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Mathematical Modeling: Issues and Challenges in Mathematics Education and Teaching

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Abstract

Mathematics education researchers and policy documents in the United States have expressed the need to improve the teaching and learning of mathematical modeling at the K–12 levels so that students can apply their knowledge of mathematics to solve real-world situations. Unfortunately, most practicing teachers (PTs) and preservice teachers (PSTs) acquire didactical and pedagogical styles that do not support effective modeling practices. To investigate these dilemmas, this study examined PTs' pedagogical experiences in and PSTs' perspectives on mathematical modeling practices. Participants included 62 PTs and 18 PSTs from a Midwestern region of the United States. Data originated from questionnaire items and open-ended questions, which were analyzed quantitatively and qualitatively. Varied participants' ideas on mathematical modeling practices were identified, recorded, and summarized. Results indicated that most of these PTs and PSTs have little to no experiences with mathematical modeling practices and associated pedagogies. Such results along with a supplemental discussion have implications for teacher education programs and professional development centered on mathematical modeling education.

Keywords: experiences, mathematical modeling, modeling practices, teachers, teacher experiences

1. Introduction

In the United States, mathematical modeling, in which mathematics is used to solve real-world problems, is a mathematical skill and practice that is widely used in STEM fields as well as specified in the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010). Mathematics education researchers and policy documents have called for the improvement in the teaching and learning of mathematical modeling at the K–12 levels so that students can apply mathematics to real-world situations (Asempapa, 2018; Asempapa, Sturgill, & Adabor, 2017; Blum, 2015; Consortium for Mathematics and Its Application [COMAP] & Society for Industrial and Applied Mathematics [SIAM], 2016; Gaston & Lawrence, 2015; National Council of Teachers of Mathematics [NCTM], 2014; National Research Council [NRC], 2013; NGA Center & CCSSO, 2010). Nevertheless, few studies have examined how to teach mathematical modeling to K–12 students, and most practicing teachers (PTs) and preservice teachers (PSTs) have pedagogical and didactical strategies that do not support effective modeling practices (Blum, 2015; Borromeo Ferri, 2018; Gaston & Lawrence, 2015; Rivera & Gallegos, 2018). Therefore, to promote creativity, problem solving, and critical thinking skills connected to mathematical modeling practices, all PTs and PSTs must be taught what mathematical modeling is, how it can be successfully integrated into their lessons, and how it can be implemented in their classrooms.

The successful implementation of mathematical modeling remains a challenge for most PTs and PSTs of mathematics (English, 2009; Rivera & Gallegos, 2018; Warwick, 2007). A fundamental reason for this challenge is that mathematical modeling requires both teachers and students to comprehend and understand complex systems of mathematics and models within a multifaceted context (Blum & Borromeo Ferri, 2009; Borromeo Ferri, 2018; English, 2009). Further compounding this challenge are teachers' misconceptions about the teaching and learning of mathematical modeling (Asempapa, Sturgill, & Adabor, 2017; Gould, 2013; Spandaw & Zwaneveld, 2010; Wolfe, 2013). Preservice teachers need to acquire special mathematical modeling didactics and content knowledge that is different from their instruction in mathematics–focused education programs (Usiskin, 2001). Nonetheless, these didactics and knowledge could be taught to PSTs during their education preparation. Regrettably, for some time now, teacher education has been criticized in supporting teachers' modeling content knowledge without its effectiveness being analyzed empirically (Kaiser, Schwarz, &Tiedmann, 2007). Thus, it is essential to develop PSTs' mathematical modeling content and pedagogical knowledge necessary to teach mathematics effectively.

Aside from mentioning or ineffectively using aspects of mathematical modeling in the classroom, most PTs have very few opportunities to implement modeling lessons and reflect on the way they present modeling tasks or activities to their students. A reflection on this prior situation and literature on teachers' education in and practice of mathematical modeling leads to questions such as:

- How are the *Common Core State Standards for Mathematics* (CCSSM) and the *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME) report supporting PTs and PSTs as they enact modeling practices?
- Is it possible for PTs to implement didactic strategies focused on mathematical modeling if they do not have the necessary training?
- How can PSTs learn about mathematical modeling practices?
- In what ways can we change teacher preparation programs and professional development so teachers' understanding of mathematical modeling is deepened and intertwined with modeling practices?

It is not enough for teachers to simply memorize the concept of mathematical modeling; their conceptions of mathematical modeling must change to guarantee effective implementation of modeling practices in the classroom. Therefore, the primary purpose of this study was to examine PTs' pedagogical experiences and PSTs' perspectives on mathematical modeling practices in order to find possible solutions or answers to these aforementioned questions.

The teaching and learning of mathematical modeling have become key competencies in most school curricula and are being addressed worldwide in most countries' educational standards. However, literature is scarce on the development of university courses and teacher training workshops for the teaching and learning of mathematical modeling (Borromeo Ferri, 2018; Rivera & Gallegos, 2018; Stohlmann & Albarracín, 2016). Although various publications accentuate how and why models and mathematical modeling are highly valued (Blum, 2015; Borromeo Ferri, 2018; Lesh, 2012; Pollak, 2011), findings show that mathematical modeling is still a relatively new topic in many United States' schools and teacher education programs (Borromeo Ferri, 2018; COMAP & SIAM, 2016; NGA Center & CCSSO, 2010; Rivera & Gallegos, 2018). Despite the recommendations from leading educational organizations and the efforts of institutions such as COMAP, SIAM, National Council of Teachers of Mathematics (NCTM), and the National Science Foundation (NSF) about the importance of modeling education, most fundamental questions remain unanswered about the effectiveness of classroom use and implementation of modeling practices. Therefore, to understand mathematical modeling practices in the classroom, we must first discuss what modeling with mathematics is, and how it is connected to teachers and their classroom practices.

2. Conceptual Framework and Literature Review

2.1 What is Mathematical Modeling?

Among the 50 plus years of topics central to mathematics education, is the relationship between mathematics and the world as we know it. This relationship is evidence that mathematical modeling is not a new phenomenon in the area of mathematics, and that the phrase mathematical modeling is used to denote any relationship between mathematics and the real world. Nonetheless, the teaching and learning of mathematical modeling has been recently predominant and prominent in K–12 mathematics education in the United States because of its importance in STEM fields and everyday-life applications. The GAIMME report defined mathematical modeling as "a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (COMMAP & SAIM, 2016, p. 8). Alternatively, the CCSSM explained modeling as when students use their knowledge of mathematics "to solve problems arising in everyday life, society, and the workplace" (NGA Center & CCSSO, 2010, p. 7). Mathematical modeling is not specific to a body of mathematical knowledge such as calculus or linear algebra, but it is an iterative process that involves the use of mathematics to solve a real-world problem (Lingefjärd, 2007).

There are two main views about mathematical modeling in teaching and learning: (a) mathematical modeling as a content area and (b) mathematical modeling as a tool to teach mathematics (Blum, 2015; Lesh, 2012). With respect to the first view, research focuses on the mathematical modeling process, its phases, and associated competencies (Blum, 2015; Blum & Borromeo Ferri, 2009). This view theorizes mathematical modeling as a framework to describe teachers' and students' behaviors while students engage in modeling, and teachers investigate students' modeling competencies as well as assess modeling activities or tasks. With respect to the second viewpoint, mathematical modeling activities can instill problem-solving skills and support the learning mathematics in relevant ways (Anhalt & Cortez, 2015; Lesh, 2012). In this light, mathematical modeling is the process of translating back and forth between the real world and mathematics (Blum & Borromeo Ferri, 2009), which embodies both perspectives of modeling in the teaching and learning of mathematics.

Because constructing a mathematical model of a real-world situation is challenging, students need access to an explicit modeling process to help them comprehend, understand, and develop strategies to solve real-world problems. This need provided other experts and researchers the rationale to view mathematical modeling as a process that links mathematics and the real world. Pollak (2003) argued that mathematical modeling is a process in which a real-world situation is identified, assumptions are made, and a mathematical formulation is created in which the results are validated for their relevancy. Alternatively, Lesh and Doerr (2003) explained that mathematical modeling involves sequences of iterative cycles in which “descriptions, explanations, and predictions are gradually refined and revised” to create meaningful representations (p. 18).

Mathematical modeling as a process uses adaptable, apportionable, and reusable conceptual tools for explaining, conjecturing, describing, predicting, creating and solving real-world situations (Doerr, Ärlebäck, & Costello Staniec, 2014). Thus, mathematical modeling is an iterative process, involves the use of mathematics, and encompasses the real world. Mathematical modeling as a process occurs only after a task is situated in a meaningful context and continues until a model is complete, and validated results are known. For this study, Figure 1 depicts the mathematical modeling process that begins with a real-world problem and includes an interactive process of these steps: (a) find a real-world problem (b) make assumptions or decide on constraints (c) create a mathematical problem, (d) make a model or a representation, (e) build a mathematical solution, and (f) interpret the solution (Immersion, 2016).

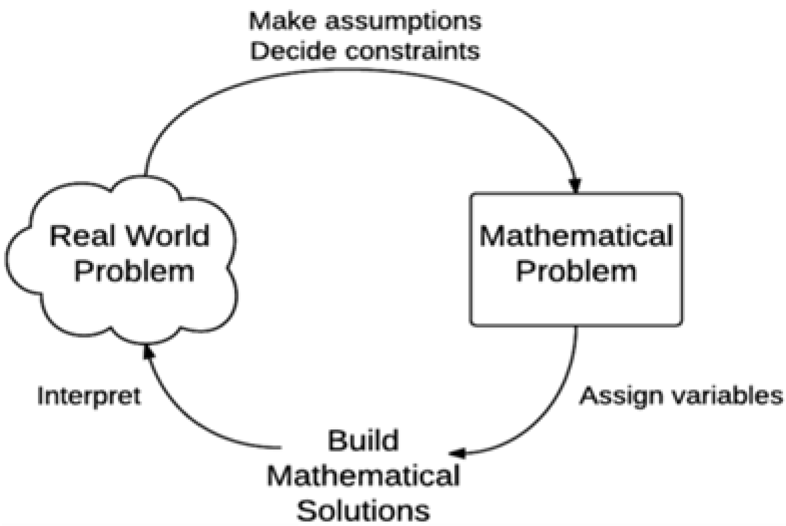

Figure 1. A diagram of the math modeling process (adapted from Immersion, 2016)

2.2 Teaching Mathematical Modeling

The teaching and learning of mathematical modeling have become an important proficiency in K–12 mathematics education and educational standards in many countries around the world. The main goal of mathematical modeling is to make mathematics education interesting and help students enjoy doing mathematics not only for their academic achievement but also to discover how they are able to connect mathematics to real-life situations. All students have potential to engage in mathematical modeling. Students’ engagement in this modeling serves as a basis for developing problem-solving skills and the appreciation for the importance and relevance of mathematics in their lives. Since 2012, concerted efforts have been made to prepare teachers for the integration of mathematical modeling in K–12 mathematics education (COMAP & SIAM, 2016; SIAM & NSF, 2012, 2014) and teacher education programs (Borromeo Ferri, 2018). With the increase emphasis on mathematical modeling from researchers and policy documents, teachers in the United States are now faced with the challenge of teaching mathematical modeling and implementing associated practices in the classroom.

Even though many teachers see the importance of mathematical modeling in their students’ education, they can be hesitant to incorporate it into their practice due to how mathematical modeling differs from their education experiences (Blum, 2015; Henn, 2010). The teaching of mathematics through modeling presents considerable challenges to current pedagogical ideas, which places new demands on teachers and can even require knowledge that goes beyond their school curricula. When students engage in modeling, teachers encounter diversity in student thinking and a reversal of roles for teachers and students—a balance of responsibility not evident in many United States’ classrooms (Blum & Borromeo Ferri, 2009; Lesh 2012). The reversal of roles is, however, important. Students need to evaluate their ideas, plans of action, and results. Opportunities where this evaluation can productively occur are essential to student learning and found in mathematical modeling.

The following four points are indicative of teachers' pedagogical strategies in teaching mathematical modeling. First, teachers need a deep and broad understanding of the modeling process and of the diverse approaches that students might take. To acquire such understanding, teachers must listen to students as they describe the situation, explain how they will solve this situation, and interpret their model that is a result of this situation (Doerr, 2007). Second, teachers need to attend to mathematical discourse by creating opportunities where students have to clarify, substantiate and evaluate the 'appropriateness' of their models. Third, a modeling environment must be created where there exists a "permanent balance between teachers' (minimal) guidance and students' (maximal) independence" (Blum & Borromeo Ferri 2009, p. 52). This environment is supported and sustained through teacher-provided and student-directed encouragement. Teachers can encourage students to solve problems independently by (a) posing purposeful questions; (b) asking questions that require explanation and justification; (c) giving focused, yet broad hints; and (d) providing support through scaffolding. Finally, teachers must create a classroom culture that supports modeling by exhibiting positive attitudes toward modeling and using rich tasks that are grounded in real-world situations.

2.3 Teachers and Mathematical Modeling

Substantial research has been done to explore how teachers of mathematics from diverse backgrounds and grade levels solve mathematical modeling problems (Blum, 2015; Blum & Borromeo Ferri, 2009), explain their conceptions and beliefs about mathematical modeling, and engage students in modeling activities in the classroom (Gould, 2013; Lesh, 2012). However, little is known about how PTs or PSTs describe or characterize mathematical modeling practices. Because requirements for teaching at different grade levels differ, and criteria for teacher licensure continues to change, the educational backgrounds of most mathematics teachers vary widely in the United States. As supported by research, teachers who have diverse disciplinary backgrounds can teach about mathematical models and modeling in different ways (Borromeo Ferri, 2018; Bautista, Wilkerson-Jerde, Tobin, & Brizuela, 2014). Thus, an important characteristic in the mathematical modeling background of teachers is their educational preparation and experiences. A better analysis of teacher backgrounds can inform teacher educators about the diversity of teacher ideas about models and modeling so that common understanding of terminology is not assumed (Bautista, Wilkerson-Jerde, Tobin, & Brizuela, 2014).

Recent studies of PSTs' knowledge and understanding of mathematical modeling identified significant difficulties in interpreting and teaching mathematical modeling and solving or creating mathematical modeling tasks (Altay, Özdemir, & Akar, 2014; Karali & Durmus, 2015). Although most participants in these studies noted that modeling pushed for a higher-level of thinking and benefited student academic achievement and growth, many struggled with the subjectivity and ambiguity of many modeling tasks. Other research stated the need for PSTs to experience mathematical modeling to develop the mathematical knowledge about modeling and connected understanding of modeling practices (Blum, 2015; Borromeo Ferri, 2018).

Mainly, the education given to both PTs and PSTs must also help them understand the following questions and provide well-developed answers: What a mathematical model is? Why mathematical modeling is essential to teaching and student learning? and What strategies and challenges are entailed in the development, use, and assessment of mathematical modeling in K–12 classrooms (Sole, 2013)? Currently, two Common Core standardized assessments exist, address modeling practices, and were created by two separate entities: one by the Partnership for Assessment of Readiness for College and Careers (PARCC) and the other by the Smarter Balanced Assessment Consortium (SBAC). Schoenfeld (2013) revealed that both assessments examine four areas: concepts and procedures, problem solving, reasoning, and modeling with mathematics. The difference between the two assessments being the area of analysis.

Presently, there are limited books or materials on teaching mathematical modeling that can be used by teachers for their teaching and by teacher educators for preparing PSTs or PTs in pre- or professional development courses for the high-quality teaching of mathematical modeling (Asempapa, 2016; Borromeo Ferri, 2018). To help teachers and teacher educators learn how to teach mathematical modeling in school and teacher education, this study became necessary. In line with this need and to achieve the purpose of this study, the authors were guided by two research questions:

> **Research Question 1**: How do PTs conceptualize mathematical modeling, and what are their experiences with mathematical modeling practices?
>
> **Research Question 2**: What are the views and experiences of PSTs with mathematical modeling practices?

3. Methods

This research study was conducted at a public-school district and a state university located in the Midwestern United States. The participants included PTs and PSTs who teach or are receiving training to teach mathematics to K–12 students. The PTs responded to the lead author-created questionnaire that contained one open-ended question and four multiple-choice questions. Similarly, the PST responded to four open-ended questions that was also created by the lead author. Both the questionnaire and open-ended questions were used to generate data to examine the participating PTs' and PSTs' familiarity and experiences with mathematical modeling pedagogies and practices.

3.1 Site and Participants

Participants in this current study were from the Midwestern United States and included both PTs and PSTs in mathematics at the K–12 level. Sixty-two PTs who answered all demographic questions self-selected to participate in this study. Of these 62 teachers, 77% were 35 years or older and almost 60% identified as White or Caucasian. In addition, 36 were Grades K–5 teachers, 9 were Grades 6–8 teachers, and 17 were Grades 9–12 teachers of mathematics. Teaching experiences ranged from 1–32 years, 85% of the sample self-identified as female, and 15% as male. Eighteen PSTs were included in this study. Of these 18 PSTs, 22% were pursuing grades 4–8 certification and 78% were pursuing K–4 certification. About 22% of the PSTs were males, and 78% were females. Because all the PSTs were traditional junior-year college students, the average age was about 22 years. In terms of race, about 88% of the PSTs identified as White or Caucasian.

3.2 Instrument

The questionnaire for the PTs contained one open-ended question and four multiple-choice questions that sought to unearth PTs' familiarity and experiences with mathematical modeling pedagogies and practices. The available answers to the multiple-choice questions were yes, no, and not sure. In this questionnaire, participants were asked to (a) define or explain what mathematical modeling means through the open-ended question and (b) share their experiences, perspectives, approaches, and practices regarding mathematical modeling by answering the following multiple-choice questionnaire questions:

- Does your mathematics textbook have mathematical modeling activities?
- Do you teach mathematical modeling activities?
- Did you take a mathematical modeling course in your teacher preparation?
- Does your school district provide you with any support in teaching mathematical modeling?

The open-ended question for the PTs were categorized and rated as excellent, good, fair, and poor by the authors and another mathematics educator. The raters are in the fields of mathematics and mathematics education, and their ratings were based on the definition of mathematical modeling exemplified in the rubric adapted from Asempapa (2018) and provided in Table 1. Because participants' responses were typed there was no ambiguity about their responses, and their statements were clear and straightforward. The ratings were coded as 4 = excellent, 3 = good, 2 = fair, and 1 = poor. The inter-rater reliability based on the intra-class correlation (ICC) was calculated to be .86 for single measures and .95 for average measures. The resulting ICC values were acceptable (Cicchetti, 1994), indicating that raters had a high degree of agreement.

Table 1. A Rubric for Evaluating the Definition of Mathematical Modeling (Adapted from Asempapa, 2018)

Category			
Excellent = 4	*Good = 3*	*Fair = 2*	*Poor = 1*
Definition demonstrates complete understanding and provides detail explanation. It states almost all steps involved in the modeling process. Links mathematics, real world situations, and the translation between the two.	Definition demonstrates basic understanding and provides minimal explanation. It mentions more steps involved in the modeling process. There is no link between mathematics and the real world.	Definition demonstrates little understanding and little to no explanation. It mentions fewer steps involved in the modeling process. There is no link between mathematics and real-world situations.	Definition shows no evidence of understanding of the phrase mathematical modeling.

For the PSTs, their questionnaire contained three questions related to their views and experiences of mathematical modeling practices that can be mapped to three domains: perception on mathematical modeling, practices of mathematical modeling, and experiences with mathematical modeling. In particular, the questions used to examine these three domains included (a) What is mathematical modeling, and what does it mean to you? (b) How would mathematical modeling be implemented in a classroom for your grade-level certification? and (c) What current or past experiences have you had with mathematical modeling practices?

4. Data Collection and Analysis

Strauss and Corbin (1990) defined the analysis of data as the operations by which data are made easy to conceptualize and reconstruct for clear interpretation. Because survey and open-ended questions were employed in the data collection process, analysis occurred both quantitatively and qualitatively as suggested by Creswell (2014). Regarding the PTs' responses to the open-ended question, data were categorized and analyzed quantitatively with descriptive statistics. Additionally, the PTs' responses to the four multiple-choice questions were analyzed using descriptive statistics such as percentages. Responses to the three questions from the PSTs were analyzed quantitatively so that the authors could comprehend and discern the PSTs' views and experiences concerning mathematical modeling practices. In particular, this analysis proceeded in three phases: (a) initial reading, (b) second reading to initiate the extraction of themes and patterns, and (c) the generation of meaningful themes and categories. During the initial reading, the data were read in detail to develop a holistic sense and to identify and determine any missing information. Before the generation of themes, participants' responses to all the three questions were read a second time. After the analysis phase, themes were developed from participants' responses for each of the three questions.

5. Results and Discussion

5.1 Research Question 1

Research Question 1 was analyzed quantitatively. This was achieved by using an open-ended item in which PTs were asked to briefly define mathematical modeling. Of the 62 PTs, only 54 responded to this open-ended item. Using the rubric from Table 1 as a guide, notable results were found during the examination of teachers' responses on their knowledge or understanding about mathematical modeling. Of the 54 teachers who responded to this question, only about 7% of the responses could be categorized as excellent responses. Thirty-three (about 61%) of the PTs had misinterpretations about the concept of mathematical modeling and confused mathematical modeling with model mathematics. Appendix A illustrates a sample of responses. Figure 2 provides the distribution of rating respondents' responses about the meaning of mathematical modeling.

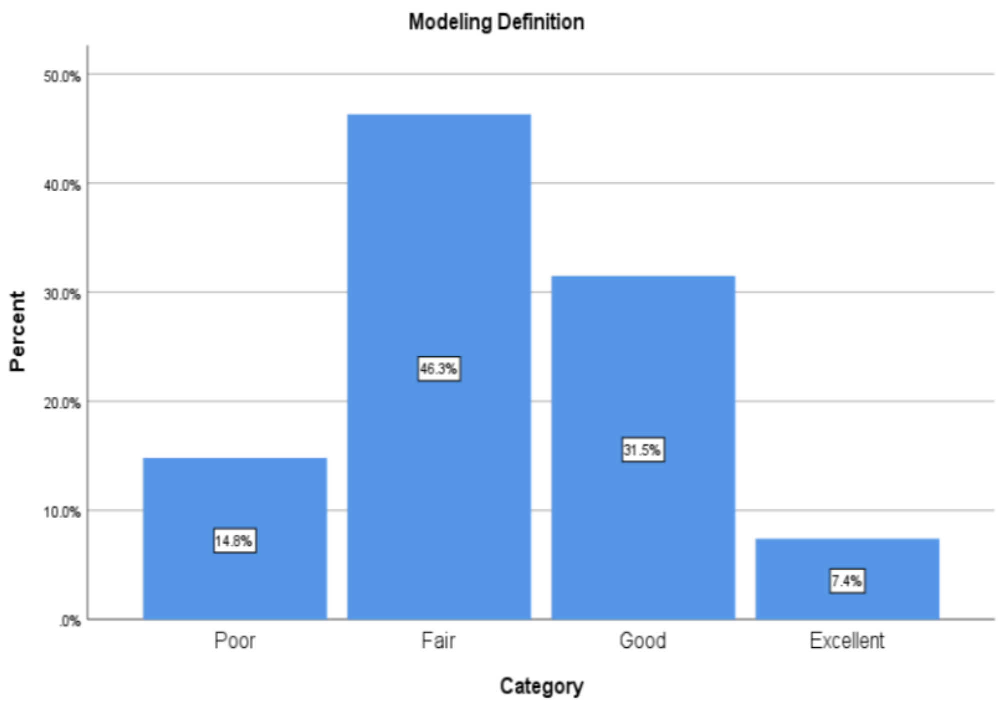

Figure 2. A bar chart showing PTs definition of mathematical modeling

Again, about 61% of the PTs' explanation or definition incorrectly assumed mathematical modeling as using physical objects, manipulatives, or representations to solve mathematics problem. Experiences shared by most of the PTs indicated the phrases *mathematical modeling* and the *modeling process* were relatively new terms, and they had little to no experience with any mathematical modeling practices. Most of their explanations failed to recognize mathematical modeling as a translation between mathematics and the real world, and that this type of modeling is an iterative process that involves choices and assumptions made by the modeler. There were notable findings in examining PTs' responses to the multiple-choice questions about their experiences and practices with mathematical modeling. Of the 62 PTs who responded to the question regarding whether they have modeling activities in their textbooks, only about 31% of the teachers responded yes.

Most of the PTs responded that they do not have modeling activities in their textbooks, and some did not have the professional knowledge to identify mathematical modeling activities. Another area of concern was the question related to whether the PTs were exposed to mathematical modeling education during their teacher preparation program. About 77% of the PTs responded not having any formal training in mathematical modeling during their teacher preparation program. Thus, most of the PTs from this study have limited exposure and experience with mathematical modeling, despite the usefulness and value in demonstrating how mathematics can help analyze and guide decision making for solving real-world problems through modeling. Table 2 provides a complete descriptive information of the participating PTs' experiences with mathematical modeling pedagogies and practices.

Table 2. PTs' Experiences with Mathematical Modeling Practices

Question	*n*	Yes (%)	No (%)	Not Sure (%)
Does your mathematics textbook have mathematical modeling activities?	62	19 (31)	26 (42)	17 (27)
Do you teach mathematical modeling activities?	62	40 (65)	5 (8)	17 (27)
Did you take a mathematical modeling course in your teacher preparation program in college?	62	6 (10)	48 (77)	8 (13)
Does your school district provide you with any support in teaching mathematical modeling?	62	30 (48)	5 (8)	27 (44)

5.2 Research Question 2

Research Question 2 was analyzed qualitatively using thematic processes to explore PSTs' views and experiences with mathematical modeling practices. Based on the analyses, the following three themes emerged: physical materials or manipulatives, step-by-step process or model mathematics, and little to no experience.

Physical materials or manipulatives. This theme encapsulated each of the PSTs' responses to the first open-ended question, which asked participants to provide a brief definition of mathematical modeling. From their responses, most PSTs believed that mathematical modeling occurs when the teacher uses manipulatives and visual representations to show students how to solve a mathematics problem. Some participants believed that this type of modeling was a way to break down the problem to the class (i.e., making the problem easier to comprehend and solve). Others believed that mathematical modeling is the process where a teacher utilizes only physical objects or materials to convey how to solve the problem to the class. An example would be when a teacher uses or shows multiple different ways to answer a specific mathematics problem using objects or manipulatives. A selection of direct quotes from the PSTs authenticate this theme: (a) "Modeling with mathematics means using manipulatives or examples to solve or express math problems," (b) "Physically showing the students how to solve mathematics problem by demonstrating different strategies with objects and how to effectively use these materials," and (c) "In grades K–8, mathematical modeling is using manipulatives such as checkers, blocks, moneys, counters and other manipulatives to demonstrate concepts."

Step-by-step process or model mathematics. This theme captured PSTs' perspective on how mathematical modeling can be implemented in K–8 classrooms. Most participants thought mathematical modeling should be taught in the form of demonstrations or by showing examples in class. They stated that repetition is key. Many continued to explain the importance for teachers to show what they did, say what they did, and explain why they did what they did. Quite literally, most of the PSTs took the meaning of *model* to indicate that mathematical modeling is about students imitating what the teacher has showed or demonstrated to them in a step-by-step process. A selection of key quotes supports this theme and includes: (a) "modeling how to do mathematics (explain why as well). Including modeling step-by-step procedures. Showing students how to properly solve problems" and (b) "by giving an example of the problem, solving it (or have them try to solve it), and then have them try on their own. Replication and repetition are important."

Little to no experience. This theme expressed the essential features of PSTs' experiences with mathematical modeling education. Most participants discussed not seeing mathematical modeling in their course work. For some, this was the first time they have encountered the phrase mathematical modeling. For others, they have heard and used the phrase *model mathematics* and have thought that mathematical modeling and model mathematics are synonyms (which, in fact, is not true). Some quotes from the PSTs to support this theme include (a) "this is my first time of seeing or hearing about mathematical modeling. Not much experience with it but use of manipulatives to demonstrate to students. Teacher or peer showing 'how' 1st then I copy," (b) "I have observed teachers using these techniques in a classroom. For example, when teaching, teachers'[sic] model to students to solve a math problem, the teacher verbalizes as she pulls parts for an equation," and (c) "I remember my teachers model how to use manipulatives when I was little. As I got older, we went to taking notes and writing out our reasoning behind what we were thinking."

6. Limitations

Although the study results are promising in terms of the rigorous methodological approaches, the data collected in this study represent the pedagogical experiences and perspectives on mathematical modeling practices of a small group of PTs and PSTs, and should be viewed with caution. The questionnaires were distributed to the study participants who only taught in public schools, and those who participated volunteered to do so. In addition, due to questionnaires being used for the data collection, another limitation is the potential for variation among participants' responses. It is unclear if issues of social desirability contributed to how each participant reacted or responded to the items on the questionnaires. Other issues such as response tendencies, nonresponse error, and sampling error and bias must be considered in the refinement of these questionnaires for future studies. These limitations should be acknowledged; yet, unearthing these limitations serves as the necessary first steps to expand the domain of mathematical modeling literature.

7. Future Research Directions

The primary purpose of this research was to determine PTs' pedagogical experiences and PSTs' perspectives on mathematical modeling practices. The results from this study present further support for both existing research and contributions toward future research and program development in mathematics teacher education centered on mathematical modeling. Further investigations of teachers' (PTs and PSTs) experiences and practices with mathematical modeling could therefore offer a valuable addition to the present findings and reveal further differences in the challenges teachers face as they engage in certain practices and processes with mathematical modeling. The methods applied in this study could be applied to future research in collaboration with a methodology that combines exchange of ideas between teachers and researchers in which continued guidance and support are provided to participants by these researchers.

In future studies, mixed-methods research designs would be a strong choice for examining PTs' pedagogical experiences and PSTs' perspectives on mathematical modeling practices. More specifically, the use of a questionnaire along with classroom observations, document analysis, interviews, and video analyses of teachers would yield insightful results. Using this research design can help other researchers understand teachers' pedagogical experiences and perspectives on mathematical modeling practices. Additionally, data collection should be expanded and continued for diverse and larger populations to provide a better understanding of teachers' pedagogical experiences and perspectives on mathematical modeling practices. Furthermore, future work could employ design or codesign research approaches in which researchers support the development of teachers' mathematical modeling practices.

8. Conclusion

This research provided insight into teachers' pedagogical experiences and perspectives on mathematical modeling and how they conceptualize modeling practices. The findings emphasized the need to consider several factors in preparing teachers to engage students meaningfully in mathematical modeling pedagogies and practices. Now, more than ever, this consideration should not be taken lightly. The CCSSM and GAIMME report called for teachers to embrace a major shift in their practices on mathematical modeling to provide opportunities for their students to be mathematically proficient, do mathematics, and engage in the practices of problem solving. The practice of modeling is consistent with the *Common Core Standards for Mathematical Practice* (SMP4)–model with mathematics and echoes the effective teaching practices and productive dispositions as explained in NCTM's *Principles to Action* (NCTM, 2014). Modeling equips teachers with effective practices that influence or impact students' awareness or perception of the role, importance, and relevance of mathematics in future careers (NCTM, 2014). Nonetheless, how can teachers be effective in supporting student learning if they, themselves, have an underdeveloped understanding of mathematical modeling and how to engage in it?

The findings presented in this study may be followed by future research in which collaboration is a key component between researchers (experts in mathematical modeling education) and teachers (PTs and PSTs). There are advantages of this type of study: the researchers will be close to the teachers in their classroom where the researchers can provide effective practices, support, and applications. Additionally, the researchers can serve as a resource that the teachers can rely on for pedagogical and theoretical applications of mathematical modeling practices. On the other hand, the teachers (PTs and PSTs) who participate in these studies can gain from the researchers the theoretical background, elements, and framework that will foster a reflection of their own mathematical modeling practices. The authors hope a shift in teacher education programs and teachers' mathematical modeling practices and pedagogies will assist students do mathematics as attributed by the standards in the CCSSM and NCTM. Such a shift becomes possible when teachers fully understand mathematical modeling and have support to engage in modeling practices.

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Appendix A

Sample Responses to the Open-ended Question

Below are selected sample responses of participants to the open-ended question. The responses were categorized into excellent (E), Good (G), Fair (F), and Poor (P) based on the rubric. The responses were coded using anonymous identifiers. For example.
E1 = represents sample 1 of the excellent responses;
G1 = represents sample 1 of the good responses;
F1 = represent sample 1 of the fair responses; and
P1 = represents sample 1 of the poor responses.

Excellent (E) responses.

E1: Mathematical modeling involves having the students apply the math that they know to the problems and to their everyday life. It also involves them changing the model as different needs arise or as they realize that they need to make improvements. It helps them come up with new assumptions and realizations.

E2: Mathematical modeling is about taking a real-world problem, applying mathematical strategies to solve the problem and then relating the solution back to the real world.

Good (G) responses.

G1: Mathematical modeling takes concepts and procedures learned in mathematics classes and make those processes function in an application type setting.

G2: Mathematical modeling is a process in which students are challenged with a problem or problems and then taken through the thinking process in order to give gradual release of thinking. It allows for more quality thinking rather than cut and dry answers.

Fair (F) responses.

F1: A mathematic model is when you are making an activity, cognitively, where you think about how an object or device behaves. It is done for fun to show students how to formulate and problem solve.

F2: Mathematical modeling is guiding students through the problem-solving process. The use of drawn or figure manipulation to explain the mathematical process or thinking to solve a problem.

Poor (P) responses:

P1: Showing your work with numbers, words, or pictures to others.

P2: Math modeling is when an instructor models the correct way or ways to solve a problem.