Journal of Mathematics Research; Vol. 14, No. 3; June 2022
ISSN 1916-9795 E-ISSN 1916-9809
Published by Canadian Center of Science and Education

Creation and Study of a Suitable Functions That Connects the Failure Rate of Exams With the Capacity of the Teaching Rooms Offered for Teaching of a Specific Course in a Greek University

Karageorgos D Athanasios
Correspondence: Department of Informatics and Telecommunications University of Thessaly Greece

Received: April 18, 2022 Accepted: May 19, 2022 Online Published: May 24, 2022
doi:10.5539/jmr.v14n3p37 URL: https://doi.org/10.5539/jmr.v14n3p37

Abstract

In this paper we are studying the connection between the exam failure rate of a course in a university and the capacity of the teaching rooms offered for the teaching of this course in this specific university

More specifically

1) We determine a function that informs us about the number of students that have the right to attend the course every moment

2) We calculate an upper bound of the exam failure rate of the course in relation with the capacity of these courses teaching rooms so that in any academic year the students have the ability to attend this class.

3) We create a mathematic function that demonstrates to the administration authorities of the university the decisions that need to be made about the enrollment the transcriptions the deletion of students and about the capacity of the teaching rooms so that in any given moment the students are able to attend the class no matter what courses exam failure rate is

1. Introduction

The motive for this particular paper is the fact that in several Greek universities there is a frequent phenomenon of students, who have the right to attend an academical course, being more than the offered positions in the teaching room. The reason of this phenomenon is the fact that the number of students that fail the course exam is such that the previous academical years failed ones (that obligingly have to select the same course again) along with the new students are more than the offered positions. So we tried to find an upper bound in the failure rate so that this phenome-non can be eradicated. In addition we create condition according to which each university administration authorities can make decisions about the capacity of the teaching rooms, the students' enrollments etc so that there are always available positions no matter what the failure rate is.

Problem Statement

More specifically lets think of a supposed university department about which we can make the following assumptions

1) C is the number of positions the department has for this course
2) The number of the students that have been accepted in the departments is stable and equal to A
3) The course is compulsory and all the A students have to select it. This means that nobody can graduate if they have not pass this courses exam
4) A number of the A students is finally positioned in other departments and a different number of students coming from other departments is positioned in the particular department
5) Let d_i , i=1,2,3……..n, $n \in N$ the courses failure rate. This mean that if Y is the number of students that have selected the course then $d_i \cdot Y$ is the number of the students that failed the exam of the year so they have to select it again the following year. Obviously $0<d_i<1 \quad \forall i = 1, 2....n \quad , n \in N$.

We do not accept the case d_i=0(which means that all the students have passed the exam) and the case d_i=1(which is means that all the students have failed the exam)

On this paper we use some mathematical tools. Mathematical tools that are necessary to prove our following claims.

More specifically, we use the sum and the product of n numbers

$$a_1 + a_2 + \ldots + a_i + \ldots + a_n = \sum_{i=1}^{n} a_i \tag{2.1}$$

$$\text{And} \quad a_1 \cdot a_2 \cdot \cdots \cdot a_i \cdots a_n = \prod_{i=1}^{n} a_i \tag{2.2}$$

Moreover we use the sum of n terms of a geometric progression

$$1 + a + a^2 + a^3 + \ldots\ldots a^n = \sum_{i=1}^{n} a^i = \frac{a^{n+1} - 1}{a - 1} \quad \text{when} \quad a \neq 1 \tag{2.3}$$

(*see*[6]) .

Also we use the limit of the sequence a^n when $0 < a < 1$ .This sequence converge to 0

$$\lim_{n \to 0} a^n = 0 \,. \tag{2.4}$$

Another mathematical tool which we use is the notion of derivative. Actually we use the following Theorem of Differential Calculus

Theorem2.1: Assume f is continuous on a closed interval l $[a,b]$ and assume that the derivative f' exist everywhere on the open interval (a,b), except possibly at a point c.

(a) *If $f'(x)$ is positive for all $x < c$ and negative for all $x > c$, then f has a relative maximum at c.*

(b) *If, on the other hand, $f'(x)$ negative for all $x < c$ and positive for all $x > c$,then f has a relative minimum at c. (see more details* [2],[6])

3. Main Results

3.1 Determination of the Function That Calculates the Number of Students that Select the Course the Year i. i=1,2,3…..n , $n \in N$

We initiate the study from the year the department was founded. Let A=the number of students and r_i =the percentage of transfers students to another university department of the country. Thus there are $(1-r_1)\cdot A$ $0<r_1<1$ left. At the same time B_1 students are coming with transcription to the department .So the total number of students is $(1-r_1)\cdot A + B_1$ d_1 =is the failure rate of the course the first year. Then $d_1\left[(1-r_1)\cdot A + B\right]$ is the number of students that fail the exams so they have to select the course again the following year. Specifically during the second year of the departments function there is :

A=the number of new students

r_2= the percentage of transfers students to another university department of the country

B_2=number of the students that come from another department

So the number of the new students that will select the course will be $(1-r_2)\cdot A + B_2$ .These student along with the ones that who fail to pass it will be $d_1 \cdot \left[(1-r_1)\cdot A + B_1\right] + (1-r_2)\cdot A + B_2$

d_2 = is the failure rate of the second year.

So the number of students that will have to try again the following year will be:

$$d_2 \cdot \left(d_1\left[(1-r_1)\cdot A + B_1\right] + (1-r_2)\cdot A + B_2\right) = d_2 \cdot d_1 \cdot (1-r_1)\cdot A + d_2 \cdot d_1 \cdot B_1 + d_2 (1-r_2)\cdot A + d_2 \cdot B_2$$

The third year of the departments function the number of the new students will be $(1-r_3)\cdot A + B_3$.

The number of students that will attend the course will be $\left(1-r_3\right)\cdot A+B_3+d_2\cdot d_1\cdot\left(1-r_1\right)\cdot A+d_2\cdot\left(1-r_2\right)\cdot A+d_2\cdot d_1\cdot B_1+d_2\cdot B_2$,

d_3 = the failure rate

So the number of students that will try again the following year will be:

$$d_3\cdot\left\{\left[\left(1-r_3\right)\cdot A+B_3\right]+d_2\cdot d_1\cdot\left(1-r_1\right)\cdot A+d_2\cdot\left(1-r_2\right)\cdot A+d_2\cdot d_1\cdot B_1+d_2\cdot B_2\right\}=$$
$$d_3\cdot d_2\cdot d_1\cdot\left(1-r_1\right)\cdot A+d_3\cdot d_2\cdot\left(1-r_2\right)\cdot A+d_3\cdot\left(1-r_3\right)\cdot A+d_3\cdot d_2\cdot d_1\cdot B_1+d_3\cdot d_2\cdot B_2+d_3\cdot B_3$$

To make this procedure more general during the n+1 year of departments function the number of students that will attend the course will be the sum of the students who failed the exams the previous n year plus with the newcomers of the year n+1, $n\in N$ So, it will be

$$d_n\cdot d_{n-1}......d_2\cdot d_1\cdot\left(1-r_1\right)\cdot A+d_n\cdot d_{n-1}......d_2\cdot\left(1-r_2\right)\cdot A+.....+d_n\cdot d_{n-1}\cdot\left(1-r_{n-1}\right)\cdot A+d_n\cdot\left(1-r_n\right)\cdot A+$$
$$d_n\cdot d_{n-1}......d_2\cdot d_1\cdot B_1+d_n\cdot d_{n-1}......d_2\cdot B_2+........+d_n\cdot d_{n-1}\cdot B_{n-1}+d_n\cdot B_n+\left(1-r_{n+1}\right)\cdot A+B_{n+1}$$

The previous sum can be written (see 2.1) and (2.2)

$$\sum_{k=1}^{n}\left[\left(\prod_{i=k}^{n}d_i\right)\cdot\left(1-r_k\right)\cdot A\right]+\sum_{k=1}^{n}\left[\left(\prod_{i=k}^{n}d_i\right)\cdot B_k\right]+\left(1-r_{n+1}\right)\cdot A+B_{n+1}$$

Therefore the function that gives us the number of students that attend the course in the year n+1 will be:

$$f=\sum_{k=1}^{n}\left[\left(\prod_{i=k}^{n}d_i\right)\cdot\left(\left(1-r_k\right)\cdot A+B_k\right)\right]+\left(1-r_{n+1}\right)\cdot A+B_{n+1}\quad .$$

The variables of f are supposed to be the failure rate $d_1, d_2, ...d_n$

3.2 Calculation of the Maximum Failure Rate That Can Be Allowed

We have mentioned that the department has a number *C* of attending positions so must $f\leq C$. However, since the failure rate is not stable we define the function having as a variable the maximum failure rate.

More specifically let us assume that $d=\max\left\{d_1, d_2, ...d_n\right\}$ the maximum failure rate during the n years of the department function. Then $\prod_{i=k}^{n}d_i\leq d^{n-k+1}$ (see$\left[14\right]$) therefore

$$f\leq\sum_{k=1}^{n}d^{n-k+1}\cdot\left(\left(1-r_k\right)\cdot A+B_k\right)+\left(1-r_{n+1}\right)\cdot A+B_{n+1}$$

We want the maximum of f to be less than C. For this reason we consider that

$r=\min\left\{r_1, r_2, ...r_n\right\}$ is the minimal rate of students who go to another department.

Then $r_k\geq r, \forall k=1,2.....n+1\Leftrightarrow 1-r_k\leq 1-r$ we obtain $\left(1-r_k\right)\cdot A\leq\left(1-r\right)\cdot A$

Consequently we assume that $B=\max\left\{B_1, B_2, ...B_{n+1}\right\}$ is the maximum number of students who come to our department by transcription from another department of the country. Obviously $B_k\leq B, \forall k=1,2.....n+1$ then we have

$$f \le \sum_{k=1}^{n} d^{n-k+1} \cdot \left((1-r) \cdot A + B \right) + (1-r_{n+1}) \cdot A + B_{n+1} = \left((1-r) \cdot A + B \right) \cdot \sum_{k=1}^{n} d^{n-k+1} + (1-r_{n+1}) \cdot A + B_{n+1}$$

However $(1-r_{n+1}) \cdot A + B_{n+1} \le (1-r) \cdot A + B$ and now we have

$f \le \left((1-r) \cdot A + B \right) \cdot \sum_{k=1}^{n} d^{n-k+1} + (1-r) \cdot A + B$ now denoting n-k+1=m when k=1,2…n then we obtain m=n,n-1,…..2,1.

Thus $f \le \left((1-r) \cdot A + B \right) \cdot \sum_{k=1}^{n} d^{m} + (1-r) \cdot A + B$ =

$$\left[(1-r) \cdot A + B \right] \cdot \frac{d^{n+1} - d}{d-1} + (1-r) \cdot A + B =$$

$$\left[(1-r) \cdot A + B \right] \cdot \frac{d^{n+1} - d + d - 1}{d-1} = \left[(1-r) \cdot A + B \right] \frac{d^{n+1} - 1}{d-1} \quad \text{(see 2.3)}$$

We have that $d \in (0,1)$ and we found that $\lim_{n \to +\infty} d^{n+1} = 0$ (see 2.4).

Thus $\max f = \left[(1-r) \cdot A + B \right] \cdot \frac{0-1}{d-1} = \left[(1-r) \cdot A + B \right] \cdot \frac{1}{1-d}$.

Must $\max f \le C \Leftrightarrow \left[(1-r) \cdot A + B \right] \cdot \frac{1}{1-d} \le C \Leftrightarrow \frac{1-d}{(1-r) \cdot A + B} \ge \frac{1}{C} \Leftrightarrow 1-d \ge \frac{(1-r) \cdot A + B}{C}$

$$\Leftrightarrow d \le 1 - \frac{(1-r) \cdot A + B}{C} \Leftrightarrow \boxed{d \le \frac{C - (1-r) \cdot A - B}{C}} \tag{1}$$

The condition (1) states that if the maximum failure rate of the course, d is less than or equal to the value $\frac{C - (1-r) \cdot A - B}{C}$ then the department will never face a capacity issue.

Example 3.1 Consider that if one year thestudent transfers to other department have been banned but this department accepts students from other department then $r = 0$ .Also we consider that the maximum number of the students who come to the department is $B_{\max} = \frac{25}{100} \cdot A$. Then according to (1) we have $d \le \frac{C - (1-0) \cdot A - \frac{25}{100} \cdot A}{C} = \frac{C - 1{,}25 \cdot A}{C} = 1 - 1{,}25 \cdot \frac{A}{C}$

If the department has 380 positions and $A = 150$ then $d \le 1 - 1{,}25 \cdot \frac{150}{380} \approx 0{,}507$

If the maximum failure rate is less than 50% then there will never come up a capacity issue but in any other case the danger is very big.

However we have to confess that the previous upper bound contains important dangers to the quality of studies and the function of the department.In particular, it is not morally accept that the examiners adjust the grades of the exam questions so that the previous rate is accomplished. They need to be independent during the exam so that they can achieve their academical duty. Thus we have to determine one function which irrespectively of the exam failure rate, can give the ability to the administration authorities of the department to change other parameters, so that there will never come up a capacity issue. We named this function *parameter determination function PaDeFu*

3.3 Creation of the Function PaDeFu

One factor that maximizes the danger that the department does not have enough space for the attention of the course is the fact that students who have not been able to pass the exam during many years are still enrolled. Thus it is normal that after m years of functions in the year m+1 the department can delete the students that select the course in the first year of function and still have not passed the exam. In the year m+2 the department deletes the students that selected the course in the second year of function and still have not passed the exam etc.

According to the thing we have mentioned before we know that in the year m $\sum_{k=1}^{m}\left(\prod_{i=k}^{m} d_i\right)\cdot\left[\left(1-r_k\right)\cdot A+B_k\right]$ students failed the exam.

Among these students the department expects $\left(\prod_{i=1}^{m} d_i\right)\cdot\left[\left(1-r_1\right)\cdot A+B_1\right]$ who are the ones that selected the course in the first year of function and keep on failing the exam.

Thus, there are $\sum_{k=2}^{m}\left(\prod_{i=k}^{m} d_i\right)\cdot\left[\left(1-r_k\right)\cdot A+B_k\right]$ left.

In the year m+1 $\sum_{k=2}^{m}\left(\prod_{i=k}^{m} d_i\right)\cdot\left[\left(1-r_k\right)\cdot A+B_k\right]$ $+\left(1-r_{m+1}\right)\cdot A+B_{m+1}$ students will select the course. In this year

$d_{m+1}\cdot\left[\left(\sum_{k=2}^{m}\left(\prod_{i=k}^{m} d_i\right)\cdot\left(\left(1-r_k\right)\cdot A+B_k\right)\right)\cdot\left(1-r_{m+1}\right)\cdot A+B_{m+1}\right]=\sum_{k=2}^{m+1}\left(\prod_{i=k}^{m+1} d_i\right)\cdot\left[\left(1-r_k\right)\cdot A+B_k\right]$ students will fail the exam.

Among these students $\left(\prod_{i=2}^{m+1} d_i\right)\cdot\left(\left(1-r_2\right)\cdot A+B_2\right)$ will be deleted. So there will be $\sum_{k+3}^{m+1}\left(\prod_{i=k}^{m+1} d_i\right)\cdot\left(\left(1-r_k\right)\cdot A+B_k\right)$ left.

In the year m+2 $(\sum_{k+3}^{m+1}\left(\prod_{i=k}^{m+1} d_i\right)\cdot\left(\left(1-r_k\right)\cdot A+B_k\right))+\left(1-r_{m+2}\right)\cdot A+B_{m+2}$ students will select the course. In this year $d_{m+2}\cdot\left(\left(\sum_{k=3}^{m+1}\left(\prod_{i=k}^{m+1} d_i\right)\cdot\left(\left(1-r_k\right)\cdot A+B_K\right)\right)+\left(1-r_{m+2}\right)\cdot A+B_{m+2}\right) = \sum_{k=3}^{m+2}\left(\prod_{i=k}^{m+2} d_i\right)\cdot\left(\left(1-r_k\right)\cdot A+B_k\right)$ students will fail the exam.

If we go on with this procedure, in the year m+n+1 $\left(\sum_{k=n+1}^{m+n}\left(\prod_{i=k}^{m+n} d_i\right)\cdot\left[\left(1-r_k\right)\cdot A+B_k\right]\right)+\left(1-r_{m+n+1}\right)\cdot A+B_{m+n+1}$ students will select the course. In order to avoid the capacity issue it is necessary that $P=$ $\left(\sum_{k=n+1}^{m+n}\left(\prod_{i=k}^{m+n} d_i\right)\cdot\left[\left(1-r_k\right)\cdot A+B_k\right]\right)+\left(1-r_{m+n+1}\right)\cdot A+B_{m+n+1}\leq C$

We consider $d=\max_{i=n+1,n+2,\ldots.n+m+1}\{d_i\}$, B= $\max_{i=n+1,n+2,\ldots..n+m+1}\{B_i\}$ and $r=\min_{i=n+1,n+2,\ldots.n+m+1}\{r_i\}$ then we have

$$P\leq\left(\sum_{k=n+1}^{m+m} d^{m+n-k+1}\cdot\left[\left(1-r\right)\cdot A+B\right]\right)+\left(1-r\right)\cdot A+B=\left(\left(1-r\right)\cdot A+B\right)\cdot\left(\sum_{k=n+1}^{m+n} d^{m+n-k+1}+1\right)$$

Nevertheless $\sum_{k=n+1}^{m+n} d^{m+n-k+1}=d^m+d^{m-1}+\ldots.+d=\sum_{k=1}^{m} d^k=\dfrac{d^{m+1}-d}{d-1}$ (see 2.3)

Therefore $P\leq\left(\left(1-r\right)\cdot A+B\right)\cdot\left(\dfrac{d^{m+1}-d}{d-1}+1\right)=\left(\left(1-r\right)\cdot A+B\right)\cdot\dfrac{d^{m+1}-d+d-1}{d-1}\Leftrightarrow$

$P\leq\left[\left(1-r\right)\cdot A+B\right]\cdot\dfrac{d^{m+1}-1}{d-1}$. We want to calculate d such that $\left(\left(1-r\right)\cdot A+B\right)\cdot\dfrac{d^{m+1}-1}{d-1}\leq C\Leftrightarrow$ $d^{m+1}-1\geq\dfrac{C}{\left(1-r\right)\cdot A+B}\cdot\left(d-1\right)$ $\Leftrightarrow d^{m+1}-\dfrac{C}{\left(1-r\right)\cdot A+B}\cdot d+\dfrac{C}{\left(1-r\right)\cdot A+B}-1\geq 0\Leftrightarrow$

$d^{m+1}-\frac{C}{(1-r)\cdot A+B}\cdot d+\frac{C-(1-r)\cdot A-B}{(1-r)\cdot A+B}\geq 0$ when $d\in(0,1)$ .Now we study the function $f(x)=x^{m+1}-\frac{C}{(1-r)\cdot A+B}\cdot x+\frac{C-(1-r)\cdot A-B}{(1-r)\cdot A+B}$.

We want that $\forall x\in(0,1)$ the $f(x)>0$ such that any failure rate would not cause capacity issue.

Now we derivative the function f $f^{'}(x)=(m+1)\cdot x^{m}-\frac{C}{(1-r)\cdot A+B}$

$f^{'}(x_0)=0\Leftrightarrow x_0=\left(\frac{C}{(m+1)\cdot((1-r)\cdot A+B)}\right)^{\frac{1}{m}}$ (does no matter if m is even or odd number because x>0).So $f^{'}(x)>0$ for every $x\in(0,x_0)$ and $f^{'}(x)<0$ for every $x\in(x_0,+\infty)$ .From the previous results we have that the function f is strictly increase in $(x_0,+\infty)$ and strictly decrease in $(0,x_0)$.(see theorem 2.1)

Thus $\lim_{x\to 0}f(x)=\frac{C-(1-r)\cdot A-B}{(1-r)\cdot A+B}>0$(we consider $A>(1-r)\cdot A+B$ because in a different case it is possible that the students will face capacity even from the first year function).Also $f(1)=1-\frac{C}{(1-r)\cdot A+B}+\frac{C-(1-r)\cdot A-B}{(1-r)\cdot A+B}=0$ so since f is strictly decrease in $(0,x_0)$ will be strictly decrease also in $(0,1)$. Therefore $\forall x\in(0,1)$ f(x)>f(1)=0. Therefore $\forall x\in(0,1)$ we have the desired result f(x)>0 so must

$$x_0>1\Leftrightarrow\left(\frac{C}{(m+1)\cdot((1-r)\cdot A+B)}\right)^{\frac{1}{m}}>1\Leftrightarrow\boxed{C>(m+1)\cdot((1-r)\cdot A+B)}$$

The last one is the PeDeFu where *C*= is the number of the offered positions

m=the time interval during which every student has to pass the course

r=maximum rate of transcriptions, *A*=the number of the new students

B=maximum number of students with transcription.

It is necessary for the departments administration authorities to be aware of the previously mentioned condition so that they will adjust the previous parameters with the intention to keep this condition valid in any case.

However it is time to refer to some commitments .More specifically the department has a function regulation that stable for a period of years and cannot be changed every year. This where the time interval m, during which every student has to pass exam, will be mentioned. So we can accept that m is stable (or that it can be changed but only after many years have passed). We can also accept that the number of positions C is stable. However the department could construct or even rent some positions, if that would be possible. But in that case the function expenses would increase and that would definitely be something not easy to do. Therefore, we accept that C is stable.

In addition the transcription rate $r\in(0,1)$ is not stable. However the department needs to be prepared for the negative possibility of r=0, which is means that not even one student leaves. Then in that case the only variables are A the number of the new students and B the number of the students with transcription.

Thus must $C>(m+1)\cdot(A+B)\Leftrightarrow\boxed{A+B<\frac{C}{m+1}}$ .Therefore the number of students that will be accepted every year has to be less than $\frac{C}{m+1}$

4. Conclusions

If we keep in mind that the students pay fees we can investigate the existence of a condition so that the department does not have a capacity issue and at the same time maximizes the income .In addition by doing a research and collecting statistical data we will investigate the creation of a condition according to which the r and B are stable and come from the data of the past. Also, since the attending of the course is not obligatory we will create a new condition which can solve the problem given that the new variable will be course attending rate

References

Alpha, C. C., & Wainwright, K. (2005). *Fundamental methods of mathematical economics*. Boston, Mass. McGraw-Hill/Irwin.

Asif, R., Merceron, A., Ali, S. A., & Haider, N. G. (2017). Analyzing undergraduate students' performance using educational data mining. *Computers & Education, 113*, 177-194.

Boylan-Ashraf, P. C., & Haughery, J. R. (2018, June). Failure Rates in Engineering: Does It Have to Do with Class Size?. In *2018 ASEE Annual Conference & Exposition*.

Egoeze, F., Misra, S., Maskeliūnas, R., & Damaševičius, R. (2018). Impact of ICT on universities administrative services and management of students' records: ICT in university administration. *International Journal of Human Capital and Information Technology Professionals (IJHCITP), 9*(2), 1-15.

Elaydi, S. N. (1999). *An introduction to difference equations*. Springer.

Evandro, B., Costa, B. F., Marcelo, A. S., Fabricia, F. de A., Joilson, R. (2017). *Evaluating the effectiveness of educational data mining techniques for early prediction of students academic failure in introductory programming courses. Computer in Human Behavior, 73*, 247-256.

Finney, R. L., Weir, M. D., & Giordano, F. R. (2018). *Thomas Calculus* 2001 tenth edition Addisson Wesley Longman Kenneth H Rosen *Discrete Mathematics and its Applications* ,Mc Graw-Hill Inc8th edition 2018.

Liu, C. L. (1977). *Elements of Discrete Mathematics.* Mc Graw-Hill Inc 2nd edition1985.

Rahal, A., & Zainuba, M. (2016). Improving students' performance in quantitative courses: The case of academic motivation and predictive analytics. *The International Journal of Management Education*, *14*(1), 8-17.

Seymour, L., & Marc, L. (2009). *Shaum's Outline of theory and problems of Discrete Mathematics* Mc Graw-Hill Inc 3nd edition 2009.

TM Apostol *Calculus.* Johm Willey and son second edition 1967.

Worthley, M. R., Gloeckner, G. W., & Kennedy, P. A. (2016). A mixed-methods explanatory study of the failure rate for freshman STEM calculus students. *PRIMUS, 26*(2), 125-142.

Yen, Y. F. (2017). Analysis of Subsidiary and Grand Policy in Public Universities and top Universities in Taiwan. *Journal of Research in Education Sciences, 62*(4), 53-88.

Zámková, M., Prokop, M., & Stolín, R. (2016). The Factors Influencing the Mathematics Exam Success Rate in the Study Programmes Taught at the College of Polytechnics Jihlava (2006-2015). *Journal on Efficiency and Responsibility in Education and Science*, *9*(3), 52-59.