# Restoring Environmental Justice: On the Coupled Dynamical Analysis of Lake Powell and Lake Mead

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# Abstract

A new omen of the current climate crisis is surfacing, threatening lives relying on Lake Powell and Lake Mead. As the largest reservoirs in Colorado River and its tributary system, the two lakes are suffering from structural water deficiency caused by persistent global warming and human related environmental destruction. This study proposes threshold values of water recycling and water saving by applying mathematical models within reality-based circumstances. The dynamic nature of environmental factors has been elucidated by applying a system of ordinary differential equations (ODE). In addition, water level predictions under various scenarios are obtained from the ODE system solutions. Our study provides a clear and simulated roadmap reflective of the scale and urgency of restoring climate justice in Lake Powell and Lake Mead.

# 1. Introduction

Water is the driving force of all lives. Ecosystems and mankind cannot survive without it. Hoover Dam, built on the Colorado River and its tributaries, is one of the iconic marvels of modern engineering and has played a crucial role in the health, well-being and economic prosperity of the Southwest in the United States. Today, the Bureau of Reclamation estimates around 40 million people are reliant on the Colorado River Basin (Baculi et. al, 2022) (see Figure 1).

When water allocations were decided by the 1922 Colorado River Compact, the Colorado River Basin was going through a period of unusually wet years (Barrett et. al, 2008). As a result, the average annual flow of the Colorado River was overshot, at 17.5 MAF. (Baculi et. al, 2022). This Compact, and later on, the Law of the River decided the allotment of the Upper and Lower Basins: each would receive 7.5 MAF annually. The current burden of Colorado River also includes the annual 1.5 MAF apportionment to Mexico (Bureau of Reclamation) and approximately 0.9 MAF of evaporation (Cerveny et. al, 2022). However, recent assessment reveals that the actual long term average natural flow of the Colorado River is 14.8 MAF - about 16% lower than what was assumed in the 1922 Compact allocations (Baculi et. al, 2022). The hardest working river in the Southwest is overexploited.

In addition to the enduring and parched drought overcasting the Basin since 1998, increasing demand on water for economic growth has further aggravated the situation. As a result, the water levels of Lake Powell and Lake Mead keep dropping. Lake Powell is just under twenty seven percent full and Lake Mead is now only about a quarter full (Barrett et al, 2008). The severe water depletion threatens access to clean water for drinking for millions of Americans. It severely impacts agricultural yields, biodiversity, and the environment of the surrounding ecosystems. Moreover, it limits the abilities of industries to operate as there is not enough water flow to produce hydroelectric power (Xu and Ramanathan 2017). In addition to electricity generation, the two lakes serve as a major source of grid resilience when full, backing up energy where solar or wind power is on shortage (Bureau of Reclamation). This function is at risk. Clearly, the long-term sustainability of Lake Powell and Lake Mead and the implications for millions people living in the Southwest is a national concern.

Built on extent literature and publicly available data, this study focuses on a unique aspect of the current crisis C the overuse of the Colorado River. Prior studies indicate that water levels failed to recover even after years of above-average snowfall occurring upstream in the River, implying human related environmental destruction as a striking impetus among other culprits. Assuming the climate trend continues, a more careful recalculation of partition of use for the agriculture, residential and industry sectors is necessary.

Grounded on prior findings, this study models the dynamic relation among the following factors: precipitation amount, evaporation rate, groundwater refill rate, water recycling rate and apportionment between stakeholders. Differently from



Figure 1. Colorado River Basin. Lake Powell holds the outflow from the Colorado River Upper Basin states, and Lake Mead is the main reservoir formed by the Hoover Dam on the border between Southern Nevada and Northwestern Arizona

other studies, these factors are applied for both water inflows and outflows. A system of ordinary differential equations (ODE) has been proposed. Then 5-year data (December 2016 C November 2021) from U.S. Bureau of Reclamation (US-BR), National Park Service and Intergovernmental Panel on Climate change has been referenced, analyzed and simulated to optimize the allotment among constituents and outflows from Lake Powell to Lake Mead. Specifically, our analyses adopts the realistic amount of total inflow of 14.8 MAF in most recent 5 years as a base. The analyses also count an additional 0.77 MAF of inflow into Lake Mead from other tributaries. The binding conditions for the ODE system are: a) at any moment, water levels have to maintain equal to or greater than the last 3-year average values b) a minimum of 1.6 MAF flow to Mexico is guaranteed. As a result, the projected water levels from my models meet the USBRs environmental sustainability goals, reflective of pragmatic and attainable solutions to various reality-based scenarios that could incur during our global climate plight. This study contributes to dynamic modeling skills in environmental research as well as serves as a roadmap to recover water levels for the two lakes.

#### 2. Method

The water volume evolution of Lake Powell can be modeled through

$$\frac{dV_p}{dt} = -f_p(t) - \alpha_p S_p + I_p(t) \tag{1}$$

where  $V_p$  is the volume of Lake Powell and  $\frac{dV_p}{dt}$  is the rate of change of  $V_p$  with respect to time t,  $f_p$  is the rate of outflow from Lake Powell into Lake Mead,  $S_p$  is the total surface area of Lake Powell, and  $\alpha_p S_p$  is the net rate of loss of water volume due to the combined influence of evaporation and precipitation through  $S_p$ . The coefficient  $\alpha_p$  is the difference of rate of evaporation and precipitation per unit area per unit time subject to Lake Powell.  $I_p$  is the net rate of water inflow to Lake Powell from Colorado River.

With the apportionment of water usage of the Upper Basin states, the inflow  $I_p$  can be further decomposed as

$$I_{p}(t) = I_{p0} - U_{p} + \gamma_{p}U_{p} - k_{p}U_{p}(t - \Delta t).$$
<sup>(2)</sup>

where  $I_{p0}$  is the rate of inflow to Lake Powell from the upstream Colorado River and various tributaries. Without human

interventions,  $I_p$  consists solely of  $I_{p0}$ , from which no human related water consumption takes place, and the underground water remains statistically saturated over a sufficient long period of time. In reality, however, certain amount of the water, denoted  $U_p$ , is taken away from the Colorado River system before it reaches Lake Powell by people living in the Upper Basin states. Some portion of  $U_p$ , characterized as  $\gamma_p U_p$ , with a proportionality factor  $\gamma_p$ , can be recycled. Furthermore, excessive water consumption by human diminishes the underground water saturation level. To describe this phenomenon, we set  $k_p U_p (t - \Delta t)$  as the s the amount of water that leaves the upstream surface river system to refill the depleted or the partially depleted groundwater reservoirs, where  $k_p$  is a proportionality coefficient C the higher the positive valued  $k_p$  the quicker the water is losing from the surface tributary system (Farsi 2022). The time lag  $\Delta t$  indicates that the current rate of inflow  $I_p$  is influenced by the underground water deficiency in the past with a time delay.

Introducing equation (2) into equation (1), we arrive at the governing equation for Lake Powell,

$$\frac{dV_p}{dt} = -f_p(t) - \alpha_p S_p(t) + I_{p0} - U_p + \gamma_p U_p - k_p U(t - \Delta t).$$
(3)

Similarly, the water volume of Lake Mead is modeled through

$$\frac{dV_m}{dt} = -f_o(t) - f_{MEX} - \alpha_m S_m(t) + f_p + I_{m0} + \gamma_m U_m - k_m U(t - \Delta t)$$
(4)

where  $f_o + f_{MEX}$  is the outflow of Lake Mead into the Colorado River that arrives in Mexico (including the water evaporated along the flow), from which  $f_o$  is the outflow that eventually arrives in the ocean, and  $f_{MEX}$  is the water outflow allocated to Mexico.  $S_m$  is the surface area of Lake Mead,  $\alpha_p$  is the net rate of water loss through  $S_m$  per unit area (caused by precipitation and evaporation).  $f_p$  is the outflow of Lake Powell which is assumed to have completely reached Lake Mead.  $I_{m0}$  is the inflow coming from tributaries between Lake Powell and Lake Mead.  $U_m$  is the rate of water use related to Lake Mead allocated to the Lower Basin states. The proportion of the Lower Basins water use that can be recycled is denoted  $\gamma_m U_m$ , where  $\gamma_m$  is a proportionality factor.  $k_m U_m(t - \Delta t)$  is the amount of water that leaves Lake Powell to refill groundwater reservoirs, where  $k_m$  is the proportionality coefficient. Equations (4) and (3) jointly determine the dynamics of the water volume for the system of reservoirs of Lake Mead and Lake Powell.

We are interested in constructing a system of equations from which the water height of Lake Mead and Lake Powell can be modeled. In order to convert the volume and the surface area in equations (3) and (4) into functions of height, we make considerations as follows.

Let  $\Delta h = h - h'$  be the maximum depth of the lake, where h is the surface elevation and h' is the elevation of the bottom, for either Lake Mead or Lake Powell. Although the 3D shape of the lake is irregular, it can be simplified, to the leading order, as a right cone.

As shown in Fig. (2), a lakes volume can be modeled as

$$V(t) = \frac{1}{3}\pi \tan^2(\theta)(h - h')^3,$$
(5)

where  $\theta$  is half the opening angle according to Fig. (2), and the lakes surface area is

$$S = \pi \tan^2(\theta)(h - h')^2, \tag{6}$$

Time differentiating equation (5) yields

$$\frac{d}{dt}V(t) = \pi \tan^2(\theta)(h - h')^2 \frac{d}{dt}h(t).$$
(7)

Substituting equations (7) and (6) into equations (3) and (4), denoting  $h_p$  and  $h_m$  for the lake surface elevation of Lake Powell and Lake Mead to replace h, respectively, and denoting  $h'_p$  and  $h'_m$  the elevation for the bottom of the Lake Powell and Lake Mead, respectively, to replace h', we obtain

$$\frac{d}{dt}h_p(t) = \frac{-f_p(t) + I_{p0}(t) - (1 - \gamma_p)U_p(t) - k_pU_p(t - \Delta t)}{\pi \tan^2(\theta_p)(h_p(t) - h'_p)^2} - \alpha_p \tag{8}$$

$$\frac{d}{dt}h_m(t) = \frac{-f_m(t) + I_{m0}(t) - (1 - \gamma_m)U_m(t) - k_m U_m(t - \Delta t)}{\pi \tan^2(\theta_m)(h_m(t) - h'_m)^2} - \alpha_m$$
(9)



Figure 2. (a) Simplified 3D shape of the lake and its cross-section. The surface elevation is *h* and the elevation of the bottom of the lake is *h*. The maximum depth of the lake is  $\Delta h = hCh$ . (b) The vertical cross section of the lake where the radius of the surface area is approximated as  $\Delta h \tan \theta$ 

were we have introduced  $f_m = f_o + f_{MEX}$  as the total outflow from Lake Mead to Mexico. Equations (8) and (9) are the governing system of ordinary differential equations for the elevations of Lake Powell and Lake Mead. We want to find the proper set of parameters  $(f_p, \gamma_p, U_p, \gamma_m, U_m)$  on the right hand side of the two equations such that the elevations,  $h_p$  and  $h_m$ , as the solutions to the system, may be sustainably obtained above the safe threshold for peoples living water supply.

# 3. Simulations and Analysis

Based on historical data, the annual net water evaporation (evaporation minus precipitation) per square feet can be estimated as  $\alpha_p = 5.7$  feet for Lake Powell (Dally 2008) and  $\alpha_m = 6.2$  feet for Lake Mead (Dally 2008). According to the Bureau of Reclamation, the bottom elevation of Lake Powell is  $h'_p = 3117$  and  $h'_m = 650$  feet for Lake Mead. The total water inflow from Colorado River upstream is approximated as  $I_{p0} = 14.8$  MAF per year. The water inflow from Colorado tributaries between Lake Powell and Lake Mead is estimated  $I_{m0} = 0.77$  MAF per year. The *tan* $\theta$  of the lake opening angle  $\theta$  as shown in Fig. (2b) is estimated to be  $tan\theta = 6$  for both lakes (Release U.S. Geological Survey Open File Report 2003). The coupled system of equations (8) and (9) can be solved numerically with finite difference by Euler method, in which we approximate the time derivative of the water height as

$$\frac{d}{dt}h(t) \approx \frac{h(t+\Delta t) - h(t)}{\Delta t},$$
(10)

where h can be either  $h_p$  or  $h_m$ ,  $\Delta t$  is a fixed finite difference time increment. The solutions of water height is viewed as functions of time and a set of human related factors:  $h_p(t) = h_p(t|f_p, U_p, \gamma_p, k_p)$  and  $h_m(t) = h_m(t|f_p, f_m, U_m, \gamma_m, k_m)$ . The notation h(t|) emphasizes that  $G = \{f_p, U_p, \gamma_p, k_p, U_m, \gamma_m, k_m\}$  generates the time-varying solution of water height. Different solutions may be obtained once scenarios of different parameter settings of G are raised. The annual inflow of water to Lake Mead  $I_{m0} + f_p$  can be estimated at 9.00 MAF, 8.23 MAF of which can be attributed to the outflow of Lake Powell (Western Resource Advocates),  $f_p = 8.23$  MAF, and the rest,  $I_{m0} = 0.77$  MAF, from downstream tributaries. Other major adjustabe factors are the Upper Basin states water apportionment  $U_p = 7.5$  MAF/year, the Lower Basin states water apportionment  $U_m = 7.5$  MAF/year, and the net outflow of Lake Mead to Mexico  $f_m = 1.5 + 0.6 = 2.1$  MAF/year (0.6 MAF is the estimated evaporation before the outflow reaches Mexico and 1.5 MAF is the actual apportionment arrives at Mexico). These numbers of  $U_p$ ,  $U_m$ , and  $f_m$  were determined based on an over-estimated inflow of the Colorado River  $I_{p0}$ into Lake Powell historically observed over an abnormal wet period in the 1920s. Modern estimates ascribe the inflow  $I_{p0}$  to a value of 14.8 MAF/year, significantly lower than what was originally estimated. The resulting overshot  $U_p, U_m$ , and  $f_m$  are the main reason behind the current water crisis. Our simulation (Fig. 3) reveals that should Lake Powell and Lake Mead continue their current water allocations to the Upper Basin and Lower Basin states, they would reach dead pool in 2-3 months from their current elevation of 3560 ft for Lake Powell and 1090 ft for Lake Mead, where dead pool is reached at ff 3530 ft for Lake Powell and 1065 ft for Lake Mead (Bureau of Reclamation). The simulation assumes the water recycling rate for Lake Powell and Lake Mead  $\gamma_p = \gamma_m = 15\%$ , and the water refilling rate  $k_p = k_m = 10\%$  (Castle et al. 2014). We set the finite difference time increment  $\Delta t = 1$  month (Leake et al. 2013). All annual inflow/outflow and apportionment are averaged to monthly amount by dividing with 12.

From 1995 to 2015, the percentage of industry and mining usage of water remain relatively small and takes only 1% - 2% of the total usage (Tang et al. 2009). We may ignore the impact of competing interests of water availability coming from industry and mining over residential and agriculture. The residential living water usage has grown from 12% in 1995 to 19% in 2015 (Tang et al. 2009).

At the same time, the irrigated agriculture, although declined from 86% to 77%, remains by the largest category of the



Figure 3. Evolution of water height for Lake Powell (top) and Lake Mead (bottom) reach the dead pool level if the annual apportionment to the Upper Basin states, Lower Basin states, and Mexico remain unchanged:  $U_p = U_m = 7.5$  MAF,  $f_m = 2.1$  MAF

water use in the Basin (Tang et al. 2009). The main factors in consideration for solving the competing demand for water by residential living and irrigated agriculture is through recycling water, represented by  $\gamma_p$ ,  $\gamma_m$  and the rate of underground water depletion and refill  $k_p$  and  $k_m$ . The water recycling rate, according to our simulation, is another important factor that may change the outcome of the water elevation completely. If we uplift the water recycling rate from 15% to 20%, while keeping the apportionment fixed (that is  $U_p = U_m = 7.5$ MAF  $f_m = 2.1$  MAF and  $f_p = 8.1$  MAF), the water elevation for Lake Mead will gradually rise (Fig. 4a) as opposed to quickly declining to the elevation of a dead pool (Fig. 3). However, at this recycling rate, the Lake Powell remains the same fate with only slightly longer lifetime extension to 7 months. If we continue to increase the recycling rate by as little as 1%. That is  $\gamma_p = \gamma_m = 21\%$ . Both Lake Mead and Lake Powell will gradually regain their water elevation over a period of 36 months and the elevation will continue to rise if the recycling rate is kept at this rate (Fig. 4). Our simulations, as shown in figures (4a) and (4) suggest that as small as 1% fluctuation of  $\gamma$  influences the resulting water level significantly.



(a) Elevation prediction at  $\gamma_p = \gamma_m = 20\%$ ,  $U_p = U_m = 7.5$  MAF/year,  $f_m = 2.1$  MAF/year.

(b) Elevation prediction at  $\gamma_p = \gamma_m = 21\%$ ,  $U_p = U_m = 7.5$  MAF/year,  $f_m = 2.1$  MAF/year.

Figure 4. Prediction of water elevation for Lake Powell and Lake Mead using slightly different recycling rate at a)  $\gamma_p = \gamma_m = 20\%$  and b)  $\gamma_p = \gamma_m = 21\%$  while the annual apportionment is kept unchanged  $U_p = U_m = 7.5$  MAF,  $f_m = 2.1$  MAF

Given the current technology status, reaching to a recycling rate of 21% is infeasible in common situations. In reality, the water recycling rate can achieve no more than 15% and the apportionment for  $U_p$ ,  $U_m$ ,  $f_m$ , and the outflow of Lake Powell (also inflow to Lake Mead)  $f_p$  must be adjusted, in order to maintain Lake Mead and Lake Power to their operational level in the next few decades. Sustainable water levels can be obtained from solutions to the system of ordinary equations (8)



Figure 5. Elevation prediction for Lake Powell and Lake Mead. Total outflow of Lake Powell is fixed at  $f_p = 7.91$  MAF/year and the total outflow of Lake Mead to Mexico is fixed at  $f_m = 1.63$  MAF/year. The apportionment for the Upper and Lower Basin states reduces from (a)  $U_p = 7.26$ ,  $U_m = 7.43$  to (b)  $U_p = 7.24U_m = 7.41$  to (c)  $U_p = 7.20$   $U_m = 7.39$  MAF/year as water elevation rises in a 36-months prediction time window

and (9) with prescribed values in  $U_p$ ,  $U_m$ ,  $f_m$ , and  $f_p$  in different scenarios. Three water levels are proposed, A) water level that is barely above the dead pool threshold, B) water level of medium height, and C) water level that is well above the threshold with a safe margin. Three corresponding scenarios of  $U_p$ ,  $U_m$ ,  $f_m$ , and  $f_p$  are documented in the sequel. We choose to reduce the Lake Powell outflow  $f_p$  from 8.20 MAF/year to 7.91 MAF/year. This helps to retain water in Lake Powell from over subscribing to Lake Mead. But at the same time, it negatively impacts the water level of Lake Mead. To compensate for Lake Mead, we further reduce the total outflow of Lake Mead  $f_m$  from 2.1 MAF/year to 1.65 MAF/year to account for the reduction of total input from upstream Colorado River. Such a reduction matches with the deficiency for the rivers actual annual snow-pack input compared with the historical over-estimation. The three water level requests, from A) to B) to C) with increasing elevations, is obtained from 36 months simulations with an overall decreasing water apportionment allocation between the Upper and the Lower Basin states. Shown in Fig. (5), the apportionment for the Upper Basin states reduces from 7.5 MAF/year to 7.26 MAF/year for level A, 7.24 for level B, and 7.08 for level C. With this reduction, the Lower Basin state apportionment can be maintained within a close range to its original value of 7.5 MAF/year to 7.50 MAF/year for level A, 7.49 MAF/year for level B, and 7.46 MAF/year for level C. The null or relative smaller reduction of apportionment for the Lower Basin states comes from the consideration that there are less alternative water sources to supply the heavy agriculture and industrial water usage in the Lower Basin states.

## 4. Conclusion

As Lake Powell and Lake Mead reach historic lows, it is time to carefully analyze the root causes of water depletion so that proper actions can be taken timely and disastrous consequences can be avoided. The coupled system of ordinary differential equations, as we propose, can be applied to model the elevations of both lakes. Predictions of water level are obtained by adjusting apportionment among Upper Basin states, Lower Basin states, and Mexico, with realistic parameters that governs the right-hand-side of Equations (8) and (9).

Our model, with considerations to evaporation, weather changing, volume and surface estimation, Colorado River tributary water inflow, and reallocation for apportionment, provides flexibilities to test various scenarios of combinations of controlling factors. With the simulation from our model, it is interesting to point out that the Lower Basin state apportionment can be largely maintained with slight reduction, whereas the Upper Basin states apportionment is strict. Our model also suggests that the water levels of both lakes are sensitive to water recycling rate. Technological advances for water recycling are long-term solutions to the current water crisis.

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# **Author Contributions**

Ms. Shen was responsible for creating the mathematical models and drafting the manuscript. Dr. Chan was responsible for reviewing the research questions and revising the manuscript. Both authors approved the final manuscript.

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# Data sharing statement

No additional data are available.

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