Journal of Mathematics Research; Vol. 15, No. 2; April 2023
ISSN 1916-9795 E-ISSN 1916-9809
Published by Canadian Center of Science and Education

Relation Between the Golden Ratio Phi and Zeta Function SUM

Shaimaa Said Soltan[1]

[1] Computer Engineer, Toronto, Canada

Correspondence: Shaimaa Soltan, 3050 Constitution Blvd, Mississauga, ON., L4Y 3X1, Canada

Received: January 4, 2023 Accepted: March 22, 2022 Online Published: April 5, 2023

doi:10.5539/jmr.v15n2p31 URL: https://doi.org/10.5539/jmr.v15n2p31

Abstract

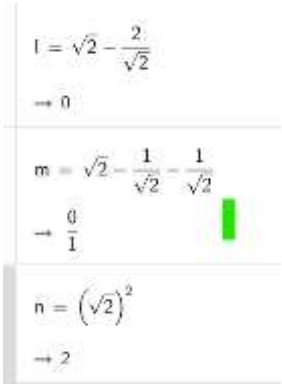
This paper will explain the relation between the golden ratio Phi and Zeta function SUM. First, we will introduce why we used Phi and its functional properties then we will go through some of Phi Properties in a complex plane. Finally, we will use this Golden ratio Phi functional formula, to find the sum of the Prime numbers in Zeta function infinite series in relation with the Golden ration Phi, and pi.

Keywords: zeta function, Riemann hypothesis, complex plane, none-trivial zeros, critical strip

1. Introduction

Some of the floating-point operations are not reversable and value depends on computer or calculator accuracy. This is a floating-point Operations approximation Issue with calculators and computers.

Try these three operations.



but

$$IFF\ \left(\sqrt{2}\right)^2 = 2\ THEN\ \left(\sqrt{2} * \sqrt{2}\right) = 2$$

Then

$$\sqrt{2} - \frac{2}{\sqrt{2}}\ it\ should\ be\ \sqrt{2} - \frac{2}{\sqrt{2}} = \sqrt{2} - \frac{\sqrt{2} * \sqrt{2}}{\sqrt{2}} = \sqrt{2} - \sqrt{2} = 0$$

Why $in\ computer\ calculators\ \sqrt{2} - \frac{2}{\sqrt{2}} = -2.3695649343384955383640735648349e - 47$

And in some other calculators = $2.2204460492503 * 10^{-16}$

$$value\ depneds\ on\ calculator\ accuracy\ although\ same\ Calculator\ shows\ \left(\sqrt{2}\right)^2 = 2$$

And this value is exactly double the value for $\sqrt{3} - \frac{3}{\sqrt{3}} = -4.4408920985006 * 10^{-16}$

$$\frac{\left(\sqrt{2}-\frac{2}{\sqrt{2}}\right)}{\left(\sqrt{3}-\frac{3}{\sqrt{3}}\right)}=-\frac{1}{2}$$

$$\frac{\left(\sqrt{13}-\frac{13}{\sqrt{13}}\right)}{\left(\sqrt{12}-\frac{12}{\sqrt{12}}\right)}=\frac{1}{2}$$

so, due to these variations between calculators we are going to use this number $\sqrt{2}$ as if it was unit (variable) for our operations so we will be able to reverse it. The only constant we know that will have $\sqrt{2}$ is $\sqrt{\varphi}=\sqrt{\frac{\sqrt{5}+1}{2}}$

2.1 $\sqrt{\varphi}$ and φ Properties

We are going to use $\sqrt{\varphi}$ *and* φ as known fixed point for all calculations to understand other functions behaviours in term at function value Phi then interpret this behaviour to the rest of the functional forms.

A) Facts and properties of $\varphi=\frac{\sqrt{5}+1}{2}$

a. Multiplication of two consecutive numbers equal one; Because $\varphi\,(\varphi-1)=1$

i. $at\ X=\varphi\,;\quad X\,(X-1)=1$

b. Reciprocal of number equal Previous number; because $(\varphi-1)=\frac{1}{\varphi}$

i. $at\ \ X=\ \varphi\,;\ (X-1)=\frac{1}{X}$

c. Difference between number and its reciprocal equal 1; because $\left(\varphi-\frac{1}{\varphi}\right)=1$

i. $at\ \ X=\ \varphi\,;\ \left(X-\frac{1}{X}\right)=1$

B) Study φ Reciprocal of number Properties

$$(\varphi-1)=\frac{1}{\varphi}$$

$divid\ both\ sides\ by\ \sqrt{\varphi};\ Then\ \left(\sqrt{\varphi}-\frac{1}{\sqrt{\varphi}}\right)=\frac{1}{\varphi\sqrt{\varphi}}$

THEN at X = φ

$$\left(\sqrt{X}-\frac{1}{\sqrt{X}}\right)=\frac{1}{X\sqrt{X}}$$

This function has some interesting, sweet points for its value.

$$f(X)=\ \sqrt{X}-\frac{1}{\sqrt{X}}$$

1- At X = 1 THEN f(x) = 0

2- At X = 2 THEN $f(X)=\frac{1}{\sqrt{2}}$

3- At X = 4 THEN $f(X)=\frac{3}{2}=1.5$

4- *At X = 16 THEN f(X) =* $\frac{15}{2} = 3.75$

5- *At X = 25 THEN f(X) = 4.8*

6- *At X = 64 THEN f(X) =* $\frac{63}{8} = 7.875$

7- *At X = 64 THEN f(X) = 9.9*

8- *At X =* φ *THEN* $f(X) = \sqrt{\varphi} - \frac{1}{\sqrt{\varphi}} = \frac{1}{\varphi\sqrt{\varphi}}$

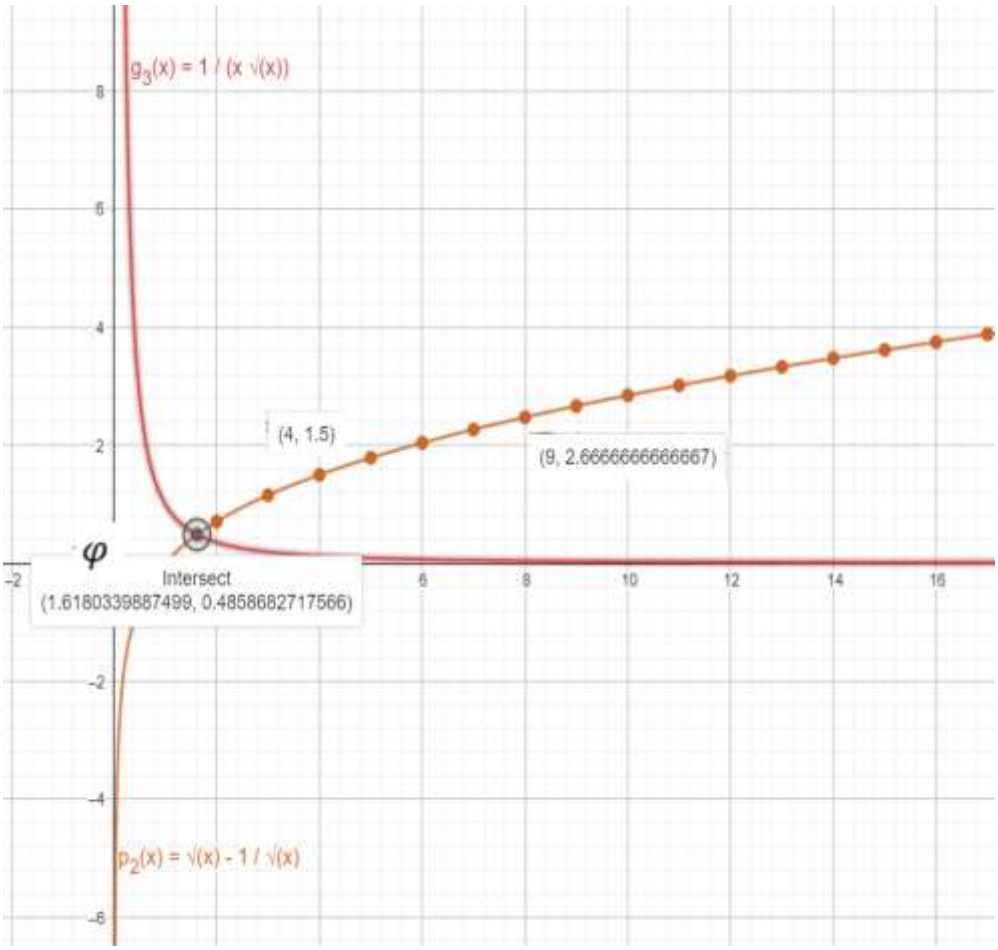
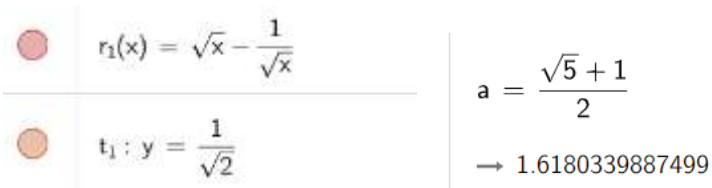


Figure 1. both functions $\left(\sqrt{X} - \frac{1}{\sqrt{X}}\right)$ *and* $\frac{1}{X\sqrt{X}}$ *intersects at* $x = \varphi$

So, this φ is the determinate for the conversion between base 10 system and power 2 system.

$$\left(\sqrt{X} - \frac{1}{\sqrt{X}}\right) = \frac{1}{X\sqrt{X}} \; ; at\ X = \varphi$$



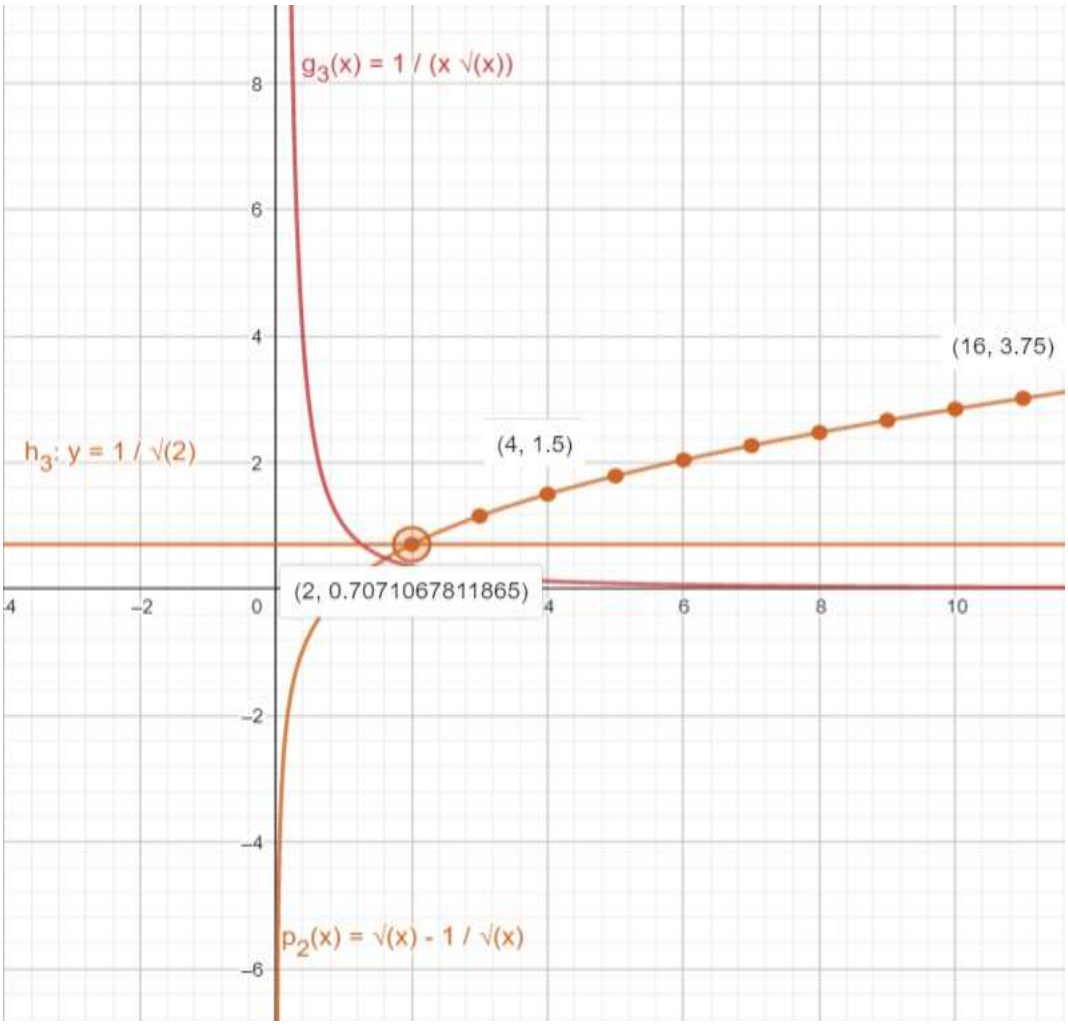


Figure 2. φ is the determinate for the conversion between base 10 and power 2 systems

C) *At* $X = \varphi = \frac{\sqrt{5}+1}{2}$

$$\left(\sqrt{X} - \frac{1}{\sqrt{X}}\right) = \frac{1}{X\sqrt{X}} \quad ; at\ X = \varphi = \frac{\sqrt{5}+1}{2}$$

$$\left(\sqrt{\varphi} + \frac{1}{\sqrt{\varphi}}\right) = \varphi\sqrt{\varphi}$$

$$\varphi^{\frac{-1}{2}} + \varphi^{\frac{1}{2}} = \varphi^{\frac{3}{2}}$$

$$\frac{1}{\sqrt{x}} + \sqrt{x} = x\sqrt{x}$$

$$\frac{1+x}{\sqrt{x}} = x\sqrt{x}$$

$$1 + x = x^2$$

$$x - x^2 = -1$$

$$\frac{1+x}{x} = x$$

$$x = \sqrt{x+1}$$

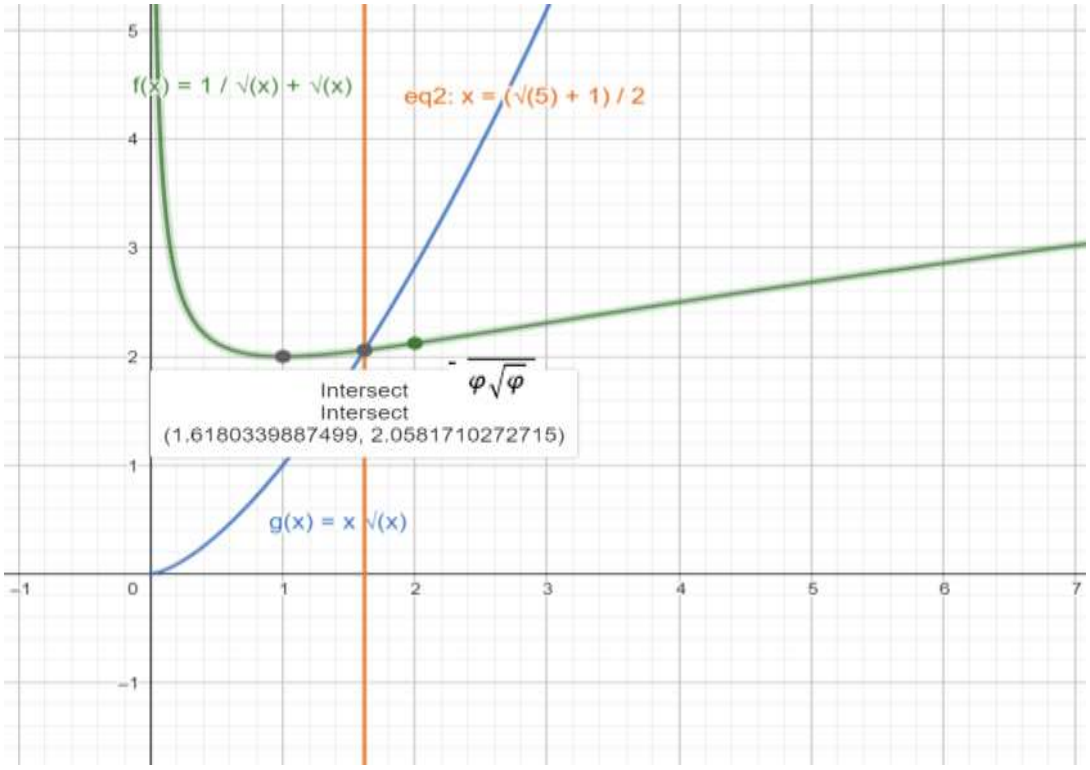


Figure 3. equivalent functions have same powers at intersection points $\varphi\sqrt{\varphi}$

Time x the same φ

$$\varphi^{\frac{3}{2}} + \varphi^{\frac{1}{2}} = \varphi^{\frac{5}{2}}$$

$$\frac{x^2}{\sqrt{x}} + \sqrt{x} = x^2\sqrt{x}$$

$$x\left(\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x}\right) = x * x\sqrt{x} = x\left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)$$

$$\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x} = x\sqrt{x}$$

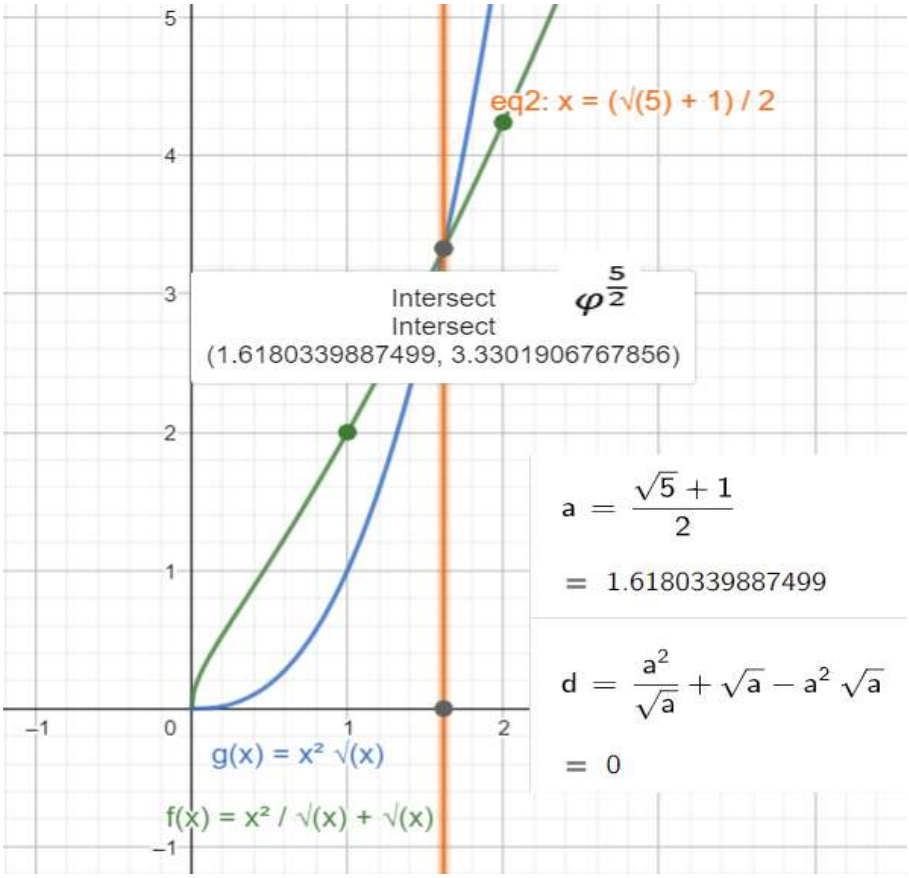


Figure 4. equivalent functions have same powers at intersection points $\varphi^2\sqrt{\varphi}$

also, can be written as

$$\varphi^{\frac{1}{2}} + \varphi^{\frac{-1}{2}} = \varphi^{\frac{5}{2}} - \varphi^{\frac{1}{2}}$$

$$2 * \varphi^{\frac{1}{2}} + \varphi^{\frac{-1}{2}} = \varphi^{\frac{5}{2}}$$

$$\frac{2x}{\sqrt{x}} + \frac{\sqrt{x}}{x} = x^2\sqrt{x}$$

$$\frac{2x}{\sqrt{x}} + \frac{\sqrt{x}}{x} = x\left(\frac{1}{\sqrt{x}} + \sqrt{x}\right) = \frac{x}{\sqrt{x}} + x\sqrt{x}$$

$$\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x} = x\sqrt{x}$$

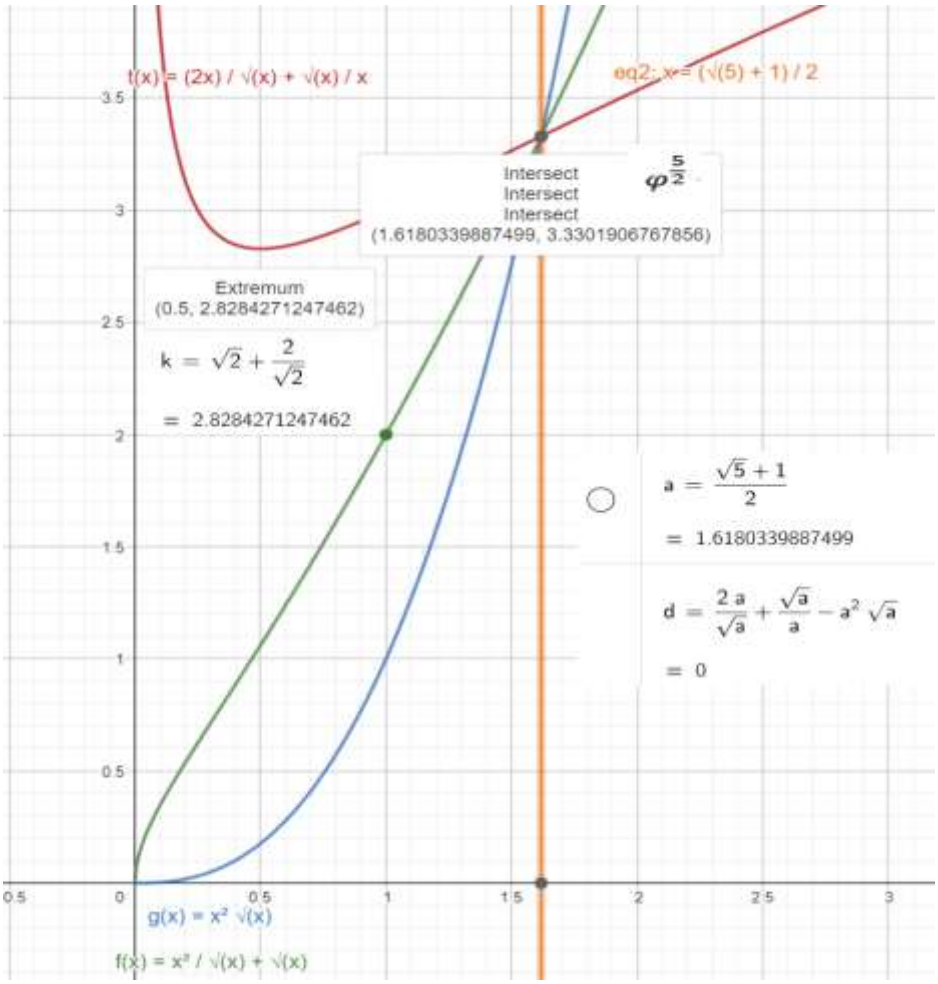


Figure 5. equivalent functions have same powers at intersection points $\varphi^2\sqrt{\varphi}$ using 2x Times x the same φ

$$\varphi^{\frac{5}{2}} + \varphi^{\frac{3}{2}} = \varphi^{\frac{7}{2}}$$

$$\frac{x^3}{\sqrt{x}} + x\sqrt{x} = x^3\sqrt{x}$$

$$x^3\sqrt{x} = x\,x^2\sqrt{x} = x\left(\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x}\right)$$

$$\frac{x^3}{\sqrt{x}} + x\sqrt{x} = x\left(\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x}\right)$$

$$\frac{x^2}{\sqrt{x}} + \sqrt{x} = \left(\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x}\right)$$

$$\frac{x^2}{\sqrt{x}} + \sqrt{x} = x^2\sqrt{x}$$

$$\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x} = x\sqrt{x}$$

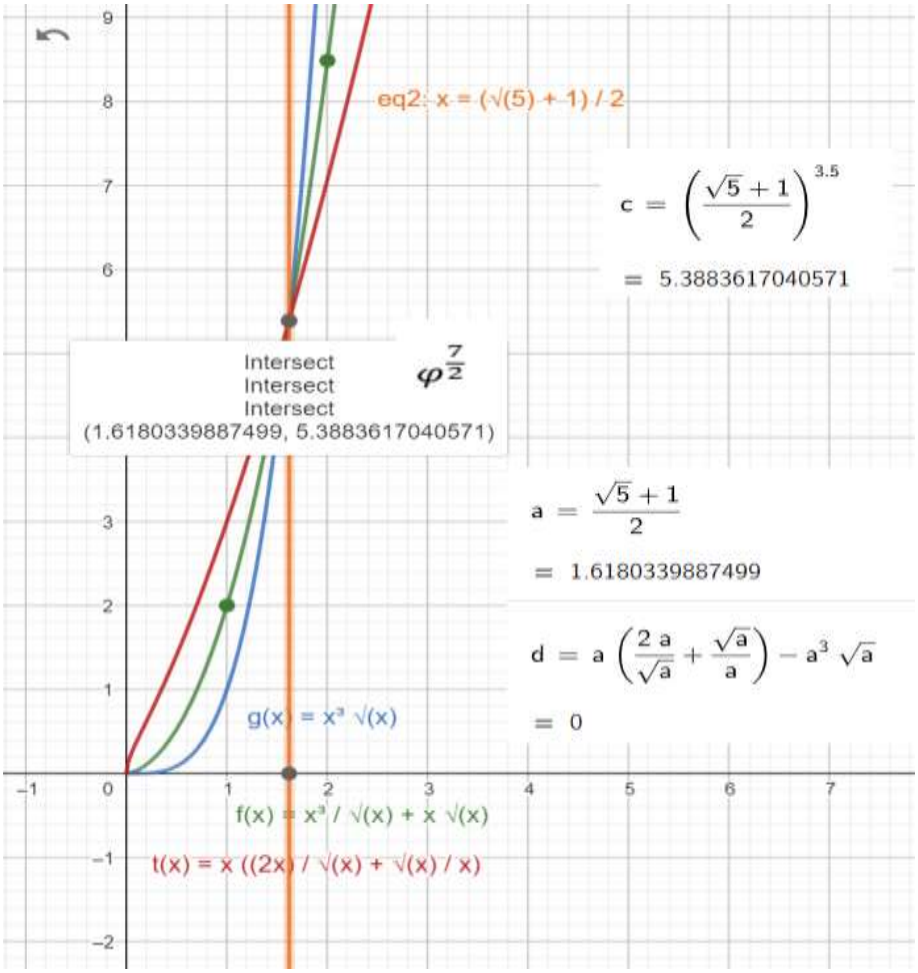


Figure 6. equivalent functions have same powers at intersection points $\varphi^3\sqrt{\varphi}$ and using 2x Times x the same φ

$$\varphi^{\frac{7}{2}} + \varphi^{\frac{5}{2}} = \varphi^{\frac{9}{2}}$$

$$\frac{x^4}{\sqrt{x}} + x^2\sqrt{x} = x^4\sqrt{x}$$

$$x^3\sqrt{x} = x\,x^2\sqrt{x} = x\left(\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x}\right)$$

$$\frac{x^3}{\sqrt{x}} + x\sqrt{x} = x\left(\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x}\right)$$

$$\frac{x^2}{\sqrt{x}} + \sqrt{x} = \left(\frac{x}{\sqrt{x}} + \frac{\sqrt{x}}{x}\right)$$

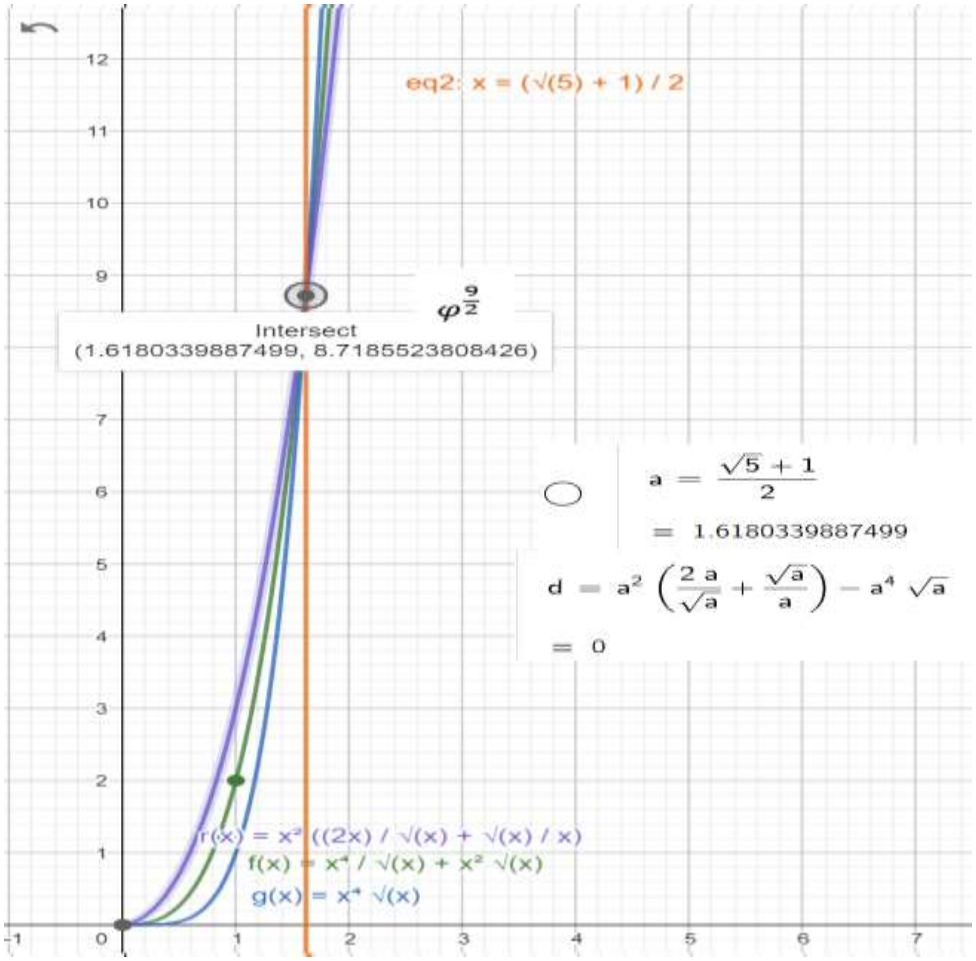


Figure 7. Equivalent functions have same powers at intersection points $\varphi^4\sqrt{\varphi}$ and using 2x

Any multiply by x will get us back to the same base equation

$$\left(\sqrt{\varphi} + \frac{1}{\sqrt{\varphi}}\right) = \varphi\sqrt{\varphi}$$

$$\varphi^{\frac{-1}{2}} + \varphi^{\frac{1}{2}} = \varphi^{\frac{3}{2}}$$

$$\varphi^{\frac{1}{2}} + \varphi^{\frac{3}{2}} = \varphi^{\frac{5}{2}}$$

$$\varphi^{\frac{3}{2}} + \varphi^{\frac{5}{2}} = \varphi^{\frac{7}{2}}$$

$$\varphi^{\frac{5}{2}} + \varphi^{\frac{7}{2}} = \varphi^{\frac{9}{2}}$$

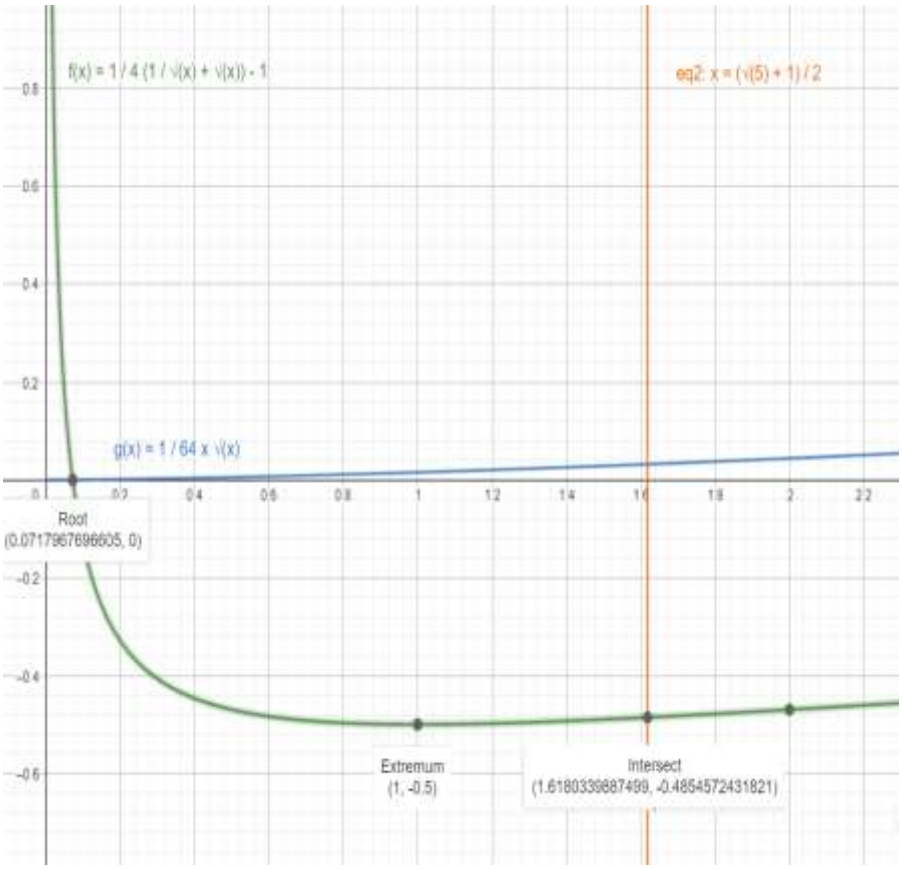


Figure 8. Equivalent functions have same powers at intersection points $\frac{1}{4} * \left(\frac{1}{\sqrt{\varphi}} + \sqrt{\varphi}\right) - 1$

D) Power Two System base ten systems intersection point

$$(\frac{1}{2}\left(\frac{1}{\sqrt{x}} - \sqrt{x}\right) + 1)^2 = (\frac{1}{2}\left(\frac{1}{\sqrt{x}} + \sqrt{x}\right))^2$$

$$\frac{1}{4}\left(\frac{1}{\sqrt{x}} - \sqrt{x}\right)^2 + 1 + \left(\frac{1}{\sqrt{x}} - \sqrt{x}\right) = (\frac{1}{2}\left(\frac{1}{\sqrt{x}} + \sqrt{x}\right))^2$$

$$\frac{1}{4}\left(\frac{1}{x} + x - 2\right) + 1 + \left(\frac{1}{\sqrt{x}} - \sqrt{x}\right) = \frac{1}{4}(\frac{1}{x} + x + 2)$$

$$\left(\frac{1}{4x} + \frac{x}{4} - \frac{1}{2}\right) + 1 + \left(\frac{1}{\sqrt{x}} - \sqrt{x}\right) = \frac{1}{4}(\frac{1}{x} + x + 2)$$

$$\frac{1}{4x} + \frac{x}{4} + \frac{1}{2} + \left(\frac{1}{\sqrt{x}} - \sqrt{x}\right) = (\frac{1}{4x} + \frac{x}{4} + \frac{1}{2})$$

$$\left(\frac{1}{\sqrt{x}} - \sqrt{x}\right) = 0$$

$$\frac{1}{4}\left(\frac{1}{x} + x + 2\right) = 1$$

$$\left(\frac{1}{x} + x - 2\right) = 0$$

$$(x^2 - 2x + 1) = 0$$

$$at\ \left(\frac{1}{\sqrt{x}} - \sqrt{x}\right) = 0\ THEN\ (x - 1)^2 = 0$$

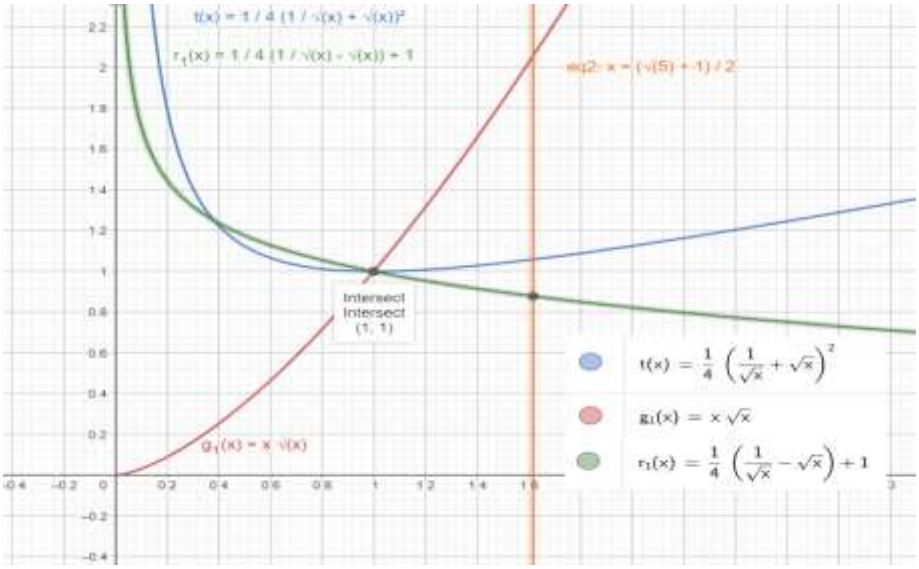


Figure 9. Equivalent functions $\frac{1}{4} * \left(\frac{1}{\sqrt{\varphi}} + \sqrt{\varphi}\right)^2$ *and* $\frac{1}{4} * \left(\frac{1}{\sqrt{\varphi}} - \sqrt{\varphi}\right) + 1$

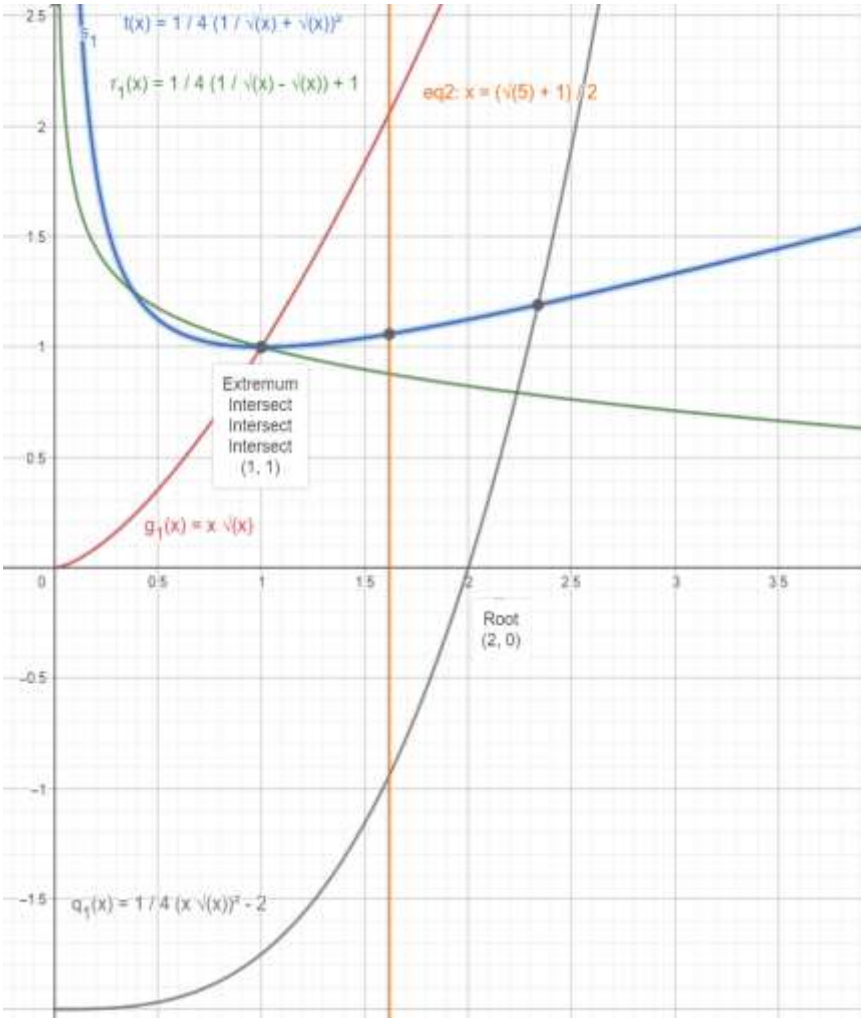


Figure 10. Equivalent functions $\frac{1}{4} * \left(\frac{1}{\sqrt{\varphi}} + \sqrt{\varphi}\right)^2$ *and* $\frac{1}{4} * \left(\frac{1}{\sqrt{\varphi}} - \sqrt{\varphi}\right)^2 + 1$ *and* $\frac{1}{4} * \left(\varphi\sqrt{\varphi}\right)^2 - 2$

2.2 $\sqrt{\varphi}$ and φ General Nth Root Power of 2

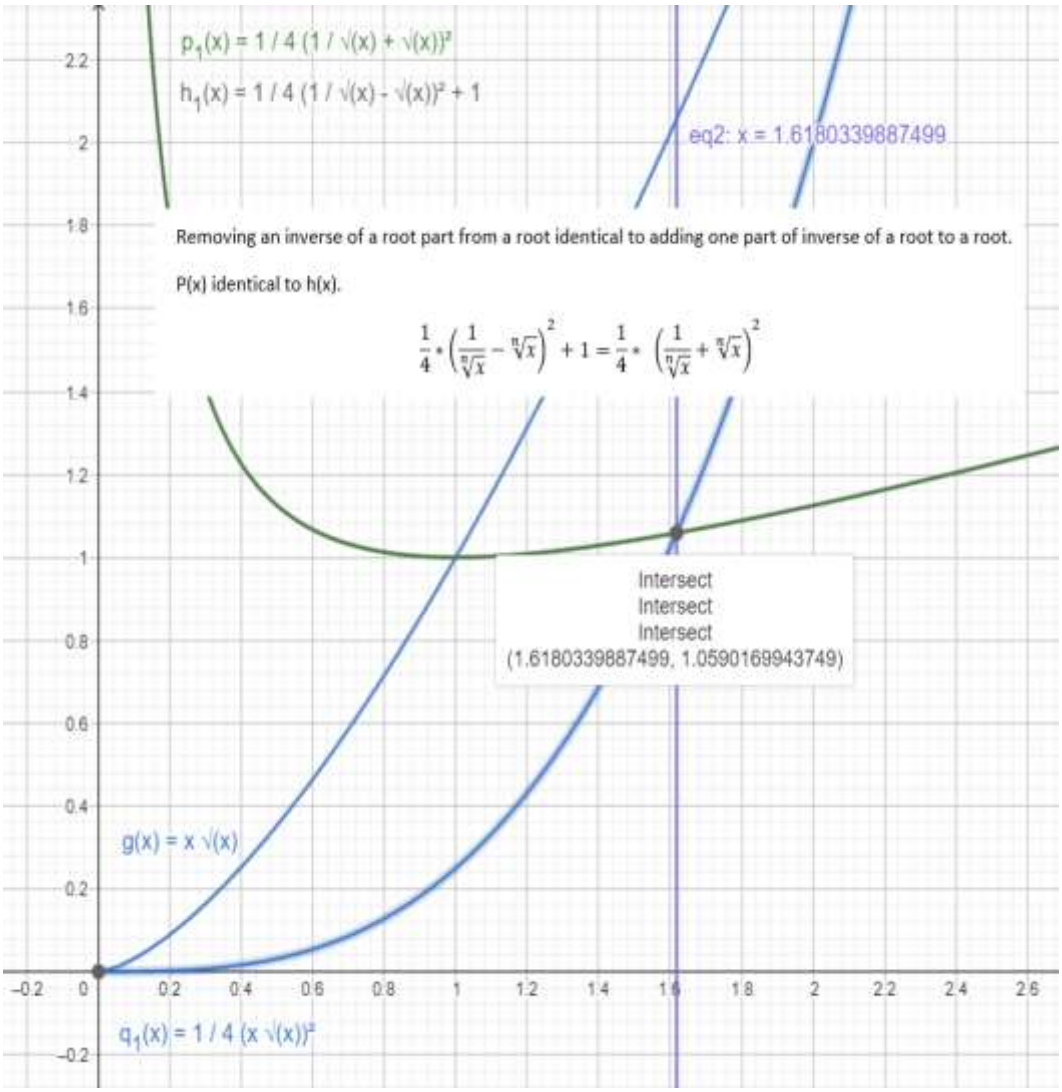


Figure 11. $\sqrt{\varphi}$ *and* φ General Nth Root power of 2

2.3 $\sqrt{x * \varphi}$ *and* $\sqrt{x}$ *Using* φ *as a Seed or Multiplier*

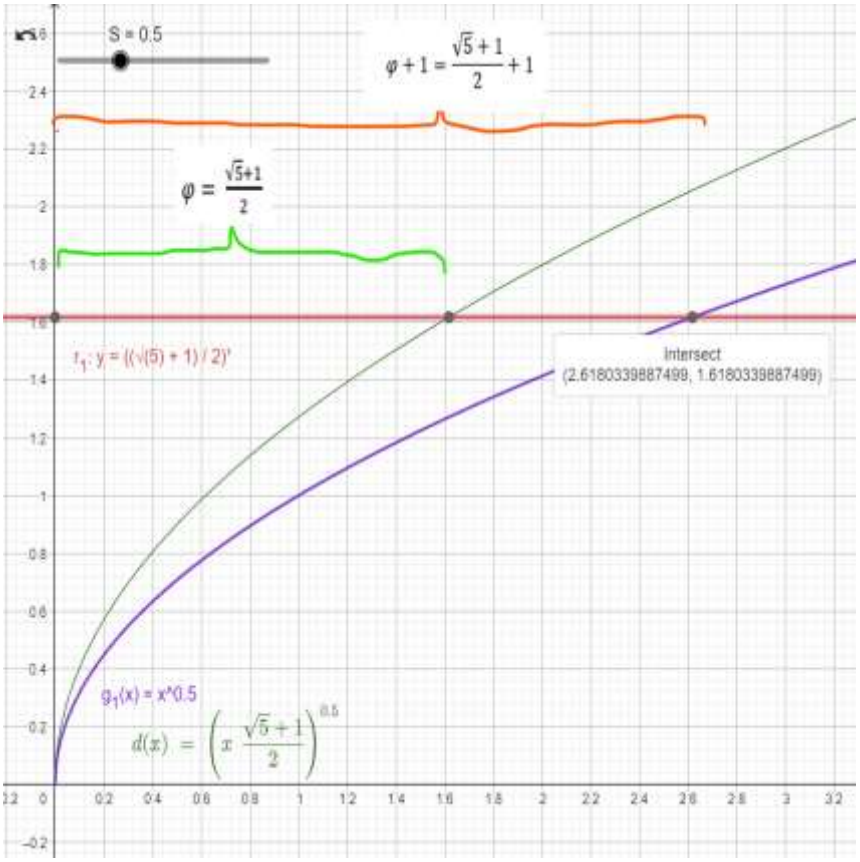


Figure 12. $\sqrt{x * \varphi}$ *and* $\sqrt{x}$ using φ as a seed or multiplier

3.1 $(-x)^{1.5}$ *and* $(x\sqrt{x})$ *Express Odd Power Functions as two Functions With Square Roots*

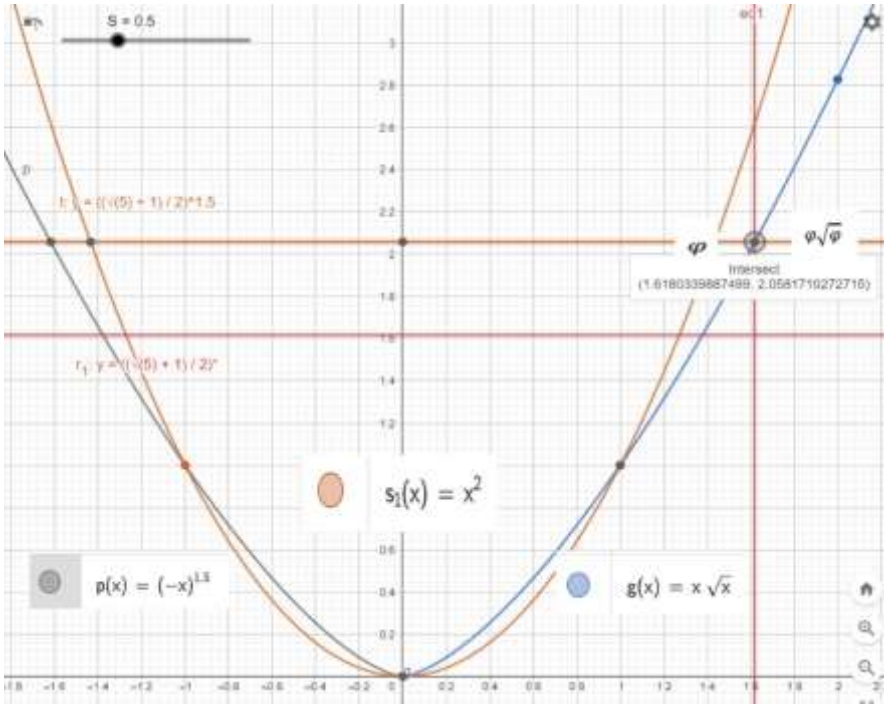


Figure 13. Express odd power functions as two functions with square roots

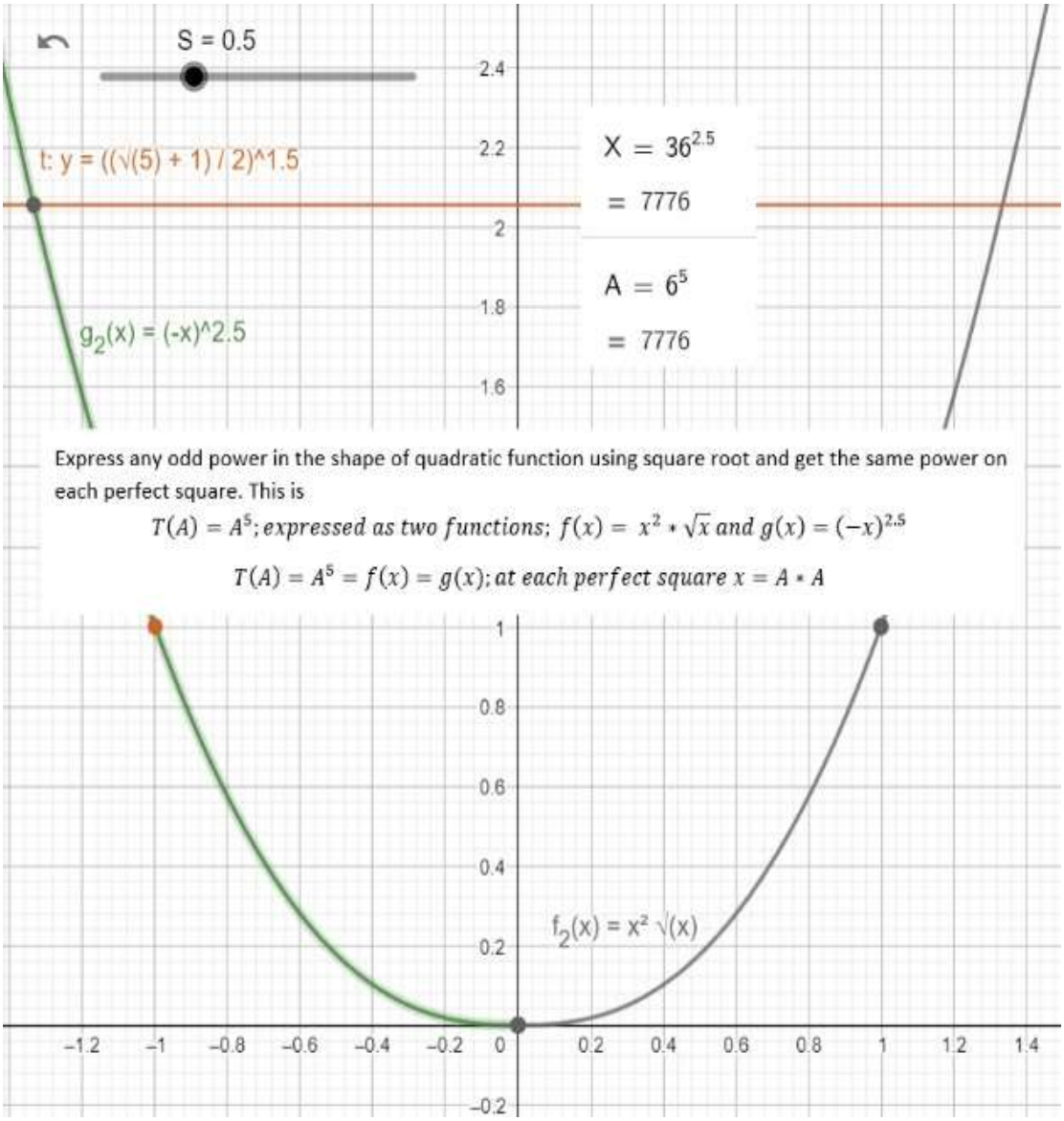


Figure 14. Express any odd power functions as two functions with square roots

Table 1. Perfect Cubic at perfect squares when using square Roots functions for odd powers

x	g(x)
3	5.1961524227066
4	8
5	11.1803398874...
8	22.6274169979...
9	27
10	31.6227766016...
11	36.4828726939...
12	41.5692193816...
13	46.8721665810...
14	52.3832034148...
15	58.0947501931...
16	64

x	g(x)
24	117.575507653...
25	125
26	132.574507353...
27	140.296115413...
29	156.169779406...
30	164.316767251...
31	172.600695247...
32	181.019335983...
33	189.570567335...
34	198.252364424...
35	207.062792408...
36	216
37	225.062213621...
38	234.247732112...

$g(x) = x^3 = Perfect\ Cubic;\ when\ \ x\ is\ a\ perfect\ square\ i.e. x = A * A$

4.1 Calculate φ Degree

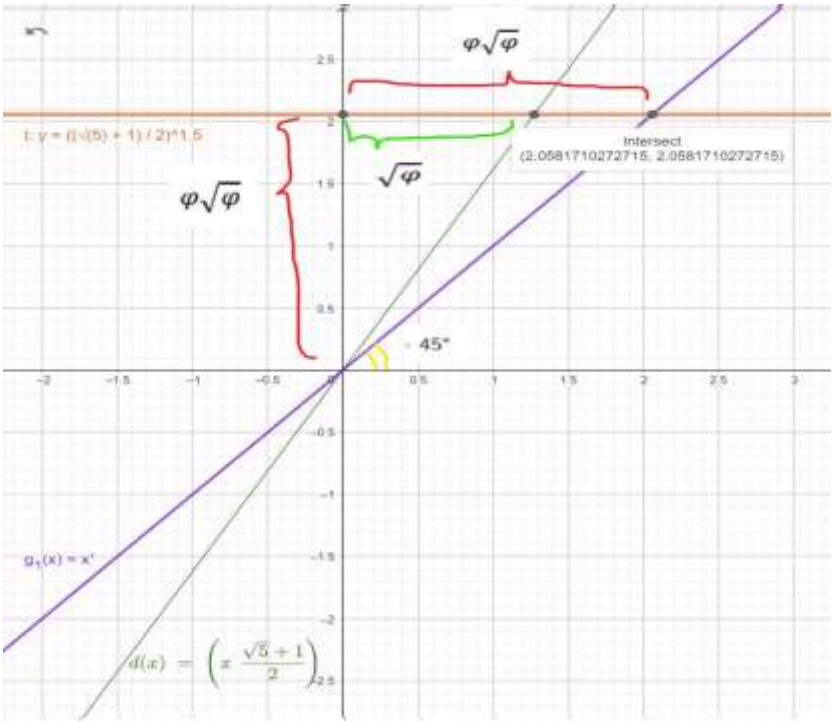


Figure 15. Using φ as multiplier for x

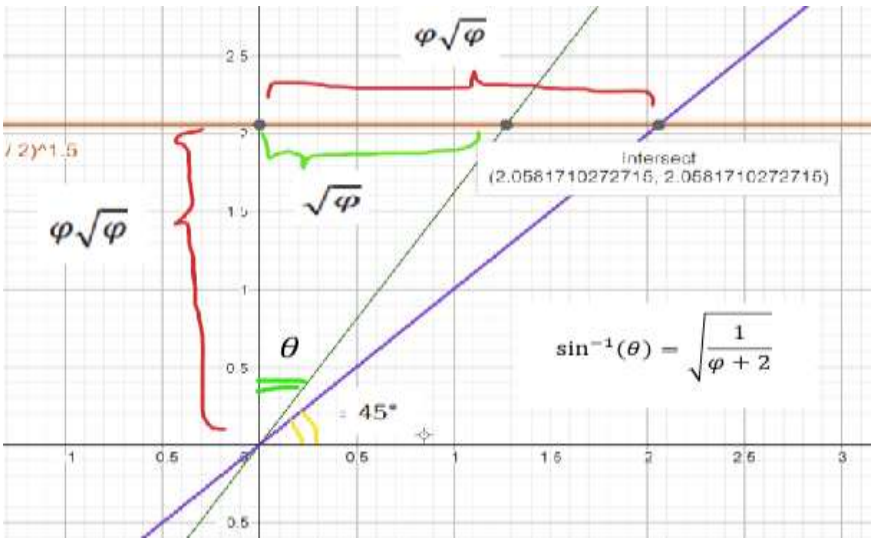


Figure 16. Using φ as multiplier for x to calculate phi degree with reference to x

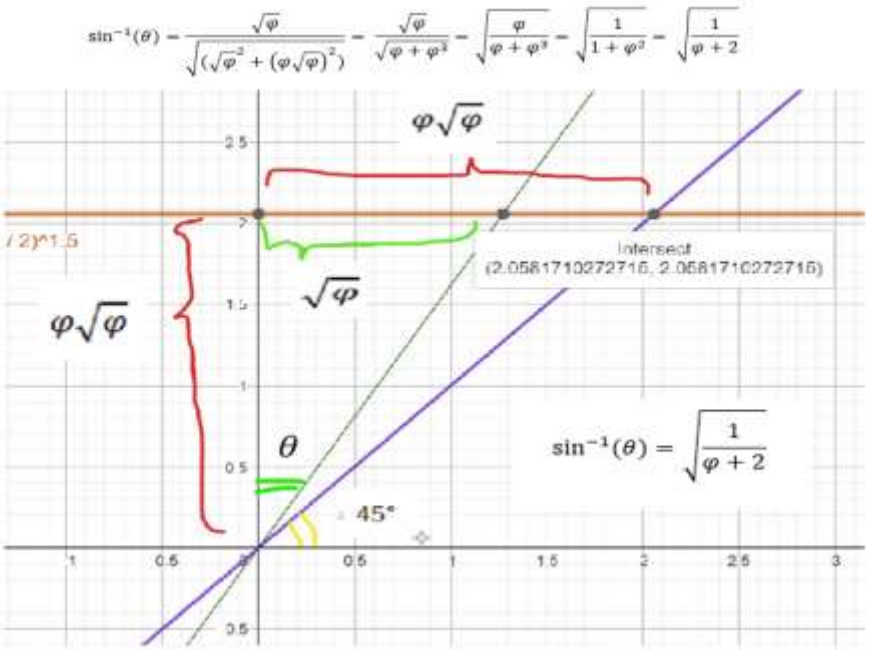


Figure 17. finding the degree using sin law

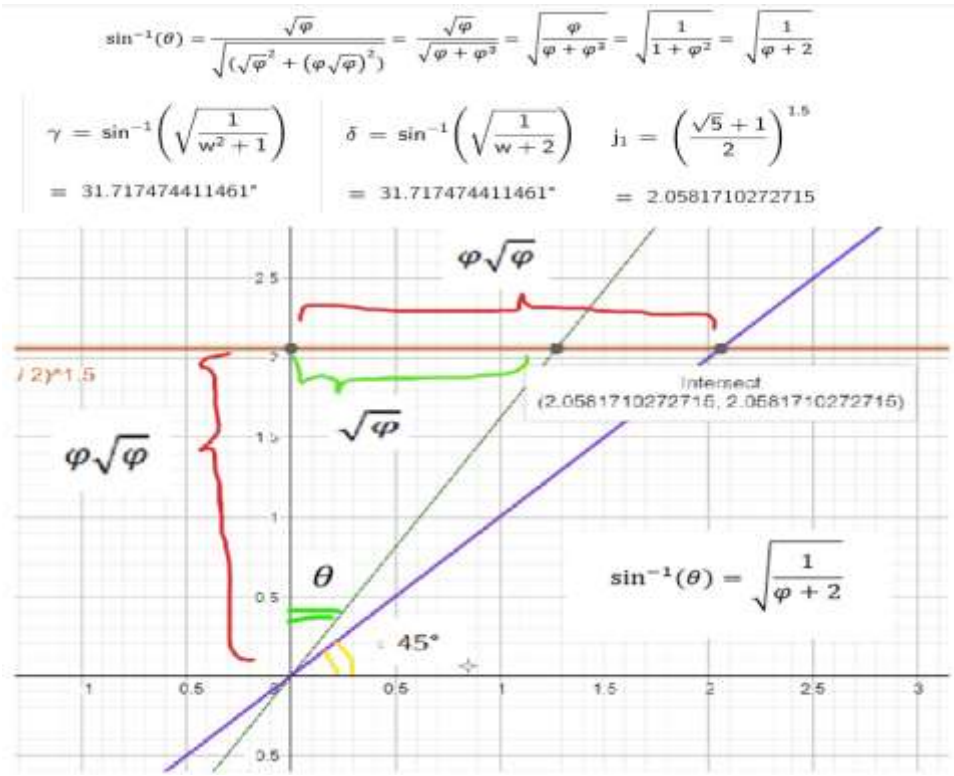


Figure 18. finding the degree value using sin law

6.1 General Golden Ratio Properties

The golden ratio properties can be generalized for any square root [1] not only for [5].

$$if\ \varphi = \frac{\sqrt{1}+1}{2};\ \sqrt{\varphi} - \frac{1}{\sqrt{\varphi}} = \frac{0}{2 * \varphi\sqrt{\varphi}} = \frac{0}{2 * \varphi} = \varphi - 1$$

$$if\ \varphi = \frac{\sqrt{2}+1}{2};\ \sqrt{\varphi} - \frac{1}{\sqrt{\varphi}} = \frac{0.5}{2 * \varphi\sqrt{\varphi}} = \frac{0.5}{2 * \varphi} = \varphi - 1$$

$$if\ \varphi = \frac{\sqrt{3}+1}{2};\ \sqrt{\varphi} - \frac{1}{\sqrt{\varphi}} = \frac{1}{2*\varphi\sqrt{\varphi}} = \frac{1}{2*\varphi} = \varphi - 1$$

$$if\ \varphi = \frac{\sqrt{5}+1}{2};\ \sqrt{\varphi} - \frac{1}{\sqrt{\varphi}} = \frac{2}{2*\varphi\sqrt{\varphi}} = \frac{2}{2*\varphi} = \varphi - 1$$

$$if\ \varphi = \frac{\sqrt{7}+1}{2};\ \sqrt{\varphi} - \frac{1}{\sqrt{\varphi}} = \frac{3}{2*\varphi\sqrt{\varphi}} = \frac{3}{2*\varphi} = \varphi - 1$$

$$if\ \varphi = \frac{\sqrt{9}+1}{2};\ \sqrt{\varphi} - \frac{1}{\sqrt{\varphi}} = \frac{4}{2*\varphi\sqrt{\varphi}} = \frac{4}{2*\varphi} = \varphi - 1$$

$$if\ \varphi = \frac{\sqrt{11}+1}{2};\ \sqrt{\varphi} - \frac{1}{\sqrt{\varphi}} = \frac{5}{2*\varphi\sqrt{\varphi}} = \frac{5}{2*\varphi} = \varphi - 1$$

$$for\ General\ N\ ;\ if\ \varphi = \frac{\sqrt{N}+1}{2}\ ;\ \frac{N-1}{4*\varphi} = \varphi - 1 \Rightarrow (A)$$

If we simplify this equation

$$4*\ \varphi^2 - 4*\ \varphi - N + 1 = 0;\ WHERE\ \varphi = \frac{\sqrt{N}+1}{2}\ and\ N\ any\ natural\ number. \Rightarrow (C)$$

From these general relations we can write [-1] or [1] in terms of any square root for any number

$$\frac{N-1}{2\,(\sqrt{N}+1)} - \frac{(\sqrt{N}+1)}{2} = -1\ \ OR\ \frac{-(N-1)}{2\,(\sqrt{N}-1)} + \frac{(\sqrt{N}-1)}{2} = 1$$

$$\frac{(\sqrt{N}+1)}{2} - \frac{\frac{N-1}{2}}{(\sqrt{N}+1)} = 1 \Rightarrow (B)$$

$$\frac{(\sqrt{N}+1)}{3} - \frac{2*\frac{N-1}{2}}{3\,(\sqrt{N}+1)} + \frac{1}{6} = \frac{1}{2} \Rightarrow (D)$$

$$\frac{(\sqrt{N}+1)}{4} - \frac{2*\frac{N-1}{2}}{4\,(\sqrt{N}+1)} = \frac{1}{2} \Rightarrow (E)$$

$$\frac{(\sqrt{N}+1)}{D} - \frac{\frac{N-1}{D}}{(\sqrt{N}+1)} + \frac{D-4}{2D} = \frac{1}{2} \Rightarrow (F)\ \ ;for\ any\ natural\ number\ N\ and\ D.where\ D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil$$

$$\frac{(\sqrt{N}+1)}{2} - \frac{\frac{N-1}{2}}{(\sqrt{N}+1)} = 1 \Rightarrow (B)$$

These nominator and denominators in red box are the factor of [2] for (N-1)

$$b_2 = \frac{\sqrt{3}+1}{2} - \frac{1}{\sqrt{3}+1}$$

$$= 1$$

$$c_2 = \frac{\sqrt{5}+1}{2} - \frac{2}{\sqrt{5}-1}$$

$$= 1$$

$$d_2 = \frac{\sqrt{7}+1}{2} - \frac{3}{\sqrt{7}+1}$$

$$= 1$$

CASE (1): - D = 2

$$\frac{(\sqrt{N}+1)}{D} - \frac{\frac{N-1}{D}}{(\sqrt{N}+1)} + \frac{D-4}{2D} = \frac{1}{2} \Rightarrow (F) \ ; for\ any\ natural\ number\ N\ and\ D. where\ D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil$$

$$\frac{(\sqrt{N}+1)}{2} - \frac{\frac{N-1}{2}}{(\sqrt{N}+1)} + \frac{2-4}{4} = \frac{1}{2}$$

$$u_2 = \frac{\sqrt{3}+1}{2} - \frac{\frac{2\cdot 1}{2}}{\sqrt{3}+1} + \frac{2-4}{4}$$

$$= 0.5$$

$$v_2 = \frac{\sqrt{5}+1}{2} - \frac{\frac{2\cdot 2}{2}}{\sqrt{5}+1} + \frac{2-4}{4}$$

$$= 0.5$$

$$w_2 = \frac{\sqrt{7}+1}{2} - \frac{\frac{2\cdot 3}{2}}{\sqrt{7}+1} + \frac{2-4}{4}$$

$$= 0.5$$

$$a_3 = \frac{\sqrt{2}+1}{2} - \frac{\frac{2\cdot \frac{1}{2}}{2}}{\sqrt{2}+1} + \frac{2-4}{4}$$

$$= 0.5$$

$$b_3 = \frac{\sqrt{4}+1}{2} - \frac{\frac{2\cdot \frac{3}{2}}{2}}{\sqrt{4}+1} + \frac{2-4}{4}$$

$$\approx 0.5$$

$$c_3 = \frac{\sqrt{6}+1}{2} - \frac{\frac{2\cdot \frac{3}{2}}{2}}{\sqrt{6}+1} + \frac{2-4}{4}$$

$$= 0.5$$

Case (1): D =3

$$\frac{\left(\sqrt{N}+1\right)}{D}-\frac{\frac{N-1}{D}}{\left(\sqrt{N}+1\right)}+\frac{D-4}{2D}=\frac{1}{2} \Rightarrow (F)\ ;for\ any\ natural\ number\ N\ and\ D.where\ D=\left\lceil\frac{\sqrt{N}}{5}\right\rceil$$

$$\frac{(\sqrt{N}+1)}{3}-\frac{2*\frac{N-1}{2}}{\left(\sqrt{N}+1\right)}+\frac{1}{3}=1$$

$$\frac{(\sqrt{N}+1)}{3}-\frac{\frac{N-1}{3}}{\left(\sqrt{N}+1\right)}-\frac{1}{6}=\frac{1}{2}$$

$$p_2=\frac{\sqrt{1}+1}{3}-\frac{\frac{2\cdot 0}{3}}{\sqrt{2}+1}-\frac{1}{6}$$

$$=0.5$$

$$l_2=\frac{\sqrt{2}+1}{3}-\frac{\frac{2\cdot 1}{3}}{\sqrt{2}+1}-\frac{1}{6}$$

$$=0.5$$

$$m_2=\frac{\sqrt{3}+1}{3}-\frac{\frac{2\cdot 1}{3}}{\sqrt{3}+1}-\frac{1}{6}$$

$$=0.5$$

$$n_2=\frac{\sqrt{5}+1}{3}-\frac{\frac{2\cdot 2}{3}}{\sqrt{5}+1}-\frac{1}{6}$$

$$=0.5$$

$$o_2=\frac{\sqrt{7}+1}{3}-\frac{\frac{2\cdot 3}{3}}{\sqrt{7}+1}-\frac{1}{6}$$

$$=0.5$$

$$q_2 = \frac{\sqrt{11}+1}{3} - \frac{\frac{2\cdot 5}{3}}{\sqrt{11}+1} - \frac{1}{6}$$
$$= 0.5$$

$$r_2 = \frac{\sqrt{13}+1}{3} - \frac{\frac{2\cdot 6}{3}}{\sqrt{13}+1} - \frac{1}{6}$$
$$= 0.5$$

$$s_2 = \frac{\sqrt{15}+1}{3} - \frac{\frac{2\cdot 7}{3}}{\sqrt{15}+1} - \frac{1}{6}$$
$$= 0.5$$

$$t_2 = \frac{\sqrt{17}+1}{3} - \frac{\frac{2\cdot 8}{3}}{\sqrt{17}+1} - \frac{1}{6}$$
$$= 0.5$$

$$u_2 = \frac{\sqrt{19}+1}{3} - \frac{\frac{2\cdot 9}{3}}{\sqrt{19}+1} - \frac{1}{6}$$
$$= 0.5$$

$$u_2 = \frac{\sqrt{2}+1}{3} - \frac{\frac{2\cdot \frac{1}{2}}{3}}{\sqrt{2}+1} - \frac{1}{6}$$
$$= 0.5$$

$$v_2 = \frac{\sqrt{4}+1}{3} - \frac{\frac{2\cdot \frac{3}{2}}{3}}{\sqrt{4}+1} - \frac{1}{6}$$
$$= \frac{1}{2}$$

$$w_2 = \frac{\sqrt{6}+1}{3} - \frac{\frac{2\cdot \frac{5}{2}}{3}}{\sqrt{6}+1} - \frac{1}{6}$$
$$= 0.5$$

$$a_3 = \frac{\sqrt{8}+1}{3} - \frac{\frac{2\cdot \frac{7}{2}}{3}}{\sqrt{8}+1} - \frac{1}{6}$$
$$= 0.5$$

$$b_3 = \frac{\sqrt{10}+1}{3} - \frac{\frac{2\cdot \frac{9}{2}}{3}}{\sqrt{10}+1} - \frac{1}{6}$$
$$= 0.5$$

Case (2): - D = [4].

$$\frac{\left(\sqrt{N}+1\right)}{D} - \frac{\frac{N-1}{D}}{\left(\sqrt{N}+1\right)} + \frac{D-4}{2D} = \frac{1}{2} \Rightarrow (F) \ ; for\ any\ natural\ number\ N\ and\ D. where\ D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil$$

$$\frac{(\sqrt{N}+1)}{4} - \frac{\frac{N-1}{4}}{(\sqrt{N}+1)} = \frac{1}{2} \Rightarrow (E)$$

$$u_2 = \frac{\sqrt{2}+1}{4} - \frac{\frac{2\cdot\frac{1}{2}}{4}}{\sqrt{2}+1}$$

$$= 0.5$$

$$v_2 = \frac{\sqrt{4}+1}{4} - \frac{\frac{2\cdot\frac{3}{2}}{4}}{\sqrt{4}+1}$$

$$= \frac{1}{2}$$

$$w_2 = \frac{\sqrt{6}+1}{4} - \frac{\frac{2\cdot\frac{5}{2}}{4}}{\sqrt{6}+1}$$

$$= 0.5$$

$$a_3 = \frac{\sqrt{8}+1}{4} - \frac{\frac{2\cdot\frac{7}{2}}{4}}{\sqrt{8}+1}$$

$$= 0.5$$

$$b_3 = \frac{\sqrt{10}+1}{4} - \frac{\frac{2\cdot\frac{9}{2}}{4}}{\sqrt{10}+1}$$

$$= 0.5$$

$$c_3 = \frac{\sqrt{12}+1}{4} - \frac{\frac{2\cdot\frac{11}{2}}{4}}{\sqrt{12}+1}$$

$$= 0.5$$

$$u_2 = \frac{\sqrt{3}+1}{4} - \frac{\frac{2\cdot 1}{4}}{\sqrt{3}+1}$$

$$= 0.5$$

$$v_2 = \frac{\sqrt{5}+1}{4} - \frac{\frac{2\cdot 2}{4}}{\sqrt{5}+1}$$

$$= 0.5$$

$$w_2 = \frac{\sqrt{7}+1}{4} - \frac{\frac{2\cdot 3}{4}}{\sqrt{7}+1}$$

$$= 0.5$$

$$a_3 = \frac{\sqrt{11}+1}{4} - \frac{\frac{2\cdot 5}{4}}{\sqrt{11}+1}$$

$$= 0.5$$

$$b_3 = \frac{\sqrt{13}+1}{4} - \frac{\frac{2\cdot 6}{4}}{\sqrt{13}+1}$$

$$= 0.5$$

$$c_3 = \frac{\sqrt{17}+1}{4} - \frac{\frac{2\cdot 8}{4}}{\sqrt{17}+1}$$

$$= 0.5$$

CASE (3): - D = [5]

$$\frac{(\sqrt{N}+1)}{D} - \frac{\frac{N-1}{D}}{(\sqrt{N}+1)} + \frac{D-4}{2D} = \frac{1}{2} \Rightarrow (F) \ ; for\ any\ natural\ number\ N\ and\ D. where\ D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil$$

$$\frac{(\sqrt{N}+1)}{5} - \frac{\frac{N-1}{5}}{(\sqrt{N}+1)} + \frac{5-4}{10} = \frac{1}{2}$$

$$u_2 = \frac{\sqrt{3}+1}{5} - \frac{\frac{2\cdot 1}{5}}{\sqrt{3}+1} + \frac{1}{10}$$
$$= 0.5$$

$$v_2 = \frac{\sqrt{5}+1}{5} - \frac{\frac{2\cdot 2}{5}}{\sqrt{5}+1} + \frac{1}{10}$$
$$= 0.5$$

$$w_2 = \frac{\sqrt{7}+1}{5} - \frac{\frac{2\cdot 3}{5}}{\sqrt{7}+1} + \frac{1}{10}$$
$$= 0.5$$

$$a_3 = \frac{\sqrt{2}+1}{5} - \frac{\frac{2\cdot \frac{1}{2}}{5}}{\sqrt{2}+1} + \frac{1}{10}$$
$$= 0.5$$

$$b_3 = \frac{\sqrt{4}+1}{5} - \frac{\frac{2\cdot \frac{3}{2}}{5}}{\sqrt{4}+1} + \frac{1}{10}$$
$$\approx 0.5$$

$$c_3 = \frac{\sqrt{6}+1}{5} - \frac{\frac{2\cdot \frac{5}{2}}{5}}{\sqrt{6}+1} + \frac{1}{10}$$
$$= 0.5$$

CASE (4): - D = [7]

$$\frac{(\sqrt{N}+1)}{D} - \frac{\frac{N-1}{D}}{(\sqrt{N}+1)} + \frac{D-4}{2D} = \frac{1}{2} \Rightarrow (F) \ ; for\ any\ natural\ number\ N\ and\ D. where\ D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil$$

$$\frac{(\sqrt{N}+1)}{7} - \frac{\frac{N-1}{7}}{(\sqrt{N}+1)} + \frac{7-4}{14} = \frac{1}{2}$$

$$u_2 = \frac{\sqrt{3}+1}{7} - \frac{\frac{2\cdot 1}{7}}{\sqrt{3}+1} + \frac{7-4}{14}$$

$$= 0.5$$

$$v_2 = \frac{\sqrt{5}+1}{7} - \frac{\frac{2\cdot 2}{7}}{\sqrt{5}+1} + \frac{7-4}{14}$$

$$= 0.5$$

$$w_2 = \frac{\sqrt{7}+1}{7} - \frac{\frac{2\cdot 3}{7}}{\sqrt{7}+1} + \frac{7-4}{14}$$

$$= 0.5$$

$$a_3 = \frac{\sqrt{2}+1}{7} - \frac{\frac{2\cdot \frac{1}{2}}{7}}{\sqrt{2}+1} + \frac{7-4}{14}$$

$$= 0.5$$

$$b_3 = \frac{\sqrt{4}+1}{7} - \frac{\frac{2\cdot \frac{3}{2}}{7}}{\sqrt{4}+1} + \frac{7-4}{14}$$

$$\approx 0.5$$

$$c_3 = \frac{\sqrt{6}+1}{7} - \frac{\frac{2\cdot \frac{5}{2}}{7}}{\sqrt{6}+1} + \frac{7-4}{14}$$

$$= 0.5$$

CASE (5): - D = [11]

$$\frac{(\sqrt{N}+1)}{D} - \frac{\frac{N-1}{D}}{(\sqrt{N}+1)} + \frac{D-4}{2D} = \frac{1}{2} \Rightarrow (F) \ ; for\ any\ natural\ number\ N\ and\ D. where\ D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil$$

$$\frac{(\sqrt{N}+1)}{11} - \frac{\frac{N-1}{11}}{(\sqrt{N}+1)} + \frac{11-4}{22} = \frac{1}{2}$$

$$u_2 = \frac{\sqrt{3}+1}{11} - \frac{\frac{2\cdot 1}{11}}{\sqrt{3}+1} + \frac{11-4}{22}$$

$$= 0.5$$

$$v_2 = \frac{\sqrt{5}+1}{11} - \frac{\frac{2\cdot 2}{11}}{\sqrt{5}+1} + \frac{11-4}{22}$$

$$= 0.5$$

$$w_2 = \frac{\sqrt{7}+1}{11} - \frac{\frac{2\cdot 3}{11}}{\sqrt{7}+1} + \frac{11-4}{22}$$

$$= 0.5$$

$$a_3 = \frac{\sqrt{2}+1}{11} - \frac{\frac{2\cdot \frac{1}{2}}{11}}{\sqrt{2}+1} + \frac{11-4}{22}$$

$$= 0.5$$

$$b_3 = \frac{\sqrt{4}+1}{11} - \frac{\frac{2\cdot \frac{3}{2}}{11}}{\sqrt{4}+1} + \frac{11-4}{22}$$

$$\approx 0.5$$

$$c_3 = \frac{\sqrt{6}+1}{11} - \frac{\frac{2\cdot \frac{5}{2}}{11}}{\sqrt{6}+1} + \frac{11-4}{22}$$

$$= 0.5$$

Using the Ceiling function give us exact values $= \frac{1}{2}$; $where\ D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil$.

$$c_3 = \frac{\sqrt{131}+1}{3} - \frac{\frac{130}{3}}{\sqrt{131}+1} + \frac{3-4}{6}$$

$$= 0.5$$

$$d_3 = \frac{\sqrt{111}+1}{3} - \frac{\frac{110}{3}}{\sqrt{111}+1} + \frac{3-4}{6}$$

$$= 0.5$$

$$e_3 = \frac{\sqrt{151}+1}{3} - \frac{\frac{150}{3}}{\sqrt{151}+1} + \frac{3-4}{6}$$

$$= 0.5$$

$$c_3 = \frac{\sqrt{131}+1}{4} - \frac{\frac{130}{4}}{\sqrt{131}+1} + \frac{4-4}{8}$$

$$= 0.5$$

$$d_3 = \frac{\sqrt{111}+1}{4} - \frac{\frac{110}{4}}{\sqrt{111}+1} + \frac{4-4}{8}$$

$$= 0.5$$

$$e_3 = \frac{\sqrt{151}+1}{4} - \frac{\frac{150}{4}}{\sqrt{151}+1} + \frac{4-4}{8}$$

$$= 0.5$$

$$D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil \; THEN \; \frac{\sqrt{N}}{D} < 5 \; ; as\ long\ as \; \frac{\sqrt{1303}}{7} > 5 \; and \; \frac{\sqrt{1303}}{8} < 5 \; then \; D = 8$$

$$g_3 = \frac{\sqrt{1303}}{7}$$
$$= 5.156727327937156$$

$$h_3 = \frac{\sqrt{1303}}{8}$$
$$= 4.512136411945011$$

$$i_3 = \frac{\sqrt{1303}+1}{7} - \frac{\frac{1302}{7}}{\sqrt{1303}+1} + \frac{7-4}{14}$$
$$= 0.500000000000001$$

$$j_3 = \frac{\sqrt{1303}+1}{8} - \frac{\frac{1302}{8}}{\sqrt{1303}+1} + \frac{8-4}{16}$$
$$= 0.5$$

$$D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil \; THEN \; \frac{\sqrt{N}}{D} < 5 \, ; as\ long\ as \; \frac{\sqrt{13031}}{22} > 5 \; and \; \frac{\sqrt{13031}}{23} < 5 \; then \; D = 23$$

$$f_3 = \frac{\sqrt{13031}}{22}$$
$$= 5.188791161629871$$

$$g_3 = \frac{\sqrt{13031}}{23}$$
$$= 4.963191545906833$$

$$i_3 = \frac{\sqrt{13031}+1}{22} - \frac{\frac{13030}{22}}{\sqrt{13031}+1} + \frac{22-4}{44}$$
$$= 0.500000000000001$$

$$j_3 = \frac{\sqrt{13031}+1}{23} - \frac{\frac{13030}{23}}{\sqrt{13031}+1} + \frac{23-4}{46}$$
$$= 0.5$$

5.1 Riemann Zeta Function

We can transform zeta function to the next form by multiplying by $\frac{5}{\pi}$ and because the Phi property $\left(\varphi - \frac{1}{\varphi}\right) = 1 \, ; which\ mean\ \varphi = (\varphi + 1)$

$$\frac{5}{\pi} * Z(S) = \frac{5}{\pi} * \sum_{n=0}^{\infty} \frac{1}{(n)^S} = \frac{5}{\pi} * \sum_{n=0}^{\infty} \frac{1}{(n)^S + 1} \quad \rightarrow (1)$$

Case (1): - ZP(S) is the Sum of T Prime numbers Terms. Multiplied by $\frac{5}{\pi}$

$$\frac{5}{\pi} * ZP(\frac{1}{2}) = \frac{T}{\pi} * \sum_{n=0}^{\infty} \frac{1}{\sqrt{n}+1} = \frac{D * \ (T+C) \prod_{T\ terms\ of\ P\ Prime} (\sqrt{P}-1)}{M * 2^K * \pi} \rightarrow (2)$$

$$; \ where\ T\ is\ number\ of\ Terms\ ;\ P\ is\ a\ prime\ number;\ and\ K\ some\ power\ for\ 2$$
$$; \ C\ is\ constant\ of\ all\ compinations\ of\ multiplications\ of\ square\ roots\ of\ the\ T\ terms$$
$$; D\ any\ Natural\ number\ mulitpier\ \ ; at\ D = M * 2^K\ the\ Denominator = \ \pi$$

And this is like multiplying $ZP\left(\frac{1}{2}\right)$ by $\varphi * \left(\sqrt{5}-1\right)$.

Case (2): - ZP(S) is the Sum of T Prime numbers Terms. Multiplied by $\varphi * \left(\sqrt{5}-1\right)$

$$\varphi * \left(\sqrt{5}-1\right) * ZP\left(\frac{1}{2}\right) = 2 * \ ZP\left(\frac{1}{2}\right) = \frac{(T+C) \prod_{T\ terms\ of\ P\ Prime} (\sqrt{P}-1)}{M * 2^K} \rightarrow (A)$$

$$; \ where\ T\ is\ number\ of\ Terms\ ;\ P\ is\ a\ prime\ number;\ and\ K\ some\ power\ for\ 2$$
$$; \ C\ is\ constant\ of\ all\ compinations\ of\ multiplications\ of\ square\ roots\ of\ the\ T\ terms$$
$$; M\ some\ Natural\ number\ mulitpier$$

Both Case (1) and Case (2) will have the same $(T+C)$.

For example, SUM of 5 Terms for both cases multiplication by $\varphi * \left(\sqrt{5}-1\right)$ and multiplication by $\frac{5}{\pi}$

$$\frac{5}{\pi} * ZP\left(\frac{1}{2}\right) = \frac{(5+C) * \left(\sqrt{2}-1\right) * \ \left(\sqrt{3}-1\right) * \left(\sqrt{5}-1\right) * \left(\sqrt{7}-1\right) * \left(\sqrt{11}-1\right)}{96 * \pi} \rightarrow (I)$$

$$\varphi * \left(\sqrt{5}-1\right) * ZP\left(\frac{1}{2}\right) = 2 * \ ZP\left(\frac{1}{2}\right) = \frac{(5+C) * \left(\sqrt{2}-1\right) * \ \left(\sqrt{3}-1\right) * \left(\sqrt{5}-1\right) * \left(\sqrt{7}-1\right) * \left(\sqrt{11}-1\right)}{240} \rightarrow (II)$$

From (I) and (II)

$$\frac{5}{\pi} * 96 * \pi * ZP\left(\frac{1}{2}\right) = \ \varphi * \left(\sqrt{5}-1\right) * 240 * ZP\left(\frac{1}{2}\right) = 2 * 240 * \ ZP\left(\frac{1}{2}\right) \rightarrow (B)$$

We can generalize Equation (B) Into a relation between $\frac{5}{\pi}\ and\ \varphi$

$$f\left(\frac{5}{\pi}\right) * ZP\left(\frac{1}{2}\right) = f(\ \varphi) * ZP\left(\frac{1}{2}\right) \rightarrow (C)$$

Example (1): Sum of first 5 Prime Terms and S = 0.5.

$$\frac{5}{(\sqrt{2}+1)\pi} + \frac{5}{(\sqrt{3}+1)\pi} + \frac{5}{(\sqrt{5}+1)\pi} + \frac{5}{(\sqrt{7}+1)\pi} + \frac{5}{(\sqrt{11}+1)\pi}$$
$$= \frac{(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-\ 1)(\sqrt{11}-\ 1)}{96\ \pi}$$

$$\frac{10}{(\sqrt{2}+1)\pi} + \frac{10}{(\sqrt{3}+1)\pi} + \frac{10}{(\sqrt{5}+1)\pi} + \frac{10}{(\sqrt{7}+1)\pi} + \frac{10}{(\sqrt{11}+1)\pi}$$
$$= \frac{2 * (5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-\ 1)(\sqrt{11}-\ 1)}{96\ \pi}$$

$$\frac{15}{(\sqrt{2}+1)\pi} + \frac{15}{(\sqrt{3}+1)\pi} + \frac{15}{(\sqrt{5}+1)\pi} + \frac{15}{(\sqrt{7}+1)\pi} + \frac{15}{(\sqrt{11}+1)\pi}$$
$$= \frac{3 * (5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-\ 1)(\sqrt{11}-\ 1)}{96\ \pi}$$

$$\frac{20}{(\sqrt{2}+1)\pi}+\frac{20}{(\sqrt{3}+1)\pi}+\frac{20}{(\sqrt{5}+1)\pi}+\frac{20}{(\sqrt{7}+1)\pi}+\frac{20}{(\sqrt{11}+1)\pi}$$

$$=\frac{4*(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}=10.1554243381\ldots$$

$$\frac{25}{(\sqrt{2}+1)\pi}+\frac{25}{(\sqrt{3}+1)\pi}+\frac{25}{(\sqrt{5}+1)\pi}+\frac{25}{(\sqrt{7}+1)\pi}+\frac{25}{(\sqrt{11}+1)\pi}$$

$$=\frac{5*(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}=10.1554243381\ldots$$

$$\frac{30}{(\sqrt{2}+1)\pi}+\frac{30}{(\sqrt{3}+1)\pi}+\frac{30}{(\sqrt{5}+1)\pi}+\frac{30}{(\sqrt{7}+1)\pi}+\frac{30}{(\sqrt{11}+1)\pi}$$

$$=\frac{6*(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$

$$=\frac{(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{16\,\pi}$$

$$\frac{35}{(\sqrt{2}+1)\pi}+\frac{35}{(\sqrt{3}+1)\pi}+\frac{35}{(\sqrt{5}+1)\pi}+\frac{35}{(\sqrt{7}+1)\pi}+\frac{35}{(\sqrt{11}+1)\pi}$$

$$=\frac{7*(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$

$$\frac{95}{(\sqrt{2}+1)\pi}+\frac{95}{(\sqrt{3}+1)\pi}+\frac{95}{(\sqrt{5}+1)\pi}+\frac{95}{(\sqrt{7}+1)\pi}+\frac{95}{(\sqrt{11}+1)\pi}$$

$$=\frac{19\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$

$$\frac{100}{(\sqrt{2}+1)\pi}+\frac{100}{(\sqrt{3}+1)\pi}+\frac{100}{(\sqrt{5}+1)\pi}+\frac{100}{(\sqrt{7}+1)\pi}+\frac{100}{(\sqrt{11}+1)\pi}$$

$$=\frac{20\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$

$$=\frac{5(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{24\,\pi}$$

$$\frac{105}{(\sqrt{2}+1)\pi}+\frac{105}{(\sqrt{3}+1)\pi}+\frac{105}{(\sqrt{5}+1)\pi}+\frac{105}{(\sqrt{7}+1)\pi}+\frac{105}{(\sqrt{11}+1)\pi}$$

$$=\frac{21\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$

$$=\frac{7(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{32\pi}$$

$$\frac{110}{(\sqrt{2}+1)\pi}+\frac{110}{(\sqrt{3}+1)\pi}+\frac{110}{(\sqrt{5}+1)\pi}+\frac{110}{(\sqrt{7}+1)\pi}+\frac{110}{(\sqrt{11}+1)\pi}$$

$$=\frac{22\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$

$$=\frac{11(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{48\pi}$$

$$\frac{115}{(\sqrt{2}+1)\pi}+\frac{115}{(\sqrt{3}+1)\pi}+\frac{115}{(\sqrt{5}+1)\pi}+\frac{115}{(\sqrt{7}+1)\pi}+\frac{115}{(\sqrt{11}+1)\pi}$$
$$=\frac{23\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$
$$\frac{120}{(\sqrt{2}+1)\pi}+\frac{120}{(\sqrt{3}+1)\pi}+\frac{120}{(\sqrt{5}+1)\pi}+\frac{120}{(\sqrt{7}+1)\pi}+\frac{120}{(\sqrt{11}+1)\pi}$$
$$=\frac{24\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$
$$=\frac{(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{12\,\pi}$$
$$\frac{120}{(\sqrt{2}+1)\pi}+\frac{120}{(\sqrt{3}+1)\pi}+\frac{120}{(\sqrt{5}+1)\pi}+\frac{120}{(\sqrt{7}+1)\pi}+\frac{120}{(\sqrt{11}+1)\pi}$$
$$=\frac{24\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$
$$=\frac{(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{12\,\pi}$$

The fractional form denominator will continue to increase, and decrease based of multiples of 5, and 2 until nominator reaches 480 and denominator reaches one pi in the fractional form.

$$\frac{480}{(\sqrt{2}+1)\pi}+\frac{480}{(\sqrt{3}+1)\pi}+\frac{480}{(\sqrt{5}+1)\pi}+\frac{480}{(\sqrt{7}+1)\pi}+\frac{480}{(\sqrt{11}+1)\pi}$$
$$=\frac{96\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$
$$=\frac{(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{\pi}$$
$$\frac{485}{(\sqrt{2}+1)\pi}+\frac{485}{(\sqrt{3}+1)\pi}+\frac{485}{(\sqrt{5}+1)\pi}+\frac{485}{(\sqrt{7}+1)\pi}+\frac{485}{(\sqrt{11}+1)\pi}$$
$$=\frac{97\,(5+c)(\sqrt{2}-1)(\sqrt{3}-1)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{96\,\pi}$$

An continue until it reaches again full multiplier of 12; 96 * 5 = {48, 32, 24, 16, 12, 8, 4 ,2 ,1}

Example (2): -The next set of 5 prime numbers will be {13 ,17, 19 ,23 ,29}; T = 5.

$$\frac{5}{(\sqrt{13}+1)\pi}+\frac{5}{(\sqrt{17}+1)\pi}+\frac{5}{(\sqrt{19}+1)\pi}+\frac{5}{(\sqrt{23}+1)\pi}+\frac{5}{(\sqrt{29}+1)\pi}$$
$$=\frac{(5+c)(13-1)(\sqrt{17}-1)(\sqrt{19}-1)(\sqrt{23}-1)(\sqrt{29}-1)}{236544\,\pi}$$

Example (3): - The next 5 prime numbers will be {31 ,37, 41, 43, 47}; T =5.

$$\frac{5}{(\sqrt{31}+1)\pi}+\frac{5}{(\sqrt{37}+1)\pi}+\frac{5}{(\sqrt{41}+1)\pi}+\frac{5}{(\sqrt{43}+1)\pi}+\frac{5}{(\sqrt{47}+1)\pi}$$
$$=\frac{(5+c)(\sqrt{31}-1)(\sqrt{37}-1)(\sqrt{41}-1)(\sqrt{43}-1)(\sqrt{47}-1)}{16692480\,\pi}$$

Example (4): -

If we took sum of 4 prime numbers each time, and add them together, we will have one term (4 + c) in the nominator.

$$\frac{5}{(\sqrt{31}+1)\pi}+\frac{5}{(\sqrt{37}+1)\pi}+\frac{5}{(\sqrt{41}+1)\pi}+\frac{5}{(\sqrt{43}+1)\pi}=\frac{(4+c)(\sqrt{31}-1)(\sqrt{37}-1)(\sqrt{41}-1)(\sqrt{43}-1)}{362880\,\pi}$$

Example (5): -

If we took sum of three prime numbers each time and add them together, we will have one term (3 + c) in the nominator.

$$\frac{1}{(\sqrt{5}+1)\pi}+\frac{1}{(\sqrt{7}+1)\pi}+\frac{1}{(\sqrt{11}+1)\pi}=\frac{(3+c)(\sqrt{5}-1)(\sqrt{7}-1)(\sqrt{11}-1)}{240\,\pi}$$

Example (6): -

This is an example for the (5 + c) term in the fraction form of the sum 5 prime terms = {31 ,37, 41 ,43, 47}

$$C=\begin{matrix}\sqrt{31}\sqrt{37}\sqrt{41}\sqrt{43}+\sqrt{31}\sqrt{37}\sqrt{41}\sqrt{47}+\sqrt{31}\sqrt{37}\sqrt{47}\sqrt{43}+\sqrt{47}\sqrt{37}\sqrt{41}\sqrt{43}+\cdots\\ +2\sqrt{31}\sqrt{37}\sqrt{41}+2\sqrt{31}\sqrt{47}\sqrt{41}+2\sqrt{31}\sqrt{43}\sqrt{41}+\cdots\ldots\ldots\\ +4\sqrt{31}+4\sqrt{37}+4\sqrt{41}+4\sqrt{43}+4\sqrt{47}\end{matrix}\quad\rightarrow(7)$$

$$\frac{5}{(\sqrt{31}+1)\pi}+\frac{5}{(\sqrt{37}+1)\pi}+\frac{5}{(\sqrt{41}+1)\pi}+\frac{5}{(\sqrt{43}+1)\pi}+\frac{5}{(\sqrt{47}+1)\pi}$$

$$*\begin{pmatrix}5+\sqrt{31}\sqrt{37}\sqrt{41}\sqrt{43}+\sqrt{31}\sqrt{37}\sqrt{41}\sqrt{47}+\sqrt{31}\sqrt{37}\sqrt{47}\sqrt{43}+\sqrt{47}\sqrt{37}\sqrt{41}\sqrt{43}+\cdots\\ +2\sqrt{31}\sqrt{37}\sqrt{41}+2\sqrt{31}\sqrt{47}\sqrt{41}+2\sqrt{31}\sqrt{43}\sqrt{41}+\cdots\ldots\ldots\\ +4\sqrt{31}+4\sqrt{37}+4\sqrt{41}+4\sqrt{43}+4\sqrt{47}\end{pmatrix}$$

$$=\frac{(\sqrt{31}-1)(\sqrt{37}-1)(\sqrt{41}-1)(\sqrt{43}-1)(\sqrt{47}-1)}{16692480\,\pi}$$

So, we can say that if we Sum all N Prime numbers at once we will have term (N + c) term in the nominator.

$$Z(S)=1+\frac{1}{2^S}+\frac{1}{3^S}+\frac{1}{4^S}+\frac{1}{5^S}+\frac{1}{6^S}+\frac{1}{7^S}\ldots..$$

$$Z(S)=\prod_{P\ Prime}\frac{1}{1-P^{-S}}$$

$$At\ S=\frac{1}{2}\ THEN\ Z\left(\frac{1}{2}\right)=1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{6}}+\cdots..$$

$$At\ S=-\frac{1}{2}\ THEN\ Z\left(\frac{-1}{2}\right)=\prod_{P\ Prime}\frac{1}{1-\sqrt{P}}$$

$$Z\left(\frac{-1}{2}\right)*\prod_{P\ Prime}(1-\sqrt{P})=1$$

THEN Sum for Primes series will be

$$ZP\left(\frac{1}{2}\right)=\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+1}+\frac{1}{\sqrt{7}+1}+\frac{1}{\sqrt{11}+1}+\frac{1}{\sqrt{13}+1}+\frac{1}{\sqrt{17}+1}\ldots..$$

$$\frac{5}{\pi}*ZP\left(\frac{1}{2}\right)=\frac{(T+c)\ \prod_2^N(\sqrt{P}-1)}{M*2^N*\ \pi}$$

Same effect if we so multipliers of φ

$$\varphi*(\sqrt{5}-1)*ZP\left(\frac{1}{2}\right)=\frac{(T+c)\ \prod_2^N(\sqrt{P}-1)}{M*2^N}$$

$;where\ C\ is\ constant\ of\ all\ compinations\ of\ multiplications$ based on our previous examples of fractional forms for adding a series of 5 terms each time

$$\frac{5}{\pi}*ZP\left(\frac{1}{2}\right)=\frac{(-1)^N(5+c)\ \prod_2^N(1-\sqrt{P})}{2^M\ \pi}\quad\rightarrow(4)$$

$$\frac{5}{\pi}*ZP\left(\frac{1}{2}\right)*Z\left(\frac{-1}{2}\right)=\frac{(-1)^N(5+c)}{2^M\ \pi}$$

$$\frac{5}{\pi} * ZP\left(\frac{1}{2}\right) * Z\left(\frac{-1}{2}\right) = \frac{(-1)^N}{\pi} * \frac{(5+c)}{2^M}$$

$$\frac{5}{\pi} * ZP\left(\frac{1}{2}\right) * Z\left(\frac{-1}{2}\right) = \frac{(-1)^N}{2^{M-1} * \pi} * \frac{(5+c)}{2}$$

$$\frac{5}{\pi} * ZP\left(\frac{1}{2}\right) * Z\left(\frac{-1}{2}\right) = \frac{(-1)^N}{2^{M-1} * \pi} * f(\varphi) \rightarrow (5)$$

$$\frac{5}{\pi} * ZP\left(\frac{1}{2}\right) * Z\left(\frac{-1}{2}\right) = \frac{(-1)^N}{2^{M-1} * \pi} * f(\varphi) \ ; WHERE\ f(\varphi)\ is\ a\ function\ of\ \left(\varphi = \frac{\sqrt{5}+1}{2}\right) \rightarrow (6)$$

Which is like expressing odd power functions as quadratic function in term of two functions with square roots.

This $f(\varphi)$ depends on the number of prime terms added in zeta series. But at the end it will have φ

And these Terms $\left(1-\sqrt{P}\right)$ do not need to be consecutive terms.

For Example, Sum 2 Terms.

$$\varphi * \frac{\left(\sqrt{5}-1\right)}{\pi} * ZP\left(\frac{1}{2}\right) = 2 * ZP\left(\frac{1}{2}\right) = \frac{\varphi * (\sqrt{5}-1)}{\left(\sqrt{5}+1\right) * \pi} + \frac{\varphi * (\sqrt{5}-1)}{\left(\sqrt{13}+1\right) * \pi} = \frac{\left(2+\sqrt{5}+\sqrt{13}\right)\left(\sqrt{5}-1\right)\left(\sqrt{13}-1\right)}{24 * \pi}$$

$$ZP\left(\frac{1}{2}\right) = \frac{\varphi}{\left(\sqrt{5}+1\right) * \pi} + \frac{\varphi}{\left(\sqrt{13}+1\right) * \pi} = \frac{\left(2+\sqrt{5}+\sqrt{13}\right)\left(\sqrt{5}-1\right)\left(\sqrt{3}-1\right)}{24 * \pi}$$

$$= \frac{(2+\sqrt{5}+\sqrt{13})}{2 * 6} * \varphi * \frac{\left(\sqrt{5}+1\right)}{4} * \prod_{2}^{3}\left(1-\sqrt{P}\right) * (-1) * (-1)$$

$$\left(\sqrt{5}-1\right) * \left(\sqrt{5}+1\right) = 4\ and\ \varphi * \left(\sqrt{5}-1\right) = \frac{4}{2} = 2 \rightarrow (D)$$

φ Which will give this 0.5 for all Prime numbers. And as we saw the golden ratio Phi have two at its denominator and as we proof all power of the main function goes back to the base relation for Phi.

$$\varphi^{\frac{-1}{2}} + \varphi^{\frac{1}{2}} = \varphi^{\frac{3}{2}}$$

So prime numbers use the power two system, which means it use the square root of two axis in complex plane and not the original complex plane axis and φ is the determinate for this transformation between the systems.

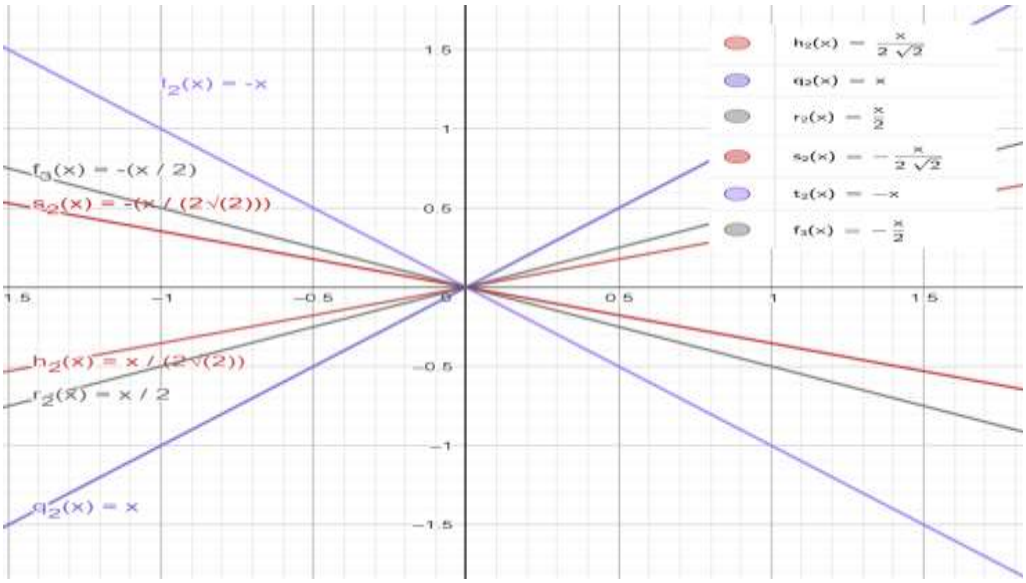


Figure 19. divide by 2 and its odd powers using tow functions with roots in case of axis are orthogonal or not

And this matches our main equation to get the benefits of phi properties

$$\left(\sqrt{X}-\frac{1}{\sqrt{X}}\right)=\frac{1}{X\sqrt{X}}\ ;at\ X=2$$

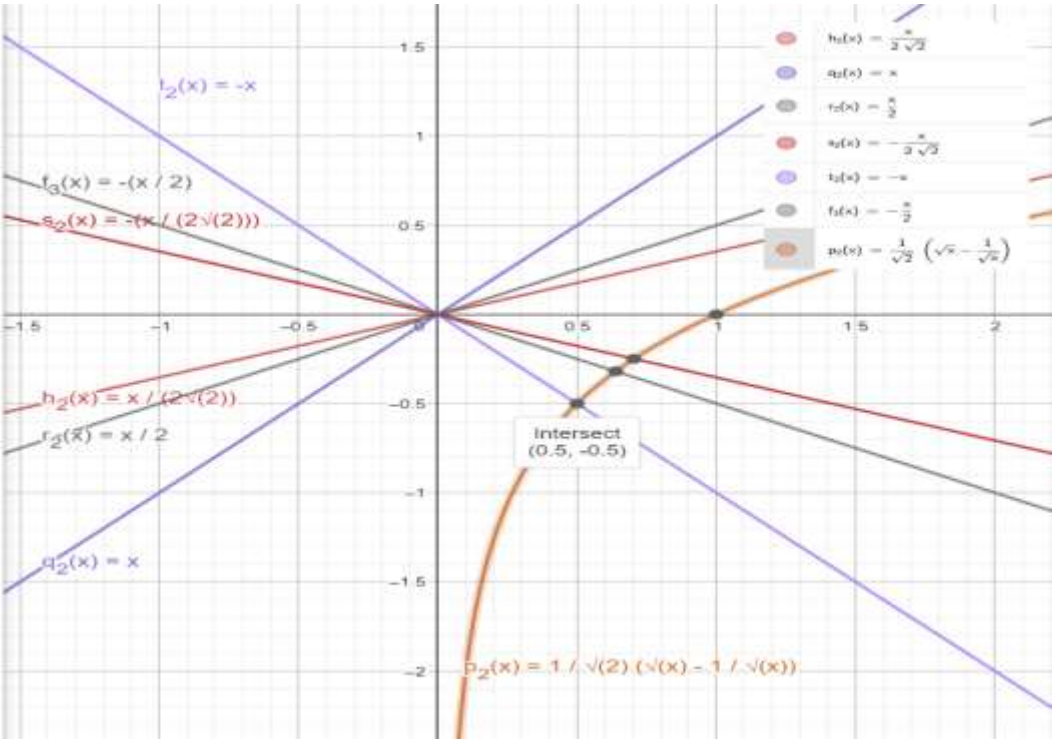


Figure 20. square root of 2 and its relation of giving Zeros to the Nth roots

$$\frac{1}{\sqrt{A}-1}*\frac{1}{\sqrt{B}-1}=\frac{\left(\sqrt{A}+1\right)\left(\sqrt{B}+1\right)}{(A-1)(B-1)}\ \rightarrow(E)$$

if we applied this to the full Zeta function formula

$$Z\left(\frac{-1}{2}\right)=\frac{1}{\prod_{P\ Prime}(1-\sqrt{P})}$$

$$Z\left(\frac{-1}{2}\right)=\frac{\prod_{P\ Prime}(1+\sqrt{P})}{2^{T}*\prod_{P\ Prime}(\frac{P}{2}-\frac{1}{2})}\ ;where\ T\ is\ the\ number\ of\ Prime\ Terms.\ \rightarrow(F)$$

6. Conclusion

It is known that square root of 2 is irrational, this means that the value of the calculation's accuracy will be different from machine to another. In this paper we tried to avoid using pure calculations and instead tried to use fractional and functional forms. we used Phi as a basic unit to reference all functions to be based on Phi as a functional form not as value. Then we used this golden ratio and its functional properties to get the sum of Prime numbers for Zeta function in term of Pi and Phi.

ZP(S) is the Sum of T Prime numbers Terms.

$$\frac{5}{\pi}*ZP(\frac{1}{2})=\frac{T}{\pi}*\sum_{n=0}^{\infty}\frac{1}{\sqrt{n}+1}=\frac{D*\ (T+C)\prod_{T\ terms\ of\ P\ Prime}(\sqrt{P}-1)}{M*2^{K}*\pi}\ \rightarrow(2)$$

$$;\ where\ T\ is\ number\ of\ Terms\ ;\ P\ is\ a\ prime\ number;\ and\ K\ some\ power\ for\ 2$$

$$;\ C\ is\ constant\ of\ all\ compinations\ of\ multiplications\ of\ square\ roots\ of\ the\ T\ terms$$

$$;D\ any\ Natural\ number\ mulitpier\ \ ;at\ D=M*2^{K}\ the\ Denominator=\ \pi$$

And this is like using multiplication by $\varphi*\left(\sqrt{5}-1\right)$

$$\varphi * \left(\sqrt{5}-1\right) * ZP\left(\frac{1}{2}\right) = 2 * \ ZP\left(\frac{1}{2}\right) = \ \frac{(T+C)\prod_{T\ terms\ of\ P\ Prime}\left(\sqrt{P}-1\right)}{M * 2^{K}} \rightarrow (A)$$

$$;\ where\ T\ is\ number\ of\ Terms\ ;\ P\ is\ a\ prime\ number;\ and\ K\ some\ power\ for\ 2$$

$$;\ C\ is\ constant\ of\ all\ compinations\ of\ multiplications\ of\ square\ roots\ of\ the\ T\ terms$$

$$;M\ some\ Natural\ number\ mulitpier$$

$$\frac{5}{\pi} * ZP\left(\frac{1}{2}\right) * \ Z\left(\frac{-1}{2}\right) = \frac{(-1)^{N}}{2^{M-1} * \ \pi} \ * \ f(\varphi)\ \ ;WHERE\ f(\varphi)\ is\ a\ function\ of\ \left(\varphi = \frac{\sqrt{5}+1}{2}\right) \rightarrow (6)$$

And We can generalize two Equations Into a relation between $\frac{5}{\pi}$ $and\ \varphi$

$$f\left(\frac{5}{\pi}\right) * ZP\left(\frac{1}{2}\right) = f(\ \varphi) * ZP\left(\frac{1}{2}\right) \rightarrow (C)$$

$$\left(\sqrt{5}-1\right) * \ \left(\sqrt{5}+1\right) = 4\ and\ \varphi * \left(\sqrt{5}-1\right) = \frac{4}{2} = 2 \rightarrow (D)$$

$$Z\left(\frac{-1}{2}\right) = \frac{\prod_{P\ Prime}\left(1+\sqrt{P}\right)}{2^{T} * \prod_{P\ Prime}\left(\frac{P}{2}-\frac{1}{2}\right)}\ ;where\ T\ is\ the\ number\ of\ Prime\ Terms. \rightarrow (F)$$

Also, we showed how we can represent any odd power function as quadratic function by using two function using square root. And from this we showed a trigonometric way to calculate phi angel.

$$\sin^{-1}(\theta) = \frac{\sqrt{\varphi}}{\sqrt{\left(\sqrt{\varphi}^{2} + \left(\varphi\sqrt{\varphi}\right)^{2}\right)}} = \frac{\sqrt{\varphi}}{\sqrt{\varphi+\varphi^{3}}} = \sqrt{\frac{\varphi}{\varphi+\varphi^{3}}} = \sqrt{\frac{1}{1+\varphi^{2}}} = \sqrt{\frac{1}{\varphi+2}}$$

$$\gamma = \sin^{-1}\left(\sqrt{\frac{1}{w^{2}+1}}\right) = 31.717474411461^{\circ} \qquad \delta = \sin^{-1}\left(\sqrt{\frac{1}{w+2}}\right) = 31.717474411461^{\circ} \qquad j_1 = \left(\frac{\sqrt{5}+1}{2}\right)^{1.5} = 2.0581710272715$$

Also, we showed how the Golden ratio properties is correct for all square roots and not only for square root of [5] and this give an easy proof for Riemann hypothesis.

$$\frac{\left(\sqrt{N}+1\right)}{D} - \frac{\frac{N-1}{D}}{\left(\sqrt{N}+1\right)} + \frac{D-4}{2D} \ = \frac{1}{2} \rightarrow (F)\ \ ;for\ any\ natural\ number\ N\ and\ D.where\ D = \left\lceil \frac{\sqrt{N}}{5} \right\rceil$$

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