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A Note on Testing Serial Correlation in Partially Linear Additive Measurement Error Models

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Abstract

The paper considers the problem of testing error serial correlation of partially linear additive measurement error model. We propose a test statistic and show that it converges to the standard chi-square distribution under the null hypothesis. Finally, a simulation study is conducted to illustrate the performance of the test approach.

Keywords: corrected profile least-squares, measurement error models, partially linear additive model, serial correlation

1. Introduction

Recently, testing serial correlation for semiparametric models have attracted more and more attention, see Li and Hsiao (1998) for semiparametric panel data models, Li and Stengos (2003) for semiparametric time series models, Liu *et al.* (2008) and Hu *et al.* (2009) for partially linear errors-in-variables models, Zhou *et al.* (2010) for partially nonlinear models, and Liu *et al.* (2011) for partially linear single-index errors-in-variables models. Recently, Yang *et al.*(2015) constructed a test statistic based on empirical likelihood method to test serial correlation for the following semiparametric partially linear additive measurement errors model

$$\begin{cases} Y = \mathbf{X}^{\mathrm{T}}\boldsymbol{\beta} + m_1(Z_1) + \cdots + m_q(Z_q) + \varepsilon, \\ \mathbf{V} = \mathbf{X} + \boldsymbol{\eta}, \end{cases} \tag{1.1}$$

where $Y, \mathbf{X}, Z_1, Z_2, \cdots, Z_q$ are response variable and explanatory variables, respectively. $\boldsymbol{\beta} = (\beta_1, \beta_2, \cdots, \beta_l)^{\mathrm{T}}$ are unknown regression coefficients, m_j is a unknown univariate component function, model error ε has mean zero and variance σ^2. We assume that $E\{m_k(Z_k)\} = 0, k = 1, 2, \cdots, q$ for identifiability. Measurement errors $\boldsymbol{\eta}$ are independent of $(Y, \mathbf{X}, Z_1, \cdots, Z_q)$, $\mathrm{E}\boldsymbol{\eta} = \mathbf{0}$, $\mathrm{Cov}\boldsymbol{\eta} = \boldsymbol{\Sigma}_{\boldsymbol{\eta}}$. In this paper, based on some results of Yang *et al.* (2015), we will propose an alternative test approach for model (1.1).

The rest is organized as follows. A new test statistic is constructed in Section 2. Section 3 conducted simulation studies to illustrate the performance of the proposed approach. Conclusion is given in Section 4.

2. Test Statistic and Its Properties

For notational simplicity, we assume $q = 2$ in model (1.1) as Liang *et al.* (2008). Suppose $\{Y_i, \mathbf{V}_i, Z_{1i}, Z_{2i}\}_{i=1}^n$ are generated from model (1.1), then we can get

$$\begin{cases} Y_i = \mathbf{X}_i^{\mathrm{T}}\boldsymbol{\beta} + m_1(Z_{1i}) + m_2(Z_{2i}) + \varepsilon_i, \\ \mathbf{V}_i = \mathbf{X}_i + \boldsymbol{\eta}_i. \end{cases} \tag{2.1}$$

For this model, we consider the problem of model error ε_i is serially uncorrelated or not. Then, The null hypothesis is $\varepsilon_i, i = 1, 2, \cdots, n$ are independent, and the alternative hypothesis is a p-th order moving average (MA(p)), or p-th order autoregression AR(p), namely,

$$\varepsilon_i = \mu_i + \alpha_1\mu_{i-1} + \cdots + \alpha_p\mu_{i-p}, \quad \mu_i \quad \text{i.i.d.} \quad (0, \sigma^2),$$

$$\varepsilon_i = \mu_i + \alpha_1\varepsilon_{i-1} + \cdots + \alpha_p\varepsilon_{i-p}, \quad \mu_i \quad \text{i.i.d.} \quad (0, \sigma^2),$$

where coefficients α_i satisfies some stationary conditions, for example, the roots of equation $\alpha(\mu) = 1 - \alpha_1\mu - \alpha_2\mu^2 - \cdots - \alpha_p\mu^p = 0$ lie outside the unit circle.

Let $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_p)^{\mathrm{T}}, \gamma_k = E\varepsilon_i\varepsilon_{i+k}, \quad k = 1, 2, \cdots, p, \quad i = 1, 2, \cdots, N$, with $N = n - p$. We can show that testing ε_i is serially uncorrelated is equivalent to testing $\boldsymbol{\gamma} = \mathbf{0}$. By the method of Liu *et al.* (2008) and Hu *et al.* (2008), denote $e_i = \varepsilon_i - \boldsymbol{\eta}_i^{\mathrm{T}}\boldsymbol{\beta}, \bar{\boldsymbol{\gamma}} = (\bar{\gamma}_1, \cdots, \bar{\gamma}_p)^{\mathrm{T}}$, with $\bar{\gamma}_k = Ee_ie_{i+k}$. By the fact that ε_i is independent of $\boldsymbol{\eta}_i$, then if the null hypothesis of no serial correlation is true, we can obtain

$$\bar{\gamma}_k = Ee_ie_{i+k} = E(\varepsilon_i - \boldsymbol{\eta}_i^{\mathrm{T}}\boldsymbol{\beta})(\varepsilon_{i+k} - \boldsymbol{\eta}_{i+k}^{\mathrm{T}}\boldsymbol{\beta}) = E\varepsilon_i\varepsilon_{i+k} = \gamma_k. \tag{2.2}$$

Let $\mathbf{U}_i = (U_{i1}, \cdots, U_{ip})^{\mathrm{T}}, U_{ik} = e_ie_{i+k}, k = 1, 2, \cdots, p, i = 1, 2, \cdots, N$, then the null hypothesis reduces to $E\mathbf{U}_i = \mathbf{0}$. To construct the test statistics based on $\mathbf{U}_i$, we can replace unknown e_i in $\mathbf{U}_i$ by its estimator. In the following, the corrected profile least-squares approach of Liang *et al.* (2008) is applied to estimate model (1.1).

If $\mathbf{X}$ can be observed exactly, and $\boldsymbol{\beta}$ is known, model (2.1) can be rewritten as the following standard bivariate additive model of Opsomer and Ruppert (1997)£

$$Y_i - \mathbf{X}_i^{\mathrm{T}}\boldsymbol{\beta} = m_1(Z_{1i}) + m_2(Z_{2i}) + \varepsilon_i, \quad i = 1, 2, \cdots, n. \tag{2.3}$$

Let $\mathbf{Y} = (Y_1, Y_2, \cdots, Y_n)^{\mathrm{T}}, \mathbf{m}_k = (m_k(Z_{k1}), m_k(Z_{k2}), \cdots, m_k(Z_{kn})), \mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_n)^{\mathrm{T}}$,

$$s_{1,z_1}^{\mathrm{T}} = \mathbf{e}_1^{\mathrm{T}}\{\mathbf{D}_1^{\mathrm{T}}\mathbf{K}_1\mathbf{D}_1\}^{-1}\mathbf{D}_1^{\mathrm{T}}\mathbf{K}_1, s_{2,z_2}^{\mathrm{T}} = \mathbf{e}_1^{\mathrm{T}}\{\mathbf{D}_2^{\mathrm{T}}\mathbf{K}_2\mathbf{D}_2\}^{-1}\mathbf{D}_2^{\mathrm{T}}\mathbf{K}_2,$$

with $\mathbf{e}_1 = (1, 0)^{\mathrm{T}}$, $\mathbf{K}_1 = \mathrm{diag}\{K_{h_1}(Z_{11} - z_1), K_{h_1}(Z_{12} - z_1), \cdots, K_{h_1}(Z_{1n} - z_1)\}$, $\mathbf{K}_2 = \mathrm{diag}\{K_{h_2}(Z_{21} - z_2), K_{h_2}(Z_{22} - z_2), \cdots, K_{h_2}(Z_{2n} - z_2)\}$, $K_{h_k}(\cdot) = K(\cdot/h_k)/h_k$, $K(\cdot)$ is a kernel function and h_k is a bandwidth, $k = 1, 2$.

$$\mathbf{S}_1 = \begin{bmatrix} s_{1,Z_{11}}^{\mathrm{T}} \\ s_{1,Z_{12}}^{\mathrm{T}} \\ \vdots \\ s_{1,Z_{1n}}^{\mathrm{T}} \end{bmatrix}, \mathbf{S}_2 = \begin{bmatrix} s_{1,Z_{21}}^{\mathrm{T}} \\ s_{1,Z_{22}}^{\mathrm{T}} \\ \vdots \\ s_{1,Z_{2n}}^{\mathrm{T}} \end{bmatrix}, \mathbf{D}_1 = \begin{bmatrix} 1 & Z_{11} - z_1 \\ 1 & Z_{12} - z_1 \\ \vdots & \vdots \\ 1 & Z_{1n} - z_1 \end{bmatrix}, \mathbf{D}_2 = \begin{bmatrix} 1 & Z_{21} - z_2 \\ 1 & Z_{22} - z_2 \\ \vdots & \vdots \\ 1 & Z_{2n} - z_2 \end{bmatrix},$$

By the backfitting method of Opsomer and Ruppert (1997), the estimators of $\mathbf{m}_1$ and $\mathbf{m}_2$ can be defined as

$$\hat{\mathbf{m}}_1 = \mathbf{W}_1(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}), \quad \hat{\mathbf{m}}_2 = \mathbf{W}_2(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}), \tag{2.4}$$

with $\mathbf{S}_k^* = (\mathbf{I}_n - \mathbf{1}\mathbf{1}^{\mathrm{T}})\mathbf{S}_k, k = 1, 2$,

$$\mathbf{W}_1 = \mathbf{I}_n - (\mathbf{I}_n - \mathbf{S}_1^*\mathbf{S}_2^*)^{-1}(\mathbf{I}_n - \mathbf{S}_1^*), \quad \mathbf{W}_2 = \mathbf{I}_n - (\mathbf{I}_n - \mathbf{S}_2^*\mathbf{S}_1^*)^{-1}(\mathbf{I}_n - \mathbf{S}_2^*).$$

Replaced $\mathbf{m}_1$ and $\mathbf{m}_2$ of model (2.3) by their estimators $\hat{\mathbf{m}}_1$ and $\hat{\mathbf{m}}_2$, respectively, we can obtain the following linear model

$$Y_i - \hat{Y}_i = (\mathbf{X}_i - \hat{\mathbf{X}}_i)^{\mathrm{T}}\boldsymbol{\beta} + \varepsilon_i - \hat{\varepsilon}_i, i = 1, 2, \cdots, n, \tag{2.5}$$

with $\hat{\mathbf{Y}} = (\hat{Y}_1, \cdots, \hat{Y}_n)^{\mathrm{T}} = \mathbf{S}\mathbf{Y}, \hat{\mathbf{X}} = (\hat{\mathbf{X}}_1, \cdots, \hat{\mathbf{X}}_n)^{\mathrm{T}} = \mathbf{S}\mathbf{X}, \hat{\boldsymbol{\varepsilon}} = (\hat{\varepsilon}_1, \cdots, \hat{\varepsilon}_n)^{\mathrm{T}} = \mathbf{S}\boldsymbol{\varepsilon}$, and $\mathbf{S} = \mathbf{W}_1 + \mathbf{W}_2, \boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n)$.

If we can observe $\mathbf{X}_i$ without measurement error, Then the profile least squares estimator of $\boldsymbol{\beta}$ can be obtained by applying the least-squares approach to linear model (2.5). To solve the problem that $\mathbf{X}_i$ cannot be observed exactly, Liang *et al.* (2008) constructed the following corrected profile least-squares estimator of $\boldsymbol{\beta}$ by the correction for attenuation technique,

$$\hat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}\in R^l}\left[(\bar{\mathbf{Y}} - \bar{\mathbf{V}}\boldsymbol{\beta})^{\mathrm{T}}(\bar{\mathbf{Y}} - \bar{\mathbf{V}}\boldsymbol{\beta}) - n\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{\Sigma}_\eta\boldsymbol{\beta}\right] = (\bar{\mathbf{V}}^{\mathrm{T}}\bar{\mathbf{V}} - n\boldsymbol{\Sigma}_\eta)^{-1}\bar{\mathbf{V}}^{\mathrm{T}}\bar{\mathbf{Y}}, \tag{2.6}$$

where $\bar{\mathbf{Y}} = \mathbf{Y} - \hat{\mathbf{Y}}, \bar{\mathbf{V}} = \mathbf{V} - \hat{\mathbf{V}}, \hat{\mathbf{V}} = (\hat{\mathbf{V}}_1, \cdots, \hat{\mathbf{V}}_n)^{\mathrm{T}} = \mathbf{S}\mathbf{V}$ and $\mathbf{V} = (\mathbf{V}_1, \cdots, \mathbf{V}_n)^{\mathrm{T}}$. Therefore, the estimators of $\mathbf{m}_1$ and $\mathbf{m}_2$ can be defined as

$$\hat{\mathbf{m}}_1 = \mathbf{W}_1(\mathbf{Y} - \mathbf{V}\hat{\boldsymbol{\beta}}), \quad \hat{\mathbf{m}}_2 = \mathbf{W}_2(\mathbf{Y} - \mathbf{V}\hat{\boldsymbol{\beta}}). \tag{2.7}$$

By the idea of Li and Stengos (2003) and Hu *et al.* (2009), we construct the following test statistic based on $\mathbf{U}_i$ with e_i replaced by $\hat{e}_i = y_i - \mathbf{V}_i^{\mathrm{T}}\hat{\boldsymbol{\beta}} - \hat{m}_1(Z_{1i}) - \hat{m}_2(Z_{2i})$ as

$$T_N = \hat{\sigma}^{-2}\mathbf{V}_N^{\mathrm{T}}\mathbf{V}_N \tag{2.8}$$

with $\hat{\sigma}^{-2} = \frac{1}{n}\sum_{i=1}^{n}\hat{e}_i^2, \mathbf{V}_N = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\boldsymbol{\xi}_i$, and $\boldsymbol{\xi}_i = (\xi_{i1}, \cdots, \xi_{ip})^{\mathrm{T}}, \xi_{ik} = \hat{e}_i\hat{e}_{i+k}$.

For $\mathbf{V}_N$ and T_N, we have the following result. The following assumptions are given by Liang *et al.* (2008) and they are can be easily satisfied.

Assumption 1. The function $K(\cdot)$ is a bounded symmetric density function with compact support.

Assumption 2. The densities $f_k(Z_k)$ of Z_k are Lipschitz continuous and bounded away from 0, and have bounded supports $\mathbf{\Omega}_k$ for $k = 1, 2$.

Assumption 3. The second derivatives of $m_k(\cdot), k = 1, 2$ exist and are bounded and continuous.

Assumption 4. As $n \to \infty$, $h_k \to 0$, $nh_k/\log n \to \infty$ and $nh_k^8 \to 0$ for $k = 1, 2$.

Theorem 2.1 *Under the assumptions 1-4, if the null hypothesis of no serial correlation is true, we have*

$$\mathbf{V}_N \xrightarrow{D} N(\mathbf{0}, \sigma_0^2 \mathrm{I}_p), \quad T_N \xrightarrow{D} \chi_p^2.$$

where $\sigma_0 = \sigma^2 + \boldsymbol{\beta}^{\mathrm{T}} \mathbf{\Sigma}_{\eta} \boldsymbol{\beta}$, χ_p^2 *is a* χ^2*-distribution with p degrees of freedom.*

Proof of Theorem 2.1 By the Lemma 4.3 of Yang *et al.* (2015), we have

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \boldsymbol{\xi}_i \xrightarrow{D} N(\mathbf{0}, \sigma_0^2), \sigma_0 = \sigma^2 + \boldsymbol{\beta}^{\mathrm{T}} \mathbf{\Sigma}_{\eta} \boldsymbol{\beta}.$$

Therefore, we can obtain $T_N \xrightarrow{D} \chi_p^2$ as $N \to \infty$. This completes the proof.

3. Simulation Studies

To illustrate the performance of our test statistic, we conduct a simulation study in this section. Following Yang *et al.* (2015), we consider the following model

$$y_i = x_i\beta + m_1(z_{1i}) + m_2(z_{2i}) + \varepsilon_i, \quad v_i = x_i + \eta_i, i = 1, 2, \cdots, n,$$

with $x_i \sim N(0, 1)$, $z_{1i} \sim U(0, 1)$, $z_{2i} \sim U(-1, 1)$, $\eta_i \sim N(0, 0.25)$, and $\beta = 1$, $m_1(z_{1i}) = 2\cos(2\pi z_{1i})$, $m_2(z_{2i}) = z_{2i}^4 + 2z_{2i}^3 + 3z_{2i}^2 - 2z_{2i} - 2$, and ε_i are generated from the following different processes:

(1) AR(1) error: $\varepsilon_i = \rho\varepsilon_{i-1} + \mu_i$; (2) MA(1) error: $\varepsilon_i = \rho\mu_{i-1} + \mu_i$;
(3) AR(2) error: $\varepsilon_i = \alpha_1\varepsilon_{i-1} + \alpha_2\varepsilon_{i-2} + \mu_i$; (4) MA(2) error: $\varepsilon_i = \alpha_1\mu_{i-1} + \alpha_2\mu_{i-2} + \mu_i$.

For model error μ_i, three different distributions are discussed, (1)$\mu_i \sim N(0, 0.5^2)$, (2)$\mu_i \sim U(-\sqrt{3}/2, \sqrt{3}/2)$, (3) $\mu_i \sim \frac{1}{8}\chi_8^2 - 1$. The Epanechnikov kernel $K(x) = 0.75(1 - x^2)\mathbf{I}_{|x|\le 1}$ and bandwidths $h_1 = h_2 = n^{-1/5}$ are used in the simulations.

Take $\rho = 0, \pm 0.2, \pm 0.5, \pm 0.8$ and $(\alpha_1, \alpha_2) = (0, 0), (0, 0.5), (0.3, 0.4), (0.2, -0.6), (-0.4, 0.5)$ and $n = 100, 200$. For each case, 1000 replications were run, we let the significance level $\alpha = 0.05$, the rejection rate at was computed as the estimated size and power of the test. The results are presented in Tables 1-4.

Table 3.1. Empirical size and power for $H_0 : \rho = 0$ when $\varepsilon_i = \rho\varepsilon_{i-1} + \mu_i$

		Error		Distribution
n	ρ	$N(0, 0.5^2)$	$U(-\sqrt{3}/2, \sqrt{3}/2)$	$\frac{1}{8}\chi_8^2 - 1$
n=100	0	0.048	0.042	0.048
	0.2	0.082	0.093	0.094
	-0.2	0.114	0.145	0.130
	0.5	0.507	0.462	0.474
	-0.5	0.570	0.588	0.552
	0.8	0.956	0.956	0.947
	-0.8	0.980	0.978	0.972
n=200	0	0.059	0.045	0.044
	0.2	0.192	0.195	0.231
	-0.2	0.246	0.260	0.239
	0.5	0.872	0.880	0.891
	-0.5	0.89	0.906	0.904
	0.8	1.000	1.000	1.000
	-0.8	1.000	1.000	1.000

From the results, we can found that if the null hypothesis is true (that is $\rho = 0$ or $(\alpha_1, \alpha_2) = (0, 0)$), the estimated sizes are quite good, and the estimated sizes converge toward their nominal sizes as n increases . Under the alternative hypothesis,

Table 3.2. Empirical size and power for $H_0 : \rho = 0$ when $\varepsilon_i = \rho u_{i-1} + \mu_i$

		Error		Distribution
n	ρ	$N(0, 0.5^2)$	$U(-\sqrt{3}/2, \sqrt{3}/2)$	$\frac{1}{8}\chi_8^2 - 1$
n=100	0	0.047	0.040	0.041
	0.2	0.080	0.100	0.091
	-0.2	0.123	0.113	0.113
	0.5	0.295	0.336	0.314
	-0.5	0.413	0.394	0.380
	0.8	0.547	0.601	0.585
	-0.8	0.626	0.649	0.658
n=200	0	0.049	0.053	0.041
	0.2	0.182	0.171	0.202
	-0.2	0.222	0.232	0.217
	0.5	0.719	0.745	0.713
	-0.5	0.781	0.792	0.761
	0.8	0.937	0.957	0.946
	-0.8	0.958	0.957	0.959

Table 3.3. Empirical size and power for $H_0 : \alpha_1 = \alpha_2 = 0$ when $\varepsilon_i = \alpha_1 \varepsilon_{i-1} + \alpha_2 \varepsilon_{i-2} + \mu_i$

		Error		Distribution
n	(α_1, α_2)	$N(0, 0.5^2)$	$U(-\sqrt{3}/2, \sqrt{3}/2)$	$\frac{1}{8}\chi_8^2 - 1$
n=100	(0,0)	0.047	0.053	0.044
	(0,0.5)	0.434	0.434	0.438
	(0.3,0.4)	0.669	0.667	0.637
	(0.2,-0.6)	0.651	0.624	0.616
	(-0.4,0.5)	0.980	0.987	0.980
n=200	(0,0)	0.050	0.041	0.056
	(0,0.5)	0.810	0.827	0.833
	(0.3,0.4)	0.956	0.961	0.966
	(0.2,-0.6)	0.960	0.957	0.964
	(-0.4,0.5)	1.000	1.000	1.000

Table 3.4. Empirical size and power for $H_0 : \alpha_1 = \alpha_2 = 0$ when $\varepsilon_i = \alpha_1 u_{i-1} + \alpha_2 u_{i-2} + \mu_i$

		Error		Distribution
n	(α_1, α_2)	$N(0, 0.5^2)$	$U(-\sqrt{3}/2, \sqrt{3}/2)$	$\frac{1}{8}\chi_8^2 - 1$
n=100	(0,0)	0.055	0.048	0.043
	(0,0.5)	0.288	0.254	0.265
	(0.3,0.4)	0.297	0.295	0.300
	(0.2,-0.6)	0.358	0.358	0.343
	(-0.4,0.5)	0.511	0.522	0.510
n=200	(0,0)	0.048	0.055	0.055
	(0,0.5)	0.617	0.615	0.631
	(0.3,0.4)	0.711	0.668	0.669
	(0.2,-0.6)	0.748	0.758	0.774
	(-0.4,0.5)	0.888	0.893	0.899

the rejection rate seems very robust to the variation of the type of error distribution, and increases rapidly as the alternative hypothesis deviates from the null hypothesis. Compared with the results of Yang *et al.* (2015), our test performs well.

4. Conclusion

To test serial correlation of partially linear additive measurement errors models, a test statistic was proposed. The test statistic is shown to have asymptotic chi-square distribution under the null hypothesis of no serial correlation. Some simulations are conducted to illustrate the performance of the proposed method and and the results are satisfactory.

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