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Cubic and Quadratic Equations and Zeta Function Zeros

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Abstract

In this paper, we will study a partial sum modulus distribution for a specific natural number set using a dynamically sliding window. Then we will construct a cubic equation from this distribution and a formula to calculate this cubic equation zero. Then we will go through some applications of this Cubic equation using the basic algebraic concepts to explain the distribution of natural numbers.

First part in this paper, we will interduce a partial sums modulus distribution for natural numbers using a dynamic sliding window as a parameter to explore the natural numbers distribution. As a simpler way of studying the distribution of a multi dynamic subsets inside natural numbers domain.

Second part in this paper, we will interpret this distribution into a quadratic and cubic equations and twin cubic equation concept clarification, then will use these two concepts to explain the distribution of zeros on the Zeta function strip line.

In the last part, we will go through some applications for this distribution one of them will be an example of getting prime number factors using a partial sum of specific series of odd numbers.

Keywords: Prime Numbers, Composite Prime Numbers, Prime Number Distribution, Zeta function

1. Introduction

1.1 Introduce the Problem

Understanding numbers distribution is not clear and is a missing part of the number system theory.

We only have two main basic concepts for natural numbers; numbers are (even numbers or odd numbers).

These main two concepts alone are not enough to get a full understanding of natural numbers distribution.

To understand natural numbers distribution more, we will study a dynamical sliding window partial sum reminder distribution to find out how numbers are behaving inside a closed sliding window then we will parametrize this window to get distribution in terms of this window size as a parameter.

Instead of studying the numbers separately, we are going to study partial sum reminder distribution by taking a partial sum using a sliding window and then find the reminder for each partial sum window to the first element in the sliding window.

0	1	2	3	5	7	9	11	13	15	17	19	21	23	25	27	29
1	2	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
2	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35
5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37
7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39
9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41
11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43
13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45
15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47
17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49
19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51
21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53
23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55
25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57
27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59
29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61
31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63
33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65
35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67
37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69
39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71
41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73
43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75
45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77
47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79
49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81
51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83
53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85
55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87
57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89
59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91
61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93
63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95

Any sliding window has size parameter called (W). this sliding window will produce a multi subsets as it moves along this natural number set N= {0,1,2,3,5,7,9,11, 13,15,17,19….}, which is odd numbers set but with number 2 (even) and number 0 added to this set.

For example, a partial sum with a sliding window of size (W = 3).

If we started at N=0 (first element in the set), we get a new set of subsets, each of size 3 and these subsets sums is a set S_0 = {(0+1+2), (1+2+3), (2+3+5), (3+5+7), (5+7+9) ….}

If we started from N=1 (second element in the set) and W=3, we get another set S_1 = {(1+2+3), (2+3+5), (3+5+7), (5+7+9) ….}

Then for each subset sum we will take the modulus to the first element in this subset for the specific window size if > 0 other else the value will be = 0.

So, For S_0 = {(0+1+2), (1+2+3), (2+3+5), (3+5+7), (5+7+9), (7+9+11), (9+11+13) ….}

The modulus set of S_0 will be S_{M0}= {0, 6mod (1), 10 mod (2), 15 mod (3), 21 mod (5), 27 mod (7), 33 mod (9), 39 mod (11), …} S_{M0}= {0, 0, 0, 0, 1, 6, 6, 6,6 …}

For S_1 = {(1+2+3), (2+3+5), (3+5+7), (5+7+9), (7+9+11), (9+11+13) ….}

The modulus set of S_{M0}= {6mod (1), 10 mod (2), 15 mod (3), 21 mod (5), 27 mod (7), 33 mod (9), 39 mod (11), …} S_{M1}= {0, 0, 0, 1, 6, 6, 6,6 …}

In Figure 1., we study window (W=3) for S_0, S_{M0}, S_1, S_{M1}, S_2, S_{M2}, S_3, S_{M3} S_5, S_{M5}, S_7, and S_{M7} from left to right.

3 Window			3 Window			3 Window			3 Window			3 Window			3 Window		
3		3	6	0		10	0		15	0		21	0		27	0	
6	0	5	10	0		15	0		21	1		27	2		33	3	
10	0	7	15	0		21	1		27	6	3	33	5		39	4	
15	0	9	21	1		27	6	2	33	6		39	3		45	0	
21	1	11	27	6	1	33	6		39	6		45	1		51	7	
27	6	0	33	6		39	6		45	6		51	12	5	57	5	
33	6	15	39	6		45	6		51	6		57	12		63	3	
39	6		45	6		51	6		57	6		63	12		69	1	
45	6		51	6		57	6		63	6		69	12		75	18	7
51	6		57	6		63	6		69	6		75	12		81	18	
57	6		63	6		69	6		75	6		81	12		87	18	
63	6		69	6		75	6		81	6		87	12		93	18	
69	6		75	6		81	6		87	6		93	12		99	18	
75	6		81	6		87	6		93	6		99	12		105	18	
81	6		87	6		93	6		99	6		105	12		111	18	
87	6		93	6		99	6		105	6		111	12		117	18	
93	6		99	6		105	6		111	6		117	12		123	18	
99	6		105	6		111	6		117	6		123	12		129	18	
105	6		111	6		117	6		123	6		129	12		135	18	
111	6		117	6		123	6		129	6		135	12		141	18	
117	6		123	6		129	6		135	6		141	12		147	18	
123	6		129	6		135	6		141	6		147	12		153	18	
129	6		135	6		141	6		147	6		153	12		159	18	
135	6		141	6		147	6		153	6		159	12		165	18	
141	6		147	6		153	6		159	6		165	12		171	18	
147	6		153	6		159	6		165	6		171	12		177	18	
153	6		159	6		165	6		171	6		177	12		183	18	
159	6		165	6		171	6		177	6		183	12		189	18	
165	6		171	6		177	6		183	6		189	12		195	18	
171	6		177	6		183	6		189	6		195	12		201	18	
177	6		183	6		189	6		195	6		201	12		207	18	
183	6		189	6		195	6		201	6		207	12		213	18	
189	6		195	6		201	6		207	6		213	12		219	18	
195	6		201	6		207	6		213	6		219	12		225	18	
201	6		207	6		213	6		219	6		225	12		231	18	
207	6		213	6		219	6		225	6		231	12		237	18	
213	6		219	6		225	6		231	6		237	12		243	18	
219	6		225	6		231	6		237	6		243	12		249	18	
225	6		231	6		237	6		243	6		249	12		255	18	
231	6		237	6		243	6		249	6		255	12		261	18	

In Figure 2., we study window (W=4) for S_0, S_{M0}, S_1, S_{M1}, S_2, S_{M2}, S_3, S_{M3} S_5, S_{M5}, S_7, and S_{M7} from left to right.

4	Window	0	4	Window	1	4	Window	2	4	Winodw	3	4	Window	5	4	Window	7
6	#DIV/0!		11	0		17	1		24	0		32	2		40	1	
11	0		17	1		24	0		32	2		40	0		48	3	
17	1		24	0		32	2		40	5		48	6		56	0	
24	0		32	2		40	5		48	3		56	2		64	1	
32	2		40	5		48	3		56	1		64	9		72	6	
40	5		48	3		56	1		64	12	3	72	7		80	2	
48	3		56	1		64	12	2	72	12		80	5		88	13	
56	1		64	12	1	72	12		80	12		88	3		96	11	
64	12	0	72	12		80	12		88	12		96	1		104	9	
72	12		80	12		88	12		96	12		104	20	5	112	7	
80	12		88	12		96	12		104	12		112	20		120	5	
88	12		96	12		104	12		112	12		120	20		128	3	
96	12		104	12		112	12		120	12		128	20		136	1	
104	12		112	12		120	12		128	12		136	20		144	28	7
112	12		120	12		128	12		136	12		144	20		152	28	
120	12		128	12		136	12		144	12		152	20		160	28	
128	12		136	12		144	12		152	12		160	20		168	28	
136	12		144	12		152	12		160	12		168	20		176	28	
144	12		152	12		160	12		168	12		176	20		184	28	
152	12		160	12		168	12		176	12		184	20		192	28	
160	12		168	12		176	12		184	12		192	20		200	28	
168	12		176	12		184	12		192	12		200	20		208	28	
176	12		184	12		192	12		200	12		208	20		216	28	
184	12		192	12		200	12		208	12		216	20		224	28	
192	12		200	12		208	12		216	12		224	20		232	28	
200	12		208	12		216	12		224	12		232	20		240	28	
208	12		216	12		224	12		232	12		240	20		248	28	
216	12		224	12		232	12		240	12		248	20		256	28	
224	12		232	12		240	12		248	12		256	20		264	28	
232	12		240	12		248	12		256	12		264	20		272	28	
240	12		248	12		256	12		264	12		272	20		280	28	
248	12		256	12		264	12		272	12		280	20		288	28	
256	12		264	12		272	12		280	12		288	20		296	28	
264	12		272	12		280	12		288	12		296	20		304	28	
272	12		280	12		288	12		296	12		304	20		312	28	
280	12		288	12		296	12		304	12		312	20		320	28	
288	12		296	12		304	12		312	12		320	20		328	28	
296	12		304	12		312	12		320	12		328	20		336	28	
304	12		312	12		320	12		328	12		336	20		344	28	
312	12		320	12		328	12		336	12		344	20		352	28	

In Figure 3., we study window (W=5) for S_1, S_{M1}, S_2, S_{M2}, S_3, S_{M3} S_5, S_{M5}, S_7, and S_{M7} from left to right.

5 Window		1	5 Window		2	5 Window		3	5 Window		5	5 window		7
11	#DIV/0!		18	0		35	2		45	0		55	1	
18	0		26	0		45	0		55	0		65	0	
26	0		35	2		55	6		65	2		75	5	
35	2		45	0		65	2		75	3		85	4	
45	0		55	6		75	9		85	8		95	7	
55	6		65	2		85	7		95	4		105	1	
65	2		75	9		95	5		105	0		115	10	
75	9		85	7		105	3		115	13		125	6	
85	7		95	5		115	1		125	11		135	2	
95	5		105	3		125	20	3	135	9		145	19	
105	3		115	1		135	20		145	7		155	17	
115	1		125	20	2	145	20		155	5		165	15	
125	20	1	135	20		155	20		165	3		175	13	
135	20		145	20		165	20		175	1		185	11	
145	20		155	20		175	20		185	30	5	195	9	
155	20		165	20		185	20		195	30		205	7	
165	20		175	20		195	20		205	30		215	5	
175	20		185	20		205	20		215	30		225	3	
185	20		195	20		215	20		225	30		235	1	
195	20		205	20		225	20		235	30		245	40	7
205	20		215	20		235	20		245	30		255	40	
215	20		225	20		245	20		255	30		265	40	
225	20		235	20		255	20		265	30		275	40	
235	20		245	20		265	20		275	30		285	40	
245	20		255	20		275	20		285	30		295	40	
255	20		265	20		285	20		295	30		305	40	
265	20		275	20		295	20		305	30		315	40	
275	20		285	20		305	20		315	30		325	40	
285	20		295	20		315	20		325	30		335	40	
295	20		305	20		325	20		335	30		345	40	
305	20		315	20		335	20		345	30		355	40	
315	20		325	20		345	20		355	30		365	40	
325	20		335	20		355	20		365	30		375	40	
335	20		345	20		365	20		375	30		385	40	
345	20		355	20		375	20		385	30		395	40	
355	20		365	20		385	20		395	30		405	40	
365	20		375	20		395	20		405	30		415	40	
375	20		385	20		405	20		415	30		425	40	
385	20		395	20		415	20		425	30		435	40	
395	20		405	20		425	20		435	30		445	40	

In Figure 4., we study window (W=6) for S_0, S_{M0}, S_1, S_{M1}, S_2, S_{M2}, S_3, S_{M3} S_5, S_{M5}, S_7, and S_{M7} from left to right.

6 Window		0	6 Window		1	6 Window		2	6 Window		3	6 Window		5	6 Window		7
18	#DIV/0!	3	27	0		37	1		48	0		60	0		72	2	
27	0	3	37	1		48	0		60	0		72	2		84	3	
37	1		48	0		60	0		72	2		84	3		96	8	
48	0		60	0		72	2		84	3		96	8		108	4	
60	0		72	2		84	3		96	8		108	4		120	0	
72	2		84	3		96	8		108	4		120	0		132	13	
84	3		96	8		108	4		120	0		132	13		144	11	
96	8		108	4		120	0		132	13		144	11		156	9	
108	4		120	0		132	13		144	11		156	9		168	7	
120	0		132	13		144	11		156	9		168	7		180	5	
132	13		144	11		156	9		168	7		180	5		192	3	
144	11		156	9		168	7		180	5		192	3		204	1	
156	9		168	7		180	5		192	3		204	1		216	30	7
168	7		180	5		192	3		204	1		216	30	5	228	30	
180	5		192	3		204	1		216	30	3	228	30		240	30	
192	3		204	1		216	30	2	228	30		240	30		252	30	
204	1		216	30	1	228	30		240	30		252	30		264	30	
216	30	0	228	30		240	30		252	30		264	30		276	30	
228	30		240	30		252	30		264	30		276	30		288	30	
240	30		252	30		264	30		276	30		288	30		300	30	
252	30		264	30		276	30		288	30		300	30	5	312	30	
264	30		276	30		288	30		300	30		312	30		324	30	
276	30		288	30		300	30		312	30		324	30		336	30	
288	30		300	30		312	30		324	30		336	30		348	30	
300	30		312	30		324	30		336	30		348	30		360	30	
312	30		324	30		336	30		348	30		360	30		372	30	
324	30		336	30		348	30		360	30		372	30		384	30	
336	30		348	30		360	30		372	30		384	30		396	30	
348	30		360	30		372	30		384	30		396	30		408	30	
360	30		372	30		384	30		396	30		408	30		420	30	
372	30		384	30		396	30		408	30		420	30		432	30	
384	30		396	30		408	30		420	30		432	30		444	30	
396	30		408	30		420	30		432	30		444	30		456	30	
408	30		420	30		432	30		444	30		456	30		468	30	
420	30		432	30		444	30		456	30		468	30		480	30	
432	30		444	30		456	30		468	30		480	30		492	30	
444	30		456	30		468	30		480	30		492	30		504	30	
456	30		468	30		480	30		492	30		504	30		516	30	
468	30		480	30		492	30		504	30		516	30		528	30	
480	30		492	30		504	30		516	30		528	30		540	30	

In Figure 5., we study window (W=1) for S_0, S_{M0}, S_1, S_{M1}, S_2, S_{M2}, S_3, S_{M3} S_5, S_{M5}, S_7, and S_{M7} from left to right.

Window	0	1	Window	1	1	Window	2	1	Window	3	1	Window	5	1	Window	7	1	Window	9	1	Window	11	1	Window	13	1	Window	15	1	Window	17	1	Window	19	1	Windov
#DIV/0!	0	2	0	1	4	0	2	6	0	3	10	0	3	14	2	3	18	0	3	22	1	3	26	2	3	30	0	3	34	1	3	38	2	3	42	
0		4	0		6	0		10	0	5	14	4	5	18	3	5	22	2	5	26	1	5	30	0	5	34	4	5	38	3	5	42	2	5	46	
0		6	0		10	0		14	0	7	18	4	7	22	1	7	26	5	7	30	2	7	34	6	7	38	3	7	42	0	7	46	4	7	50	
0		10	0		14	0		18	0	9	22	4	9	26	8	9	30	3	9	34	7	9	38	2	9	42	6	9	46	1	9	50	5	9	54	
0		14	0		18	0		22	0	11	26	4	11	30	8	11	34	1	11	38	5	11	42	9	11	46	2	11	50	6	11	54	10	11	58	
0		18	0		22	0		26	0		30	4		34	8		38	12	13	42	3	13	46	7	13	50	11	13	54	2	13	58	6	13	62	1
0		22	0		26	0		30	0		34	4		38	8		42	12		46	1	15	50	5	15	54	9	15	58	13	15	62	2	15	66	
0		26	0		30	0		34	0		38	4		42	8		46	12		50	16	17	54	3	17	58	7	17	62	11	17	66	15	17	70	
0		30	0		34	0		38	0		42	4		46	8		50	12		54	16		58	1	19	62	5	19	66	9	19	70	13	19	74	1
0		34	0		38	0		42	0		46	4		50	8		54	12		58	16		62	20	21	66	3	21	70	7	21	74	11	21	78	1
0		38	0		42	0		46	0		50	4		54	8		58	12		62	16		66	20		70	1	23	74	5	23	78	9	23	82	1
0		42	0		46	0		50	0		54	4		58	8		62	12		66	16		70	20		74	24	25	78	3	25	82	7	25	86	1
0		46	0		50	0		54	0		58	4		62	8		66	12		70	16		74	20		78	24		82	1	27	86	5	27	90	
0		50	0		54	0		58	0		62	4		66	8		70	12		74	16		78	20		82	24		86	28	29	90	3	29	94	
0		54	0		58	0		62	0		66	4		70	8		74	12		78	16		82	20		86	24		90	28		94	1	31	98	
0		58	0		62	0		66	0		70	4		74	8		78	12		82	16		86	20		90	24		94	28		98	32	33	102	
0		62	0		66	0		70	0		74	4		78	8		82	12		86	16		90	20		94	24		98	28		102	32		106	
0		66	0		70	0		74	0		78	4		82	8		86	12		90	16		94	20		98	24		102	28		106	32		110	3
0		70	0		74	0		78	0		82	4		86	8		90	12		94	16		98	20		102	24		106	28		110	32		114	3
0		74	0		78	0		82	0		86	4		90	8		94	12		98	16		102	20		106	24		110	28		114	32		118	3
0		78	0		82	0		86	0		90	4		94	8		98	12		102	16		106	20		110	24		114	28		118	32		122	3
0		82	0		86	0		90	0		94	4		98	8		102	12		106	16		110	20		114	24		118	28		122	32		126	3
0		86	0		90	0		94	0		98	4		102	8		106	12		110	16		114	20		118	24		122	28		126	32		130	3
0		90	0		94	0		98	0		102	4		106	8		110	12		114	16		118	20		122	24		126	28		130	32		134	3
0		94	0		98	0		102	0		106	4		110	8		114	12		118	16		122	20		126	24		130	28		134	32		138	3
0		98	0		102	0		106	0		110	4		114	8		118	12		122	16		126	20		130	24		134	28		138	32		142	3
0		102	0		106	0		110	0		114	4		118	8		122	12		126	16		130	20		134	24		138	28		142	32		146	3
0		106	0		110	0		114	0		118	4		122	8		126	12		130	16		134	20		138	24		142	28		146	32		150	3
0		110	0		114	0		118	0		122	4		126	8		130	12		134	16		138	20		142	24		146	28		150	32		154	3
0		114	0		118	0		122	0		126	4		130	8		134	12		138	16		142	20		146	24		150	28		154	32		158	3
0		118	0		122	0		126	0		130	4		134	8		138	12		142	16		146	20		150	24		154	28		158	32		162	3
0		122	0		126	0		130	0		134	4		138	8		142	12		146	16		150	20		154	24		158	28		162	32		166	3
0		126	0		130	0		134	0		138	4		142	8		146	12		150	16		154	20		158	24		162	28		166	32		170	3
0		130	0		134	0		138	0		142	4		146	8		150	12		154	16		158	20		162	24		166	28		170	32		174	3
0		134	0		138	0		142	0		146	4		150	8		154	12		158	16		162	20		166	24		170	28		174	32		178	3
0		138	0		142	0		146	0		150	4		154	8		158	12		162	16		166	20		170	24		174	28		178	32		182	3
0		142	0		146	0		150	0		154	4		158	8		162	12		166	16		170	20		174	24		178	28		182	32		186	3
0		146	0		150	0		154	0		158	4		162	8		166	12		170	16		174	20		178	24		182	28		186	32		190	3
0		150	0		154	0		158	0		162	4		166	8		170	12		174	16		178	20		182	24		186	28		190	32		194	3
0		154	0		158	0		162	0		166	4		170	8		174	12		178	16		182	20		186	24		190	28		194	32		198	3

Conclusion:

1- The cubic value for each window (W^3) will be in the window that contains W^2 as one of its elements.

   And [Sum (window elements) mod (window first element) = 0].

2- Modules set for a sliding window if N >=W will contain the same odd numbers set before N in a reversed order as the sum increases until it reaches a steady modulus number. (Highlighted in green in figure 1. And figure 2. And figure 3.)

3- As window size [W] increases; more elements of the reversed N set will start to be shown up as remainder for our partial sum.

4- Modules set for any sliding window W will reach a Steady value such that for each set S_N; will be a steady value = W * N if N > 3 and steady value = W (W-1) if 0<= N and N <=3; where W is window size and N is a start number for the set from original set N.

In figure 1., For example, for window (W =3) and N=0; so W^3 = 27 which is the sum of window elements (7,9,11) where 9 is the square of W and one of the window elements and [27 mod (7) = 0]

The main point for this distribution is that this partial sum reminder will reach a steady value no matter what the window size is used to do the partial sum at W^3 for S0, S1, S2, and S3 the steady point will be at the partial sum = W^3

2. Distribution Cubic Equation Solution

2.1 Cubic Equation Solution Formula

Based on our partial sum distribution study in point 1; we constructed a new set

C = {all steady values in modules sets for all sliding windows with size W_i}

C = {steady value for W=1, steady value for W=2, steady value for W=3, ….}

C = {0,2,6,12,20,30,42,56,72, 90…}

Table 1. Cubic Equations and steady values

A	A * (A-1)	A^2	$(A-1)^2$	A^3	$X^3+dX^2+dX+f = (X-a)(X-b)(X-c)$ $(X-A)(X^2-(A-1)X+(A*(A-1)+1)$
1	0	1	0	1	$(X-1)(X^2+1)$
2	2	4	1	8	$(X-2)(X^2-X+2)$
3	6	9	4	27	$(X-3)(X^2-2X+7)$
4	12	16	9	64	$(X-4)(X^2-3X+13)$
5	20	25	16	125	$(X-5)(X^2-4X+21)$
6	30	36	25	216	$(X-6)(X^2-5X+31)$
7	42	49	36	343	$(X-7)(X^2-6X+43)$
8	56	64	49	512	$(X-8)(X^2-7X+57)$
9	72	81	64	729	$(X-9)(X^2-8X+73)$
10	90	100	81	1000	$(X-10)(X^2-9X+91)$
11	110	121	100	1331	$(X-11)(X^2-10X+111)$
12	132	144	121	1728	$(X-12)(X^2-11X+133)$
13	156	169	144	2197	$(X-13)(X^2-12X+157)$
14	182	196	169	2744	$(X-14)(X^2-13X+183)$
15	210	225	196	3375	$(X-15)(X^2-14X+211)$
16	240	256	225	4096	$(X-16)(X^2-15X+241)$
17	272	289	256	4913	$(X-17)(X^2-16X+273)$
18	306	324	289	5832	$(X-18)(X^2-17X+307)$
19	342	361	324	6859	$(X-19)(X^2-18X+243)$
20	380	400	361	8000	$(X-20)(X^2-19X+281)$
21	420	441	400	9261	$(X-21)(X^2-20X+421)$
22	462	484	441	10648	$(X-22)(X^2-21X+263)$

W= {0, 1, 3, 4, 5, 6 ,7, 8, 9, 10 ….}

The difference between each element in these set are the even number set = {2,4,6,8,10,12,14, 16…}

So, as we increase the Window size to add an odd new number to the window; the remainder from the partial sum will increase by an even number positional to the even ((W+1) W - (W-1) W) = 2*W

Now let us relate these steady values to cubic of a natural number set and squares of a natural number set.

A	A * (A-1)	A^3	$X^3+dX^2+dX+f = (X-a)(X-b)(X-c)$ $(X-A)(X^2-(A-1)X+(A*(A-1)+1)$	$X^3+(A+A-1)X^2+(2*A*(A-1)+1)X+$ $(A^3-A*(A-1))$
1	0	1	$(X-1)(X^2+1)$	X^3-X^2+X-1
2	2	8	$(X-2)(X^2-X+2)$	X^3-3X^2+5X-6
3	6	27	$(X-3)(X^2-2X+7)$	$X^3-5X^2+13X+21$
4	12	64	$(X-4)(X^2-3X+13)$	$X^3-7X^2+25X+52$
5	20	125	$(X-5)(X^2-4X+21)$	$X^3-9X^2+41X+105$
6	30	216	$(X-6)(X^2-5X+31)$	$X^3-11X^2+61X+186$
7	42	343	$(X-7)(X^2-6X+43)$	$X^3-13X^2+85X+301$
8	56	512	$(X-8)(X^2-7X+57)$	$X^3-15X^2+113X+456$
9	72	729	$(X-9)(X^2-8X+73)$	$X^3-17X^2+...$
10	90	1000	$(X-10)(X^2-9X+91)$	$X^3-19X^2+...$
11	110	1331	$(X-11)(X^2-10X+111)$	$X^3-21X^2+...$
12	132	1728	$(X-12)(X^2-11X+133)$	$X^3-23X^2+...$
13	156	2197	$(X-13)(X^2-12X+157)$	$X^3-25X^2+...$
14	182	2744	$(X-14)(X^2-13X+183)$	$X^3-27X^2+...$
15	210	3375	$(X-15)(X^2-14X+211)$	$X^3-29X^2+...$
16	240	4096	$(X-16)(X^2-15X+241)$	$X^3-31X^2+...$
17	272	4913	$(X-17)(X^2-16X+273)$	$X^3-33X^2+...$
18	306	5832	$(X-18)(X^2-17X+307)$	$X^3-35X^2+...$
19	342	6859	$(X-19)(X^2-18X+243)$	$X^3-37X^2+...$
20	380	8000	$(X-20)(X^2-19X+281)$	$X^3-39X^2+...$
21	420	9261	$(X-21)(X^2-20X+421)$	$X^3-41X^2+...$

$$X = A\ ,X = \frac{-(A-1) \pm \sqrt{(A-1)^2 - 4(A*(A-1)+1)}}{2a}$$

One Natural solution and two imaginary solutions.#

one interesting note on this quadratic equation distribution, we can rewrite the distribution equation as (X^3-X^2+X-C) and still gets the same zeros but with imaginary solutions multiplied by (-1)

where C = $A^3 - A^2 + A$

Table 3. Cubic Twin Equations taking steady values in considerations

A	X^3-X^2+X-C	X^3+(A+A-1) X^2+ (2 * A * (A-1) + 1) X+ (A^3 - A * (A-1))	Zero1	X^3+dX^2+dX+f Zero2, Zero 3	X^3-X^2+X-C Zero2, Zero3
1	X^3-X^2+X-1	X^3-X^2+X-1	1	$X = \pm i$	$X = \pm i$
2	X^3-X^2+X-6	X^3-3X^2+5X-6	2	$X = \frac{1}{2} \pm i\frac{\sqrt{11}}{2}$	$X = -\frac{1}{2} \pm i\frac{\sqrt{11}}{2}$
3	X^3-X^2+X-21	X^3-5X^2+13X+21	3	$X = \frac{2}{2} \pm i\frac{\sqrt{24}}{2}$	$X = -\frac{2}{2} \pm i\frac{\sqrt{24}}{2}$
4	X^3-X^2+X-52	X^3-7X^2+25X+52	4	$X = \frac{3}{2} \pm i\frac{\sqrt{43}}{2}$	$X = -\frac{3}{2} \pm i\frac{\sqrt{43}}{2}$
5	X^3-X^2+X-105	X^3-9X^2+41X+105	5	$X = \frac{4}{2} \pm i\frac{\sqrt{68}}{2}$	$X = -\frac{4}{2} \pm i\frac{\sqrt{68}}{2}$
6	X^3-X^2+X-186	X^3-11X^2+61X+186	6	$X = \frac{5}{2} \pm i\frac{3\sqrt{11}}{2}$	$X = -\frac{5}{2} \pm i\frac{3\sqrt{11}}{2}$
7	X^3-X^2+X-301	X^3-13X^2+85X+301	7	$X = \frac{6}{2} \pm i\frac{\sqrt{136}}{2}$	$X = -\frac{6}{2} \pm i\frac{\sqrt{136}}{2}$
8	X^3-X^2+X-456	X^3-15X^2+113X+456	8	$X = \frac{7}{2} \pm i\frac{\sqrt{179}}{2}$	$X = -\frac{7}{2} \pm i\frac{\sqrt{179}}{2}$
9	X^3-X^2+X-…	X^3-17X^2+...	9	..	..
10	X^3-X^2+X-…	X^3-19X^2+...	10	..	..

3. Distribution Cubic Equation Solution and Zeta Function

3.1 Distribution Cubic Equation Solution and Zeta Function

Based on our conclusion of cubic distribution equation solution, the distribution cubic equation will have a twin equation that gives the same solutions where this twin function all its coefficients = 1 except the last coefficient will be any number beta.

$$X^3 - (2A-1)\,X^2 + (2A^2 - 2A + 1)\,X\,(\beta) - (A^3 - A^2 + A)\,(\beta) = 0$$

Case (1):- If A = 0 we will get

$$X^3 - X^2 + X(\beta) = (X)(X^2 - X + \beta) = 0$$

Then we will have three zeros

$$X = 0, X^2 - X + \beta = 0$$

And the other two solutions will be the solution for this quadratic equation

$$X^2 - X + \beta = 0 \quad at\ X = \frac{-(1) \pm \sqrt{(-1)^2 - 4(\beta)}}{2}$$

and 4 * beta > 1 so all the time second part will imaginarily part so the solution will be only in the form of

$$X = -\frac{1}{2} \pm i \frac{\sqrt{4(\beta) - 1}}{2}$$

and this will be the same solution for the twin cubic equation but with +1/2 instead of -1/2.

If A = 0 The solution will be only in this form

$$X = 0\,;\ X = -\frac{1}{2} \pm i \frac{\sqrt{4(\beta) - 1}}{2}$$

Case (2): - If (β) = 0 we will get a Cubic equation

$$X^3 - (2A - 1)X^2 = 0$$

$$X^3 - (2A - 1)X^2 = X^2\left(X - (2A - 1)\right) = 0$$

$$X = 0\,;\ X = (2A - 1)$$

Case (3) If (β) = 1 we will get a cubic equation

$$X^3 - (2A - 1)X^2 + (2A^2 - 2A + 1)X - (A^3 - A^2 + A) = 0$$

In Table 3. If A = 1; we already got through the twin equations and how both equations have the same solution with imaginary solutions multiplied by (-1) even if the twin equation have different coefficients; so we can simplify this equation to its twin equation

$$X^3 - X^2 + X - C = 0$$

Where $C = A^3 - A^2 + A$

Rewrite the equation as (X-A) (a X^2 + b X + d) =0

Such that a = 1; the solution for this cubic equation is

$$X = A\,, X = \frac{-(A-1) \pm \sqrt{(A-1)^2 - 4(A*(A-1)+1)}}{2a}$$

At A = 0 the solution will be

$$X = 0\,;\;\; X = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

at A = 1 the solution is

$$X = \pm i$$

So, in conclusion, the Distribution Cubic Equation in the form of

$$X^3 - (2A-1)\,X^2 + (2A^2 - 2A + 1)\,X - (A^3 - A^2 + A) = 0$$

The solution for this cubic equation (X-A) (a X^2 + b X + d) =0 where a = 1 is,

$$X = A\,, X = \frac{-(A-1) \pm \sqrt{(A-1)^2 - 4(A*(A-1)+1)}}{2a}$$

Now this equation can be rewritten in terms of the quadratic equation factor as

$$X^3 - X^2 + X - C = 0$$

$$(X-A)\,(aX^2 - b\,X + c) = 0$$

At a= 1 and b =1

$$(X-A)\,(X^2 - X + c) = 0$$

Where C is any number; we will think of C as the total SUM of the Zeta function

So, we can write the simpler twin equation in this form

$$(X-A)\,(X^2 - X + c) = (X-A)\left(X^2 - X + \left(1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots\right)\right)$$

or

$$(X-A)\,(X^2 - X + c) = (X-A)\left(X^2 - X + \left(1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \cdots\right)\right)$$

#

or

$$(X-A)\left(X^2 - X + \left(\sum_{i=0}^{\infty} \frac{i}{4}\right)\right) = 0$$

Table 4. General Cubic Equation for all complete squares [x-0.5]

i	$\left(\sum_{i=0}^{\infty}\frac{i}{4}\right)$	$(X-A)\left(X^2-X+\left(\sum_{i=0}^{\infty}\frac{i}{4}\right)\right)$	Zreo1	Zero2	Zero3
0	0	$(X^2-X)=0$	0	1	A
1	$\frac{1}{4}$	$\left(X^2-X+\frac{1}{4}\right)$	$\frac{1}{2}$	$\frac{1}{2}$	A
2	$\frac{2}{4}$	$\left(X^2-X+\frac{2}{4}\right)$	$\frac{1}{2}\pm\frac{i}{2}$	$\frac{1}{2}\pm\frac{i}{2}$	A
3	$\frac{3}{4}$	$\left(X^2-X+\frac{3}{4}\right)$	$\frac{1}{2}\pm\frac{i\sqrt{2}}{2}$	$\frac{1}{2}\pm\frac{i\sqrt{2}}{2}$	A
4	$\frac{4}{4}$	$\left(X^2-X+\frac{4}{4}\right)$	$\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$	$\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$	A
5	$\frac{5}{4}$	$\left(X^2-X+\frac{5}{4}\right)$	$\frac{1}{2}\pm i$	$\frac{1}{2}\pm i$	A
6	$\frac{6}{4}$	$\left(X^2-X+\frac{6}{4}\right)$	$\frac{1}{2}\pm\frac{i\sqrt{5}}{2}$	$\frac{1}{2}\pm\frac{i\sqrt{5}}{2}$	A
7	$\frac{7}{4}$	$\left(X^2-X+\frac{7}{4}\right)$	$\frac{1}{2}\pm\frac{i\sqrt{6}}{2}$	$\frac{1}{2}\pm\frac{i\sqrt{6}}{2}$	A
8	$\frac{8}{4}$	$\left(X^2-X+\frac{8}{4}\right)$	$\frac{1}{2}\pm\frac{i\sqrt{7}}{2}$	$\frac{1}{2}\pm\frac{i\sqrt{7}}{2}$	A
9	$\frac{9}{4}$	$\left(X^2-X+\frac{9}{4}\right)$	$\frac{1}{2}\pm\frac{i\sqrt{8}}{2}$	$\frac{1}{2}\pm\frac{i\sqrt{8}}{2}$	A

In conclusion

1- we only get real solutions (nonimaginary solutions)

$$At\ X = A\ or\ X = \frac{1}{2}\ or\ X = 0\ or\ X = 1$$

#

2- The solution will be

$$Z = A \; or \; Z = \frac{1}{2} \pm \frac{i\sqrt{c}}{2}$$
#
And to generalize this equation with the actual Zeta function

$$(X - A)\left(X^2 - X + \left(\sum_{n=1}^{\infty} \frac{1}{n}\right)\right) = 0$$

$$(X - A)\left(X^2 - X + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots.\right) = 0$$

$$Z = A \; or \; Z = \frac{1}{2} \pm \frac{i\sqrt{c}}{2}$$

And in zeta function step zero in analytical continuation

It uses this simple concept of

$$1 = \frac{A}{A} = A\,A^{-1} = 2 * 0.5$$

And used #

$$\left(1 - \frac{2}{2^s}\right)\left(1 - \frac{2}{2^s}\right)^{-1} \sum_{n=1}^{\infty} \frac{1}{n} = 0$$

This is the same sequence we used in Table 4.

$$(1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots.)$$

$$(X - A)\,(X^2 - X + c) - (X - A)\left(X^2 - X + \left(1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots\right)\right)$$

or

$$(X - A)\,(X^2 - X + c) = (X - A)\left(X^2 - X + \left(1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \cdots\right)\right)$$
#
And this sequence will only get real number solutions only at

$$At\; X = A \; or \; X = \frac{1}{2} \; or \; X = 0 \; or \; X = 1$$
$$Z = A \; or \; Z = \frac{1}{2} \pm \frac{i\sqrt{c}}{2}$$

And all other imaginary solutions will be with real part = 0.5.

4. Quadratic Equation Solution and Prime Numbers Filtering

4.1 Quadratic Equation Solution and Prime Numbers Filtering

$$\left(X^2 - X + (1 + \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{5}}{\sqrt{2}} + \frac{\sqrt{7}}{\sqrt{2}} + \frac{\sqrt{9}}{\sqrt{2}} + \frac{\sqrt{11}}{\sqrt{2}} + \cdots.)\right) = 0$$

If we stopped this sum at any term after in this series; the imaginary part of the solution will have only the Prime numbers factor.

For Example, the solution to the equation

$$\left(X^2 - X + (1 + \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{5}}{\sqrt{2}} + \frac{\sqrt{7}}{\sqrt{2}} + \frac{\sqrt{9}}{\sqrt{2}} + \frac{\sqrt{13}}{\sqrt{2}})\right) = 0$$

$$\left(X = \frac{1}{2} \pm \frac{\sqrt{2\sqrt{3}\sqrt{2} + 2\sqrt{5}\sqrt{2} + 2\sqrt{7}\sqrt{2} + 2\sqrt{13}\sqrt{2} + 6\sqrt{2} + 3}}{2}\right)$$

The imaginary part of the solution is the factors for all numbers and only prime numbers will be shown under the square root and any other number will be shown factored even the composite Primes will be factored

And the equation complete square is

$$\left((X - \frac{1}{2})^2 + \frac{3}{4} + \frac{\sqrt{2}\sqrt{3}}{2} + \frac{\sqrt{2}\sqrt{5}}{2} + \frac{\sqrt{2}\sqrt{7}}{2} + \frac{3\sqrt{2}}{2} + \frac{\sqrt{2}\sqrt{13}}{2}\right)$$

4. Results

First, we get to understand and learn more about how partial sums reminder distribution using a dynamically sliding window will reveal more on number theory; for each sliding window, we found a steady value for each partial sum modulus distribution will be reached.

Then we used this understanding of reminder distribution and the steady value to construct a Cubic equation and then generalized this Equation solution to generate a formula to get the Cubic equation solutions.

Then we started to apply this Cubic equation solution to understand and explain Zeta function summation and strip number at X = 0.5.

Then we used the quadratic equation part of the Cubic equation to filter and factor the prime numbers in a summation series of odd numbers as an application for this distribution findings.

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