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Implication of Reducing the Angle at the Centre to 10^{-n} to the Relationship Between the Regular Polygon and the Circle Circumscribing the Polygon

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Abstract

The purpose of the study was to establish the relationship between the regular polygon and the circle circumscribing it. The study revealed that the circle is a special polygon if the angle at the centre is reduced to 10^{-n} where *n* is the number of decimal places of the angle at the centre of the polygon after finding the sum of areas of the triangles in the regular polygon. It was also discovered that if the angle at the centre of the polygon is reduced to 10^{-n} and has same number of decimal places as π, then the circle circumscribing it would have the same area with the polygon if the two answers are rounded off to $(n – 3)$ decimal places and a polygon with infinite number of sides is a circle. This relationship also proved that $\pi = \sin \theta \, (1.8 \times 10^{n+2})$ where n is the number of decimal places of π.

Keywords: regular polygon, circumscribed polygon, area, segment, relationship, circle, π

1. Introduction

Mathematics is one of the subjects that every school going child should learn because it is widely recognized to occupy a special position in the intellectual development of man because it provides a powerful universal language and intellectual toolkit for abstraction, generalization and synthesis (https://www.ampleforth.org.uk/college-welcome/academics/departments/mathematics). With the help of mathematics man is able to probe and have a full understanding of the universe and his environment with the help of numbers and geometrical figures (Kwenge, 2014). Without numbers and geometrical figures, technology cannot advance to the level that man can see beyond what the naked eyes cannot see. Mathematics, like everything else that man has created, exists to fulfil not only certain needs and desires of the society but also helps in accelerating the rate of development in this digital age and acquisition of knowledge. The whole range of human activities and institutions are conceptualized and regulated numerically with the help of geometrical figures, a situation which has resulted in the Mathematisation of modern society and modern life. Mathematisation is the organizing of reality using mathematical ideas and concepts which are used to acquire knowledge and skills to discover unknown regularities, relations and structures (Treffers & Goffree, 1985). It is mathematics which is shaping the future of humanity. Man is able to know where he came from, where he is and where he shall be in future because of the role that mathematics is playing in the intellectual development of man. With the help of mathematics, man is able to understand natural phenomena which are responsible for determining his destiny. To have a full understanding of life, man needs numbers and geometrical figures such as circles, triangles and polygons. When people hear the word geometry, they tend to think about shapes. These shapes surround their lives each and every day. Many of these shapes, or polygons, can be described as flat closed figures with 3 or more sides. Polygons are two-dimensional objects, not solids. It is mathematics which helps man to understand reality in terms of relationships and structures of objects, nature and man. It is difficult to understand the relationship which exists between the circle and the polygons without mathematics.

1.1 The Purpose

The purpose of the study was to show that there is a special relationship between the circle and the regular polygon when the angle at the centre of the triangle of a regular polygon is reduced to 10^{-n}, where *n* is the number of decimal place of π, and help the learners, teachers and mathematicians develop more critical thinking ability and appreciate the role of mathematical reasoning in real world situations.

1.2 Literature Review

The main reason why mathematics is compulsory at both primary and secondary schools is to enable the learners to develop proficiency with mathematical skills, improve logical thinking, expand understanding of mathematical concepts and promote success in what they intend to be in future (Foster A.G, Rath J.N and Winters L.J., 1986). Learning mathematics without the learner developing mathematical reasoning, full understanding of the relationships between numbers and geometrical figures is incomplete because at one point or another, one will meet the situation that would demand the knowledge of deductive or inductive reasoning (Kwenge, Mwewa & Mulenga, 2015). If a circle passes through the corners of a polygon then this polygon is called inscribed polygon and the circle is called the circumscribed circle because it is outside the polygon. A polygon is a plane shape with straight sides. Regular polygons are 2-dimensional closed shapes. They are made of straight lines, and the shape is "closed". More precisely, no internal angle can be *more than* 180 °(*https://www.mathsisfun.com/geometry/polygons.html*).

If the inscribed polygon is a regular polygon the centre of the circle is also the centre of the polygon. In the diagram below O is the centre of the polygon and OA is the radius of the circumscribed Hexagon as shown in figure 1.

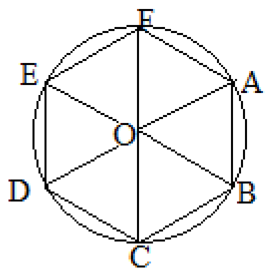


Figure 1. Circumscribed Hexagon

A polygon which is drawn outside a circle so that the sides touch circumferences of the circle then the circle is called inscribed circle because it is inside the polygon. If the polygon is regular then the centre of the circle is also the centre of the polygon. The radius of the hexagon below is denoted by OP in the figure 2.

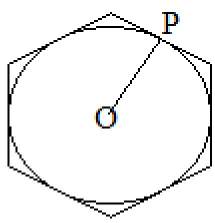


Figure 2. Inscribed Hexagon

According to Kwenge et al (2015), there is a special relationship between perfect numbers and consecutive odd numbers as can be seen in the illustration below which can help to understand the full meaning of the root of any perfect number.

Square root	odd numbers right-angled triangle	perfect square
1	1	1
2	1 3	4
3	1 3 5	9
4	1 3 5 7	16
5	1 3 5 7 9	25
6	1 3 5 7 9 11	36
7	1 3 5 7 9 11 13	49
8	1 3 5 7 9 11 13 15	64
n	$[n^{(t-1)} - (n-1)]$ ……….$[n^{(t-1)} + (n-1)]$	n^2

where *n* is the square root and *t* the power.

If *n* is the root and *t* the power of *n*,

then (i) the first term = $[n^{(t-1)} - (n-1)]$

(ii) the last term in the sequence = $[n^{(t-1)} + (n-1)]$

Power 5 and odd numbers

Fifth root	consecutive odd numbers	n^5
1	1	1
2	15, 17	32
3	79, 81, 83	243
4	253, 255, 257, 259	1024
n	$[n^{(t-1)} - (n-1)]$, ………,$[n^{(t-1)} + (n-1)]$	n^t

If *n* is the root and *t* the power of *n*, then

the first term in the sequence = $[n^{(t-1)} - (n-1)]$,

the last term in the sequence = $[n^{(t-1)} + (n-1)]$,

The general formula for the number of *n* consecutive odd numbers that can add up to n^t where first term in the sequence = $[n^{(t-1)} - (n-1)]$ and last term = $[n^{(t-1)} + (n-1)]$ and the common difference for the arithmetic progression is 2. According to Kwenge-Mwewa root theory, the meaning of the root of any perfect number is that there are n consecutive odd numbers that should be added to get that perfect number. For example, the root of 4 is 2, which means that there are 2 consecutive odd numbers which if added will give 4 and the 2 odd numbers are 1 and 3.

Given that *b* is a perfect number, *n* the root of *b* and *t* the power of *n*. The relationship between *b*, *n* and *t* is an AP (Arithmetic Progression). If $b = n^t$

(a) Expand *b* given that $b = n^t$ in terms of *n* and *t*.

(b) Show that $b = n^t$

Solution

(a) Since *b* is a perfect number,

then $n^t = [n^{t-1} - (n-1)] + [n^{t-1} - (n-1) + 2] + [n^{t-1} - (n-1) + 4] + [n^{t-1} - (n-1) + 6] + \ldots\ldots + [n^{t-1} + (n-1)]$, where *n* is the root of *b* and *t* the power of *n*

(b) Since *b* is a perfect number,

then $b = [n^{t-1} - (n-1)] + [n^{t-1} - (n-1) + 2] + [n^{t-1} - (n-1) + 4] + [n^{t-1} - (n-1) + 6] + \ldots\ldots$

$\ldots\ldots + [n^{t-1} + (n-1)]$, where *n* is the root of *b* and *t* the power of *n*

$$b = \frac{[n^{t-1} - (n-1) + n^{t-1} + (n-1)] \times n}{2}$$

$$= \frac{[n^{t-1} + n^{t-1} - (n-1) + (n-1)] \times n}{2}$$

$$= \frac{2n^{t-1} \times n}{2}$$

$$= \frac{2n^{t-1+1}}{2}$$

$$= n^t$$

or

$b = [n^{t-1} - (n-1) + n^{t-1} + (n-1)] \text{ x } n \text{ x } \frac{1}{2}$

$= 2\, n^{t-1} \text{ x } n \text{ x } \frac{1}{2}$

$= n^{t-1} \text{ x } n$

$= n^{t-1+1}$

$b = n^t$

Mathematics should not be seen as the body of mathematical knowledge, but the activity of solving problems and looking for problems and, more generally, the activity of organizing matter from reality or mathematical matter – which is called ‘Mathematisation’ (Freudenthal, 1971). In very clear terms, Freudenthal clarified what mathematics is about: “There is no mathematics without mathematician” (Freudenthal, 1973).This activity-based interpretation of mathematics has also important consequences for how mathematics education is conceptualized. More precisely, it affects both the goals of mathematics education and the teaching methods. According to Freudenthal (1973), mathematics can best be learned by doing and mathematising is the core goal of mathematics education: What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematising reality and if possible even that of mathematising mathematics.

1.3 Conceptual Framework

The study was guided by the theory of learning based on inductive reasoning. In inductive reasoning conclusion is made from specific observations to broader generalizations and theories. Informally, it is sometimes called a "bottom up" approach. In inductive reasoning, the process of establishing the relationship begin with specific observations and measures, detecting patterns and regularities, formulating some tentative hypotheses that can be explored, and ending up by developing some general conclusions or theories. For Singh (2011), inductive reasoning is a process of reasoning which is used for developing more general rules from specific observations. The study was also guided by the theory of learning based on vision which according to Miller (1983) possess two stages of learning with the first stage being differentiation in which categorization and classification of information take place and the second stage being interpretation which involves synthesizing of knowledge for making conclusions and judgments about the newly integrated information. For Lawler (1981), acquisition of mathematical knowledge requires deep understanding of procedural and conceptual knowledge and relationship of variables involved. Relationships can be established on two levels; understanding which originates from the ideas presented within the context and the relationships that are understood in an environment where appropriate abstractions have been made like the case of the circle and the regular polygon.

Number patterns and geometrical figures do help in defining some concepts in mathematics at the same time make learners be involved in creativity which is part of active participation in learning. According to Sara Katz1 & Moshe Stupel (2015), creativity is viewed as a property of cognitive process which tends to focus on analysing the steps involved in creative thinking or in teaching creative cognitive processing. Creativity is about generating new and useful ideas and rules which is supported by the theory of constructivism. Engagement in creativity makes learners be involved in producing something new and useful with respect to the previous knowledge (Kwenge et al, 2015), creative work requires applying and balancing three abilities. The three abilities are; synthetic, the analytic, and the practical abilities: Synthetic ability is the ability to generate novel and interesting ideas (Sara et al, 2015). Using number patterns and geometrical figures would make learners be creative and good synthetic thinkers and at the same time be able to make connections between things that other learners would not recognize without much involvement of the reasoning ability. Analytic ability is typically considered to be critical thinking ability which makes learners to analyse and evaluate ideas. According to social constructivist scholars, learning is viewed as an active process where learners learn to discover principles, concepts and facts for themselves, hence the importance of encouraging guesswork and intuitive thinking in learners (Brown et al, 1989).

2. Methodology

Drawing the polygons, circumscribing them and finding the areas of the circle, polygon, triangles within the polygon and the segment opposite to the angle at the centre. Observing the shape of the polygon after increasing the number of sides and reducing the angle at the centre to 10^{-n} which cannot be drawn without the help of technology. Deriving formulae from the results obtained as a result of reducing the angle at the centre and comparing the area of the circle and the polygon which is circumscribed.

3. Presentation of Results

When the angle at the centre of the polygon is reduced 10^{-n}, the side subtending this angle also reduces resulting in the increase of the number of sides and triangles in the polygon.

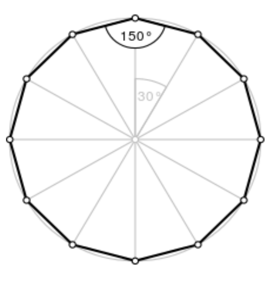


Figure 3. (a) The polygon has 11 sides Figure 3. (b) the polygon has 12 sides

The source of all the polygons is Wikipedia, the free encyclopedia.

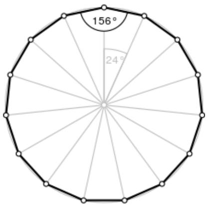


Figure 4(a). the polygon has 15 sides

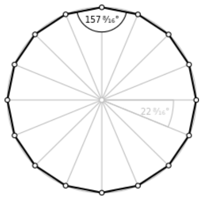


Figure 4(b). the polygon has 16 sides

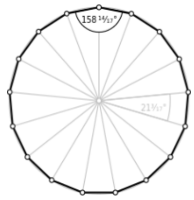


Figure 4(c). the polygon has 17sides

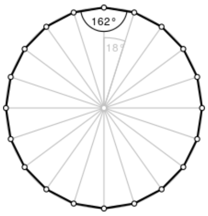


Figure 4(d). The polygon has 20 sides

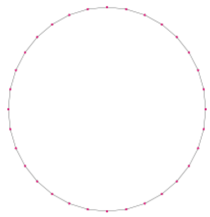


Figure 4(e). the polygon has 32 sides

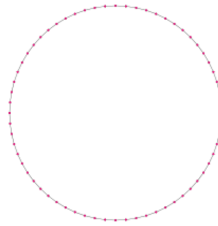


Figure 4 (f). the polygon has 64 sides

The source of all the polygons is Wikipedia, the free encyclopaedia.

When the size of the angle at the centre of the polygon is reduced, the area of the segment subtending the angle is also reduced. As the angle at the centre approaches zero degrees, the area of the segment is reduced to a dot resulting in the polygon becoming a **special circle** with infinite number of sides and line of symmetry as can be seen from the polygons above.

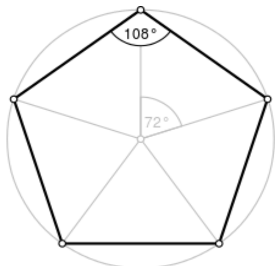


Figure 5. Relationship among the angle at the centre, the side opposite to this angle and the area of the segment opposite to the same angle

Source of diagram is Wikipedia, the free **encyclopaedia.**

When the angle at the centre is reduced the side opposite to it also gets smaller or reduced while the other two equal sides remain the same. These two sides of the isosceles triangle are the radii of the circle circumscribing the polygon. The area of the segment opposite to this angle is also reduced. The reduction of the angle at the centre of the polygon results in the side becoming a dot turning the polygon into a **special circle**. For example if 72^0 is reduced to less than $(10^{-6})^0$, then the polygon will look like the circle with naked eyes like figure 6 below. If θ is the angle at the centre of the isosceles triangle in the polygon, then as θ approaches 0^0 ($\theta \rightarrow 0^0$), the side of the triangle subtending this angle becomes a dot resulting in a polygon becoming a circle with naked eyes. This relationship among the angle at the centre, the side opposite to the angle and the area of the triangle can help to derive the Kwenge-Mwewa formula for π.

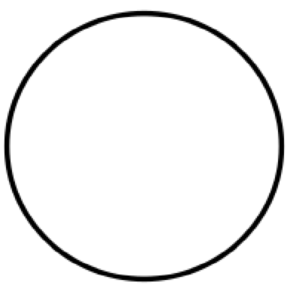
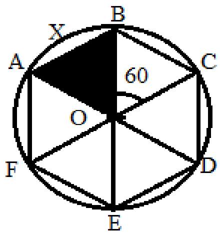
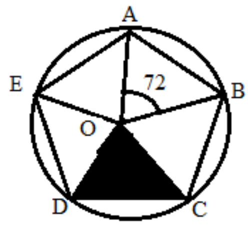


Figure 6. The polygon has 1000 000 sides

The source of the polygon is Wikipedia, the free encyclopaedia

3.1 Area of the Triangle and the Segment Between the Polygon and the Circle





Find the area of (i) ΔBOC (ii) ΔBOA, if the radius of OB is 10 cm.

(i) The area of ΔBOC = $\frac{1}{2}$ bc sin θ

$= \frac{1}{2} \text{ x } 10 \text{ x } 10 \sin 60^0$

$= 43.30127019 \text{ cm}^2$

(ii) the area of ΔBOA= $\frac{1}{2}$ ab sin θ

$= \frac{1}{2} \text{ x } 10 \text{ x} 10 \sin 72$

$= 47.55282581 \text{ cm}^2$

Find the area of sector (i) BOC (ii) AOB

(i) Area of sector BOC = $\frac{\theta}{360} \pi r^2$

$= \frac{60}{360} \text{ x } 10 \text{ x} 10\ \pi$

$= 52.35987756 \text{ cm}^2$

(ii) Area of sector AOB = $\frac{\theta}{360} \pi r^2$

$= \frac{72}{360} \text{ x } 10 \text{ x } 10\ \pi$

$= 62.83185307 \text{ cm}^2$

Find the area of the segment opposite to (i) 60^0 (ii) 72^0

(i) Area of segment opposite to 60^0 = area of sector BOC – area of ΔBOC

$= 52.35987756 – 43.30127019$

$= 9.05860737 \text{cm}^2$

(ii) Area of segment opposite to 72^0 = area of sector BOA – area of ΔBOA

$= 62.83185307 – 47.55282581$

$= 15.27902726 \text{ cm}^2$

Find the area of the segment opposite to 30^0 if the radius of the circle is 10 cm.

Let the triangle be X and the sector be P and segment be Y.

Area of segment Y = area of sector P – area of triangle X

$= \frac{30}{360} \text{ x } 10 \text{ x } 10\ \pi - \frac{1}{2} \text{ x } 10 \text{ x } 10 \sin 30^0$

$= 26.17993878 – 25.00$

$= 1.17993878 \text{ cm}^2$

Find the area of the segment opposite to 20^0 if the radius of the circle is 10 cm.

Let the triangle be A, sector be B and the segment be C.

Area of segment C = area of sector B – area of triangle A

$= \frac{20}{360} \text{ x } 10 \text{ x} 10\ \pi - \frac{1}{2} \text{ x } 10 \text{ x } 10 \sin 20^0$

$= 17.45329252 – 17.10100717$

$= 0.35228535\ cm^2$

Find the area of the segment opposite to 10^0 if the radius is 10 cm.

Let the triangle be Q, sector be R and segment be S

Area of segment R = area of sector S – area of triangle Q

$$= \frac{10}{360} \text{ x } 10 \text{ x } 10\ \pi - \frac{1}{2} \text{ x } 10 \text{ x } 10 \sin 10^0$$

$= 8.72664626 – 8.68408883$

$= 0.04255743cm^2$

Find the area of the segment opposite to 1^0 if the radius is 10 cm.

Let the triangle be K, sector be M and segment be E

Area of segment E = area of sector M – area of triangle K

$$= \frac{1}{360} \text{ x } 10 \text{ x } 10\ \pi - \frac{1}{2} \text{ x } 10 \text{ x } 10 \sin 1^0$$

$= 0.872664626 – 0.872620321$

$= 0.000044305\ cm^2$

Find the area of the segment opposite to 0.1^0 if the radius is 10 cm.

Let the triangle be H, sector be J and segment be F

Area of segment F = area of sector J – area of triangle H

$$= \frac{0.1}{360} \text{ x } 10 \text{ x } 10\ \pi - \frac{1}{2} \text{ x } 10 \text{ x } 10 \sin 0.1^0$$

$= 0.087266462 – 0.087266418$

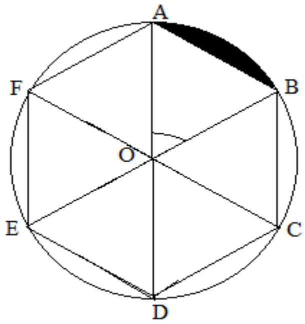
$= 0.000000044\ cm^2$

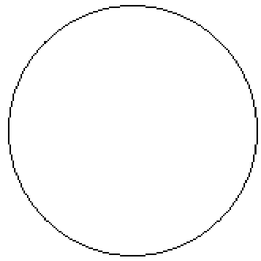
Table 1. Summary of the relationship between the angle at the Centre and other affected Geometric Parts

S/N	**Angle at the centre of polygon (θ)**	**Number of sides of polygon**	**Area of sector cm^2**	**Area of triangle cm^2**	**Area of segment cm^2**
1	72^0	5	62.83185307	47.55282581	15.27902726
2	60^0	6	52.35987756	43.30127019	9.05860737
3	30^0	12	26.17993878	25.00000000	1.17993878
4	20^0	18	17.45329252	17.10100717	0.35228535
5	10^0	36	8.72664626	8.68408883	0.04255743
6	1^0	360	0.872664626	0.872620321	0.0000443048
7	0.1^0	3600	0.087266462	0.087266418	0.0000000443048
8	0.01^0	36 000	0.00872664626	0.008726646216	0.0000000000443048

If θ is the angle at the centre of the isosceles triangle in the polygon, then as θ approaches 0^0

($\theta \to 0^0$), the side of the triangle subtending this angle becomes a dot resulting in a polygon becoming a circle with naked eyes.



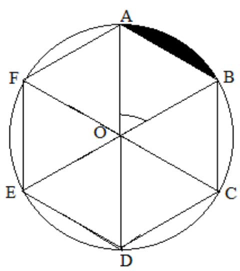


As θ approaches 0^0 (θ $\rightarrow 0^0$), the polygon becomes a circle with infinite number of sides. The relationship between the angle at the centre of the polygon and the side subtending it when the angle is reduced closer to 0^0 makes the polygon to become a special circle.

If θ is the angle of the isosceles triangle of the polygon and it is at the centre, and $\theta = 10^{-n}$ where *n* is the number of decimals of θ, then the area of the segment opposite θ = 4.43048 x $10^{-(5 + 3n)}$ when the answer is written correct to 5 decimal places.

3.2 Area of the Segment Between the Circle and Polygon

Area of segment = $\frac{r^2}{2}\left(\frac{\theta\pi - 180\sin\theta}{180}\right)$ where θ is the angle at the centre of the polygon. In this case angle AOB = θ .



If angle AOB = θ and OA = 10 cm, then the area of the shaded segment using the formula above is;

(i) Area = **0.000044304**13298914749089645842849673 3 cm^2 when θ is 1

(ii) Area = **0.00000004430480**10370572744023819281622 73 cm^2, when θ is 0.1

(iii) Area = **0.00000000004430480**77175848196902137803350 99 cm^2, when θ is 0.01

(iv) Area = **0.00000000000004430480 77**84390099987857873280707 cm^2, when θ is 0.001

(v) Area = **0.00000000000000004430480 77**85058152791318790874723 cm^2, when θ is 0.0001

(vi) Area = **0.00000000000000000004430480 77**85064833319353835607797 cm^2, when θ is 0.00001

When $\theta = 10^{-n}$ where n is the number of decimals of θ, then the area of any segment between the circle and polygon = 4.43048 x $10^{-(5 + 3n)}$, correct to 5 decimals places. If the angle at the centre is 0.00001^0 then the polygon has $\frac{360}{0.00001}$ isosceles triangles inside and if simplified, it gives 36 000 000 isosceles triangle.

The area of triangle with the angle at the centre of the polygon 0.00001^0 and radius being 10cm; Let area of the triangle inside the polygon be A.

$$A = \frac{1}{2}r^2\sin\theta \quad \text{where } \theta \text{ is the angle at the centre.}$$

$$= \frac{1}{2} \times 10^2 \sin 0.00001$$

$$= 50\sin 0.00001$$

$= 0.0000087266462599716035798106687776097\ cm^2$

Since there are 36 000 000 isosceles triangles inside the polygon, the area of the polygon

$= 36\ 000\ 000 \times 0.0000087266462599716035798106687776097\ cm^2$

$= 314.159265359cm^2$ (correct to 5 d.p since the angle at the centre has 5 d.p)

Area of the polygon = $\frac{360}{\theta} \times \frac{1}{2} r^2 \sin\theta$

$$A = \frac{180}{\theta} r^2 \sin\theta$$

Area of the circle with radius 10cm,

$A = \pi r^2$

$= 10^2 \pi$

$= 100\pi$

$= 314.15926535897932384626433832795$ (correct answer to 5d.p since the angle at the

$= 314.159265359\ cm^2$ centre of polygon has 5 d.p).

Using the formula above, we can derive the formula for π since the polygon has become a special circle.

Area of the circle = πr^2

Area of the polygon = $\frac{180}{\theta} r^2 \sin\theta$ (circumscribed polygon where θ is angle at centre of polygon)

Find the area of the regular polygon with the angle at the centre being equal to $(10^{-25})^0$ and circumscribed by the circle with radius 10 cm.

(a) Area of the circle = πr^2

$= 100\pi$

$= 314.15926535897932384626433832795\ cm^2$

(b) Area of the polygon. Let the angle at the centre of the polygon be θ

Area of the polygon = $\frac{360}{\theta} \times \frac{r^2}{2} \sin\theta$ where θ is $(10^{-25})^0$

$= \frac{360}{10^{-25}} \times \frac{100}{2} \sin(10^{-25})^0$

$= \frac{360}{10^{-25}} \times 50 \sin 0.0000000000000000000000001$

$= 314.15926535897932384626433832795\ cm^2$

Since the circumscribed polygon has the same area with the circle if rounded off to 29 decimal places then,

Area of the polygon = Area of the circle because the polygon has become a special circle

$$\frac{360}{\theta} \times \frac{r^2}{2} \sin\theta = \pi r^2$$

$\frac{180}{\theta} r^2 \sin\theta = \pi r^2$, divide both sides by r^2

$$\frac{180}{\theta} \sin\theta = \pi .$$

Since θ = 10^{-n} where *n* is the number of decimal places of π required according to Kwenge (2013), then π = $\frac{180}{\theta}$ sin θ

$$\pi = \sin\theta\,(180 \times 10^{n})$$
$$\pi = \sin\theta\,(1.8 \times 10^{2} \times 10^{n})$$
$$\pi = \sin\theta\,(1.8 \times 10^{n+2})$$
$$\pi = \sin\theta\,(1.8 \times 10^{n+2}).$$ This is Kwenge-Mwewa formula for π

3.3 θ and π With Same Number of Decimal Places, Where θ = 10^{-n}

If θ and π have the same number of decimal places where θ is the angle at the centre of the regular polygon circumscribed by the circle and θ = 10^{-n}, find the area of the following

(i) Given that π has 6 decimal places and radius 4cm;

(a) Circle

(b) The polygon circumscribed by the circle

(a) Area of the circle = πr^2. π = 3.141593 since it has 6 decimal places.

= 4 x 4 x 3.141593

= 50.265488 cm^2

(b) Since π has 6 decimal places then θ = 10^{-6} = 0.000001.

Area of regular polygon with angle at the centre being θ;

Area = $\frac{360}{\theta} \times \frac{r^2}{2}\sin\theta$

$= \frac{180 \times 4 \times 4\, sin 0.000001}{0.000001}$

= 50.265482457436689263445365712734 cm^2.

If both answers are written correct to 3 decimal places, the two answers will be the same.

(a) 50.265488 correct to 3 decimal places. is 50.265

(b) 50.265482457436689263445365712734 correct to 3 decimal places is 50.265

(ii) Given that π has 9 decimal places and radius of the circle 5cm, find the area of;

(a) circle

(b) regular polygon

(a) Area of the circle = πr^2 , π = 3.141592654 correct to 9 decimal places

= 5 x 5 x 3.141592654

= 78.53981635 cm^2

(b) Area of the regular polygon = $\frac{360}{\theta} \times \frac{r^2}{2}$ sinθ, θ = 0.000000001 since π has 9 decimal places.

Area of the regular polygon = $\frac{180 \text{ x } 5 \text{ x } 5 \text{ x } \sin 0.000000001}{0.000000001}$

= 78.539816339744830961562097149287 cm^2

If both answers are written correct to 6 decimal places, then the two answers will be the same.

(a) 78.53981635 correct to 6 decimal places is 78.539816.

(b) 78.539816339744830961562097149287 correct to 6 decimal places is 78.539816

(iii) Given that π has 7 decimal places and radius of the circle 3cm, find the area of,

(a) Circle

(b) Regular polygon

(a) Area of the circle = πr^2 , π = 3.1415927 correct to 7 decimal places

= 3x3x3.1415927

= 28.2743343 cm^2

(c) Area of the regular polygon = $\frac{360}{\theta}$ x $\frac{r^2}{2}$ sinθ, where θ = 0.0000001 since π has 7 decimal places.

Area of the regular polygon = $\frac{180 \text{ x } 3 \text{ x } 3 \sin 0.0000001}{0.0000001}$

= 28.274333882308139131809032727154 cm^2

If both answers are written correct to 4 decimal places, then the two answers will be the same.

(a) 28.2743343 correct to 4 decimal places is 28.2743.

(b) 28.274333882308139131809032727154 correct to 4 decimal places is 28.2743.

From the three examples, it can be concluded that if θ is the angle at centre of the polygon and it has the same number of decimal places (*n*) as π and $\theta = 10^{-n}$, where *n* is the number of decimal places of π, then the area of the regular polygon will be the same as the circle circumscribing it provided the two answers are rounded of correct to (*n* – 3) decimal places. This shows that there is a special relationship between the regular polygon and the circle circumscribing it, this special relationship according to Kwenge is the circle-polygon theorem which is in terms of area and the shape of the polygon with increased number of sides as a result of reducing the angle at the centre to less than 10^{-n}.

3.4 Kwenge-Mwewa Formula for π

(a) Find the value of π correct to 18 decimal places (d.p) using Kwenge-Mwewa formula for π

$\pi = \sin\theta$ (1.8 x 10^{n+2}) Since π should have 18 decimal places, then n= 18.

= sin 10^{-18}(1.8 x 10^{18+2})

= sin 0.000 000 000 000 000 001(1.8 x 10^{20})

= 3.14159265358979323846264338327950 (write the answer correct to18 d.p)

= 3.141 592 653 589 793 238 correct to 18 decimal places.

(b) Find the value of π correct to 27 decimal places using Kwenge-Mwewa formula for π .

$\pi = \sin\theta$ (1.8 x 10^{n+2}) Since π should have 27 decimal places, then n= 27.

= sin 10^{-27}(1.8 x 10^{27+2})

= sin 0.000000000000000000000000001(1.8 x 10^{27+2})

= sin 0.000000000000000000000000001(1.8 x 10^{29})

= 3.14159265358979323846264338327950 (write the answer correct to 27 d.p).

= 3.141 592 653 589 793 238 462 643 383

4. Discussion of the Findings

The study revealed that there is a special relationship between the circle and the regular polygon. When the number of sides of the polygon is increased, the shape of the polygon starts looking like the circle. For example the polygon with 1 000 000 sides looks exactly like the circle as can be seen in the diagram below.

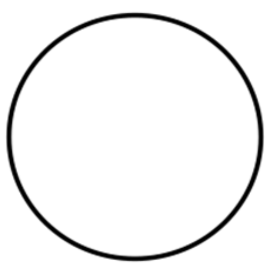


Figure 7. The polygon has 1000 000 sides (Wikipedia free encyclopedia)

The more number of sides the polygon has, the more it looks like a circle. If the number of sides are increased to infinite, then the polygon will become a **special circle** and that is why we say that the circle has infinite number of lines of symmetry and it is true that a polygon with infinite number of sides is a circle. This relationship which is in terms of areas of the polygon and the circle circumscribing it, helps in finding the value of π correct to the required number of decimal places and it is known as **Kwenge-Mwewa circle-polygon theorem**. According to Kwenge & Mwewa, if the angle at the centre is reduced to 10^{-n} and has the same number of decimal places with π, then the area of the polygon and the circle circumscribing it will be the same if both answers are rounded of correct to (n – 3) decimal places where *n* is the number of decimal places of π. From the same relationship, it was concluded that all **circles are special polygons** but not all polygons are circles. This relationship also explains why π is not a repeating decimal number because there is an extent to which the angle at the centre of the polygon can be reduced and the polygon would look like a circle or becomes a circle. The purpose of teaching mathematics is to enable learners to develop and use investigating patterns which will allow them to experience the excitement and satisfaction of mathematical discovery (Kwenge, 2017), like in the case of the circle and the polygons. There is need for the teacher to help learners acquire the 21st century skills in learning so that they can be active participants in the teaching-learning process that takes place within the social context of the classroom. In addition to this, Freudenthal (1991), emphasized that the process of re-invention should be a guided one. Students should be offered a learning environment in which they can construct mathematical knowledge and have possibilities of coming to higher levels of comprehension. This implies that scenarios should be developed by educators that have the potential to elicit this growth in understanding. An important role of the teacher is to provide students with working arrangements that are responsive to their needs bearing in mind that the constructivist classroom is an environment in which student will have enough time to develop mental models of the content, which will assist in moving that knowledge away from primary content area, so that it can be applied elsewhere (Spiro 2006).

All students need some time to think and work quietly by themselves, away from the varied and sometimes conflicting perspectives of other students (Sfard & Keiran, 2001). This implies that the teacher must be knowledgeable about relationships among geometrical figures so that there could be effective teaching of mathematics. According to Walshaw & Anthony (2009), partners or peers in groups can provide the context for sharing ideas and for learning with and from others. Group or partner arrangements are useful not only for enhancing engagement but also for exchanging and testing ideas and generating a higher level of thinking (Ding, Li, Piccolo, & Kulm, 2007). In supportive, small-group environments, students learn to make conjectures and learn how to engage in mathematical argumentation and validation (O‘Conner & Michaels, 1996). In particular, when groups are mixed in relation to academic achievement, insights are provided at varying levels within the group, and these insights tend to enhance overall understandings. However, teachers need to clarify expectations of participation and ensure that roles for participants, such as listening, writing, answering, questioning, and critically assessing, are understood and implemented (Hunter, 2008).

Learning about the geometrical shapes such as triangles, circles and polygons makes learners develop the mathematical skills which makes them be risk-takers, inquirers and critical thinkers (Kwenge, 2014). The use of mathematical investigation skills gives learners the opportunity to apply mathematical knowledge and problem-solving techniques to investigate a problem, generate ideas or rules, analyse information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them. Mathematics should be visualized as the vehicle to train learners to think, reason, analyse and articulate issues or problems logically. Analysis of geometrical shapes can help learners and teachers acquire skills in thinking, reasoning, analysing and articulating issues logically because the higher aim of teaching mathematics is to help the learner to develop the ability to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction which includes the way of doing things, and the ability and the attitude to formulate and solve problems (National Council of Education Research and Training, 2005). The results of the study showed that doing activities involving shapes can help the learners and the teachers of mathematics develop the ability to think creatively and mathematically (Brian, 1986). Creativity is about generating new and useful ideas and rules. Engagement in creativity makes learners be involved in producing something new and useful with respect to the previous knowledge. Creative work requires applying and balancing the abilities such as synthetic, analytic and practical ability. Synthetic ability is the ability to generate novel and interesting ideas (Sara & Moshe, 2015).

5. Conclusion

The polygon with infinite number of sides has a special relationship with the circle circumscribing it. Such a polygon with naked eyes will look like a circle. This relationship can help in deriving the formula for π which is known as Kwenge-Mwewa formula for π. Critical analysis of polygons with at least 20 sides can help learners develop interest in inductive reasoning and appreciate the importance of mathematical reasoning in real world situations. Exposing learners to analysing of the shapes of the polygons as they increase the number of sides can make learners discover more theories in mathematics and appreciate the power of mathematics and at the same time make the learners develop critical and analytical thinking abilities. This type of learning can help learners have full understanding of reality and the reoccurrence of the natural phenomena. Studying this special relationship between the polygon and the circle would make the learners and teachers of mathematics be able to appreciate abstraction and the nature of mathematics. Through the analysis of patterns of numbers, the shape of the polygon after the angle at the centre is reduced to less than 10^{-n} , and areas of the sector, segment and triangle generated within the circumscribed polygon the learner will become more knowledgeable.

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