Data Envelopment Analysis and Bootstrap Approaches for Efficiency Measure of the Autonomous Port of Dakar

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Abstract

In this article, we measure the efficiency of the Autonomous Port of Dakar (APD) and identify the causes of inefficiency for the year 2021. Measuring port efficiency is an important factor in strengthening its competitiveness and stimulating national development. In the literature, the most widely used methods for measuring efficiency in the port sector are parametric and non-parametric methods. Our objective is to apply Data Envelopment Analysis (DEA) with two models, namely: the Charnes, Cooper and Rhodes (CCR) and Banker, Charnes and Cooper (BCC) to determine the efficiency scores and the bootstrap approach the Simar and Wilson in 2007 to correct the errors and determine the confidence intervals. Numerical simulations are performed in the two distinct zones separated by a Fishing Port (FP), Naval Repair Workshops (NRW) and the Military Zone (MZ), while the others zones are the Decision-Making Units (DMUs), detailed in table 2. The results show that APD obtains six (6) effective DMUs with CCR model (average score of 0.858) and ten (10) effective DMUs with BCC model (average score of 0.951). The average scale efficiency is 0.897. With the bootstrap approach, we obtain an average bias-corrected of 0.700 for CCR model (with confidence intervals of [0.324; 0.1291]) and 0.870 for BCC model (with confidence intervals of [0.620; 1.197]). These results will allow the decision makers of the Dakar port authority to improve its performance and competitiveness at the national and international levels.

Keywords: data envelopment analysis, optimization, efficiency, port, bootstrapping

1. Introduction

Ports play a major role in the competitiveness of international supply chains and hence in the competitiveness of countries and regions. As well, global trade, supply chains, production processes, and economic integration depend heavily on the existence of efficient port systems and associated logistics. According to the United Nations Conference on Trade and Development (UNCTD) held in Geneva in 2018, more than 80% of the volume of goods traded in the world is transported by sea and that shipping and ports are an integral part of any door-to-door transport solution (Ripoll, 1973). Based on these assumptions, we can say that it is therefore important to monitor and measure the operational, financial, economic, social and environmental efficiency of ports. This will be an important factor in promoting the competitiveness of ports and stimulating global development. The APD is based on the six (6) major ports in West Africa, and it is also based on the twenty-five (25) major ports in Africa. When we look at the world ranking of ports, we see that ports are far from the ranking relative to the rest of the world. We can say that in Africa we have a lot of challenges in the port sector.

Efficiency measurement methods can be classified into two categories according to the one-dimensional and multidimensional approaches (Farah, 2018). The technique used by the one-dimensional approach to assess efficiency is the partial ratios, whereas for the multidimensional approach, the techniques used are the methods of the frontier and the methods of the mean. Speaking of the frontier method, we have parametric methods and non-parametric methods, all developed by Farrell in 1957. In the literature, the most widely used methods for measuring efficiency in the port sector are parametric and non-parametric methods (Carine, 2015; Nguyen, Nguyen, Chang, Chin & Tongzon, 2016; Simões & Marques, 2010; Tovar & Wall, 2019) we we all used the frontier method to assess efficiency in the port sector.

The DEA method was introduced by Charnes et al. to measure the efficiency of a U.S federal program of resource allocation to schools (Follow Through Program). The use of the DEA method was then generalized in public and private structures (Health System, Human Resources, Unemployment Offices, Power Plants, Police Units, Waste treatment Plants, logistic etc.). The DEA method is a tool for analysis and decision support in the areas mentioned above and allows:
• calculate an efficiency score for each DMU, indicating whether an organization has a margin for improvement;
• set target values for each DMU, indicating how much the inputs need to be reduced and the outputs increased for a DMU to become efficient;
• identify reference peers for each DMU, indicating which organizations have the best practices to analyze.

The question of improving the efficiency and competitiveness of the APD (infrastructure, logistics, etc.) is a hard subject which has been concerning the local authorities for several years, and on which a research work was launched in the last decade. The challenges are: (i) port congestion, which stifles the efficiency of various zone area (ii) modernization of infrastructure to make of the APD among one of in the sub-region, in Africa and even in the world hubs of reference; (iii) ship movements. Our research addresses these challenges, focusing on the three economic, social and environmental aspects to measure the efficiency of the APD in two stages. First, we use the two main DEA models (CCR and BCC) proposed by Charnes et al. and Banker et al. respectively. Second, we apply the bootstrap method proposed by Simar and Wilson to correct the bias and determine the confidence intervals.

This article is organized as follows: in section 2 we present a review of the literature on port efficiency. In section 3 we briefly describe the DEA method and bootstrap approach used in this article. In section 4 we introduce the DMUs, the input and output variables, and dataset. In section 5 we provide numerical results for both the proposed DEA/bootstrap formulations applied to APD and comment on the results. Finally, in section 6 we provide conclusions and future works.

2. Literature Review

In this section, we give a general overview of the literature review of the DEA method, DEA bootstrap approach and Stochastic Frontier Analysis (SFA) method, before focusing on our case study: the application of the DEA method and the bootstrap approach in the port sector. The literature review on the application of the DEA, SFA and DEA bootstrap methods in the port sector is summarized in the table 1.

The non-parametric efficiency measuring method is due to Farrell by only considering one variable (input and output). Charnes et al. took up this method while considering several variables (input and output) at the same time, also by using the technique of linear programming, hence the exponential progress of their method, called DEA. The DEA bootstrap approach was introduced by Simar and Wilson, and today several authors have used this approach to evaluate efficiency in different sectors.

The literature on the efficiency of the port industry is relatively new (the first studies appeared in the mid-1990s). The DEA method has been applied for the first time in the port sector by Roll and Hayuth and assumes that the convexity hypothesis is verified. In the case where this hypothesis is not verified, Tulkens and Deprins developed a model called Free Disposal Hull (FDH).

Emrouznejad and Yang identified over more than 900 articles related to the application of the method in different sectors and in different countries around the world. According to their study, the most common areas of application of the DEA method are: agriculture, banking, supply chain, transportation sector. Georgiadis et al. have analyzed the performance of 34 multimodal public transport networks worldwide to investigate whether the service characteristics of their metro components significantly affect bus performance and vice versa as well as whether their operational environment exerts the same impact on metro and bus public transport modes. They used the DEA bootstrap approach to determine efficiency scores and correct errors as a first step. In the second step, they used robust condition efficiency estimators of order-m to identify the factors that could explain these performance rankings.

Novickytė and Droždz used the DEA method to examine the performance of Lithuania Banks. On average, they achieved an efficiency score of 86% for the VRS model and 60% for the CRS model. As mentioned in section 1, the DEA method and its components are used in several areas. For example, in education, Mahmudah and Lola applied the Fuzzy Data Envelopment Analysis (FDEA) method to measure the performance of the 25 Indonesian Universities. In the health care system, Hamidi has used the SFA technique to measure technical efficiency of governmental hospitals in Palestine. Numerous authors had applied the DEA method to assess efficiency in the port sector in different countries of the world (Al-Eraqi, Mustafa, Khader & Barros, 2008; Ashraf Malabika Deo, 2014; Trujillo & Tovar, 2007; Wanke & Barros, 2016). For more details on the application of the DEA method see Al-Eraqi et al.. In table 1, column 1 concerns the authors of the article and the country where the study was conducted. Column 2 concerns the DMUs and the methods used. In columns 3 and 4 are the input and output variables. Finally, column 5 is dedicated to a short summary relating to the study.
In this section, we briefly describe the DEA method and the bootstrap approach adapted to our study.

### 3. Methodology

In this section, we introduce DEA mathematical both models (CCR and BCC), the scale efficiency (SE), the returns to scale (RTS) and the bootstrap approach. The DEA method is one of the most widely used methods to evaluate efficiency a
unit of a multiple-inputs and multiple-outputs simultaneously. The bootstrap approach allows us to correct measurement errors and determine the confidence intervals. The motivation of using the DEA method is that it is non-parametric and deterministic. In addition, it takes into account several inputs and outputs. The disadvantage is that it does not take into account hazards. To overcome this last point, we used the bootstrap approach to correct errors and determine the confidence intervals. The framework of the methodology is composed of four (4) main phases. Firstly, we present the CCR mathematical model in 3.1, which assumes constant returns to scale (CRS). Through this model, we determine the CCR efficiency scores. Secondly, we present the BCC mathematical model in 3.2, which assumes variable returns to scale (VRS). Through this model, we determine the BCC efficiency scores. Thirdly, present the SE and RTS in 3.3. Finally, we develop the bootstrap approach in 3.4. Note that we have presented five (5) mathematical problems, both models (CCR and BCC) which are: ($P_{f_{-CCR}}$): Fractional problem of the CCR model; ($P_{l_{-CCR}}$): Linear problem of the CCR model; ($P_{d_{-CCR}}$): Dual problem of the CCR model; ($P_{l_{-BCC}}$): Linear problem of the BCC model and ($P_{d_{-BCC}}$): Dual problem of the BCC model.

3.1 DEA-CCR Model

So et al. defined the DEA approach as a linear programming, based on a deterministic and non-parametric method, by evaluating the relative efficiency of a decision-making unit to transform inputs into outputs. This tool makes it possible to empirically determine the production frontier, without first having to define the form of this function. Charnes et al. developed a model assuming constant returns to scale (CRS). The CCR model is appropriate when all DMUs operate at their optimum size. The notion of efficiency measure is defined by Charnes et al., as being the maximum value of the ratio (weighted outputs by weighted inputs), under the constraints that the similar ratio for each DMU are greater than or equal to the unit. Taking the reverse, i.e. the minimum of the ratio (weighted inputs by weighted outputs), the mathematical form of the CCR model for the selected entity $k$ using $m$ inputs to produce $s$ outputs is given by the ($P_{f_{-CCR}}$). Note that in this article, we have three (3) input variables, two (2) output variables and eighteen (18) DMUs. In other words, $m = 3, s = 2$ and $n = 18$. For more information on the data, see section 4. The relation (1) is an objective function that minimises quantity of inputs $m$ to be used to produce a given quantity of $s$ for DMU $k$; The relation (2) is the constraint that states the weighted sum of inputs in relation to the weighted sum of outputs must be greater than or equal to unity. The constraints (3) and (7) states that the input and output variables must be strictly positive.

\[
\begin{align*}
\text{Minimize} & \quad \sum_{i=1}^{m} \frac{v_i x_{ik}}{\sum_{r=1}^{s} u_r y_{rk}} \\
(P_{f_{-CCR}}) & \quad \text{subject to:} \\
& \quad \sum_{i=1}^{m} \frac{v_i x_{ij}}{\sum_{r=1}^{s} u_r y_{rj}} \geq 1, \quad j = 1, \ldots, n; \\
& \quad u_r, v_i > \epsilon > 0, \quad \forall r = 1, \ldots, s; i = 1, \ldots, m. 
\end{align*}
\]

Where: $x_{ik}$ is the quantity of input $i$ consumed by the DMU of $k$; $y_{rk}$ is the quantity of output $r$ produced by the DMU of $k$. $s, m,$ and $n$ are the number of outputs, the number of inputs and number of DMUs respectively and $\epsilon$ is a non-Archimedean element (small positive value). ($P_{f_{-CCR}}$) is non-linear, fractional and admits an infinite number of solutions. To make it linear, we use the transformations of Charnes and Cooper, this leads us to ($P_{l_{-CCR}}$). The ($P_{l_{-CCR}}$) is the multiplier form of the CCR mathematical model. The relation (4) is an objective function. The inequation (5) is the constraint of the variables (input and output), the relation (6) is the normalising constraint.

\[
\begin{align*}
\text{Minimize} & \quad \sum_{i=1}^{m} v_i x_{ik} \\
(P_{l_{-CCR}}) & \quad \text{subject to:} \\
& \quad \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0, \quad j = 1, \ldots, n; \\
& \quad \sum_{r=1}^{s} \mu_r y_{rk} = 1, \quad r = 1, \ldots, s; \\
& \quad \mu_r, v_i > \epsilon > 0, \quad r = 1, \ldots, s; \quad i = 1, \ldots, m.
\end{align*}
\]
The orientation of the DEA model must be chosen according to the variables (inputs, outputs) and according to which decision-makers exercise the greatest management power. According to Pallis et al., port efficiency is a multi-dimensional concept that refers to operational performance, particularly the maximization of the produced output or the production of a given output with limited possible resources. Given that the objective of port decision-makers is to maximize production with the limited resources available on the one hand. On the other hand, given the assertion of Pallis et al., we use output orientation DEA, which maximize outputs for a given level of input. Using the techniques of linear programming in the (P_{l-CCR}) in other words, the notions of duality, we obtain the (P_{d-CCR}), which is the envelope form of the CCR mathematical model. We use the (P_{d-CCR}) for the numerical simulations, because it possesses m + s constraints, whereas the (P_{l-CCR}) possesses n + 1 constraints. The relation (8) is an fonction objective.

\[
(P_{d-CCR}) : \begin{align*}
\text{Maximize} & \quad \theta_k \\
\text{subject to} & \quad \sum_{j=1}^{n} A_j y_{rj} - \theta_k y_{rk} \leq 0 \quad r = 1, \ldots, s; \\
& \quad \sum_{j=1}^{n} A_j x_{ij} - x_{ik} \leq 0 \quad i = 1, \ldots, m; \\
& \quad \lambda_j \geq 0 \quad j = 1, \ldots, n.
\end{align*}
\]

where: \( \frac{1}{\theta_k} \) is the technical efficiency for DMU k. If \( \frac{1}{\theta_k} = 1 \), the observed DMU is on the boundary, that is, it is efficient in the sense of Farrell, otherwise if \( 0 < \frac{1}{\theta_k} \leq 1 \), this shows the existence of technical inefficiency. The inequations (9) and (10) are, respectively, the constraints of the outputs and the inputs. The inequation (11), where \( \lambda_j = (\lambda_1, \lambda_2, \ldots, \lambda_n) \) is a n-vector of constants represent the constraint multipliers of CCR model. In other words, the weights associated with the outputs and inputs of DMU j.

3.2 DEA-BCC Model

The assumption of the CRS is only really appropriate if all DMUs operates on an optimal scale. This is not always the case (imperfect competition, financial constraints, etc.). Banker et al. have proposed a model that can be used to determine if production is in an area of constant, increasing or decreasing returns. The (P_{l-BCC}) mathematical model from Banker et al. is obtained by adding a free variable vo to the (P_{l-CCR}). The (P_{d-BCC}) mathematical model from Banker et al. is obtained by adding a the convexity constraint \( \sum_{j=1}^{n} A_j = 1 \) to the (P_{d-CCR}). We recall that we use the (P_{d-BCC}) for numerical simulations, because it has fewer constraints than the (P_{l-CCR}). In addition, Coelli et al. proposed another model, called the non-increasing returns to scale model (Non-Increasing Returns to Scale model-NIRS) to identify the nature of scale efficiency. This model is obtained by replacing the convexity constraint \( \sum_{j=1}^{n} A_j = 1 \) in the BCC model by the constraint \( \sum_{j=1}^{n} A_j \leq 1 \).

3.3 Scale Efficiency (SE) and Returns To Scale (RTS)

According to Färe et al., scale efficiency is defined as the ratio of the CRS efficiency score to the VRS efficiency score. In other words, it is the ratio of the CCR efficiency score to the BCC efficiency score. Mathematically, this is expressed by the problem (12), where \( 0 < SE \leq 1 \).

\[
SE = \frac{\theta_{CCR}}{\theta_{BCC}}
\]

We have a scale efficiency, if \( SE = 1 \), if \( 0 < SE < 1 \), we have a scale inefficiency. The RTS are considered to be increasing if a proportionate increase in all the inputs results in a more than proportionate increase in the single output (Banker & Thrall, 1992). The RTS are increasing, if a proportional increase in all inputs leads to a more than proportional increase in outputs. The RTS are decreasing, if a proportional increase in all inputs results in a less than proportional increase in inputs. We use the approach in Coelli et al. to determine the nature of the returns to scale. For more details in relation to DEA mathematical models, see Cooper, Seiford & Zhu, 2011; Charnes, Cooper & Rhodes, 1978; Banker, Charnes & Cooper, 1984.

3.4 DEA-bootstrap Approach

Any efficiency measure will depend on the sample selected, the time period chosen, and the data used. Taking these factors into account, there may be erroneous choices. The DEA method also sometimes leads to erroneous results, for
example, a DMU may be on the efficiency frontier due to measurement error. To account for this, here we use the bootstrap approach to correct the errors and determine the confidence intervals. The bootstrap can be either parametric or non-parametric see Jal, 2003. We are interested in the non-parametric case because the DEA method is a non-parametric and deterministic method. Simar and Wilson proposed a bootstrap strategy for analyzing the sensitivity of the efficiency measures to sampling variation, providing confidence intervals and corrections for the bias inherent in the DEA procedure (Simar & Wilson, 2007). The principle of the bootstrap method is as follows: first, build a number \( B \) of samples of size \( n \), coming from the initial sample. In our case \( B \) is the efficiency scores obtained by the DEA method (CCR and BCC). Second, take a series of simple random samples with submission of \( n \) observations in the initial sample. Let \( \hat{\theta}_1^*, \hat{\theta}_2^*, \ldots, \hat{\theta}_B^* \), where \( b = 1, 2, \ldots, B \) the sample taken from the initial sample. Efron and Tibshirani suggests to take \( B \) equal to at least 200 in order to get a decent estimate, in our study we take \( B = 2000 \).

The bootstrap estimation is given by the problem (13).

\[
\hat{\theta}_{\text{boot}} = \frac{1}{B} \sum_{b=1}^{B} \hat{\theta}_{(b)} 
\]

(13)

The problem (14) is the variance of the bootstrap estimate.

\[
\text{Var}_2^{\text{boot}}(\hat{\theta}) = \frac{1}{B-1} \sum_{b=1}^{B} (\hat{\theta}_{(b)} - \hat{\theta}_{\text{boot}})^2 
\]

(14)

Problems (15) and (16) concern the bias and the bias corrected.

\[
\text{Bias}_{\text{boot}}(\hat{\theta}) = \hat{\theta}_{\text{boot}} - \hat{\theta} 
\]

(15)

\[
\hat{\theta}^* = \hat{\theta} - \text{Bias}_{\text{boot}}(\hat{\theta}) = 2\hat{\theta} - \hat{\theta}_{\text{boot}} 
\]

(16)

Finally, the problem (17) concerns the confidence intervals.

\[
\Pr(\hat{z}_{\frac{1}{2}} < \hat{\theta}_b^* - \hat{\theta} < \hat{z}_{1-\frac{1}{2}}|\hat{\theta} = \hat{T}) = 1 - \alpha 
\]

(17)

where \( \hat{T} \) is the estimated DEA technology \( \hat{z}_{\frac{1}{2}} \) and \( \hat{z}_{1-\frac{1}{2}} \) the estimated upper and lower quantiles, respectively. For more information on bootstrapping, see Barros & Managi, 2008; Nguyen, Nguyen, Chang, Chin & Tongzon, 2016.

4. Case Study

In this section, the DMUs is described in subsection 4.1, while the data description and decision variables are reported in subsection 4.2. The Table 2 shows the DMUs and table 3 the statistical description and the Spearman correlation matrix of variables (inputs and output).

4.1 Decision Making Units (DMUs)

The APD in Senegal (west Africa) is the first deep water port for northern shipments and the last transit port to come up from the south. It covers 10km and has 40 posts for ships, 11 meters maximum, 80900\( m^2 \) of unmarked land for short-term storage, 170600\( m^2 \) of gross area (container yard) and 60597\( m^2 \) of covered area. The APD has terrestrial infrastructures spread over two distinct zones separated by a fishing port (FP), naval repair workshops (NRW) and the military zone (MZ). The south zone is composed of three moles (M1, M2, and M3). The north zone is composed of four moles (M4, M5, M6, and M8), Container Terminals (CT1, CT2, CT3, CT4, and CT5), Container Terminal Extension (CTE) and petroleum wharf (PW). We consider the different parts of each area listed above as DMUs. We have a total of 18 DMUs with: 13 in the north zone, 3 in the south zone, 1 in the fishing port and 1 in military zone. The container terminals are managed by the Dubai Ports Word (DPW) group in Dakar, the Ro-Ro Terminal (RRT) by the Bollor group and the bulk terminal (BT) is managed by the Necotrans group. The first column in table 2 concerns the number of DMUs and the second the name of the DMUs. The columns, three and four, are the area where the DMUs are located and the handling companies, respectively. The figure 1 illustrate the some DMUs of the different areas of APD (DMUs 1, 6, 7, 9, 14, 15, 17 and 18) of APD. For more information on the APD, see Dakar, 2021; Sureté, 2008.
Table 2. Descriptive of Decision Making Units (DMUs)

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Name of DMUs</th>
<th>Zone</th>
<th>Material handling company</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Container Terminal (CT1)</td>
<td>North</td>
<td>DPW</td>
</tr>
<tr>
<td>2</td>
<td>Container Terminal 2 (CT2)</td>
<td>North</td>
<td>DPW</td>
</tr>
<tr>
<td>3</td>
<td>Container Terminal 3 (CT3)</td>
<td>North</td>
<td>DPW</td>
</tr>
<tr>
<td>4</td>
<td>Container Terminal 4 (CT4)</td>
<td>North</td>
<td>DPW</td>
</tr>
<tr>
<td>5</td>
<td>Container Terminal 5 (CT5)</td>
<td>North</td>
<td>DPW</td>
</tr>
<tr>
<td>6</td>
<td>Ro-Ro Terminal (RRT)</td>
<td>North</td>
<td>Necotrans</td>
</tr>
<tr>
<td>7</td>
<td>Bulk Terminal (BT)</td>
<td>North</td>
<td>Bollor</td>
</tr>
<tr>
<td>8</td>
<td>GCO Terminal (GCOT)</td>
<td>North</td>
<td>Mixte</td>
</tr>
<tr>
<td>9</td>
<td>Mole 4 (M4)</td>
<td>North</td>
<td>Mixte</td>
</tr>
<tr>
<td>10</td>
<td>Mole 5 (M5)</td>
<td>North</td>
<td>Mixte</td>
</tr>
<tr>
<td>11</td>
<td>Mole 6 (M6)</td>
<td>North</td>
<td>Mixte</td>
</tr>
<tr>
<td>12</td>
<td>Mole 8 (M8)</td>
<td>North</td>
<td>Mixte</td>
</tr>
<tr>
<td>13</td>
<td>Military Zone (MZ)</td>
<td>–</td>
<td>Mixte</td>
</tr>
<tr>
<td>14</td>
<td>Petroleum Wharf (PW)</td>
<td>–</td>
<td>Mixte</td>
</tr>
<tr>
<td>15</td>
<td>Fishing Port (FP)</td>
<td>–</td>
<td>Mixte</td>
</tr>
<tr>
<td>16</td>
<td>Mole 1 (M1)</td>
<td>South</td>
<td>Mixte</td>
</tr>
<tr>
<td>17</td>
<td>Mole 2 (M2)</td>
<td>South</td>
<td>Mixte</td>
</tr>
<tr>
<td>18</td>
<td>Mole 3 (M3)</td>
<td>South</td>
<td>Mixte</td>
</tr>
</tbody>
</table>

4.2 Data Description and Variables (Input, Output)

The data were collected through reports from the National Agency for Statistics and Demography (NASD) and the Dakar port authority. The choice of input and output variables, is a key element of DEA method. The DEA method does not provide criteria for defining input and output variables, it’s up to the decision makers. The choice is made according to the decision-makers and the objectives of the study. According to Cooper et al., the number of DMU must be greater than or equal to the maximum between the number of inputs and outputs or three times the number of inputs plus the number of outputs.

\[ N \geq \max\{s \times m, \ 3(s + m)\} \]  

Figure 1. Some DMUs of the different areas of APD

where \( N \) is a number of DMUs, \( s \) is a number of outputs and \( m \) is a number of inputs. However, several authors have done studies on the choice of input and output variables (Nataraja & Johnson, 2011; and references inside). Most of these studies are based on statistical tests. In this article, we have chosen our input and output variables based on their availability and the objectives. In addition, the total number of berths, terminal area and number of handling equipment in a port are very appropriate, and can be considered as input variables. The number of tonnages treated per year, the annual flux of containers, the number of ships treated per year, can be considered as output variables. For more details regarding the choice of variables input and output in the general case, see Coelli et al.. Based on previous studies relating to the measurement of efficiency in the port sector (Cullinane, Song & Wang, 2005; Cullinane, Wang, Song & Ji, 2006), we use, in this article, three input variables and two output variables.

The input variables are: the length of the quay (\( m \)), the total area of land (ha) and the total number of handling equipment (tugs, cranes, forklifts, etc.). Where: the length of the quay captures the nautical capacities of the port and makes it
possible to integrate the size and number of vessels that can be received simultaneously by the port. The output variables are: the number of tonnages and the number of ships treated per year; with the number of tonnages handled per year measures the amount of traffic of containerized goods. It is expressed in thousands of tonnes and is composed of the tonnages handled for import and export per year. In the following, we denote by \( X_1, X_2 \) and \( X_3 \) the input variables, \( Y_1 \) and \( Y_2 \) the output variables, with:

- \( X_1 := \) the length of the quay (m);
- \( X_2 := \) the total area of land (ha);
- \( X_3 := \) the total number of machines (tugs, cranes, forklifts, etc.);
- \( Y_1 := \) the number of tonnages (metric tons);
- \( Y_2 := \) the number of ships (number).

In table 3, we have the statistical description and the matrix of Spearman correlation coefficients of our input and output variables. Given the inputs and outputs are not strongly correlated our choices of variables are valid according to the criteria of selection of variables in the literature.

Table 3. Descriptive statistics and Spearman matrix the correlation of variables on the period 2021

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td>350</td>
<td>1000</td>
<td>647.222</td>
<td>224.520</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>4.5</td>
<td>13.8</td>
<td>10.011</td>
<td>3.020</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>12</td>
<td>54</td>
<td>34.666</td>
<td>13.758</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_1 )</td>
<td>650000</td>
<td>1000000</td>
<td>836611.111</td>
<td>99524.244</td>
</tr>
<tr>
<td>( Y_2 )</td>
<td>200</td>
<td>800</td>
<td>478.611</td>
<td>164.261</td>
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Spearman matrix correlation

<table>
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<tr>
<th></th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( X_3 )</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
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<tr>
<td>( X_2 )</td>
<td>0.448</td>
<td>1.000</td>
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<tr>
<td>( X_3 )</td>
<td>0.044</td>
<td>0.446</td>
<td>1.000</td>
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<tr>
<td>( Y_1 )</td>
<td>-0.007</td>
<td>-0.192</td>
<td>0.062</td>
<td>1.000</td>
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</tr>
<tr>
<td>( Y_2 )</td>
<td>0.552*</td>
<td>0.448</td>
<td>0.300</td>
<td>-0.083</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Source: authors
* Significant correlations at a 0.05 level

5. Numerical Simulations

We use DEAP software version 2.1, developed by Coelli for the DEA method and the rDEA version 1.2-5 package, for the bootstrap method. Here, the condition (18) is verified. In sub-sections 5.1 and 5.2, the results of the CCR and BCC models are presented and discussed. In the sub-section 5.3, we find interpretation of the efficiency of scale and the nature of returns to scale, where: SE: is the scale efficiency, - : is the constant returns to scale, irs: the increasing returns to scale and drs: is the decreasing returns to scale. In sub-section 5.4, the results related to the bootstrap DEA approach are presented and discussed. The figure 2 illustrates the efficiency scores and bias-corrected of both models (CCR and BCC), and figure 3 shows the confidence intervals and bias-corrected of both models (CCR and BCC).

5.1 Results of DEA-CCR

We recall that a DMUs said to be efficient if it solves problem (8), and \( \theta \) obtains an efficiency score, which is equal to 1 (\( \theta = 1 \)). In table 4, line 2 illustrates the results of the CCR model, with six (6) DMUs (2, 4, 5, 8, 9 and 10) efficient. The CCR model assumes that all DMUs run in a situation of constant returns to scale (CRS). This happens, when all DMUs reach their optimal size. On average, the port has an efficiency score of 0.858. For the port to become 100 percent efficient with this CCR model, production will have to be increased by 14.2 percent. When we take the fishing port (FP), DMU 15, in table 4, line 2 of column 16 it got an efficiency score of 0.670. To improve its performance and become 100 percent efficient, it will have to increase its efficiency score by 0.33. To be able to increase this score, we propose a solution...
through the projected values obtained during the numerical simulations. To obtain this 33 percent, it will be necessary to increase production by 33.046 percent for $Y_1$ and 33.046 for $Y_2$. The projected values obtained are 1269539.646 and 821.467, respectively. The improvement margins are obtained by the following calculations: $[((1269539.646 - 850000)/1269539.646) \times 100]$ for $Y_1$ (where 850000 is the original value) and $[((821.467 - 550)/821.467) \times 100]$ for $Y_2$ (where 550 is the original value). Moreover, the DEA method identifies for each inefficient DMU, which comes closest to its production function. Under the CCR model, in table 4, column 3, the DMUs (2, 4, 5, 8, 9 and 10) are located on the efficiency frontier. Therefore, they are identified as the reference pairs. If inefficient DMU want to improve their efficiency, they need to analyze the best practices developed by their respective peers. For example, the FP (DMU 15) has a peer the CT4, GCOT and M4 (DMUs 4, 8 and 9). To be efficient, it must refer to these reference peers (CT4, GCOT and M4).

5.2 Results of DEA-BCC

In table 4, line 4, we have a results BCC model, with nine (9) DMUs (1, 2, 3, 4, 5, 8, 9, 10 and 13) efficient. The BCC model assumes that the DMUs, evaluate in a situation of variable returns to scale (VRS). This happens, when not all DMUs operate at their optimal size. The efficiency scores of the BCC model are always higher or equal to the scores of the CCR model ($\theta_{BCC} \geq \theta_{CCR}$). The port obtained an average efficiency score of 0.951, this average verifies the hypothesis ($\theta_{BCC} \geq \theta_{CCR}$), because with the CCR model we obtained an average efficiency score of 0.858. To be efficient, on average, production must be increased by 4.9 percent.

Ro-Ro Terminal (RRT) has an efficiency score of 0.878. It must increase its production by 12.2 percent with an improve margin of 12.154 percent on $Y_1$ and 17.526 percent on $Y_2$. The improvement margins of 12.154 percent and 17.526 percent are obtained from the projected values provided by the DEA method. The projected values obtained are 967600 and 485, respectively. The improvement margins are obtained by the following calculations: $[((967600 - 850000)/967600) \times 100]$ for $Y_1$ where 850000 is the original value and $[((485 - 400)/485) \times 100]$ for $Y_2$ where 400 is the original value. In table 4, column 5, the DMUs (1, 2, 3, 4, 5, 8, 9, 10 and 13) are located on the efficiency frontier and are identified as reference pairs. RRT (DMU 6), has two reference pairs, the CT1 and M4 (DMUs 2 and 9). To improve its performance in order to be efficient, it must analyze these benchmarks. Figure 2, asserting the theory that the efficiency scores of the BCC model from Coelli et al. The points at which the CCR and BCC efficiency scores are equal, coincide. We have a coincidence between the DMUs (CT2, CT4, GCOT, M4, M5 and M8), whose CCR and BCC lines coincide in one line. It was found that all the corrected efficiency scores of the CCR model are always less than or equal to the efficiency scores of the BCC model from Coelli et al.. The points at which the CCR and BCC efficiency scores are equal, coincide. We have a coincidence between the DMUs (CT2, CT4, GCOT, M4, M5 and M8), whose CCR and BCC lines coincide in one line. It was found that all the corrected efficiency scores of the CCR model are always lower than or equal to the corrected efficiency scores of the BCC model. In table 4, $\theta_{CCR}$: is the original DEA score for model CCR, $\theta_{BCC}$ the original score DEA for model BCC, SE the scale efficiency and RTS the returns to scale. Figures 2 and 3 are simulated by using Python version 3.8.10 (Python, 2021).

### Table 4. Results of DEA (CCR, BCC), Benchmarks (CCR, BCC), scale efficiency and RTS

<table>
<thead>
<tr>
<th>DMUs</th>
<th>CCR</th>
<th>BCC</th>
<th>SE</th>
<th>RTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.840</td>
<td>1.000</td>
<td>0.840</td>
<td>drs</td>
</tr>
<tr>
<td>2</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.873</td>
<td>1.000</td>
<td>0.873</td>
<td>irs</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.873</td>
<td>0.878</td>
<td>2, 9</td>
<td>0.993</td>
</tr>
<tr>
<td>7</td>
<td>0.972</td>
<td>1.000</td>
<td>0.972</td>
<td>irs</td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>0.445</td>
<td>0.674</td>
<td>9, 1</td>
<td>0.661</td>
</tr>
<tr>
<td>12</td>
<td>0.864</td>
<td>0.882</td>
<td>8, 9</td>
<td>0.880</td>
</tr>
<tr>
<td>13</td>
<td>0.832</td>
<td>1.000</td>
<td>0.832</td>
<td>drs</td>
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<tr>
<td>14</td>
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<td>0.787</td>
<td>9, 1</td>
<td>0.810</td>
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<tr>
<td>15</td>
<td>0.670</td>
<td>0.889</td>
<td>1, 9</td>
<td>0.753</td>
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<tr>
<td>16</td>
<td>0.731</td>
<td>0.954</td>
<td>9, 13</td>
<td>0.766</td>
</tr>
<tr>
<td>17</td>
<td>0.809</td>
<td>0.949</td>
<td>13, 9</td>
<td>0.852</td>
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<tr>
<td>18</td>
<td>0.901</td>
<td>0.997</td>
<td>1, 2</td>
<td>0.904</td>
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</tbody>
</table>

Mean 0.858 0.951 0.897
5.3 Results of Scale Efficiency (SE) and Returns to Scale (RTS)

In table 4, columns 4 and 5, are respectively the scale efficiency and the nature of the returns to scale. In total, we obtained ten (10) DMUs (1, 6, 11, 12, 13, 14, 15, 16, 17, 18) evolving in a situation of decreasing returns to scale (drs), two (2) DMUs (3, 7) evaluating in a situation of increasing returns to scale and six (6) DMUs (2, 4, 8, 9, 10) evolving in a situation of constant returns to scale (–). The DEA method allows us to identify the two sources of inefficiency by breaking down technical efficiency into pure technical efficiency and scale efficiency. Pure technical efficiency refers to inefficiency related to poor management, while scale efficiency refers to inefficiency related to non-optimal size. The mole 8 (DMU12, zone north) in table 4, has a pure efficiency score of 86.4% and an efficiency scale of 88% and evolves in a situation of decreasing returns to scale (drs). By improving mole management (DMU 12, zone north), production could be increased by 1.8% (100-98.2); and by adjusting mole size, production could be increased by 12% (100-88).

5.4 Results of Bootstrap

In table 5 \( \theta_{CCR} \) is the original score, \( B_{CCR} \) the bias-corrected, \( Bias_{CCR} \) the bias, \( LB_{CCR} \) the lower bounds and \( UB_{CCR} \) the upper bounds for CCR model. \( \theta_{BCC} \) is the original score, \( B_{BCC} \) the bias-corrected, \( Bias_{BCC} \) the bias, \( LB_{BCC} \) the lower bounds and \( UB_{BCC} \) the upper bounds for BCC model. In total, with the DEA method, we have six (6) efficient DMUs with the CCR model, ten (10) with the BCC model. After using the bootstrap approach, we find that none of the DMUs are efficient with both models (CCR and BCC). This show the advantage and the necessity of using the bootstrap approach before proposing the solution to the decision makers (port authority). Comparing the results of the classical DEA method in table 4 and the combination of DEA and bootstrap approach in table 5, we observe average bias scores of 0.159 for the CCR model and 0.077 for the BCC model. Concerning the bias-corrected, we obtain an average score of 0.700 for CCR model (with confidence intervals of [0.324; 1.291]) and 0.870 for BCC model (with confidence intervals of [0.620; 1.197]). The figure 3 illustrates the confidence intervals and bias-corrected of the both models (CCR and BCC), where the DMU 11 admits the lower bound and DMU 8 admits the highest upper bound [0.620; 1.197] for CCR. The DMU 11 admits the lower bound and DMU 7 admits the highest upper bound [0.324; 1.291] for CCR. The corrected bias and confidence interval of the BCC model of DMU 11 are very close values (0.919, 0.874 and 0.999) in terms of efficiency.

In table 4 and the combination of DEA and bootstrap approach in table 5, we observe average bias scores of 0.159 for the CCR model and 0.077 for the BCC model. Concerning the bias-corrected, we obtain an average score of 0.700 for CCR model (with confidence intervals of [0.324; 1.291]) and 0.870 for BCC model (with confidence intervals of [0.620; 1.197]). The figure 3 illustrates the confidence intervals and bias-corrected of the both models (CCR and BCC), where the DMU 11 admits the lower bound and DMU 8 admits the highest upper bound [0.620; 1.197] for CCR. The DMU 11 admits the lower bound and DMU 7 admits the highest upper bound [0.324; 1.291] for CCR. The corrected bias and confidence interval of the BCC model of DMU 11 are very close values (0.919, 0.874 and 0.999) in terms of efficiency scores (see the figure 3).

Table 5. Results DEA bootstrap (original scores, bias-corrected, bias and the confidence intervals of both models (CCR, BCC)

<table>
<thead>
<tr>
<th>DMUs</th>
<th>( \theta_{CCR} )</th>
<th>( B_{CCR} )</th>
<th>( Bias_{CCR} )</th>
<th>( \theta_{BCC} )</th>
<th>( B_{BCC} )</th>
<th>( Bias_{BCC} )</th>
<th>( LB_{CCR} )</th>
<th>( UB_{CCR} )</th>
<th>( LB_{BCC} )</th>
<th>( UB_{BCC} )</th>
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<td>0.698</td>
<td>0.142</td>
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<td>0.663</td>
<td>0.992</td>
<td>0.814</td>
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<td>0.097</td>
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<td>0.882</td>
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Mean 0.858 0.700 0.159 0.951 0.870 0.077 0.591 0.875 0.807 1.024
6. Conclusion and Perspectives

In this article, we proposed an applied research work to propose a more adequate tool for the port of Dakar. The DEA (CCR and BCC) models and the bootstrap approach have been successfully used to measure its efficiency. The CCR, BCC and SE scores were first determined, then the corrected scores and confidence intervals of the CCR and BCC models were determined through the bootstrap approach. The efficient units are 33.33% for the CCR model and 55.55% for the BCC model. It confirms that the CCR model scores are all less than or equal to the BCC model scores as reported in the literature. The results may have implications for economic policy at the APD level by improving the units of some container terminals. The terminals to be improved are Container Terminals (DMUs 1, 3), Moles (DMUs 11, 12, 16, 17, 18), the Bulk Terminal (DMU 6), the Ro-Ro Terminal (DMU 7), the Military zone (DMU 13), the Fishing Port (DMU 15) and the Petroleum wharf (DMU 14) for CCR model. In addition, with BCC model, the terminals to be improved are the Bulk Terminal (DMU 6), the moles (DMU 11, 16, 17, 18), the Petroleum wharf (DMU 14) and the Fishing Port (DMU 15). At least some of the commodity DMUs are effective. Therefore, the implications are, among others, all the DMUs mentioned above are inefficient and already, the study gave projected values to make them efficient. Given the focus of the article on output orientation, these values are nothing more than the number of outputs (number of ships and tonnages) to be produced annually to make them efficient.

In future work, we plan to study the same problem using fuzzy logic, stochastic DEA and dynamic DEA taking into account quasi-fixed input and output variables, and many other factors.
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Conflict of Interest

The authors declare that they have no conflict of interest.

Author contributions

Methodology, P.M. and B.M.N.; resources, K.S.D. and G.D.; software, K.S.D. and B.M.N.; data curation, K.S.D. and G.D.; conceptualization, K.S.D., P.M. and B.M.N., funding acquisition, B.M.N. and G.D., supervision B.M.N.. All authors have read and agreed to the published version of the manuscript.

References


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