 7 \exp(x\mu_1 + y\mu_2 + z\mu_3 - kt) + 4) \end{aligned} \right)$$

$$\nu \nabla^2 \mathbf{u} = -\nu \nabla \times (\nabla \times \mathbf{u}) = -4\nu^2 \begin{pmatrix} -\mu_1\mu_2^2 \\ \mu_2 (\mu_1^2 + \mu_3^2) \\ -\mu_3^2\mu_3 \end{pmatrix} P^2 (1 - P) ((1 - P)^2 - 7P(1 - P) + 4P^2)$$

$$\nu \nabla^2 \mathbf{u} = -\nu \nabla \times (\nabla \times \mathbf{u}) = -4\nu^2 P^2 (1 - P) (12P^2 - 9P + 1) \begin{pmatrix} -\mu_1 \mu_2^2 \\ \mu_2 (\mu_1^2 + \mu_3^2) \\ -\mu_2^2 \mu_3 \end{pmatrix}$$

**Pressure integral terms,**  $\int_{P_0}^P dp = p - p_0$

$$\int_{P_0}^P dp = p - p_0 = \int_{P_0}^P \frac{\partial p}{\partial x} dP + \int_{P_0}^P \frac{\partial p}{\partial y} dP + \int_{P_0}^P \frac{\partial p}{\partial z} dP$$

Taking into consideration that each of the integrals of (19) are large we will do it one by one to finally join the partial results.

$$\int_{P_0}^P \frac{\partial p}{\partial x} dP = \int_{P_0}^P \frac{2\rho_0 \nu \mu_1 (12\nu P^2 \mu_2^2 - 12\nu P \mu_3^2 + 2kP + 2\nu \mu_2^2 - k)}{-\mu_1} dP =$$

$$-2\nu \rho_0 (P - P_0) (4\nu P^2 \mu_2^2 + 4\nu P \mu_2^2 P_0 - 6\nu P \mu_2^2 + kP + 4\nu \mu_2^2 P_0^2 - 6\nu \mu_2^2 P_0 + 2\nu \mu_2^2 + kP_0 - k)$$

$$\int_{P_0}^P \frac{\partial p}{\partial y} dP = \int_{P_0}^P \frac{-2\rho_0 \nu \mu_2 (12\nu P^2 \mu_1^2 + 12\nu P \mu_3^2 - 12\nu P \mu_1^2 - 12\nu P \mu_3^2 + 2kP + 2\nu \mu_1^2 + 2\nu \mu_3^2 - k)}{-\mu_2} dP =$$

$$2\nu \rho_0 (P - P_0) (4\nu P^2 \mu_1^2 + 4\nu P^2 \mu_3^2 + 4\nu P \mu_1^2 P_0 - 6\nu P \mu_1^2 + 4\nu P \mu_3^2 P_0 - 6\nu P \mu_3^2 + kP + 4\nu \mu_1^2 P_0^2 - 6\nu \mu_1^2 P_0) + \dots$$

$$\dots + (2\nu \mu_1^2 + 4\nu \mu_3^2 P_0^2 - 6\nu \mu_3^2 P_0 + 2\nu \mu_3^2 + kP_0 - k)$$

$$\int_{P_0}^P \frac{\partial p}{\partial z} dP = \int_{P_0}^P \frac{2\rho_0 \nu \mu_3 (12\nu P^2 \mu_2^2 - 12\nu P \mu_3^2 + 2kP + 2\nu \mu_2^2 - k)}{-\mu_3} dP =$$

$$-2\nu \rho_0 (P - P_0) (4\nu P^2 \mu_2^2 + 4\nu P \mu_2^2 P_0 - 6\nu P \mu_2^2 + kP + 4\nu \mu_2^2 P_0^2 - 6\nu \mu_2^2 P_0 + 2\nu \mu_2^2 + kP_0 - k)$$

**Appendix B**

**Derivatives of sine,**  $P = \sin(\mu_1 x + \mu_2 y + \mu_3 z - kt)$

Para  $P = \sin(\mu_1 x + \mu_2 y + \mu_3 z - kt)$ , we verify that the Navier Stokes 3D equations are also met.

$$P = \sin(\mu_1 x + \mu_2 y + \mu_3 z - kt)$$

$$\mathbf{u} = \begin{pmatrix} -2\nu \mu_1 \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) \\ 2\nu \mu_2 \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) \\ -2\nu \mu_3 \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) \end{pmatrix} = -2\nu \sqrt{1 - P^2} \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$$

We will only work with the positive roots when we clear breast and cosine. That is, if

$$P = \sin(\mu_1 x + \mu_2 y + \mu_3 z - kt), \text{ then } \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) = \sqrt{1 - P^2}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x} \sin(\mu_1 x + \mu_2 y + \mu_3 z - kt) = \mu_1 \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) = \mu_1 \sqrt{1 - P^2}$$

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} \sin(\mu_1 x + \mu_2 y + \mu_3 z - kt) = \mu_2 \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) = \mu_2 \sqrt{1 - P^2}$$

$$\frac{\partial P}{\partial z} = \frac{\partial}{\partial z} \sin(\mu_1 x + \mu_2 y + \mu_3 z - kt) = \mu_3 \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) = \mu_3 \sqrt{1 - P^2}$$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial t} \sin(\mu_1 x + \mu_2 y + \mu_3 z - kt) = -k \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) = -k \sqrt{1 - P^2}$$

$$\frac{\partial \sqrt{1 - P^2}}{\partial x} = \frac{\partial}{\partial x} \cos(\mu_1 x + \mu_2 y + \mu_3 z - kt) = -\mu_1 \sin(x\mu_1 + y\mu_2 + z\mu_3 - kt) = -\mu_1 P$$

$$\frac{\partial \sqrt{1 - P^2}}{\partial y} = \frac{\partial}{\partial y} \cos(\mu_1 x + \mu_2 y + \mu_3 z - kt) = -\mu_2 \sin(x\mu_1 + y\mu_2 + z\mu_3 - kt) = -\mu_2 P$$

$$\frac{\partial \sqrt{1 - P^2}}{\partial z} = \frac{\partial}{\partial z} \cos(\mu_1 x + \mu_2 y + \mu_3 z - kt) = -\mu_3 \sin(x\mu_1 + y\mu_2 + z\mu_3 - kt) = -\mu_3 P$$

$$\frac{\partial \sqrt{1 - P^2}}{\partial t} = \frac{\partial}{\partial t} \cos(\mu_1 x + \mu_2 y + \mu_3 z - kt) = k \sin(x\mu_1 + y\mu_2 + z\mu_3 - kt) = kP$$

$$\nabla \cdot \mathbf{u} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot (-2\nu \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt)) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix}$$

$$\nabla \cdot \mathbf{u} = -2\nu \left( \mu_1 \frac{\partial \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{\partial x} - \mu_2 \frac{\partial \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{\partial y} + \mu_3 \frac{\partial \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt)}{\partial z} \right)$$

$$\nabla \cdot \mathbf{u} = 2\nu (\sin(x\mu_1 + y\mu_2 + z\mu_3 - kt)) (\mu_1^2 - \mu_2^2 + \mu_3^2) = 2\nu (\mu_1^2 - \mu_2^2 + \mu_3^2) P$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = \left( -2\nu \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \right) \mathbf{u}$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = -2\nu \sqrt{1 - P^2} (\mu_1 \frac{\partial}{\partial x} - \mu_2 \frac{\partial}{\partial y} + \mu_3 \frac{\partial}{\partial z}) \left( -2\nu \cos(x\mu_1 + y\mu_2 + z\mu_3 - kt) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} \right)$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = 4\nu^2 \sqrt{1 - P^2} P (-\mu_1^2 + \mu_2^2 - \mu_3^2) \begin{pmatrix} \mu_1 \\ -\mu_2 \\ \mu_3 \end{pmatrix} = -4\nu^2 \sqrt{1 - P^2} P (\mu_1^2 - \mu_2^2 + \mu_3^2)$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = 2\nu P (\mu_1^2 - \mu_2^2 + \mu_3^2) (-2\nu \sqrt{1 - P^2}) = (\nabla \cdot \mathbf{u}) (-2\nu \sqrt{1 - P^2}) = 0$$

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