

Jacobi Manifolds, Contact Manifolds and Contactomorphism

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Abstract

Let M be a smooth manifold and let $\mathcal{D}(M)$ be the module of first order differential operators on M . In this work, we give a link between Jacobi manifolds and Contact manifolds. We also generalize the notion of contactomorphism on M and thus, we characterize the Contact diffeomorphisms.

Keywords: Jacobi manifolds, contact manifolds, differential operators, contactomorphism

1. Introduction

1.1 Jacobi Manifolds

The notion of Jacobi manifolds has been introduced by A. Lichnerowicz (Lichnerowicz, 1978). These manifolds are a generalization of the notion of locally conformally symplectic manifolds and of contact manifolds (which can be used in the representation of energy in thermodynamics). A Jacobi structure on a manifold M is a pair (Λ, E) where Λ is a contravariant skew-symmetric 2-tensor and E is a vector field on M such that

$$[\Lambda, \Lambda] = 2E \wedge \Lambda, \theta_E \Lambda = [E, \Lambda] = 0 \quad (1)$$

with \wedge the Grassmann wedge product and θ_E the Lie derivative with respect to the vector field E . The bracket defined in (1) is the Schouten-Nijenhuis bracket on multivectors (Koszul, 1985).

Using the classical Darboux's theorem, around every point of M there exist canonical coordinates (x_1, \dots, x_{2n}) and a local smooth function f on M such that

$$E = e^{-f} \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \frac{\partial}{\partial x_{n+i}} - \frac{\partial f}{\partial x_{n+i}} \frac{\partial}{\partial x_i} \right), \Lambda = e^{-f} \sum_{i=1}^n \frac{\partial}{\partial x_{n+i}} \wedge \frac{\partial}{\partial x_i}. \quad (2)$$

Defining a Jacobi structure on (Λ, E) is equivalent to defining an internal composition law $\{\cdot, \cdot\}$ on $\mathcal{C}^\infty(M)$ such that for all $f, g \in \mathcal{C}^\infty(M)$

$$\{f, g\} = \Lambda(df, dg) + f \cdot E(g) - g \cdot E(f) \quad (3)$$

where d is the exterior differentiation operator (Okassa, 2007). The bracket $\{\cdot, \cdot\}$ is skew-symmetric and satisfies the Jacobi identity if and only if (1) is fulfilled. Furthermore, one has the locality condition

$$\text{supp}\{f, g\} \subseteq \text{supp}(f) \cap \text{supp}(g). \quad (4)$$

A natural generalization of Poisson bracket is Jacobi bracket. The only difference is that we only replace the Leibniz rule by

$$\{f, g \cdot h\} = \{f, g\} \cdot h + g \cdot \{f, h\} - g \cdot h \cdot \{f, 1\} \quad (5)$$

for all $f, g, h, 1 \in \mathcal{C}^\infty(M)$, where 1 is the unit of $\mathcal{C}^\infty(M)$.

Proposition 1. *The Jacobi bracket $\{\cdot, \cdot\}$ defines a Lie algebra structure on $\mathcal{C}^\infty(M)$, if and only if (1) is satisfied.*

The vector field E is the hamiltonian vector field associated with the constant function 1. We have $E(f) = \{1, f\}$, whence the Jacobi identity for $(1, f, g)$ yields

$$E(\{f, g\}) = \{E(f), g\} + \{f, E(g)\}$$

which equivalent to $\theta_E \Lambda = 0$. If the vector field E vanishes, we recover the Poisson bracket.

Remark 2. (Lichnerowicz, 1977) *Let (Λ, E) be a Jacobi structure on a manifold M and consider a product manifold $M \times \mathbb{R}$, the bivector $\tilde{\Lambda}$ given by $\tilde{\Lambda} = e^{-t} \left(\Lambda + \frac{\partial}{\partial t} \wedge E \right)$, where t is the usual coordinate on \mathbb{R} . Then, $\tilde{\Lambda}$ defines a Poisson structure on $M \times \mathbb{R}$. The manifold $M \times \mathbb{R}$ with the structure $\tilde{\Lambda}$ is called the poissonization of the Jacobi manifold (M, Λ, E) .*

Note that a smooth manifold M is a Jacobi manifold when the algebra of numerical functions of class \mathcal{C}^∞ on M , $\mathcal{C}^\infty(M)$ admits a Jacobi algebra structure (Okassa, 2007).

1.2 Contact Manifolds

By a contact manifold in this paper, we mean a \mathcal{C}^∞ manifold M^{2n+1} together with a 1-form α such that $\alpha \wedge (d\alpha)^n \neq 0$. In particular, $\alpha \wedge (d\alpha)^n$ is a volume element on M^{2n+1} , so that a contact manifold is orientable. Also $(d\alpha)^n$ has rank $2n$ on the Grassmann algebra ΛT_p^*M at each point $p \in M$, and thus we have a 1-dimensional subspace,

$$\{X \in T_pM / d\alpha(X, T_pM) = 0\}$$

on which $\alpha \neq 0$ and which is complementary to the subspace defined by $\alpha = 0$.

Therefore choosing R_p in this subspace normalized by $\alpha(R_p) = 1$, we have a global vector field R satisfying $\alpha(R) = 1$ and $d\alpha(R, X) = 0$. In this case, the $\mathcal{C}^\infty(M)$ -module of vector fields $\mathfrak{X}(M)$ admits a inner sum direct decomposition

$$\mathfrak{X}(M) = Ker\alpha \oplus \mathcal{C}^\infty(M) \cdot R,$$

where, R is called Reeb vector field of the contact structure α . We deduce that $\theta_R\alpha = 0$ and $\theta_Rd\alpha = 0$.

One denotes by \mathcal{D} the contact distribution defined by the space

$$\mathcal{D}_p = \{X \in T_pM / \alpha(X) = 0\} = ker(\alpha).$$

Roughly speaking, the meaning of the contact condition, $\alpha \wedge (d\alpha)^n \neq 0$, is that the contact subbundle is as far from being integrable as possible.

By a smooth manifold, we will mean a paracompact connected finite dimensional \mathcal{C}^∞ manifold without boundary.

Here one lists a few examples of contact manifolds and Reeb vector field.

(i) The pair $(\mathbb{R}^{2n+1}, \alpha = dz + \sum_{i=1}^n x_i dy_i)$ is a contact manifold with Reeb vector field $\frac{\partial}{\partial z}$. Indeed, if $R = a \frac{\partial}{\partial z} + b_i \frac{\partial}{\partial x_i} + c_i \frac{\partial}{\partial y_i}$,

then $1 = \alpha(R) = a - b_i y_i$. Also $0 = d\alpha(R, \frac{\partial}{\partial x_i}) = \sum_{i=1}^n (dx_i \wedge dy_i)(R, \frac{\partial}{\partial x_i})$ gives $c_i = 0$. Similarly, $0 = d\alpha(R, \frac{\partial}{\partial y_i}) = \sum_{i=1}^n (dx_i \wedge dy_i)(R, \frac{\partial}{\partial y_i})$ gives $b_i = 0$. Thus $a = 1$ and $R = \frac{\partial}{\partial z}$.

(ii) Consider \mathbb{R}^3 with the contact form $\alpha = \sin(y)dx + \cos(y)dz$. Then the Reeb vector field is $R = \sin(y) \frac{\partial}{\partial x} + \cos(y) \frac{\partial}{\partial z}$.

Given two contact manifolds (M, α) and (M', α') , let $h : M \rightarrow M'$ be a smooth map, that is, differentiable.

The map h is called a morphism of contact manifolds, so long as $h^* \alpha' = \alpha$ (Gatsé, 2016).

Given a contact diffeomorphism h , h^{-1} is also contact, and h is called a contactomorphism, where by definition of the pullback of a 1-form α

$$(h^* \alpha)(x)(u) = \alpha(h(x))(dh(x)u)$$

for x element of M and for any u element of T_xM .

The main goal of this paper is to show that the contact manifolds are the Jacobi manifolds. We also will give the generalization of the notion of contactomorphism and will characterize the contact diffeomorphisms.

In what follows, M denotes a paracompact and connected smooth manifold, $\mathcal{C}^\infty(M)$ the algebra of numerical functions of class \mathcal{C}^∞ on M , $\mathcal{D}(M)$ the $\mathcal{C}^\infty(M)$ -module of first order differential operators on $\mathcal{C}^\infty(M)$ and δ (Okassa, 2008) the cohomology operator associated with the identically map

$$id : \mathcal{D}(M) \rightarrow \mathcal{D}(M).$$

The term "differential operator" will mean "first order differential operator".

2. Link Between Jacobi Manifolds and Contact Manifolds

Let $\Omega^1(M)_R$ be the $\mathcal{C}^\infty(M)$ -module of differential forms of degree 1 on M which vanish on R . Then the map

$$ker \alpha \rightarrow \Omega^1(M)_R, X \mapsto i_X d\alpha$$

is an isomorphism of $\mathcal{C}^\infty(M)$ -modules. Moreover, for any function $f \in \mathcal{C}^\infty(M)$, we obtain the following properties:

(i) $\theta_{f \cdot R} \alpha = df,$

(ii) $\theta_{f \cdot R} d\alpha = 0,$

- (iii) $\theta_R(f\alpha) = R(f) \cdot \alpha + df,$
- (iv) $\theta_R d(f\alpha) = (dR(f)) \wedge \alpha + R(f) \cdot d\alpha.$

The differential form $df - R(f) \cdot \alpha$ vanishes on R and we have easily

$$[R, \ker \alpha] \subset \ker \alpha.$$

One notes X_f the unique vector field on M belonging to $\ker \alpha$ such that

$$i_{X_f} d\alpha = df - R(f) \cdot \alpha.$$

We notice that

- (i) $R \in \ker(i_{X_f} d\alpha).$
- (ii) If $f = 0$, then $X_f = 0.$
- (iii) If $f = 1$, then $X_f = R.$

A differential operator on M is a \mathbb{R} -linear map

$$\varphi : \mathcal{C}^\infty(M) \longrightarrow \mathcal{C}^\infty(M)$$

such that

$$\varphi(f \cdot g) = \varphi(f) \cdot g + f \cdot \varphi(g) - f \cdot g \cdot \varphi(1)$$

for any $f, g \in \mathcal{C}^\infty(M)$. We verify that the map

$$\varphi_f = R(f) + X_f - f \cdot R : \mathcal{C}^\infty(M) \longrightarrow \mathcal{C}^\infty(M)$$

is a differential operator on M . We set

$$\{f, g\} = \varphi_f(g),$$

we deduce the following results.

Theorem 3. For all f, g and h in $\mathcal{C}^\infty(M)$, we get

- (i) $[X_f, X_g] = X_{\{f, g\}} + f \cdot X_{R(g)} - g \cdot X_{R(f)} + X_f(g) \cdot R.$
- (ii) $\{f, g \cdot h\} = \{f, g\} \cdot h + g \cdot \{f, h\} - g \cdot h \cdot \{f, 1\}.$
- (iii) $[R, \varphi_f] = \varphi_{R(f)}.$
- (iv) $R\{f, g\} = \{R(f), g\} + \{f, R(g)\}.$
- (v) $[\varphi_f, \varphi_g] = \varphi_{\{f, g\}}.$

Proof. (i) We check that

$$i_{[X_f, X_g] - X_{\{f, g\}} - f \cdot X_{R(g)} + g \cdot X_{R(f)} - X_f(g) \cdot R} d\alpha = 0.$$

(ii) The proof of this assertion is obvious.

(iii) For any $f \in \mathcal{C}^\infty(M)$, we get

$$\begin{aligned} [R, \varphi_f] &= [R, R(f) + X_f - f \cdot R] \\ &= [R, R(f)] + [R, X_f] - [R, f \cdot R] \\ &= R(R(f)) + X_{R(f)} - R(f) \cdot R \\ &= \varphi_{R(f)}. \end{aligned}$$

(iv) Since

$$R(X_f(g)) = X_f(R(g)) + X_g(R(f)),$$

we deduce that

$$R\{f, g\} = \{R(f), g\} + \{f, R(g)\}.$$

(v) We calculate

$$\begin{aligned} [\varphi_f, \varphi_g] &= [R(f) + X_f - f \cdot R, R(g) + X_g - g \cdot R] \\ &= X_f(R(g)) - f \cdot R(R(g)) - X_g(R(f)) + g \cdot R(R(f)) \\ &\quad + [X_f, X_g] - X_f(g) \cdot R + g \cdot [R, X_f] + f \cdot [R, X_g] \\ &\quad + X_g(f) \cdot R + (f \cdot R(g) - g \cdot R(f)) \cdot R \\ &= R\{f, g\} + X_{\{f, g\}} - \{f, g\} \cdot R \\ &= \varphi_{\{f, g\}} \end{aligned}$$

That ends the proof. □

Proposition 4. *The map*

$$\mathcal{C}^\infty(M) \times \mathcal{C}^\infty(M) \longrightarrow \mathcal{C}^\infty(M), (f, g) \longmapsto \{f, g\},$$

defines a real Lie algebra structure on the contact manifold M .

Proof. We write

$$\begin{aligned} \{f, \{g, h\}\} + \{g, \{h, f\}\} + \{h, \{f, g\}\} &= \{f, \{g, h\}\} - \{g, \{f, h\}\} - \{\{f, g\}, h\} \\ &= \varphi_f[\varphi_g(h)] - \varphi_g[\varphi_f(h)] - \varphi_{\{f, g\}}(h) \\ &= ([\varphi_f, \varphi_g] - \varphi_{\{f, g\}})(h) \\ &= 0. \end{aligned}$$

Hence the assertion is proved. □

Give the foregoing, it is concluded that a contact manifold is a Jacobi manifold.

3. Contactomorphism

If $\mathcal{D}(M)$ is the set of differential operators on M and

$$\omega : \mathcal{D}(M) \times \mathcal{D}(M) \longrightarrow \mathcal{C}^\infty(M)$$

is a nondegenerate skew-symmetric 2-form on M , then the pair (M, ω) is a contact manifold (fore more details, see Okassa, 2011). Let

$$i_1 \omega : \mathcal{D}(M) \longrightarrow \mathcal{C}^\infty(M)$$

be the 1-form on M such that

$$i_1 \omega|_{\mathcal{C}^\infty(M)} = 0.$$

Proposition 5. (Okassa, 2008) *If ω is a nondegenerate skew-symmetric 2-form such that $\delta\omega = 0$, then $\omega = \delta(i_1\omega)$.*

Proof. For all φ and ψ in $\mathcal{D}(M)$, we have

$$\begin{aligned} 0 &= (\delta\omega)(1, \varphi, \psi) \\ &= \omega(\varphi, \psi) - \varphi[(i_1\omega)(\psi)] + \psi[(i_1\omega)(\varphi)] + (i_1\omega)[\varphi, \psi] \\ &= [\omega - \delta(i_1\omega)](\varphi, \psi). \end{aligned}$$

We deduce that $\omega = \delta(i_1\omega)$. □

Proposition 6. (Gatsé, 2019) Suppose that f is a diffeomorphism on M . Let φ be a differential operator of $\mathcal{C}^\infty(M)$. Then the map

$$f_*\varphi = (f^{-1})^* \circ \varphi \circ f^* : \mathcal{C}^\infty(N) \longrightarrow \mathcal{C}^\infty(N)$$

is a differential operator of $\mathcal{C}^\infty(N)$.

Proposition 7. When φ is a differential operator, when f is a diffeomorphism then for any g element of $\mathcal{C}^\infty(M)$, we obtain

$$\varphi(g \circ f) = [(f_*\varphi)(g)] \circ f.$$

Proof. Let x be an element of M , we write

$$\begin{aligned} [\varphi(g \circ f)](x) &= [\varphi[f^*(g)]](x) \\ &= [f_*\varphi(g)](f(x)) \\ &= [[(f_*\varphi)(g)] \circ f](x). \end{aligned}$$

Thus $\varphi(g \circ f) = [(f_*\varphi)(g)] \circ f$. □

Proposition 8. If f is a diffeomorphism and let ξ be a multilinear p -form on N then for all $\psi_1, \psi_2, \dots, \psi_p \in \mathcal{D}(M)$,

$$(f^*\xi)(\psi_1, \psi_2, \dots, \psi_p) = [\xi(f_*\psi_1, f_*\psi_2, \dots, f_*\psi_p)] \circ f.$$

Proof. For x element of M , we obtain

$$\begin{aligned} (f^*\xi)(\psi_1, \psi_2, \dots, \psi_p)(x) &= (f^*\xi)(x)(\psi_1(x), \psi_2(x), \dots, \psi_p(x)) \\ &= \xi[f(x)][(f_*\psi_1, f_*\psi_2, \dots, f_*\psi_p)(f(x))] \\ &= [\xi(f_*\psi_1, f_*\psi_2, \dots, f_*\psi_p)](f(x)) \\ &= ([\xi(f_*\psi_1, f_*\psi_2, \dots, f_*\psi_p)] \circ f)(x), \end{aligned}$$

then

$$(f^*\xi)(\psi_1, \psi_2, \dots, \psi_p) = [\xi(f_*\psi_1, f_*\psi_2, \dots, f_*\psi_p)] \circ f.$$

That ends the proof. □

Theorem 9. When f is a diffeomorphism then δ and f^* commute.

Proof. We have

$$\begin{aligned} &[\delta(f^*\xi)](\psi_1, \psi_2, \dots, \psi_p, \psi_{p+1}) \\ &= \sum_{i=1}^{p+1} (-1)^{i+1} \psi_i [(f^*\xi)(\psi_1, \psi_2, \dots, \widehat{\psi}_i, \dots, \psi_p, \psi_{p+1})] \\ &\quad + \sum_{i < j} (-1)^{i+j} (f^*\xi)([\psi_i, \psi_j], \psi_1, \psi_2, \dots, \widehat{\psi}_i, \dots, \widehat{\psi}_j, \dots, \psi_p, \psi_{p+1}) \\ &= \left[\sum_{i=1}^{p+1} (-1)^{i+1} (f_*\psi_i) [\xi(f_*\psi_1, f_*\psi_2, \dots, f_*\widehat{\psi}_i, \dots, f_*\psi_{p+1})] \right] \circ f \\ &\quad + \left[\sum_{i < j} (-1)^{i+j} \xi([f_*\psi_i, f_*\psi_j], f_*\psi_1, \dots, f_*\widehat{\psi}_i, \dots, f_*\widehat{\psi}_j, \dots, f_*\psi_p, f_*\psi_{p+1}) \right] \circ f. \end{aligned}$$

On the other hand, we get

$$\begin{aligned} &[f^*(\delta\xi)](\psi_1, \psi_2, \dots, \psi_p, \psi_{p+1}) \\ &= [(\delta\xi)(f_*\psi_1, f_*\psi_2, \dots, f_*\psi_{p+1})] \circ f \\ &= \left[\sum_{i=1}^{p+1} (-1)^{i+1} (f_*\psi_i) [\xi(f_*\psi_1, f_*\psi_2, \dots, f_*\widehat{\psi}_i, \dots, f_*\psi_{p+1})] \right] \circ f \\ &\quad + \left[\sum_{i < j} (-1)^{i+j} \xi([f_*\psi_i, f_*\psi_j], f_*\psi_1, \dots, f_*\widehat{\psi}_i, \dots, f_*\widehat{\psi}_j, \dots, f_*\psi_{p+1}) \right] \circ f. \end{aligned}$$

That ends the proof. □

Proposition 10. *If f is a contactomorphism i.e. $f^*i_1\omega_{/\mathcal{X}(M)} = i_1\omega_{/\mathcal{X}(M)}$ then $f^*i_1\omega = i_1\omega$.*

Proof. Suppose that f is a contactomorphism then for any vector field X on M , we obtain

$$[(i_1\omega)(f_*X)] \circ f = (i_1\omega)(X).$$

For any $\varphi \in \mathcal{D}(M)$, $\varphi = \varphi(1) + X$, and

$$\begin{aligned} (i_1\omega)(f_*\varphi) &= (i_1\omega)(f_*X) \\ &= [(i_1\omega)(X)] \circ f^{-1} \\ &= [(i_1\omega)(\varphi)] \circ f^{-1}, \end{aligned}$$

then $[(i_1\omega)(f_*\varphi)] \circ f = (i_1\omega)(\varphi)$ so $f^*i_1\omega = i_1\omega$. □

Theorem 11. *The map f is a contactomorphism if and only if $f^*\omega = \omega$.*

Proof. Suppose that f is a contactomorphism, for all $\varphi, \psi \in \mathcal{D}(M)$, we have

$$\begin{aligned} \omega(f_*\varphi, f_*\psi) &= [\delta(i_1\omega)(f_*\varphi, f_*\psi)] \\ &= (f_*\varphi)[(i_1\omega)(f_*\psi)] - (f_*\psi)[(i_1\omega)(f_*\varphi)] - (i_1\omega)[f_*\varphi, f_*\psi] \\ &= \varphi[(i_1\omega)(\psi)] \circ f^{-1} - \psi[(i_1\omega)(\varphi)] \circ f^{-1} - (i_1\omega)[\varphi, \psi] \circ f^{-1} \\ &= \omega(\varphi, \psi) \circ f^{-1}, \\ [\omega(f_*\varphi, f_*\psi)] \circ f &= \omega(\varphi, \psi). \end{aligned}$$

Thus $f^*\omega = \omega$.

Conversely let $f^*\omega = \omega$. For $\varphi = 1$, we have $f_*\varphi = 1$. For φ, ψ elements of $\mathcal{D}(M)$, $\omega(f_*\varphi, f_*\psi) = \omega(\varphi, \psi) \circ f^{-1}$, then

$$\begin{aligned} \omega(1, f_*\psi) &= \omega(1, \psi) \circ f^{-1} \\ (i_1\omega)(f_*\psi) &= (i_1\omega)(\psi) \circ f^{-1}. \end{aligned}$$

In particular if $\psi = Y$, we have

$$[(i_1\omega)(f_*Y)] \circ f = (i_1\omega)(Y),$$

then $f^*i_1\omega_{/\mathcal{X}(M)} = i_1\omega_{/\mathcal{X}(M)}$.

That ends the proof. □

Let $\mathcal{D}iff_\omega(M)$ be the group of $\mathcal{C}^\infty(M)$ -diffeomorphisms of M , which preserve ω . This group is the automorphism group of the contact geometry of (M, ω) . It acts transitively on the contact manifold (provided that is connected). Hence the contact manifold (M, ω) can be viewed as a homogeneous space of $\mathcal{D}iff_\omega(M)$.

Theorem 12. *Let (M, ω) and (M', ω') be two smooth contact manifolds equipped with contact structures ω and ω' respectively. Let $\gamma : M \rightarrow M'$ be a bijective map such that for any map $f : M \rightarrow M$, $\gamma f \gamma^{-1} \in \mathcal{D}iff_{\omega'}(M')$ iff $f \in \mathcal{D}iff_\omega(M)$. Then γ is a $\mathcal{C}^\infty(M)$ -diffeomorphism and $\gamma^*\omega' = \omega$.*

Proof. We use the Taken's theorem (see Takens, 1979) and the same arguments as in (Banyaga, 1986). □

Theorem 13. *Suppose a group isomorphism $\tau : \mathcal{D}iff_\omega(M) \rightarrow \mathcal{D}iff_{\omega'}(M')$ is also a homeomorphism when $\mathcal{D}iff_\omega(M)$ and $\mathcal{D}iff_{\omega'}(M')$ are endowed with the point-open topology. Then there exists a $\mathcal{C}^\infty(M)$ -diffeomorphism $\gamma : M \rightarrow M'$ such that $\tau(f) = \gamma f \gamma^{-1}$, for any $f \in \mathcal{D}iff_\omega(M)$ and $\gamma^*\omega' = \omega$.*

Proof. Note that, these groups are γ -transitive. There exists a homeomorphism $\gamma : M \rightarrow M'$ such that $\tau(f) = \gamma f \gamma^{-1}$, for any $f \in \mathcal{D}iff_\omega(M)$. Using the above theorem, we have $\gamma^*\omega' = \omega$. □

Corollary 14. *When an automorphism of the group of contact diffeomorphisms $\mathcal{D}iff_\omega(M)$ of contact manifold (M, ω) is also a homeomorphism for the point-open topology, then it is an inner automorphism.*

4. A Characterization of Contact Diffeomorphisms

Let

$$id : \mathcal{D}(M) \longrightarrow \mathcal{D}(M)$$

be the identically map of $\mathcal{D}(M)$. One notes $\delta = d_{id}$, the cohomology operator associated with the representation id . Let

$$\omega : \mathcal{D}(M) \times \mathcal{D}(M) \longrightarrow \mathcal{C}^\infty(M)$$

be a nondegenerate skew-symmetric 2-form on M .

Proposition 15. *When*

$$f : M \longrightarrow M$$

satisfies $f^ \omega = \omega$, then for every function $u \in \mathcal{C}^\infty(M)$, we have*

$$f_* \Phi_u = \Phi_{u \circ f}.$$

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