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Algorithm on Trees With Knots and Their Folding

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Abstract

In this paper, we introduced an algorithm on trees with knots when the trees are weighted. The folding of the algorithm on tree with knots in many cases such as: folding of the edges and folding of the knots are discussed .Some theorems related to these results are obtained. Also, some applications are introduced.

Keywords: algorithm, knots, folding, trees, weighted

1. Introduction and Background

In mathematics and computer science, an algorithm is an effective method expressed as a finite list of well-defined instruction for calculating a function. In simple words an algorithm is a step-by-step procedure for calculations.

Graph algorithms are one of the oldest classes of algorithms and they have been studied for almost 300 years (in 1736) which solve problems related to graph theory. There are some of important algorithms for solving these problems.

Weighted graph is a graph for which each edge has an associated real number weight.

In Kruskal's algorithm, the edges of weighted graph are examined one by one in order. What will of increasing weight. At each stage the edge being examined is added to become the minimum spanning tree, provided that this addition doesn't create a circuit.

After n -1 edges have been added (where n is the number of vertices of the graph), these edges, together with the vertices of the graph form a minimum spanning tree for the graph.

The weighted Kruskal's algorithm

How it works:

Input: G (weighted connected undirected graph with n vertices).

Algorithm body:

Build a subgraph T of G to consist of all the vertices of G with edges added at each stage.

1. Initialized T (empty graph) to have all vertices of G.
2. Let E be the set of all edges of G.
3. Find an edge e in E of least weight.
4. Delete e from E.
5. If addition of e to edge set of T doesn't produce a circuit.

Then add e to the edge set of T.

T is a minimum spanning tree of G.

Tree is a connected graph which contains no cycles or loops is called a tree.

Minimal spanning tree for a weighted graph is a spanning tree that has at least possible total weight compared to all other spanning trees for the graphs.

It is minimum spanning tree in a connected weighted graph with $n \geq 1$ vertex carry out the following procedure:

Step (1) Find an edge of least weight and call this e^1. Set k=1

step (2) While k<n, if there exists an edge e such that $\{e\} \cup \{e^1,\dots,e^k\}$ does not contain a circuit, let e^{k+1} be such an edge of least weight replace k by k+1, else output $e^1,e^2,\dots,e^k$ and stop.

End while.

In mathematics a knot is a subset of 3- space homeomorphic to the unit circle, while the link is a union of finitely many disjoint knots. The individual knots that make up a link are called its components (so a knot is a link with just one component, i.e. a connected link).

A singular knot is a knot with self-intersection. Figure (1.1) shows left handed trefoil (with anticlockwise direction), right handed trefoil (with clockwise direction), Hopf link and singular trefoil knot.

Left hand trefoil Right hand trefoil Hopf link Singular trefoil knot

Figure (1.1)

We define the tree with knot which is a connected graph that contains number of knots. The field of folding began with S.A. Robertson's work , in 1977 , on isometric folding of Riemannian manifold M into N , which send any piecewise geodesic path in M to a piecewise geodesic path with the same length in N. Loop it is edge joining a vertex to itself.

2. Main Results

El-Ghoul, M. submitted the work of a knot with trees and also algorithms work on some types of graphs, in this paper we will introduce algorithm on trees with knots when the trees are weighted and their folding.

2.1 Algorithm on Tree With Knots

We will compute the algorithm on tree with knots when the tree weighted by Kruscal's Algorithm.

Let G be a tree with knots have four vertices v^0,v^1,v^2,v^3 and three edges e^0,e^1,e^2 with two knots k^0, k^1 then we can compute its by Kruscal's algorithm.

The weight of the tree is knowing, such as: G ($v^0v^1= \varepsilon^0$, $v^1v^2= \varepsilon^1$, $v^1v^3= \varepsilon^2$, $k^0= \varepsilon^3$, $k^1= \varepsilon^4$), where ($\varepsilon^0 > \varepsilon^1$, $\varepsilon^1 > \varepsilon^2$, $\varepsilon^3 < \varepsilon^1$, $\varepsilon^4 < \varepsilon^2$), see Figure (2.1.1).

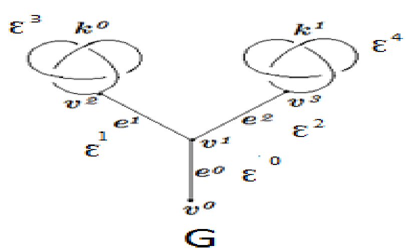

Figure (2.1.1)

By using Kruscal's algorithm we find the minimum spanning tree as follows in table (2.1.1).

Table (2.1.1)

Iteration no.	Considered	Weight	Action taken
1	K^1	ε^4	Not added
2	K^0	ε^3	Not added
3	V^1-v^3	ε^2	Added
4	V^1-v^2	ε^1	Added
5	V^1-v^0	ε^0	Added

The minimum spanning tree is a tree without knots, shown in Figure (2.1.2).

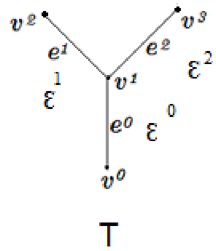

Figure (2.1.2)

Theorem (1)

The algorithm on weighted tree with knots goes to weighted normal tree.

Proof

The proof is clear from the above discussion, see Figure (2.1.1) and Figure (2.1.2).

2.2 Folding of Algorithm on Tree With Knots

In this section we will discusses the folding of algorithm on tree with knots, the folding of algorithm on tree with knots have many cases.

First: Folding of the edges:

Case (1)

Here, the folding of algorithm decrease the length of the edges e^0,e^1,e^2. The limit of the folding of algorithm give a graph without edges, see Figure (2.2.1).

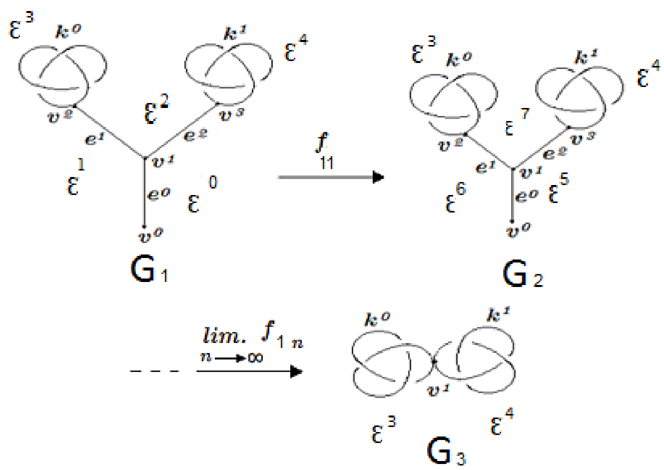

Figure (2.2.1)

Here, the weight of $G_1(v^0v^1=\varepsilon^0, v^1v^2=\varepsilon^1, v^1v^3=\varepsilon^2, k^0=\varepsilon^3, k^1=\varepsilon^4)$, $G_2(v^0v^1=\varepsilon^5, v^1v^2=\varepsilon^6, v^1v^3=\varepsilon^7, k^0=\varepsilon^3, k^1=\varepsilon^4)$, $G_3(k^0=\varepsilon^3, k^1=\varepsilon^4)$, where $(\varepsilon^0>\varepsilon^1, \varepsilon^1>\varepsilon^2, \varepsilon^5<\varepsilon^0, \varepsilon^6<\varepsilon^1, \varepsilon^7<\varepsilon^2, \varepsilon^3<\varepsilon^6, \varepsilon^4<\varepsilon^5)$.

By using Kruscal's algorithm we find the minimum spanning tree of the result of the folding

As follows in table (2.2.1).

Table (2.2.1)

Iteration no.	Considered	Weight	Action taken
1	K^1	ε^4	Not added
2	K^0	ε^3	Not added

The minimum spanning tree is a vertex v^1 (null graph), shown in Figure (2.2.2).

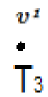

Figure (2.2.2)

Case (2)

The folding of algorithm acts on the edge e^0 and change it to a knot k^2, where $f_2(v^0) = v^1$,
$f_2(e^0) = k^2$, see Figure (2.2.3).

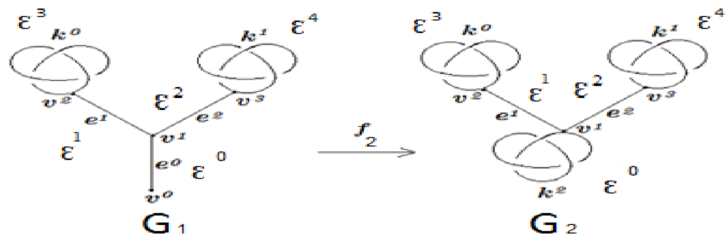

Figure (2.2.3)

Here, the weight of $G_1(v^0v^1=\varepsilon^0, v^1v^2=\varepsilon^1, v^1v^3= \varepsilon^2,\ k^0=\varepsilon^3,\ k^1=\varepsilon^4)$, $G_2\,(k^2 = \varepsilon^0,\ v^1v^2 = \varepsilon^1,\ v^1v^3= \varepsilon^2,\ k^0=\varepsilon^3,\ k^1=\varepsilon^4)$, where $(\varepsilon^0 > \varepsilon^1, \varepsilon^1 > \varepsilon^2, \varepsilon^3 < \varepsilon^1, \varepsilon^4 < \varepsilon^2)$.

By using Kruscal's algorithm we find the minimum spanning tree of the result of the folding as follows in table (2.2.2).

Table (2.2.2)

Iteration no.	Considered	Weight	Action taken
1	K^1	ε^4	Not added
2	K^0	ε^3	Not added
3	V^1-v^3	ε^2	Added
4	V^1-v^2	ε^1	Added
5	K^2	ε^0	Not added

The minimum spanning tree is a graph, shown in Figure (2.2.4).

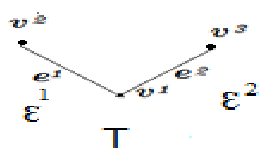

Figure (2.2.4)

Theorem (2)

The folding of algorithm on weighted tree with knots (in case folding the edges) goes to weighted graph.

Proof

The proof is clear from the above discussion, see Figure (2.2.1) to Figure (2.2.4).

Second: Folding of the knots:

Case (1)

In this case, the folding acts on the length of the knots k^0, k^1 until it reaches the null knots which represent the limit of folding. The result is the original tree, see Figure (2.2.5).

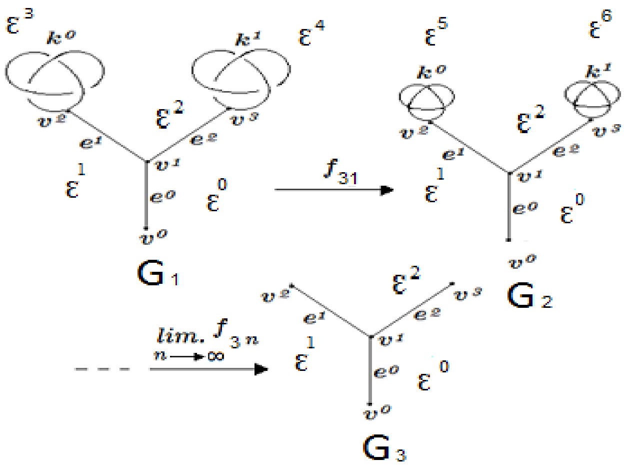

Figure (2.2.5)

Here, the weight of $G_1(v^0v^1= \varepsilon^0, v^1v^2= \varepsilon^1, v^1v^3= \varepsilon^2, k^0= \varepsilon^3, k^1= \varepsilon^4)$, $G_2(v^0v^1= \varepsilon^0, v^1v^2= \varepsilon^1, v^1v^3= \varepsilon^2, k^0= \varepsilon^5, k^1= \varepsilon^6)$, $G_3(v^0v^1= \varepsilon^0, v^1v^2= \varepsilon^1, v^1v^3= \varepsilon^2)$,where $(\varepsilon^0 > \varepsilon^1, \varepsilon^1 > \varepsilon^2, \varepsilon^3 < \varepsilon^1, \varepsilon^4 < \varepsilon^2, \varepsilon^5 < \varepsilon^3, \varepsilon^6 < \varepsilon^4)$.

By using Kruscal's algorithm we find the minimum spanning tree of the result of the folding as follows in table (2.2.3).

Table (2.2.3)

Iteration no.	Considered	Weight	Action taken
1	V^1-v^3	ε^2	Added
2	V^1-v^2	ε^1	Added
3	V^1-v^0	ε^0	Added

The minimum spanning tree is a normal tree, shown in Figure (2.2.6).

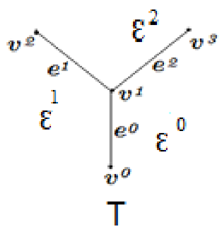

Figure (2.2.6)

Case (2)

In this case, the folding lead a changing in the knots k^0, k^1 from 3-dimensional knot into 2-dimension. The number of the vertices and the edges will increase, and these edges have weights.

Here, the weight of $G_1(v^0v^1= \varepsilon^0, v^1v^2= \varepsilon^1, v^1v^3=\varepsilon^2, k^0= \varepsilon^3, k^1= \varepsilon^4)$, $G_2(v^0v^1= \varepsilon^0, v^1v^2= \varepsilon^1, v^1v^3=\varepsilon^2)$, and the knot at v^2 consist of four vertices v^2, v^4, v^5, v^6 and seven edges $e^3, e^4, e^5, e^6, e^7, e^8, e^9$, the weight of this knot $k^0(v^2v^4=e^3=\varepsilon^5, v^4v^5=e^4=\varepsilon^6, v^4v^5=e^5=\varepsilon^7, v^5v^6=e^6=\varepsilon^8, v^5v^6=e^7=\varepsilon^9, v^6v^2=e^8=\varepsilon^{10}, v^4v^6=e^9=\varepsilon^{11})$, and the knot at v^3 consist of four vertices v^3, v^7, v^8, v^9 and seven edges $e^{10}, e^{11}, e^{12}, e^{13}, e^{14}, e^{15}, e^{16}$, the weight of this knot $k^1(v^3v^7=e^{10}=\varepsilon^{12}, v^7v^8=e^{11}=\varepsilon^{13}, v^7v^8=e^{12}=\varepsilon^{14}, v^8v^9=e^{13}=\varepsilon^{15}, v^8v^9=e^{14}=\varepsilon^{16}, v^9v^3=e^{15}=\varepsilon^{17}, v^7v^9= e^{16}=\varepsilon^{18})$,where $(\varepsilon^0 > \varepsilon^1, \varepsilon^1 > \varepsilon^2, \varepsilon^3 < \varepsilon^1, \varepsilon^4 < \varepsilon^2, \varepsilon^3 = \varepsilon^5+\varepsilon^6+ \varepsilon^7+\varepsilon^8+\varepsilon^9+\varepsilon^{10}+\varepsilon^{11}$ and $\varepsilon^6, \varepsilon^9 > \varepsilon^7, \varepsilon^8, \varepsilon^{11} > \varepsilon^5, \varepsilon^{10}$ and $\varepsilon^4=\varepsilon^{12}+\varepsilon^{13}+\varepsilon^{14}+ \varepsilon^{15}+\varepsilon^{16}+\varepsilon^{17}+\varepsilon^{18}$ and $\varepsilon^{13}, \varepsilon^{16}>\varepsilon^{14}, \varepsilon^{15}, \varepsilon^{18}>\varepsilon^{12}, \varepsilon^{17})$,see Figure (2.2.7).

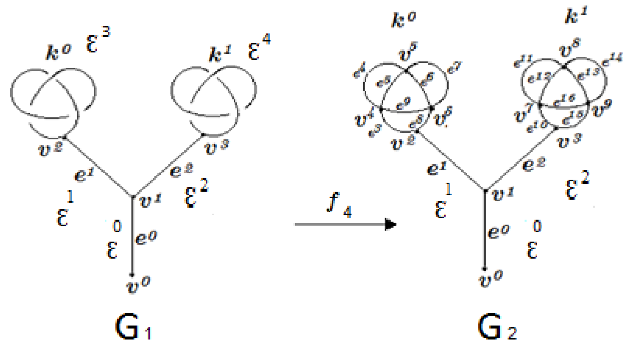

Figure (2.2.7)

By using Kruscal's algorithm we find the minimum spanning tree of the result of the folding as follows in table (2.2.4).

Table (2.2.4)

Iteration no.	Considered	Weight	Action taken
1	V^3-v^7	ε^{12}	Added
2	V^3-v^9	ε^{17}	Added
3	V^7-v^8	ε^{14}	Added
4	V^8-v^6	ε^{15}	Not added
5	V^9-v^7	ε^{18}	Not added
6	V^7-v^8	ε^{13}	Not added
7	V^8-v^9	ε^{16}	Not added
8	V^2-v^4	ε^{5}	Added
9	V^2-v^6	ε^{10}	Added
10	V^4-v^5	ε^{7}	Added
11	V^5-v^6	ε^{8}	Not added
12	V^6-v^4	ε^{11}	Not added
13	V^4-v^5	ε^{6}	Not added
14	V^5-v^6	ε^{9}	Not added
15	V^3-v^1	ε^{2}	Added
16	V^1-v^2	ε^{1}	Added
17	V^1-v^0	ε^{0}	Added

The minimum spanning tree is a tree, shown in Figure (2.2.8).

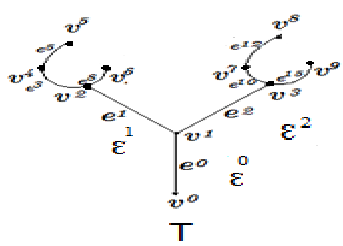

Figure (2.2.8)

Case (3)

Here, the folding on the knots leads a , a′ and b , b′ to coincide on each other on the vertex and change the knots k^0,k^1 into loops, where $f(k^i)=l^j$, $i=0,1,j=0,1,2,3$.

Here, the weight of $G_1(v^0v^1=\varepsilon^0, v^1v^2=\varepsilon^1, v^1v^3=\varepsilon^2, k^0=\varepsilon^3, k^1=\varepsilon^4)$, $G_2(v^0v^1=\varepsilon^0, v^1v^2=\varepsilon^1, v^1v^3=\varepsilon^2, f(k^0)=l^0, l^1, l^1, l^3$ and $l^0=\varepsilon^5, l^1=\varepsilon^6, l^2=\varepsilon^7, l^3=\varepsilon^8$,$f(k^1)=l^0, l^1, l^2, l^3$ and $l^0=\varepsilon^9, l^1=\varepsilon^{10}, l^2=\varepsilon^{11}, l^3=\varepsilon^{12})$,where $(\varepsilon^0>\varepsilon^1, \varepsilon^1>\varepsilon^2, \varepsilon^3<\varepsilon^1, \varepsilon^4<\varepsilon^2, \varepsilon^3=\varepsilon^5+\varepsilon^6+\varepsilon^7+\varepsilon^8$ and $\varepsilon^4=\varepsilon^9+\varepsilon^{10}+\varepsilon^{11}+\varepsilon^{12})$, see Figure (2.2.9).

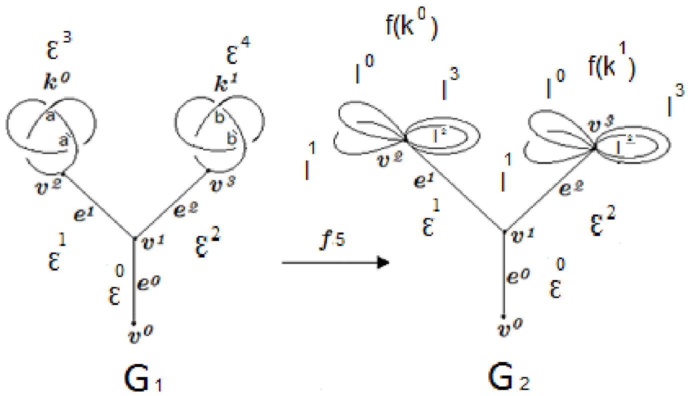

Figure (2.2.9)

By using Kruscal's algorithm we find the minimum spanning tree of the result of the folding as follows in table (2.2.5).

Table (2.2.5)

Iteration no.	Considered	Weight	Action taken
1	l^0 at $f(k^1)$	ε^9	Not added
2	l^1at $f(k^1)$	ε^{10}	Not added
3	l^2at $f(k^1)$	ε^{11}	Not added
4	l^3at $f(k^1)$	ε^{12}	Not added
5	l^0 at $f(k^0)$	ε^5	Not added
6	l^1at $f(k^0)$	ε^6	Not added
7	l^2at $f(k^0)$	ε^7	Not added
8	l^3at $f(k^0)$	ε^8	Not added
9	V^3-v^1	ε^2	Added
10	V^1-v^2	ε^1	Added
11	V^1-v^0	ε^0	Added

The minimum spanning tree is a tree, shown in Figure (2.2.10)

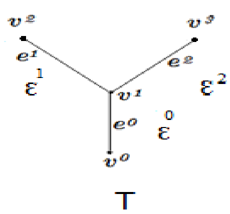

Figure (2.2.10)

Theorem (3)

The folding of algorithm on weighted tree with knots (in case folding the knots) goes to weighted tree.

Proof

The proof is clear from the above discussion, see Figure (2.2.5) to Figure (2.2.10).

3. Examples of Life

1-Trees laden with fruit and vegetables represent algorithm on weighted tree, and harvest this trees represent algorithm on weighted tree by Kruscal's algorithm and the minimum spanning tree is a tree without fruit and vegetables ,see Figure (3.1).

Figure (3.1)

2- Grape leaves trees represent algorithm on weighted trees with knot, see Figure (3.2)

Figure (3.2)

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