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Mathematical Modeling Through the Eyes of Elementary and Middle Preservice Teachers

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Abstract

Mathematical modeling is a useful pedagogy in mathematics education, but preservice teachers (PSTs) conceptualization of the teaching and learning of modeling practices is an ongoing concern. This research study examined 31 elementary and middle grades PSTs' conceptualization of mathematical modeling and their experiences with such modeling. The study participants were recruited from two, four-year university mathematics methods courses located in northeastern and midwestern United States. Data for this study was collected through a questionnaire and analyzed using qualitative and quantitative methods. Results indicated that most of the PSTs had little understanding of the intersection of mathematical modeling, and the teaching and learning of mathematics. As such, many perceived mathematical modeling as an exclusive action reserved for teachers. Additionally, the results revealed that most participants had minimal, if any, experience with mathematical modeling. These limited experiences portrayed mathematical modeling as a show and tell method or step-by-step explanation. Our results expand the inadequate research into elementary and middle grades PSTs' knowledge of and experiences in mathematical modeling and warrant the exigency for the inclusion of extensive mathematical modeling practices into methods and content courses for PSTs. The directions for future research and implications for researchers and teacher education programs are also discussed.

Keywords: preservice teachers, mathematical modeling, conceptualization, experiences, elementary and middle grades

1. Introduction

For more than a decade, mathematical modeling has been prevalent in standards, documents, and curricula in the United States of America (U.S.) (Consortium for Mathematics and Its Applications [COMAP] & Society for Industrial and Applied Mathematics [SIAM], 2019; Kaiser et al, 2011; National Council of Teachers of Mathematics [NCTM], 2014; National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010). Additionally, mathematical modeling has emerged and shown to be an effective approach to enable students to use mathematics to solve real-world problems. Based on research and the literature in mathematics education, mathematical modeling can be described as a process that uses mathematical representations to model real-world behaviors or characteristics so questions can be answered, and these answers validated in the context in which they were investigated (Borromeo Ferri, 2018; Cai et al., 2014; COMAP & SIAM, 2019; Gould, 2013). Moreover, students' engagement in mathematical modeling not only offers opportunities for interpretation, argumentation, and problem solving but also impacts their dispositions and attitudes toward mathematics (English, 2021; Hernández et al., 2016; Meyer, 2015). Unfortunately, researchers have found areas of concern in elementary and middle grades teachers' mathematical modeling and missed opportunities in the engagement and discussion of mathematical modeling (Blum, 2015; Blum & Borromeo Ferri, 2009; Spandaw & Zwaneveld, 2010).

Even though there have been considerable gains on research in the teaching and learning of mathematical modeling (Blum & Borromeo Ferri, 2009; Borromeo Ferri, 2018; Galbraith et al., 2007; Gaston & Lawrence, 2015), the inclusion and infusion of mathematical modeling into U.S. teacher preparation programs has been a slow process (Asempapa & Sturgill, 2019; Borromeo Ferri, 2018; Hamson, 2003; Phillips, 2016; Tan & Ang, 2016; Zbiek, 2016). In teacher preparation programs mathematical modeling receives little attention on how it can be used in elementary and middle grades education as a support of pedagogical knowledge (Asempapa, 2018; Mathews & Reed, 2007). This slow transition as well as limited support translates to missed experiences and opportunities for preservice teachers (PSTs) to learn mathematical modeling efficiently and consistently (Borromeo Ferri, 2018; Phillips, 2016). Therefore, this research study became necessary to close the research gap and to understand elementary and middle grades PSTs' conceptualization of mathematical modeling and their experiences surrounding it so to inform practitioners, researchers, policy makers, and stakeholders.

Although curricula can provide students with opportunities to learn mathematical modeling, classroom instruction is arguably the most important influence on what students learn about mathematical modeling (Phillips, 2016; Zbiek, 2016). Baumert and his colleagues (2010) stated that "The repertoire of teaching strategies and the pool of mathematical representations and explanations available to teachers in the classroom are largely dependent on the breadth and depth of their conceptual understanding [and experiences] of the subject" (p. 138). However, teachers' initial training and the curricular contexts of schooling have not readily provided opportunities to make mathematical modeling as an integral part of daily lessons (Phillips, 2016; Zbiek & Conner, 2006). Thus, it is essential that PSTs have a well-developed understanding of mathematical modeling and how to engage students in modeling activities when applying their prior knowledge to new mathematical concepts. To reflect this and understand how elementary and middle grades PSTs grapple with mathematical modeling as a concept and in their future practice, we investigated two research questions (RQs):

RQ1. How do elementary and middle grades PSTs conceptualize mathematical modeling?

RQ2. What are elementary and middle grades PSTs' experiences with mathematical modeling?

2. Related Literature Review

The topics that have been important to mathematics education during the past 50 years are the relationship between mathematics and the real-world. Thus, mathematical modeling cuts across all levels of education, and this includes teacher preparation programs. For the purposes of this study, we define mathematical modeling as the "process that uses mathematics to represent, analyze, make predictions or otherwise provide insights into real-world phenomena" (COMAP & SIAM, 2016, p. 8). In retaining the aspect of mathematical modeling, this section examines mathematical modeling conceptualization and definition, mathematical modeling problems, and mathematical modeling in teacher preparation programs.

2.1 Mathematical Modeling Definition and Conceptualization

Researchers in the mathematics education community have offered varying definitions for mathematical modeling. Due to the nature of the problem and context in which it originates, modeling requires translations between reality and mathematics (Borromeo Ferri, 2018). Mathematical modeling presents students with realistic problem-solving experiences that involve critical thinking skills, use of prior knowledge, and strategizing which include testing and revising solutions in a real-world context (Blum & Borromeo Ferri, 2009; Lesh et al., 2010). Delving deeper, mathematical modeling involves identifying a real-world situation, making assumptions to better understand that situation, using a mathematical model to represent some or all the situation, and then applying mathematical techniques to obtain reasonable, real-world results (Pollak, 2003). For the purpose of this study, we define and conceptualize mathematical modeling as real-life problem-solving process that involves mathematical processes such as persevering, using tools, seeing structure, thinking critically, and communicating effectively.

Mathematical modeling has also been described as a process that uses mathematics to represent, analyze, make predictions, or otherwise provide insights into reality through a set of formal structures and representations (English et al., 2005; COMAP & SIAM, 2016). Those engaged in mathematical modeling must move fluidly between the real world and the mathematized world (Blum & Borromeo Ferri, 2009; Galbraith et al., 2007) and develop useful ways to interpret the nature of givens, goals, possible solution paths, and patterns and regularities beneath the surface of things (English, 2021). The use of mathematical modeling to solve real-world problems has been and continues to be a vital means that students use to investigate phenomena that affect their lives as well as society in and beyond their neighborhoods (NCTM, 2009; NGA Center & CCSSO, 2010). Mathematical modeling then becomes a student-initiated (teacher-supported) process for students to make sense of the world around them. With mathematical modeling, the challenging situations provide students opportunities to justify strategies and representations, share and critique these strategies and representations, and discover relationship among strategies and representations (English, 2021; Suh et al., 2017). However, to construct meaningful models and representations of the given situations, students—including PSTs—must understand and make sense of complex and authentic information (Lesh et al., 2010; Kaiser et al., 2010).

Mathematical modeling requires a well-connected set of mathematical representations, structures, and skills that, when used flexibly, supports problem solving and a better understanding of the real-world. Success with routines or traditional tasks do not imply being proficient in mathematical modeling because the process of moving from assumptions to outcomes may not be obvious (Doerr & Lesh, 2011). This is especially true when one has little to no experience with modeling. Jung and Newton (2018) found that PSTs' definitions and understanding of modeling evolved after experiencing a modeling activity and searching for modeling problems in textbooks; however, their experiences with the mathematical process and problems were concerning during their teaching demonstration. Likewise, Paolucci and Wessels (2017) explained that PSTs struggled developing contextualized problems for students in elementary school, and concerns were raised about PSTs' proficiency with the curriculum content and capacity to create content-specific mathematical modeling problems.

2.2 Mathematical Modeling Problems

All mathematics problems are not created equal–different problems require different levels and kinds of student thinking (NCTM, 2014; Stein et al., 2009). Mathematical modeling problems are real-life applications that are of high cognitive demand. Borromeo Ferri (2018) explained that a mathematical modeling problem is cognitively demanding because of the context and transitions between reality and mathematics, but similarly self-differentiated. Thus, mathematical modeling problems give students access to mathematics as an effective way of practicing critical thinking skills and making mathematics more motivating, meaningful, and relevant to students. Gains in student learning are the greatest in classrooms where problems are of high cognitive demand, and students learn to explain as well as apply their thinking and reasoning (Stein et al. 2009). In the interest of this study, two mathematical modeling problems from other studies are presented and discussed.

The first modeling problem, the "*Lighthouse Problem*," was adapted from Blum and Ferri (2009, p. 48): In the bay of Sandusky, in Ottawa county, Ohio, directly on the coast, a lighthouse called "Marblehead Lighthouse" was built in 1819, measuring 15.0 m in height. Its beacon was meant to warn ships that they were approaching the coast. How far, approximately, was a ship from the coast when it saw the lighthouse for the first time? Explain your solution. The second modeling problem, the "*Concert Party Problem*," was adapted from the Organisation for Economic Co-operation and Development [OECD] PISA 2012 Assessment and Analytical Framework (2012, p. 52): To hold a rock concert, a 100 m by 50 m rectangular field was reserved for the audience. The concert was completely sold out, and the field was packed with a diverse group of fans. What is the best estimate of the total number of people attending the concert? Both modeling problems are contextualized, considered to be high cognitive demand, promote reasoning and critical thinking, and encourage problem-solving skills. Consequently, modeling problems pose a challenge in constructing a mathematical model for the real-world situation. Therefore, with the support of teachers, students may better conceptualize and understand the modeling process so that they can develop strategies to solve the modeling problem.

The challenges of engaging in mathematical modeling extend beyond a single group, and it necessitates skill sets students may initially lack. Mathematical modeling problems "are only a little simplified and … recognized by people working in this field as being a problem they might meet in their daily work" (Kaiser et al., 2011, p. 592). As such, these problems can be presented in an authentic, yet complex or messy context. Moreover, mathematical modeling problems require creative thinking, perseverance, and problem solving. They have multiple points of entry and often need novel approaches to authentically connect real-world situations to mathematics (Borromeo Ferri, 2018; Maaß, 2007). In this regard, mathematical modeling problems are different (and even more complex) from traditional word problems typically seen in elementary and middle grades textbooks, or resources, or materials. Mathematical modeling problems are essential and meaningful thought-revealing problems (Lesh et al., 2010) that require students to articulate and modify their current ways of thinking to interpret and create useful models to solve the real-life, complex situations. In the mathematics education literature, there are many characteristics of mathematical modeling problems (e.g., Borromeo Ferri, 2018; Galbraith, 2012; Lesh et al., 2010; Maaß, 2007); however, a critical component in the process of modeling is the one who introduces the problem and supports those involved (Borromeo Ferri, 2018).

Mathematical modeling problems present unique challenges for PSTs who are not typically required to take courses on modeling as part of their preparation. Challenges stem from working with open-ended problems, making and validating assumptions, and interpreting the mathematical results in the context of the given situation (Anhalt & Cortez, 2016; Blum & Borromeo Ferri 2009). On the other hand, the use of mathematical modeling problems provides PSTs opportunities to engage students in authentic mathematical problems that foster critical thinking, mathematical understanding, innovation, and communication needed to advance 21st century skills (Paolucci & Wessels, 2017; Zbiek & Conner, 2006). These types of problems are regarded as high-leverage problem-solving situations and practices (Ball et al., 2008) and are an important process in the development of PSTs' knowledge on the teaching and learning of mathematics using high cognitive demand situations (Paolucci & Wessels, 2017; Schukajlow et al., 2018). Consequently, in teacher preparation programs, it is relevant to address what counts as a mathematical modeling problem as well as the actors involved and their roles. Particularly, it is important for teacher preparation programs to add incrementally over time to what is known about how PSTs can best be prepared to effectively teach and provide support to students as they engage in mathematical modeling.

2.3 Mathematical Modeling and Teacher Preparation Programs

There is broad consensus that teachers need to have strong knowledge of the subject content, as this influences both what they teach and how they teach it (Ponte & Chapman, 2016). Because teaching is a complex profession, teachers draw upon and apply their past experiences, expertise, and knowledge to pedagogical practices or in the moment decisions. Both elementary and secondary PSTs' knowledge of teaching mathematics has received much attention in mathematics education research (Borromeo Ferri, 2018; Ponte & Chapman, 2016). Studies on PSTs' mathematics knowledge, including mathematical modeling, indicate serious issues that teacher preparation programs ought to address (Borromeo Ferri, 2018; Ponte & Chapman, 2016). Issues include misconceptions for different topics of school mathematics, such as modeling literacy, problem solving, and quantitative reasoning. Therefore, it is no surprise that PSTs' mathematical modeling knowledge continues to be a central theme in teacher preparation program research.

Teacher preparation programs have been examined and marked regarding mathematical modeling without its effectiveness being analyzed empirically (Kaiser et al., 2010). PSTs need to acquire special mathematical knowledge different from core college-level instruction, and mathematical modeling is no exception (Borromeo Ferri, 2018; Usiskin, 2015). Most teacher preparation programs have been unable to provide extensive modeling experiences for PSTs, although these PSTs would be expected to teach their future students modeling-based mathematics (Borromeo Ferri, 2018; Usiskin, 2015). Moreover, researchers have found that PSTs have limited understanding about the classroom teaching of mathematical modeling (Jung & Newton, 2018; Paolucci & Wessels, 2017; Phillips, 2016). Knowledge and experiences do impact individuals' actions as well as form the foundation from which PSTs build upon to become high-quality teachers. To better prepare future teachers for the current classroom, they need to engage in mathematical modeling to support student learning and help students develop related competencies (Philips, 2016). Many of these experiences as developing educators should confront their conceptions about modeling, structure of modeling problems, pedagogical modeling-focused ideas, and understanding of students' modeling competencies (Gould, 2013; Zbiek, 2016).

Research has found that most PSTs' understanding of mathematical modeling in teacher preparation programs is limited or scarce, and this is impacted by how teacher preparation programs have addressed mathematical modeling based on the Common Core State Standards for Mathematics [CCSS–M] (Corum & Garofalo, 2019; Jung et al., 2016; Jung & Newton, 2018). Although the CCSS–M emphasizes the need for teachers to teach mathematical modeling, many teachers have not experienced mathematical modeling as learners (Phillips, 2016) and most teachers are unfamiliar with mathematical modeling and accompanying practices (Usiskin, 2015). Researchers have argued and explained that teacher preparation programs must provide PSTs experiences, skills, and educational attitudes that are needed to teach mathematical modeling to their future classes (Akkan et al., 2018). Yet, few teacher preparation programs provide meaningful courses, course modules, or projects on issues relating to mathematical knowledge for teaching mathematical modeling (Akkan et al., 2018; Borromeo Ferri, 2018). Therefore, it is important that we support teacher preparation programs where such modeling experiences engage PSTs in reasoning and constructing mathematical models; assessing the extent to which a mathematical argument is valid; and developing, comparing, and evaluating alternative solution processes.

The need exists for understanding PSTs' knowledge of and experiences in mathematical modeling so steps can be taken to better prepare them for their own classroom in which this modeling is used as a teaching and learning tool. However, most research on the intersection of teachers and mathematical modeling focuses on practicing teachers or secondary PSTs. Yet, mathematical modeling and the use of it in the teaching and learning of mathematics reaches beyond these two groups. As teachers' conceptualizations and experiences can affect their mathematics instruction, it is important to understand them in relation to mathematical modeling, especially in case of PSTs', whose conceptualizations are open to examination and change. Therefore, this present study sought to understand elementary and middle grades PSTs' knowledge of and experiences in mathematical modeling.

3. Method

3.1 Context and Participants

In this research study, we used qualitative methods to examine and understand how PSTs grapple with the concept of mathematical modeling and their experiences with such modeling. The research team consisted of three faculty from different universities. Collected expertise includes mathematical modeling, representation, and quantitative and qualitative analysis. We used purposive sampling (Leedy & Ormrod, 2019) to determine interested participants. Participants included 31 elementary and middle grades PSTs (mostly sophomores and juniors) who were recruited from their methods courses taught at two, four-year universities located in the northeastern and midwestern U.S. These participating PSTs sought licensure for elementary or middle school grade levels and were in the middle of their teacher preparation program (not completers). The PSTs had enrolled in other mathematics content or methods courses as well.

3.2 Data Collection

Data collection consisted of participants' responses from a two-prompt questionnaire (Appendix A) addressing PSTs' knowledge of and experiences in mathematical modeling. We administered the questionnaire in two different mathematics methods courses during class time. Participants were given 20 to 30 minutes to complete the questionnaire. Their written responses were collected anonymously, converted into PDFs, and analyzed using mainly qualitative techniques and a few quantitative approaches. In the next section, we provide the description of how the data was analyzed and coding schemes used for the data analysis.

3.3 Data Analysis

An inductive content analysis approach (Grbich, 2007) was used to address the two research questions. Data analysis began by applying a rubric adapted from Asempapa (2018) designed to support the thematic analysis of participants' knowledge of and experiences in mathematical modeling (see appendix B). The steps involved in mathematical modeling process include understanding the problem, mathematically formulating the real-life situation, performing operations to solve the model, and interpretation of the solution and its application. We used an existing rubric to assess participants conceptualization of mathematical modeling. The rubric's reliability and validity information can be found in Asempapa (2018). To establish a consensus on the ratings applied, we individually examined 10% of our participants' work (i.e., four responses) and then came together to discuss the ratings. Once we reached a consensus on the ratings across categories based on the rubric (Appendix B), we then separately rated all 31 participants, discussing and resolving any disparities. The rubric we used had a 4-point Likert-type scale to rate PST's definition of mathematical modeling as poor (1), fair (2), good (3), and excellent (4) as shown in Appendix B. Due to the nature of the data, intraclass correlation coefficient (ICC) was chosen to measure our rating consistency (Liljequist et al., 2019; Shweta et al., 2015). ICC was helpful with this data set because it provides information about both inter-rater reliability and inter-rater agreement (Landers, 2015; LeBreton & Senter, 2008; Shweta et al., 2015). The ICC ratings were all acceptable and above 0.89 (Liljequist et al., 2019).

We developed codes deductively and inductively. Deductive in that we could not fully distance ourselves from mathematical modeling literature and prior studies. Inductive in that PSTs' own experiences and our understanding of these experiences led to the creation of codes. The process for coding transitioned from initial coding to focused coding (Saldaña, 2013). During initial coding, we collectively read participants' responses discussing potential themes and making memos. Example of memos include how the PSTs incorporated manipulatives in the description of mathematical modeling or the teacher-centered focus when PSTs recounted their modeling experiences. Initial codes were adapted and combined so distinctions among them were recognizable and applicable. These actions flowed into a focused coding of the data in which HyperResearch© software was used. Additionally, this software enabled us to retrieve, build theories, and conduct analyses as shown in Table 1.

Table 1. An Example of the Coding Evolution

Initial Codes	Focused Subcodes
Characteristic of math modeling	Collaboration
	Explanation/justification
	Manipulative
Experiences of math modeling	Main actor is the teacher or instructor
	As a method of instruction
	Have no experience

The goal of the coding evolution from Table 1 was to provide a sample from the initial coding with connected subcodes derived from the focused coding, which enables data analysis and successive steps to serve the purpose of the study. Validity was maintained due to the cyclic nature of coding, memo writing, collective agreement, and attributes of our research team.

4. Results

In this section, we present the results of our investigation into PSTs' knowledge of and experiences in mathematical modeling. We begin by summarizing our analysis of participants' responses and then discuss common themes. In the following paragraphs, we build a case for findings that address our two research questions: (a) RQ1: How do elementary and middle grades PSTs conceptualize mathematical modeling? and (b) RQ2: What are elementary and middle grades PSTs experiences with such modeling?

4.1 RQ1: Mathematical Modeling Conceptualization

RQ1 examined how PSTs conceptualize mathematical modeling. All but one of 31 participants responded to this prompt: "What is mathematical modeling?" Twenty-six (87%) PSTs had a "Poor" or "Fair" understanding of mathematical modeling, while only four (13%) had a good understanding. Example quotes rated as "Poor" or "Fair" include "using manipulatives to represent math/thinking" (Participant 1), "I believe it will be like the graphic organizers we made for the article review" (Participant 20), and "showing examples to students of similar problems and then to solving the[se] problems" (Participant 6). However, as previously mentioned, four PSTs had a "Good" understanding of modeling: "mathematical modeling is taking concepts, formulas, etc. … and applying them to the real world" (Participant 29). Per anecdotal evidence, we were not surprised that no participating PST had an excellent understanding of mathematical modeling. Given that many participants had an inaccurate or underdeveloped understanding of mathematical modeling, we will now discuss the common themes.

4.1.1 Who Uses Mathematical Modeling

Mathematical modeling is not exclusive. Both teachers and students can actively engage in this process and benefit from it as well. Nevertheless, about 30% of participating PSTs placed the act of modeling on the teacher who is then described as being engaged in traditional instruction. "Mathematical modeling is when the instructor demonstrates mathematical strategies through example[s]" (Participant 4). Echoed by other PSTs, Participant 6 stated that modeling occurred when one "show[s] examples to students of similar problems and the steps to solving the problems." However, Participant 11's response affirmed that mathematical modeling is not an exclusive action: "Mathematical modeling is a visual support that the teacher or student can use to help support the process and answer the problem."

4.1.2 How to and Why Use Mathematical Modeling

As described previously, most participants understood modeling as an action reserved for teachers. Yet, in the opinion of our participants, how and why was modeling being used? Half of the participating PSTs explained that teachers use mathematical modeling to solve examples in front of students, show them strategies, or both. Participant 13 provided a list on how teachers use modeling: "demonstrating [to] students a strategy … for them to apply in their problems, repetition, [and] seeing different strategies." Another PST simply stated that modeling occurs when one "model[s] mathematics with examples" (Participant 30). However, mathematical modeling involves transitioning between mathematics and the real world while making sense of one's actions and assumptions. This view was partially shared by three PSTs who explained that mathematical modeling involves "taking math and applying it to real-world scenarios" (Participant 23).

As noted, many participants saw mathematical modeling as a method of instruction, and underlying their responses is presumably the use of modeling to support student learning. Three of the 30 PSTs explained the benefit as either directed at student learning or for accessibility. Modeling "is taking concepts that are sometimes abstract and making a concrete model so students can better understand" (Participant 18). Not only is modeling being applied to the abstract but also through "representing parts of a math problem" and as "concepts as way for students to learn" (Participants 18 & 10, respectively). What might be considered rare, in our study, is how one PST ended the description of modeling as a method to engage students: "I believe mathematical modeling is when a teacher models to their students good strategies for mathematics and sparks an interest" (Participant 20).

4.1.3 Representations in Mathematical Modeling

Participants also described the action of mathematical modeling as visual (n = 10 or 33%) or with symbolic representation (n = 6 or 20%). An example of visual would be how Participant 10 described modeling as a visual support to support student learning. A different PST added that visuals would "help support the [modeling] process and answer the problem" (Participant 11). Other participants explained visual in terms of "showing through visuals how to solve a problem" or "demonstrating [to] students a strategy" to solve problems (Participants 12 & 13, respectively). In particular, Participant 4 added these demonstrations were mediated through examples in which the teacher "visually show[ed] each step" in solving them.

The second common referral to modeling was how it was used as a representation in problem solving—a notion that is seen in the next finding as well. Three of six PSTs stated that mathematical modeling occurs when representations are used to help solve a problem. As Participant 15 explained, these representations are used to make the "problem … more understand[able]." As such, this act could occur when elements of the problem are represented by "concrete items" (Participant 17). However, the other three viewed the application of mathematical modeling in a broader sense. Modeling "is taking concepts that are sometimes abstract and making a concrete model so students can understand" (Participant 18). According to a different, yet similar view of a PST, this model could also be applicable in other subjects, not just mathematics.

Only four participants characterized mathematical modeling with the use of manipulatives; however, three of four did include other qualities, such as visuals and demonstrations. Participant 12 summarized the steps needed to engage in mathematical modeling as below:

- Occurring in a whole-class setting in which a lesson is taught or concept demonstrated,
- Modeling the ideas/concepts with manipulatives, and
- Providing students with much practice.

Accordingly, one could use "manipulatives to model an addition or subtraction problem" said Participant 8. For the four PSTs who described mathematical modeling involving manipulatives, their use of it centered on how to "represent math/thinking" (Participant 1).

4.2 RQ2: Mathematical Modeling Experiences

RQ2 explored PSTs' experiences about mathematical modeling. In doing so, we critically examined their knowledge gained and lived through experiences of mathematical modeling. We first quantitatively summarize our results for RQ2 and then thematically present our findings. The data showed that 12 participants (39%) received a rating of 1 (i.e., no experience or not applicable experience), while the others (61%) earned a rating of 2 (i.e., one or two accurate experiences). Furthermore, slightly less than one-fourth of the participants (n = 7) explained that they never experienced or were unaware of such an experience. Such a revelation did not surprise us but speaks volumes of the need to better inform and train teachers to become facilitators of mathematical modeling and how it can be used as a teaching and learning tool. Overall, all PSTs in this study had little to no experience with mathematical modeling. We next summarize the common themes among their responses.

4.2.1 Focus on the Teacher or Instructor

When asked about their experiences with mathematical modeling, one-third (n = 10) of the participants described settings in which the teacher or instructor was the main actor. Such a result was also noted in how the PSTs described mathematical modeling. In addition, many of these 10 PSTs either spoke positively or negatively (not neutral) about the outcome of these classroom practices. "In school, math teachers will bring items to help model how a problem works to make it easier to understand" (Participant 18). Unfortunately, not all had a positive view of mathematical modeling in terms of examples. "My middle school teacher always modeled problems on the board without any variation, and this made it somehow difficult to understand" (Participant 2). One PST remarked that "even if the teacher did various examples, I would still be confused" (Participant 13). That is, some PSTs have different perceptions on their teacher's use of a variety of examples.

4.2.2 Teacher or Instructor Use in Practice

In general, PSTs saw mathematical modeling as problem solving initiated and completed by the teacher or instructor. Some quotes from the participants supporting this theme include "showing and explaining to students what you know" (Participant 26), "My teachers would always show how to solve problems on an overhead" (Participant 3), and "My instructor would show a formula then go through each step explaining how and why we had to answer/solve the problem using that method" (Participant 4). Four participants, however, discussed a differing viewpoint of mathematical modeling. Two participants included small groups or pairs of students in their description. As noted by Participant 14, "[the instructor] would write multiple problems on the board that relate to the concept and have all students, in pairs, work to solve each one." Participant 1 described that mathematical modeling could occur in small groups, stations, and student-led activities. As such, and as explained by another PST, these activities should be "relatable to engage young learners" (Participant 23). In addition, setting was found as a theme as explained in the next section.

4.2.3 Setting

What is interesting to note is that among the participants' experiences with mathematical modeling, not one PST mentioned two or more educational settings. That is, the PSTs described such classroom practices in elementary school, middle school, high school, or college but never once added that mathematical modeling could span grade levels or traverse from secondary education into higher education.

5. Discussion

The research questions that guided this study sought to examine how PSTs grapple with the concept of mathematical modeling and their experiences with such modeling. The CCSS-M expects elementary and middle grade students to engage in mathematical modeling (i.e., worthwhile, authentic, and open-ended tasks in a real-world context) as emphasized in the standard of mathematical practice–model with mathematics. Engaging students in mathematical modeling experiences provides opportunities for interpretation, argumentation, and justification, that are not common in the word problems typically used in early grades (English, 2021). This study serves as a critical reference about modeling standards, associated practices, and the CCSS–M within the context of mathematics teacher preparation; particularly, given that mathematical modeling is an important concept in the teaching and learning of mathematics. The findings from this study reveal important information for consideration that impact teacher preparation programs and future classroom practices when discussing PSTs' conceptualization of mathematical modeling and their experiences with such modeling. In the next section, we discuss our findings and their significance in relation to the research questions.

5.1 Mathematical Modeling Conceptualization

In this current study, having PSTs shared their conceptualization and experiences in mathematical modeling was deemed critical for teacher preparation programs. As indicated by the literature on mathematical modeling (Lesh et al., 2021), PSTs should be provided with learning opportunities in which they can develop their conceptions and foster knowledge about mathematical modeling in mathematics education (Borromeo Ferri, 2018; Sevis, 2016). The first part of our research question investigated how PSTs conceptualize mathematical modeling in relation to the teaching and learning of mathematics. The findings for this question indicated that almost all the participants in this study had little to no understanding of what mathematical modeling is in relation to the teaching and learning of mathematics. When it came to PSTs' mathematical modeling conception and knowledge, other studies have showed comparable findings or effects (Jung et al., 2019; Paolucci & Wessels, 2017), but with secondary grades. The findings from previous studies also reveal that many teachers, including PSTs, have a limited understanding of mathematical modeling. A possible explanation for this trend of little attention to mathematical modeling in teacher preparation programs could be the lack of mathematical modeling and integrated modeling courses within teacher preparation programs (Borromeo Ferri, 2018; Kuntze et al., 2013). This means that most learning on how to integrate modeling in mathematics instruction is done after PSTs have entered the profession as seen in some research studies on developing professional development courses on mathematical modeling for teachers (Phillips, 2016; Zbiek, 2016). Providing more support and attention to mathematical modeling in teacher preparation programs might make this learning process and experience of PSTs more efficient and effective.

Additionally, most PSTs perceived mathematical modeling as an exclusive action reserved for only teachers. This perception is contrary to the CCSS–M modeling standards: "mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGA Center & CCSSO, 2010, p. 7). Additionally, most participants interpreted mathematical modeling as an instructional tool, and the use of modeling to promote student learning was probably the basis of their responses. Moreover, PSTs described the act of mathematical modeling to include a visual element or symbolic representation. Again, this notion carried by the participants about mathematical modeling shows their inadequate understanding of what mathematical modeling is about: a process by which students consider and make sense of an everyday situation that will be analyzed using mathematics for the purpose of understanding, explaining, or predicting the situation (Association of Mathematics Teacher Educators [AMTE], 2020; Borromeo Ferri, 2018, COMAP & SIAM, 2016; Hirsch & Roth McDuffie, 2016; NGA Center & CCSSO, 2010).

5.2 Experiences With Mathematical Modeling

The second research question explored the experiences that PSTs have with mathematical modeling. The findings for this question showed that most participants had little to no experience with mathematical modeling. As previously discussed, these experiences were second-hand in nature because the main actors were the teachers themselves, not the PSTs. These limited experiences may be attributed to the teacher preparation programs whose focus on mathematical modeling was limited or nonexistent (Asempapa & Sturgill, 2019; Borromeo Ferri, 2018; Phillips, 2016). Moreover, research studies have found that several teacher preparation programs' support for PSTs to teach or integrate mathematical modeling is limited in its effectiveness (Borromeo Ferri, 2018; Widjaja, 2013). Differing pedagogies emerged as PSTs described their experiences on how their former teachers used mathematical modeling: from show and tell, to small-group relevant activities. Nonetheless, the prominent descriptors used to explain the practice of mathematical modeling were characteristics of traditional instruction. As such, these descriptions may be influenced by PSTs' scarce mathematical modeling experiences and the method of instruction experienced by our PSTs when they were in school. Many participants discussed mathematical modeling experiences in one setting (e.g., elementary school) and, as such, could have simply forgotten; especially, if such experiences were distant from daily life.

6. Limitations

In any research study, it is important to acknowledge the limitations. This study had a few limitations and some opportunities for expansion. First, using two questions on the questionnaire without other follow-up questions is inadequate to capture PSTs' in-depth conceptualization and experiences of mathematical modeling. Even though we intended to ask open-ended questions for more in-depth responses, our PSTs' responses were generally shallow with short answers or yes/no comments. In assessing PSTs' conceptualization of mathematical modeling and having in-depth understanding of their modeling experiences, employing qualitative techniques such as interviews and focus groups might be helpful in future studies. Second, we were unable to connect their understanding of mathematical modeling to their implementation or engagement in mathematical modeling. Thus, in addition to collecting quantitative data, collecting qualitative data (e.g., asking them to solve a mathematical modeling task and analyzing their solution process) might help us to truly understand our participants' mathematical modeling conceptualizing and experiences. Finally, our sample size was not enough to generalize the results in a broader context of the U.S. elementary and middle grade levels teacher education population.

7. Conclusion

The purpose of this study was to investigate elementary and middle grades PSTs' conception of mathematical modeling in relation to their experiences with modeling practices. This study contributes to the current literature on mathematical modeling and the knowledge base of mathematics teacher education. This suggests that teacher educators need to work on developing PSTs' knowledge of and conception of mathematical modeling in line with recommendations of current reform documents and of researchers who are interested in furthering understanding of teachers' knowledge, conception, and experiences of mathematical modeling. In teacher preparation programs, a mathematical modeling-infused approach can provide the concurrent and authentic content and pedagogy, supporting PSTs to teach mathematics to help students see the relevance of mathematics and make it more meaningful (Borromeo Ferri, 2018; Hirsch & Roth McDuffie, 2016; Jung et al., 2019). Mathematical modeling provides an effective context for developing students' problem-solving and creative thinking skills (Cirillo et al., 2016; Jung & Newton, 2018). Additionally, modeling practices highlight mathematical connections, addresses effective strategies of teaching and learning, and reinforces students' use and understanding of mathematics around us (Borromeo Ferri, 2018; Cai et al., 2014; Paolucci & Wessels, 2017). Although mathematical modeling may not supplant traditional approaches to the teaching and learning of mathematics, it does provide teachers with additional tools of reaching and motivating students. We recommend that teacher preparation programs, and teacher educators need to provide opportunities for PSTs to relearn mathematics through mathematical modeling. Although we cannot change PSTs' past experiences with mathematical modeling, teachers of mathematics methods or content courses can help them experience mathematical modeling by focusing more on the processes and metacognition than on the end-product. Therefore, we believe PSTs not only need to develop a rich sense of mathematical modeling but also grow a repertoire of effective classroom practices that will help their future students to develop mathematical modeling competencies (Philips, 2016).

The findings of this study showed that our elementary and middle grades PSTs' conceptions of mathematical modeling were minimal. Most participants indicated that either they had little or no experience with mathematical modeling. In the context of teaching, many PSTs explained modeling as using physical manipulatives or showing a step-by-step process. Notwithstanding the suggestions by researchers that students should engage in mathematical modeling early in their mathematics education, PSTs need more training and support in meeting this challenge. A possible reason may be the lack of mathematical modeling courses or integration of modeling within teacher preparation programs (Asempapa & Sturgill, 2019; Borromeo Ferri, 2018) and this study demonstrates and advocates the ongoing need for it to be better addressed both in teacher preparation courses and with more comprehensive and widely accessible resources in these early grades mathematics instruction. Moreover, the findings suggest that teacher preparation programs, as emphasized by the Common Core 2010 and AMTE 2020 standards documents, should integrate or infuse mathematical modeling practices, modules, and if possible, courses into their curricula to optimize PSTs' mathematical modeling competencies. Finally, we believe that having PSTs engage or participate in experiences such as attending workshops, seminars, and conferences about mathematical modeling will develop their modeling practices and skills, so that their conceptual models could be revised and extended.

Although this study was unable to unearth other aspects of PSTs' conceptualization and experiences of mathematical modeling because participants were in the middle of their teacher preparation program (i.e., not completers) and had not yet enrolled in other mathematics content or methods courses, the findings provide a strong and reasonable starting point for building future research around PSTs and mathematical modeling. First, future studies should examine PSTs' knowledge in mathematical modeling practices who have completed their program within a year or two (i.e., completers). Second, future studies should include other forms of qualitative methods of collecting data such as observations, interviews, case studies, and document analysis that can provide in-depth insights into how PSTs conceptualize mathematical modeling and specifics on what their experiences are with mathematical modeling. Finally, future research should investigate the dimensions of mathematical modeling competencies among elementary and middle grades PSTs in various settings with the focus of describing the pedagogy of mathematical modeling and ways of improving PSTs' competencies of mathematical modeling practices.

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Appendix A

Mathematical Modeling Questionnaire

The following two-item questionnaire was used to capture elementary and middle grades preservice teachers' (PSTs) conceptualization and experiences of mathematical modeling. The questionnaire has been edited to fit within this page.

Instructions: This activity is designed to explore PSTs' practices on mathematical modeling. Because this activity is voluntary and by participation, these are no correct or incorrect responses to these questions. So, feel free to answer the following prompts to the best of your ability.

1. What is mathematical modeling?
2. What experiences have you had with mathematical modeling practices?

Appendix B

Mathematical Modeling Questionnaire Rubrics

These rubrics were used to rate the questionnaire.

Prompt 1: What is mathematical modeling?

Category			
Excellent = 4	*Good = 3*	*Fair = 2*	*Poor = 1*
Definition demonstrates complete understanding and provides detailed explanation. It states almost all steps involved in the modeling process—links mathematics, real world situations, and the translation between the two.	Definition demonstrates basic understanding and provides minimal explanation. It mentions more steps involved in the modeling process. There is a link between mathematics and the real world.	Definition demonstrates little understanding and little to no explanation. It mentions fewer steps involved in the modeling process. There is no link between mathematics and real-world.	Definition shows no evidence of understanding of the phrase mathematical modeling.

Note: Rubric categories adapted from Asempapa, 2018; and Blum & Borromeo Ferri, 2009. A rating of 0 will be given for no response.

Prompt 2: What past experiences have you had with mathematical modeling practices?

Category		
Has Experience = 3	*Somewhat Experience = 2*	*No Experience = 1*
Explanation provides three or more accurate experiences.	Explanation provides one or two accurate experiences.	Explanation provides no experiences or nonapplicable experiences.

Note: Experiences considered included a workshop, seminar, training, courses taken, teaching observation, fieldwork, professional development, K–12 past school experience, outside school experiences (e.g., playing school, grocery shopping).