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Revisiting the Question of Atmospheric Predictability

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Abstract

This study seeks to demystify the claim that the 'atmospheric chaos' imposes a two-week limit on reliable weather forecasts. 'Deterministic chaos' indeed occurs due to the use of nonlinear numerical models for these forecasts. This 'deterministic chaos' does impose time limits on valid predictions, but it also facilitates, through the ensemble forecasting technique, the use of interesting statistical indicators that define regions and the duration these predictions are more or less reliable. Recently published articles show that the 'uncertainties' in the initial conditions are an inherent difficulty in meteorological observations and have nothing to do with the atmospheric behavior. These studies demonstrate two important aspects regarding 'uncertainties' in data used to initialize models. First, to achieve improvements in numerical weather forecasts, these 'uncertainties' must be skillfully introduced in the large scale and not in the small scale. Secondly, the numerical models must include equations or parameterizations that reproduce nature's ways that let different scales 'interact', that is, the models should reproduce how the energy of different atmospheric modes 'travels'.

Keywords: weather prediction, atmospheric predictability, deterministic chaos

1. Introduction

Since dynamic models were first used for numerical weather forecasting, it was known that those predictions were limited due to the inadequate observations of the atmospheric physical processes. Nevertheless, the predictability theme is always relevant when considering the high demand for weather forecasts and for increasingly extended periods. Therefore, weather forecasting research is primarily focused not only on increased reliability but also on longer validity.

In the 1960s and 1970s the academic meteorology community debated if more degrees of freedom could reduce the system instabilities in forecast model equations. With this line of reasoning the articles of Charney (1963) and Lorenz (1963) should be highlighted. Charney thought that with more degrees of freedom, the system of equations could stabilize, and thus extend the effective forecast limits. However, at that time, Lorenz, using a very simple convection model based on an approximate system of nonlinear ordinary differential equations, discovered that two runs of the model starting from slightly different initial conditions gave surprisingly divergent responses after a non-long period of integrations. Lorenz called this unexpected result 'deterministic chaos'. Reinforcing the Lorenz results, Kalnay (2003) stated that nothing could be done to ameliorate the models because, as the atmosphere was 'chaotic', the fourteen-day predictability limit could not be surpassed.

Nowadays, nonlinear models are used instead of linear to forecast weather, because nonlinear models produce more 'realistic' or substantiated results. This is probably because nonlinear models take into account the disturbance products in the advection and others nonlinear terms, which are important to simulate the real atmosphere and which have aperiodic solutions, contrary to linear models. This study revisits the question of atmospheric predictability, suggesting the research community should invest its effort in two approaches. The first should endeavor to find modeling strategies that better reproduce the realistic ways large-scale interact with the small-scale in geophysical fluid systems, especially the atmosphere, in what are here named 'better models'. The second approach, more evident to the meteorological community, is to strongly invest to obtain more and 'better information' about the actual atmospheric state and also in more effective forms to assimilate this information in models, in what here is called 'better observational data'.

2. Chaos and Atmospheric Predictability

There is a common belief that weather prediction model output is unreliable after two weeks due to the 'chaotic' atmospheric condition. However, this problem is related to the nonlinear model equations, in which slightly different initial conditions produce uncontrolled divergence in the solutions after some time steps. It is unsuited suppose that atmosphere should work like models, but instead, good models should behave like the atmosphere. In other words, the 'chaotic' response of the atmospheric models used today should not impede the search for better ways to simulate atmospheric behavior. Nonlinear meteorological models have their own inherent difficulties, from the physical-mathematical point of view, and these difficulties have no connection with the atmospheric 'behavior'. The allegation that weather predictions have only two-week validity should not be imputed to the atmosphere, because this limitation is a problem generated by the non-linear models employed.

According to Lorenz (1987) the vast majority of atmospheric phenomena have not a single cause. Therefore, to explain physically the existence of a given phenomenon, it is needed to associate it with various multiple physical factors as causes acting simultaneously. In his 1987 article, Lorenz studied 'deterministic chaos' in association with a nonlinear ordinary differential equation system in which solutions were aperiodic and sensitive to 'uncertainties' in the initial conditions. Then Lorenz compared these with the solutions of a 'non-chaotic' linear ordinary differential equation system, where solutions were periodic or quasi-periodic. In this second case, were included stochastic terms and prescribed forcing, and both solutions show no dependence on initial value influences.

Kalnay (2003) asserts that "even with perfect models and perfect observations, the 'chaotic' nature of the atmosphere would impose a finite limit of about two weeks to the predictability of the weather", and she attributes this statement to four articles published by Lorenz between 1963 and 1968. Santos & Buchmann (2013) associate the "finite limit of about two weeks to the predictability of the weather" with the 'deterministic chaos', which should be considered an attribute of the models, not of the atmosphere being simulated. All arguments related to 'deterministic chaos' refer to non-linear models that rely on equations to represent the atmospheric behavior. It is 'obvious' that these models are 'chaotic'.

Santos & Buchmann (2013) disagree with the term 'atmospheric chaos' which was considered different from 'deterministic chaos'. This in turn is a problem related with the initial value 'inconsistencies', the nonlinear models, and the iterative methods to integrate the equations. The weather prediction by nonlinear numerical models used today should be considered a 'deficient way' to accomplish this purpose, although they are the best tool available to do this. 'Deterministic chaos' is not an atmospheric defect but a model limitation to predict the atmospheric behavior and thus, with better initial conditions and 'better models' one could extend the weather forecasts beyond two weeks.

Dr. Isidoro Orlansky working at NOAA (Personal Communication, 2014) read the article of Santos & Buchmann (2013) and agreed with some conclusions. He agrees that models themselves reproduce a 'chaotic' system, but even in those systems there is a large external force that maintains some periodicity that could be predicted for longer periods. According to Orlansky the 'chaos' of nonlinear model is not linked with the atmosphere. Even the atmosphere being 'chaotic', climate studies for long periods can be obtained beyond two weeks and the 'deterministic chaos' should not impede further research, spite the inherent nonlinearity of the models and the initial value 'uncertainties'. The ensemble forecast provides the basic tool to extend forecasts beyond Lorenz's limit of two weeks for the weather predictability.

3. Linear and Nonlinear Models

The issue of atmospheric model instability was studied by Charney (1963) while trying to discover effective prediction limits. He thought models with greater degrees of freedom would be less unstable or could stabilize 'quickly'. However, after the Lorenz (1963) discovery of the 'deterministic chaos', Charney's ideas, have been put aside or even abandoned, with an end to investigations. The Lorenz model was made up of an approximate system of three nonlinear ordinary differential equations, involving three variables related to the hydrodynamic nonlinear equations used to describe the convection phenomenon; when using initial values with little differences, the model integration answers 'quickly' diverge. This result was extended to more complex models, but was not proved for each separate model, taking into account different initial conditions (observational errors) and thus verifying the limits of the predictability. In 1956, in a meeting in Wisconsin, Lorenz decided abandon the linear models because the present wave-type solutions were considered less 'realistic' when compared with the results of nonlinear models. However, at this time probably Lorenz had no knowledge of the problem of 'deterministic chaos' related to 'inconsistencies' in the initial value (Lewis, 2005).

A linear equation system is obtained by applying the perturbation method to nonlinear hydrodynamic equations, which are linearized with respect to a basic state at rest. Then, using the variables separation method, a set of linear equations is produced consisting of the horizontal and vertical structure functions, the latter of which, with convenient boundary conditions, constitutes a Sturm-Liouville problem. The horizontal structure can be constituted of a non-homogeneous or homogeneous equations system, being that the last system has the same form as the linearized equations of the shallow-water model (Matsuno, 1966) which are often referred to as Laplace's tidal equations discussed by Longuet-Higgins (1968), which admit wave-type solutions (Kasahara, 1977 and 1978; Kasahara & Puri, 1981 and Kasahara, 1984). The linearized equations of the shallow-water model are used to partition the nonlinear model response into two kinds of waves, gravity-inertia and Rossby modes, obtained by dynamical, physical and mathematical associations and filtered one from the other at each stage (Matsuno, 1966). Actually the output of the linear models to simulate the atmosphere is expressed mathematically through wave-type solutions. On the other hand, instead of waves, the actual atmosphere 'transfers' energy physically, and then the wave frequencies can be associated with the energy that propagates in a 'similar way' with those theoretical waves.

The general solution of these linear equations system is obtained by a combination of the horizontal and vertical solutions, which can be a wave-type transient (periodic or quasi periodic), which neither amplifies, nor diverge, nor iterates (Kasahara & Puri, 1981; Kasahara, 1984). Thus 'deterministic chaos' only emerges in the nonlinear models and doesn't result from an atmospheric 'chaotic' behavior. The linear equations system can be used for weather simulations, but care must be taken in using this kind of model, instead of nonlinear, bearing in mind it does not take into account advection and other nonlinear physical terms, dropped by the linearization. The horizontal and vertical structure equations are linear and analytical and thus have solutions independent of the initial value 'uncertainties'. Some of these models can be integrated analytically, or the solution can be specified or estimated if they lack known analytical solutions.

Lorenz (1987) proposed using nonlinear analytical (non-meteorological) equations to represent the 'deterministic chaos', and small errors are introduced as initial state. Due the iterative scheme to solve these equations, after some steps the predictions begin to diverge due to the growth of the error. Using the formula of Lorenz (1987) and also the first-order quadratic differential equation [13.71] by Holton (2004), the Appendix was constructed with Tables I, II and III. In each table, the first and second columns refer to the temporal solutions of the indicated formula, and the third column shows the difference between these solutions at each 'time step'. Two different analytical expressions were used in each table. In Table I the error in the initial data is 0.00001, in Table II the error is 0.0001, and in Table III the error is 0.001. As can be seen in these three tables, the suggested equations become unstable soon after the iterative process starts. In the Table I with the first formula, the significant divergence (>+1 or <-1) occurs at step 37; with the second formula, the significant divergence occurs at step 17. In the Table II, with the first formula, the significant divergence occurs at step 28 and with the second formula at step 14. In Table III with the first formula significant divergence occurs at step 16 and with the second formula at step 10. These examples indicate that the discrepancy begins at different steps depending on the equation and the error in the initial condition.

Nowadays it is well accepted that the actual models used to predict the weather are 'chaotic' with only a finite predictability and it is believed that the source of chaos in those atmospheric models is their nonlinearities. Since high-resolution global modeling approaches have become a current trend for weather prediction and climate projection, Shen (2014) propose numerical experiments with the objective to understand the role of the increased resolutions in the predictability of the models, as one might expect that solutions to the equations with more nonlinear modes would become more chaotic. This rationale is equivalent to stating that the appearance of small-scale features and their nonlinear effects, resolved by additional modes, should make the system less predictable. Numerical tests are proposed by Shen (2014) with different Lorenz models leading to conclude that the inclusion of new modes introduces terms that have collective impact on the increase of solution stability. The additional nonlinear terms are mainly associated with the improved vertical advection of temperature. While Lorenz demonstrated the association of the nonlinearity with the existence of the nontrivial critical points and strange attractors, Shen (2014) emphasizes the importance of the nonlinearity in enabling subsequent negative feedback to improve solution stability. He concluded that the chaotic responses that appear in the Lorenz's models could be suppressed by the inclusion of additional modes, producing stable solutions. A macroscopic view of these results suggests that in some way the additional modes enable the transfer of domain-averaged potential energy at different scales.

Although chaos may appear in the presence of nonlinearity as well as a heating term in Lorenz's models, the increased degree of nonlinearity with additional dissipative terms in Shen's experiments could reduce chaotic responses, that is, the appearance of small-scale processes that involve the nonlinear interactions may help stabilize solutions. The butterfly effect exists in the numerical solutions in Shen's experiments as they display sensitive dependence on initial conditions after some time steps. However, the appearance of this chaotic behavior should not directly lead to the conclusion that small perturbations can alter large-scale structure, namely the butterfly effect associated to the imperfect initial conditions. Therefore, Shen's results demonstrate that stable solutions can be obtained in "sufficiently high-resolution" approach with additional modes in atmospheric models.

4. Initial Value Problem

The problem of 'deterministic chaos' is due to the inherent nonlinearity of the atmospheric models, and also arises from the large number of iterative operations performed during the process of model integration. As regards the initial value problem, the 'uncertainty' is ours as users and does not come from the atmosphere, that is, it comes from the doubt of knowing exactly what data should be used to initiate the model integration process.

During the atmospheric parameters measurement phase, small differences associated to user 'uncertainty' produces 'deterministic chaos' because the models are sensitive to small errors in the initial conditions, as Lorenz (1963) discovered. The measurement instruments are liable to errors, as with human interference on the data communication system, and these errors result in 'uncertainties' during the construction of the model's initial input. The 'deterministic chaos' produced by those errors is a mathematical problem due to the sensitivity of nonlinear models to variations at the integration starting point. The 'deterministic chaos' has nothing to do with the atmospheric behavior, but comes from the 'uncertainties' in the input data.

Two atmospheric parameter measurements cannot be taken in one determinate instant and one determinate point, even with a minimal difference. This could be possible only if there was a 'parallel universe'. In other words, atmospheric 'uncertainties' cannot be detected. Thus, it must be assumed that the real atmosphere will follow its own course and behave as a hydrodynamic 'continuous' system, running differently from the model in time and direction. The doubts result from the choices made after imprecise observations and then the 'deterministic chaos' of the model appears.

Lorenz (1969) said that as small errors generally require about five days to double, it should be possible to increase the range of predictability by five days simply by reducing by half the initial field errors, although this would be difficult to achieve. Lorenz (1969) considers the energy spectra under power-law behavior ($k^{(-p)}$, where k is wave number magnitude) for high wave numbers and discusses results for p = 5/3, 7/3 and 3. Using the vorticity equation which governs two-dimensional incompressible flow and integrating it numerically starting from two states differing as little as the "observational errors", Lorenz (1969) showed that, for different scales of motion, the range of predictability changes about one hour for "cumulus-scale", a few days for "synoptic-scale" and a few weeks in advance for motions in the largest scales. He found that each motion scale possesses an 'intrinsic' finite predictability range. These results application to real physical systems, including the earth's atmosphere, is considered.

Nonlinear models can be initiated by normal modes decomposition, providing one or more kind of waves of interest and filtering out the others. When the integration process begins, 'intrinsically' interactions will occur between the diverse motion scales, generating thereby new and different modes in response, as opposed to what happens with linear models (Buchmann et al., 1995). In more realistic numerical experiments as in numerical weather prediction, the energy of the atmospheric waves should follow 'trajectories' apparently established, such as the 'ray-path' explained by Hoskins & Ambrizzi (1993).

To better understand how different atmospheric motion scales should 'interact', reproducing in the models what nature probably does, Rotunno & Snyder (2008) have generalized the Lorenz model using a two-dimensional vorticity equation (with a "-3" energy spectrum) and equations of quasi-geostrophic dynamics at the surface (with a "-5/3" energy spectrum). They produced flows with unlimited and limited predictability, respectively. Later the Rotunno & Snyder (2008) model was modified by Durran & Gingrich (2014) using a smoother nonlinear saturation approach to investigate the error growth from different initial error distributions. Consistent with the Lorenz findings, the predictability loss generated by initial errors of small but fixed absolute magnitude is essentially independent of their spatial scale when the kinetic energy spectrum background is proportional to the "-5/3" wave number power. Thus, as the background kinetic energy increases with the scale, very small relative errors at long wavelengths have similar impacts on the perturbation error growth as do large relative errors at short wavelengths. To the extent this model applies to practical meteorological forecasts, the influence of initial perturbations generated by 'butterflies' would be swamped by unavoidable tiny relative errors in the large scales.

5. Improving the Atmospheric Predictability

Santos & Buchmann (2013) suggest that the term of the atmospheric predictability is not necessarily limited to two weeks. It is more reasonable to think that, in view of all the advances in weather and climate numerical prediction obtained in the last decades, 'better models' and 'better observational data' should lead to more accurate predictions, instead of worrying if a 'butterfly' beating its wings in Brazil could or not form a 'tornado' in Texas. The 'butterflies' cannot randomly form 'tornadoes' in the atmosphere, since the energy propagates through some preferential 'trajectories' or 'ray path', that suffer refraction, reflection or even is absorbed in some region, as Hoskins & Ambrizzi (1993) explain with great clarity. An example of these preferential trajectories is the 'critical latitude' proposed by Dickinson (1971). The 'butterfly' can be 'symbolically' understood as a cloud, in which the energy contained is distributed or dispersed, or even spread in the form of waves that are known and identified in the meteorological literature as the modes of gravity, Rossby and acoustic. The sound waves are not considered as having meteorological significance, although Kasahara & Qian (2000) consider its importance, mentioning they could be taken into account in future global numerical prediction models with high-resolution. The modes of Matsuno (1966) and the acoustic mode, associated with the energy, propagate at the group velocity, which is controlled by the large scale.

Durran & Gingrich (2014) point out this issue, saying that *"initial small-scale errors, including those at length scales far larger than the size of 'butterflies' do not matter when minor relative errors are present in the largest scales."* The basic explanation for the difference between the experiments they present (initial errors for k < 400 km and initial errors for k > 400 km) is that downscale error propagation in turbulence is very fast, and therefore downscale error propagation is much faster than upscale propagation. The relative non-importance of small scale errors was actually included in Lorenz (1969) but seems to have been largely overlooked both in his conclusions and in some subsequent research.

In the more famous Lorenz numerical experiment, the initial error was placed only at the shortest retained wavelength. Actually Lorenz placed the initial error at the second-to-shortest wavelength, and because he extended his model to much smaller scales, the initial error appears at a much shorter wavelength. In a less well-known Lorenz second experiment, the same absolute initial error was placed at the longest retained wavelength. Lorenz found that predictability was lost just as rapidly in both experiments and commented: *"Evidently, when the initial error is small enough, its spectrum has little effect upon the range of predictability".* Durran & Gingrich (2014) comment that although the Lorenz model modified by Rotunno & Snyder (2008) is a very simplified representation of the actual dynamic atmospheric flows, and being not as theoretically advanced as more recent turbulence models, it proved capable of estimating the ensemble error growth evolution in the simulations of two East Coast snow storms with surprising fidelity.

The impact of large-scale initial errors in the ensembles implemented by Durran & Gingrich (2014) suggests a need to revisit the idea that mesoscale motions typically inherit extended predictability from the large-scale flow. Mesoscale motions are indeed generated when large-scale circulations create fronts or 'interact' with small scale features such as topography, but there is no guarantee the large scales can be specified with sufficiently small relative errors to ensure the correct mesoscale response. Previous research has identified instances where very small differences in the large-scale flow rapidly produce significant differences in the mesoscale response on flow over topography and on the rain-snow line position. Thus more extensive use of well-calibrated ensemble forecasts may provide one way of addressing the 'uncertainty' associated with initial errors at all scales.

It is well known the large-scale flow presents some different kinds of wave motions, basically the low-frequency Rossby waves, which are characterized by a near geostrophic behavior, and the high-frequency inertio-gravity waves, apart from the mixed Rossby-gravity and Kelvin waves. Because in middle and high latitudes most part of the energy of the large-scale motions is in quasi-geostrophic modes, many initialization methods used in global models of primitive equations filter out inertio-gravity oscillations, and other schemes attempt to separate the solutions into slow and fast modes. An important point to be considered in any filtering procedure is whether vital information is lost on the system's dynamics with the filtering processes.

Raupp & Silva Dias (2010) made several studies to understand how Rossby slow waves can 'interact' with fast waves and also if these slow modes can be significantly affected by the propagating fast modes. Based on arguments from the fluid dynamic resonance theory, they demonstrate that the only way for a Rossby mode to 'interact' with fast waves is by entering in resonance with two inertio-gravity waves with nearly equal or opposite temporal frequencies. They also show in this sort of resonant 'interaction' that the Rossby mode essentially acts as a catalyst for the energy exchanges between the two high-frequency modes, in the sense that it enables the resonance conditions to be satisfied and controls the 'interaction' period through its amplitude, although the slow waves energy (amplitude) is not significantly affected by the fast propagating waves. This property inclusively explains the non-linear Rossby adjustment observed in mid-latitudes even for unbalanced initial data under strong rotation.

Branstator (2014) presents modeling evidences supporting the notion that when considering the influence of tropical rainfall anomalies on the extratropical conditions, this influence on midlatitudes overcomes the two-week limit. He found that for typical pulses of tropical heating of transient events, the effect on midlatitudes is strong for more than a week after the heating occurrence. When the pulse has amplitude similar to the amplitude of the commonly observed equatorial rainfall anomalies, on average its effect persists for at least two weeks and is even longer in certain regions. A consequence of this remote, delayed impact is that the adequate assimilation of the tropical heating produced by observed rainfall can lead to enhanced predictability in midlatitudes upper-tropospheric and surface conditions. Branstator (2014) results also indicate that it is not necessary to successfully predict the tropical heating for a week or longer in order to benefit extratropical predictions at ranges longer than a week. If one accurately predicts heating for a day or two, that will affect the mid-latitude prediction for perhaps two weeks. Therefore, an implication of the delayed impact of tropical heating is that if one took observed tropical precipitation in account during data assimilation, the initial conditions would be better and the predictions in extratropics would improve.

As can be gathered from the examples presented here, improvements in weather and climate prediction using nonlinear numerical models could be obtained by including algorithms that simulate the ways the real atmosphere connects the different scales and also how some regions, like the tropics, affect other regions like the extra-tropics.

6. The Ensemble Prediction

Determinism was the basic fundamental of physics since the time of Newton (late 1600s until the late 1800s). The science postulate was: the future state of a system is completely determined by the present system state, and the state evolution is governed by causal relationships such as the Newtonian equations of motion. The test bed for determinism was orbital mechanics, especially its application in predicting the heavenly body's motions. The numerical weather prediction initially followed the path of determinism. The theoretical treatment of the motion scales in the atmosphere proposed by Jule Charney was the foundation for the first successful weather prediction by numerical methods. After the success of Princeton's prediction reached by Charney and his team, the numerical weather prediction 'quickly' spread worldwide, but the meteorological community soon began to deal with the crucial question of temporal limits for deterministic predictions.

In the early 1960s it was evident that initial state generally erroneous and imperfect models should conduct to the perception that the essential character of the causal laws – unstable models characterized by non-periodicity – places limits on predictability. Actually, the Lorenz entry into atmospheric predictability study was fortuitous. When he inadvertently has introduced a truncation error into an atmospheric model, the subsequent forecast was surprisingly different only due to changes in the accuracy of the retaining digits in model's initial data. (Lewis, 2005).

Today meteorologists know that, in weather forecasting, as the information flows from the initial conditions to a later prediction, any 'uncertainty' in the initial conditions implies that such an information flow should be quantified with tools from probability theory. Lorenz (1963 and 1965) showed that the forecast skill of the atmospheric models depends not only on the initial conditions accuracy and model realism, but also depends on the instabilities of the simulated flow.

From this great discovery by Lorenz, the community of atmospheric predictors realized that the ensemble prediction could take advantage of the 'deterministic chaos' of the models in benefit of the weather prediction. Lorenz demonstrated that any nonlinear dynamical system with instabilities, as with models to simulate atmospheric behavior, show errors that grow during the prediction. Today it is known that those errors' growth is due to instabilities associated to small imperfections of the models and to the tiniest initial condition error. Lorenz's discovery made inevitable the realization that in numeric weather prediction one needs to take in account the stochastic nature of the atmospheric evolution in the simulation made by models.

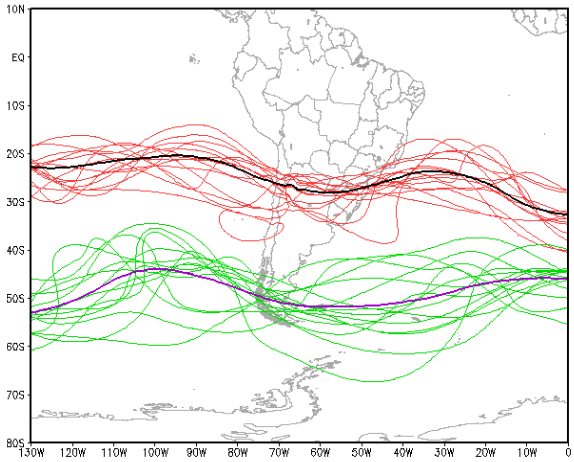
As explained by Kalnay (2003), the first attempt to explicitly acknowledge the 'uncertainties' in atmospheric models prediction was the introduction of the stochastic-dynamic forecasting concept, where a continuity equation of probability density for a model solution is treated. Subsequently researchers proposed several stochastic methods that evolved to the present operational *methods of ensemble prediction*. In this technique, the weather prognostics started employing initial conditions slightly disturbed, in a method named by Kalnay as *breeding growing perturbations*.

After some days of predictive model integration, the ‘deterministic chaos’ appears, and the divergence of the solutions can be statistically evaluated to indicate “*when”* and “*where”* the weather forecast has greater or smaller confidence (Zang & Krishnamurti, 1999; Krishnamurti et al., 2014). Although a great number of members in the ensemble prediction should lead to greater solution variability, it is important to delineate the level of confidence in the atmospheric prognostic, considering the average of the several solutions in each location of the model’s domain *(“where”*) in association to the number of days the prediction is being considered acceptable *(“when”*).

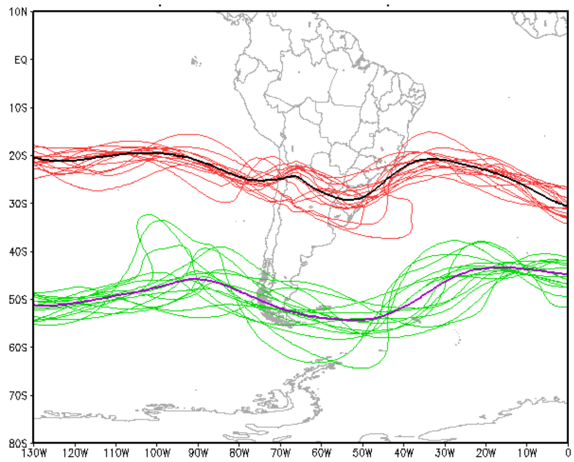
As can be seen throughout the present discussion, there are atmospheric states that are more predictable than others. This is what operational meteorologists see daily, and also some theoretical studies suggest. Actually there are situations in which the predictability, at least for the greater scales, goes beyond the fifteen days. Obviously if the predictability metric is rain occurrence in a specific location, the predictability will be for a very short period of time. From a practical point of view, the indication of the expectation degree related to the weather prediction is itself a strong signal of predictability.

Below Figure 1 shows the temporal evolution of ensemble predictions made operationally by CPTEC (Center for Weather Forecasting and Climate Studies), the Brazilian principal weather forecasting institution (Cunningham et al., 2014). As an example, spaghetti plots are presented with predictions for 15, 12, 10, 7, 5, and 3-days for September 17, 2014 at 12 UTC.

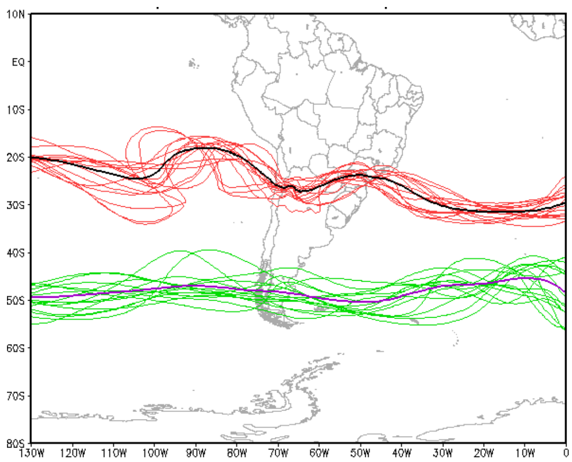
(a) Ensemble Forecast for 15 days



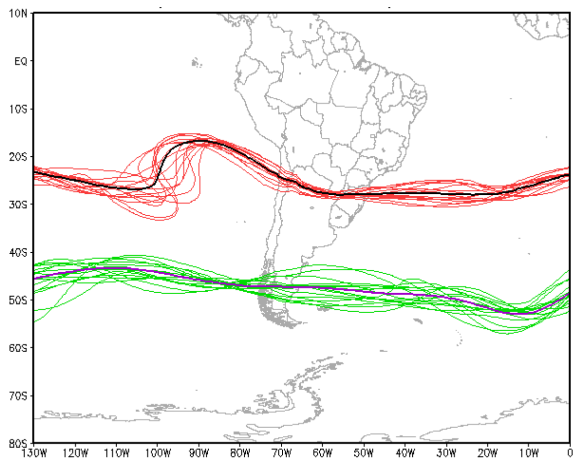
(b) Ensemble Forecast for 12 days



(c) Ensemble Forecast for 10 days



(d) Ensemble Forecast for 7 days



(e) Ensemble Forecast for 5 days (f) Ensemble Forecast for 3 days

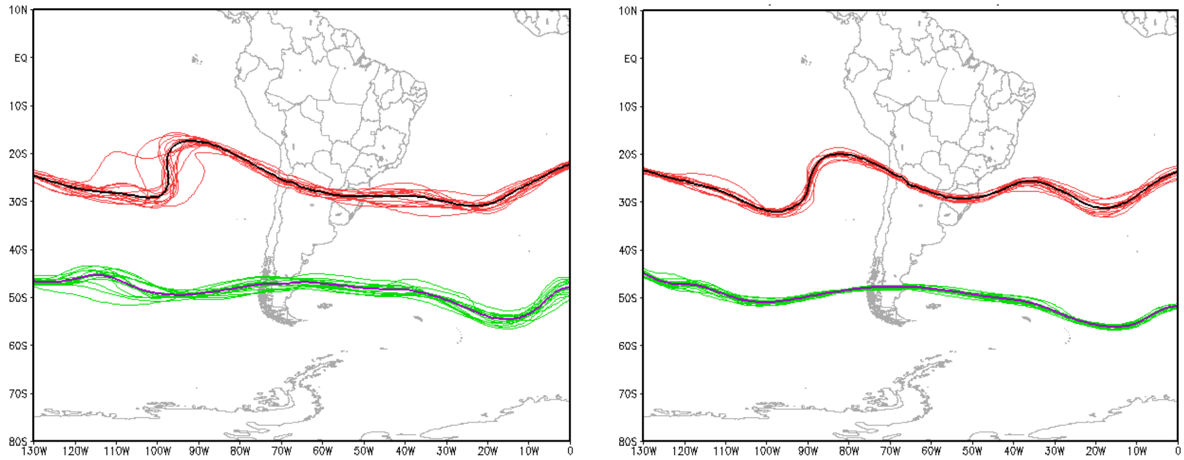


Figure 1. Spaghetti plots of ensemble predictions of 500 hPa geopotential heights of 5800m and 5600m around South America, for September 17, 2014 at 12 UTC: (a) 15-day forecast; (b) 12-day forecast; (c) 10-day forecast; (d) 7-day forecast; (e) 5-day forecast; (f) 3-day forecast

Source: CPTEC/INPE

As expected, in Figure 1, the more distant the prediction, the greater the uncertainty indicated by the divergence of the geopotential heights of 5800m and 5600m at 500 hPa level, over the Pacific and Atlantic Oceans and over the South American continent, at sub-tropical and temperate latitudes respectively. Considering that 500 hPa troughs and ridges indicate respectively cold and warm air intrusions below this level, these examples show the degree of 'uncertainties' in the frontal evolution ensemble prediction expected for 15 days ahead until forecasts is as close as 3 days.

Figure 2 refers to the 500 hPa geopotential height analysis at 12UTC of September 17, 2014. Comparing this 500 hPa analysis with the predictions shown in Figure 1, it is easy to see the subtropical 5800m geopotential height shows a very well predicted trough in the Pacific Ocean and another trough in the Atlantic Ocean, although not so well predicted before 5 days. By Figure 3, it can be seen the Atlantic trough is associated with a frontal system that puts cold air in its SW sector and warm air in its NE sector. In the case of the Pacific trough shown in Figure 2, Figure 3 indicates there was a cold advection, which is common in that region, and this cold sector is blocked by the Andes. Therefore, the continental warm air advances in the South American interior, coming from the Amazon region and going southward until the Argentina sector. This warm advection explains the South American 500 hPa continent ridge feature, well predicted with 15 days in advance.

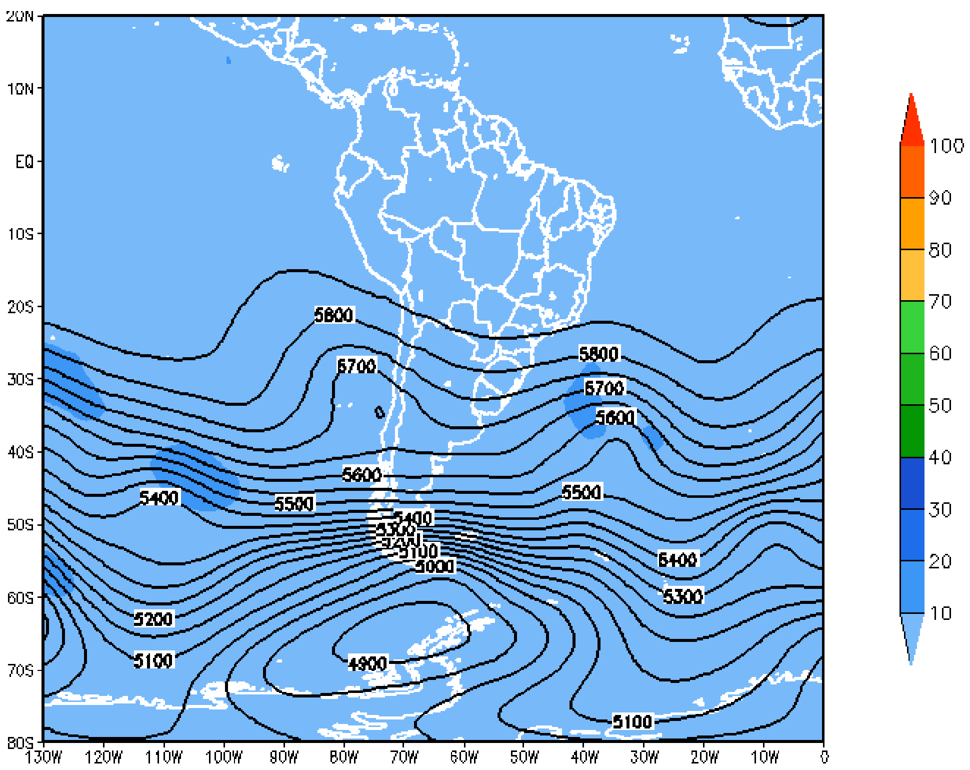


Figure 2. Verification of the 500 hPa geopotential height for September 17, 2014 at 12UTC

Source: CPTEC/INPE

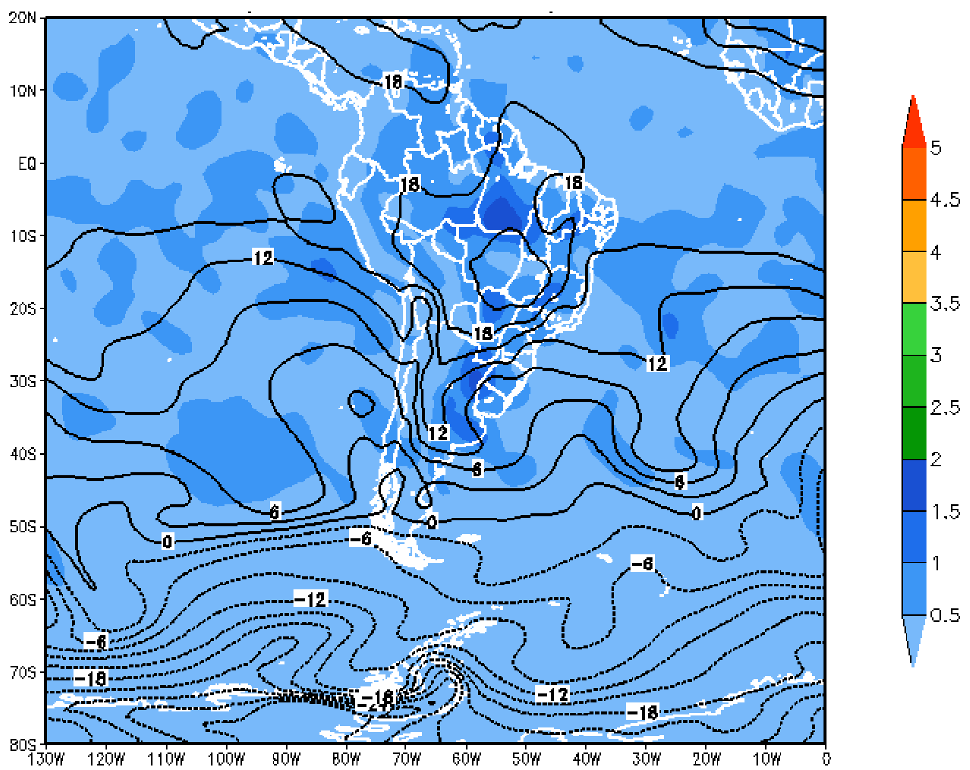


Figure 3. Verification of the 850 hPa temperature on September 17, 2014 at 12UTC

Source: CPTEC/INPE

7. Concluding Remarks

This article's principal objective was to demystify the confusion between the expressions 'atmospheric chaos' and 'deterministic chaos'. The expression 'atmospheric chaos' led to the belief nothing could extend predictions beyond two weeks, because the 'atmospheric chaos' imposes an 'intrinsic' physical limitation on predictions. On the other hand, actually, the models used today by meteorologists to predict atmospheric behavior produce 'deterministic chaos', as discovered by Lorenz (Lorenz, 1963). This confusion between the two expressions lead to an erroneous conclusion: if the atmosphere is 'chaotic', nothing could extend the forecasts beyond two weeks, because this was a limitation physically imposed by the atmosphere itself.

The arguments presented here took a different line of thought: the models to predict atmospheric behavior are 'chaotic' due the nonlinear character of its equations. Due to this characteristic, very small input changes in initial values lead to great divergence in the outputs some days later, and these changes are associated with the difficulty to know what is the 'real atmospheric state'. Since their prior studies, the authors suggest that possibly 'better models' and 'better observational data' could lead to atmospheric simulations and predictions more accurate, including to extend the limit of useful forecasts to beyond two weeks (Santos & Buchmann, 2013).

In the beginning of the numerical weather prediction in early 1950s, linear models were used to simulate atmospheric behavior, using principally the geostrophic and quasi-geostrophic concepts, but the very important advection issue was not considered due the mathematical difficulty to solve the advection (nonlinear) terms. Meteorological experience demonstrates that a nonlinear equations system simulates better geophysical fluids than a linear system, but the nonlinear models bring 'deterministic chaos' as a byproduct. Therefore, minimal errors in the initial state makes these errors grow after some steps, that is, the predictions 'quickly' begin to diverge. Lorenz himself proposed numerical experiments demonstrating it is practically impossible to avoid the 'chaos' emerge in nonlinear equations systems.

An equivocal conclusion emerged from Lorenz's digressions considering that the predictability of certain turbulent systems could not be extended beyond some finite threshold. This leads to the conclusion that weather forecasts could be limited by perturbations as trivial as the flapping wings of 'butterflies'. The question of the errors in the initial condition is elucidated in the recent article by Durran & Gingrich (2014), who explain the key-factor limiting the predictability of such systems as being the large scale impact of initial errors that originate in the small scales and then grow with the process repetition. After more appropriate numerical experiments than the Lorenz's tests, they conclude that in any real-world event, the contributions of the 'butterflies' to the 'uncertainties' in the initial conditions would be completely dwarfed by errors in the larger scales. The study of Durran & Gingrich (2014) conclude there is a mathematical 'intrinsic' problem when nonlinear models are affected by differences in the "observational error" related physically with the energy flows in many scales of motion. Thus, small relative errors at long wavelengths cause similar impacts on error growth as large relative errors at short wavelengths. Therefore, considering the influence of 'butterflies' in the initial conditions as a means to improve meteorological forecasts would not be important due to relative small errors that exist in the larger scales.

The musing of Lorenz that a 'butterfly' beating its wings in Brazil could form a 'tornado' in Texas was applauded by the academy after its proposal in 1965. However, more recently it has been realized that this concern could cause delays in scientific development as it suggests that with the 'uncertainties' inherent in meteorological observation (these 'uncertainties' could be the 'butterflies') the predictions become practically worthless after 15 days. In fact, more careful studies using numerical simulations have pointed to an interesting conclusion: the atmospheric modes, whether fast (high frequency) or slow (low frequency), which are associated with energy, propagate at the group velocity, which is controlled by the large scale. Therefore, small-scale initial errors, including those at scale-lengths far larger than the 'butterfly' size do not matter when minor relative errors are present in the larger scales (Durran & Gingrich, 2014).

Meteorologists' intuition suggests that even millions 'butterflies' would not cause an important effect on weather prediction because the energy of small perturbations propagates through preferential trajectories. Actually the ability to predict atmospheric behavior will only improve when models can include the ways nature transfers energy of different scales. Brazilian researchers have followed this way to address the question of predictability by searching how fast atmospheric modes, such as high-frequency inertio-gravity waves, 'interact' with Rossby slow waves. This search to understand how the energy of different atmospheric modes 'travels', can lead to what are herein called 'better models' (Raupp & Silva Dias, 2010).

In the beginning of the 1960s, Lorenz discovered that any nonlinear dynamical system with instabilities, like the models for weather forecasts, presents errors that grow during the prediction process. This 'deterministic chaos' discovered by Lorenz led to the realization that numerical weather predictions should take into account the stochastic nature of the error growth that occurs while running the model. While the integration of the predictor models happens, the 'deterministic chaos' appears as a surprising divergence in the solutions. Fortunately, it is possible associate the divergence to the model's predictability by identifying geographic sectors and also periods in which the prediction can be considered more or less accurate. The stochastic treatment of the errors growth during the weather prediction is called today 'ensemble forecast' and brings to meteorologists a great additional tool, because it indicates "*where*" and "*when*" the model predicts with greater or smaller confidence future atmospheric conditions.

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Appendix

Table I. Shows two different analytical expressions, with an error of the order of 0,00001. Both formulas are unstable with diverge occurring at the steps 37 and 17, respectively

Formula: $x^{(n+1)}=3.75*x^{(n)}-(x^{(n)})^2$				Formula: $x^{(n+1)}=(x^{(n)}-2)^2$			
step	cond 1	cond 2	difference	step	cond 1	cond 2	difference
0	0.50000	0.50001	-0.00001	0	0.50000	0.50001	-0.00001
1	1.62500	1.62503	-0.00003	1	2.25000	2.24997	0.00003
2	3.45312	3.45314	-0.00001	2	0.06250	0.06248	0.00002
3	1.02515	1.02510	0.00004	3	3.75391	3.75396	-0.00006
4	2.79337	2.79330	0.00008	4	3.07619	3.07639	-0.00020
5	2.67221	2.67235	-0.00014	5	1.15818	1.15862	-0.00044
6	2.88007	2.87985	0.00022	6	0.70866	0.70792	0.00074
7	2.50545	2.50590	-0.00045	7	1.66755	1.66946	-0.00191
8	3.11816	3.11759	0.00057	8	0.11052	0.10926	0.00127
9	1.97019	1.97159	-0.00140	9	3.57013	3.57491	-0.00479
10	3.50656	3.50629	0.00027	10	2.46530	2.48035	-0.01506
11	0.85362	0.85450	-0.00088	11	0.21650	0.23074	-0.01424
12	2.47241	2.47421	-0.00179	12	3.18087	3.13028	0.05059
13	3.15872	3.15657	0.00215	13	1.39445	1.27753	0.11692
14	1.86769	1.87319	-0.00551	14	0.36669	0.52196	-0.15527
15	3.51557	3.51562	-0.00005	15	2.66769	2.18459	0.48310
16	0.82415	0.82399	0.00017	16	0.44580	0.03407	0.41173
17	2.41134	2.41099	0.00035	17	2.41552	3.86487	**-1.44935**
18	3.22796	3.22834	-0.00037	18	0.17266	3.47774	-3.30508
19	1.68512	1.68410	0.00101	19	3.33917	2.18371	1.15546
20	3.47957	3.47918	0.00039	20	1.79339	0.03375	1.75964
21	0.94098	0.94222	-0.00124	21	0.04269	3.86614	-3.82345
22	2.64324	2.64555	-0.00231	22	3.83107	3.48246	0.34861
23	2.92544	2.92188	0.00356	23	3.35282	2.19769	1.15513
24	2.41220	2.41966	-0.00746	24	1.83013	0.03908	1.79104
25	3.22704	3.21897	0.00807	25	0.02886	3.84520	-3.81634
26	1.68761	1.70938	-0.02176	26	3.88540	3.40475	0.48066
27	3.48051	3.48819	-0.00768	27	3.55475	1.97331	1.58144
28	0.93796	0.91323	0.02473	28	2.41725	0.00071	2.41653

29	2.63758	2.59062	0.04695	29	0.17409	3.99715	-3.82306
30	2.93410	3.00351	-0.06941	30	3.33393	3.98862	-0.65468
31	2.39393	2.24209	0.15184	31	1.77938	3.95459	-2.17522
32	3.24634	3.38087	-0.13453	32	0.04868	3.82043	-3.77175
33	1.63506	1.24799	0.38707	33	3.80767	3.31396	0.49371
34	3.45805	3.12248	0.33557	34	3.26767	1.72649	1.54117
35	1.00957	1.95942	-0.94985	35	1.60698	0.07481	1.53217
36	2.76666	3.50850	-0.74184	36	0.15446	3.70637	-3.55191
37	2.72057	0.84731	**1.87327**	37	3.40600	2.91171	0.49429
38	2.80063	2.45947	0.34115	38	1.97684	0.83122	1.14561
39	2.65884	3.17402	-0.51518	39	0.00054	1.36604	-1.36551
40	2.90123	1.82819	1.07304	40	3.99785	0.40190	3.59595

Table II. Shows two different analytical expressions, with an error of the order of 0,0001. Both formulas are unstable with diverge occurring at the steps 28 and 14, respectively

Formula: $x^{(n+1)}=3.75*x^{(n)}-(x^{(n)})^2$				Formula: $x^{(n+1)}=(x^{(n)}-2)^2$			
step	cond 1	cond 2	difference	step	cond 1	cond 2	difference
0	0.50000	0.50010	-0.00010	0	0.50000	0.50010	-0.00010
1	1.62500	1.62528	-0.00028	1	2.25000	2.24970	0.00030
2	3.45312	3.45326	-0.00014	2	0.06250	0.06235	0.00015
3	1.02515	1.02471	0.00043	3	3.75391	3.75449	-0.00058
4	2.79337	2.79264	0.00074	4	3.07619	3.07822	-0.00204
5	2.67221	2.67357	-0.00135	5	1.15818	1.16257	-0.00439
6	2.88007	2.87791	0.00216	6	0.70866	0.70129	0.00737
7	2.50545	2.50979	-0.00434	7	1.66755	1.68664	-0.01909
8	3.11816	3.11267	0.00549	8	0.11052	0.09819	0.01233
9	1.97019	1.98380	-0.01362	9	3.57013	3.61687	-0.04675
10	3.50656	3.50379	0.00278	10	2.46530	2.61428	-0.14898
11	0.85362	0.86268	-0.00905	11	0.21650	0.37734	-0.16084
12	2.47241	2.49083	-0.01841	12	3.18087	2.63304	0.54783
13	3.15872	3.13638	0.02234	13	1.39445	0.40073	0.99371
14	1.86769	1.92455	-0.05686	14	0.36669	2.55765	**-2.19096**
15	3.51557	3.51317	0.00240	15	2.66769	0.31098	2.35671
16	0.82415	0.83202	-0.00787	16	0.44580	2.85280	-2.40699
17	2.41134	2.42783	-0.01648	17	2.41552	0.72726	1.68826
18	3.22796	3.21001	0.01795	18	0.17266	1.61986	-1.44720
19	1.68512	1.73338	-0.04826	19	3.33917	0.14451	3.19467
20	3.47957	3.49557	-0.01600	20	1.79339	3.44285	-1.64946
21	0.94098	0.88939	0.05160	21	0.04269	2.08182	-2.03913
22	2.64324	2.54419	0.09904	22	3.83107	0.00669	3.82438
23	2.92544	3.06781	-0.14237	23	3.35282	3.97327	-0.62045
24	2.41220	2.09283	0.31937	24	1.83013	3.89378	-2.06366
25	3.22704	3.46817	-0.24114	25	0.02886	3.58641	-3.55756
26	1.68761	0.97742	0.71020	26	3.88540	2.51671	1.36870
27	3.48051	2.70997	0.77054	27	3.55475	0.26699	3.28776
28	0.93796	2.81845	**-1.88049**	28	2.41725	3.00334	-0.58609
29	2.63758	2.62554	0.01204	29	0.17409	1.00668	-0.83259
30	2.93410	2.95232	-0.01822	30	3.33393	0.98668	2.34725
31	2.39393	2.35501	0.03892	31	1.77938	1.02682	0.75256
32	3.24634	3.28522	-0.03888	32	0.04868	0.94709	-0.89841
33	1.63506	1.52691	0.10815	33	3.80767	1.10863	2.69904
34	3.45805	3.39446	0.06359	34	3.26767	0.79455	2.47312
35	1.00957	1.20687	-0.19730	35	1.60698	1.45312	0.15386
36	2.76666	3.06923	-0.30257	36	0.15446	0.29908	-0.14461
37	2.72057	2.08944	0.63113	37	3.40600	2.89314	0.51286
38	2.80063	3.46964	-0.66901	38	1.97684	0.79770	1.17914
39	2.65884	0.97275	1.68609	39	0.00054	1.44553	-1.44500
40	2.90123	2.70157	0.19966	40	3.99785	0.30743	3.69042

Table III. Shows two different analytical expressions, with an error of the order of 0,001. Both formulas are unstable with diverge occurring at the steps 16 and 10, respectively

Formula: $x^{(n+1)}=3.75*x^{(n)}-(x^{(n)})^2$				Formula: $x^{(n+1)}=(x^{(n)}-2)^2$			
step	cond 1	cond 2	difference	step	cond 1	cond 2	difference
0	0.50000	0.50100	-0.00100	0	0.50000	0.50100	-0.00100
1	1.62500	1.62775	-0.00275	1	2.25000	2.24700	0.00300
2	3.45312	3.45449	-0.00137	2	0.06250	0.06101	0.00149
3	1.02515	1.02083	0.00432	3	3.75391	3.75968	-0.00578
4	2.79337	2.78602	0.00736	4	3.07619	3.09649	-0.02030
5	2.67221	2.68567	-0.01346	5	1.15818	1.20229	-0.04411
6	2.88007	2.85844	0.02163	6	0.70866	0.63635	0.07232
7	2.50545	2.54847	-0.04302	7	1.66755	1.85955	-0.19200
8	3.11816	3.06206	0.05609	8	0.11052	0.01973	0.09080
9	1.97019	2.10651	-0.13632	9	3.57013	3.92148	-0.35135
10	3.50656	3.46203	0.04454	10	2.46530	3.69209	**-1.22679**
11	0.85362	0.99697	-0.14334	11	0.21650	2.86317	-2.64667
12	2.47241	2.74468	-0.27227	12	3.18087	0.74506	2.43581
13	3.15872	2.75928	0.39945	13	1.39445	1.57488	-0.18043
14	1.86769	2.73368	-0.86600	14	0.36669	0.18073	0.18597
15	3.51557	2.77829	0.73728	15	2.66769	3.30975	-0.64206
16	0.82415	2.69969	**-1.87554**	16	0.44580	1.71544	-1.26964
17	2.41134	2.83551	-0.42417	17	2.41552	0.08097	2.33455
18	3.22796	2.59305	0.63492	18	0.17266	3.68266	-3.51000
19	1.68512	3.00004	-1.31492	19	3.33917	2.83134	0.50783
20	3.47957	2.24992	1.22965	20	1.79339	0.69113	1.10226
21	0.94098	3.37506	-2.43408	21	0.04269	1.71313	-1.67044
22	2.64324	1.26544	1.37780	22	3.83107	0.08229	3.74878
23	2.92544	3.14406	-0.21862	23	3.35282	3.67760	-0.32478
24	2.41220	1.90511	0.50709	24	1.83013	2.81434	-0.98422
25	3.22704	3.51472	-0.28768	25	0.02886	0.66316	-0.63430
26	1.68761	0.82695	0.86067	26	3.88540	1.78715	2.09826
27	3.48051	2.41721	1.06330	27	3.55475	0.04531	3.50944
28	0.93796	3.22163	-2.28367	28	2.41725	3.82083	-1.40358
29	2.63758	1.70221	0.93537	29	0.17409	3.31542	-3.14133
30	2.93410	3.48577	-0.55167	30	3.33393	1.73033	1.60360
31	2.39393	0.92105	1.47288	31	1.77938	0.07272	1.70665
32	3.24634	2.60560	0.64074	32	0.04868	3.71440	-3.66573
33	1.63506	2.98185	-1.34679	33	3.80767	2.93918	0.86849
34	3.45805	2.29051	1.16754	34	3.26767	0.88206	2.38561
35	1.00957	3.34297	-2.33340	35	1.60698	1.24979	0.35719
36	2.76666	1.36068	1.40598	36	0.15446	0.56282	-0.40835
37	2.72057	3.25110	-0.53052	37	3.40600	2.06550	1.34050
38	2.80063	1.62198	1.17865	38	1.97684	0.00429	1.97255
39	2.65884	3.45161	-0.79277	39	0.00054	3.98286	-3.98232
40	2.90123	1.02994	1.87129	40	3.99785	3.93173	0.06612