Journal of Education and Learning; Vol. 6, No. 3; 2017
ISSN 1927-5250 E-ISSN 1927-5269
Published by Canadian Center of Science and Education

Middle Grade Students' Concept Images of Algebraic Concepts

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Received: April 9, 2017 Accepted: May 3, 2017 Online Published: May 9, 2017

doi:10.5539/jel.v6n3p299 URL: http://doi.org/10.5539/jel.v6n3p299

Abstract

This study investigates middle school students' concept images of algebraic concepts which are term, constant term, variable, and coefficient. Also, the study aimed to explore their performances in defining these concepts correctly. A phenomenological method was used to support methodological perspective and to reveal the findings of the study. Twenty seven 7th grade students completed a questionnaire, which aims to assess students' concept images of the algebraic concepts. Twelve of them were selected for semi-structured interviews on the basis of purposeful sampling method to clarify and expand on their responses to the questionnaire. The data was analyzed via the content analysis approach. The analysis of the data indicated that students have various concept images of basic algebraic concepts. More interestingly, this study showed that although students' performances on defining the algebraic concepts were above 50% for each concept, 7th grade students' concept images of the algebraic concepts is not so strong for learning algebra conceptually.

Keywords: algebraic concepts, concept image, concept definition, middle grade students

1. Introduction

Algebra has a vital role and is considered to be a gatekeeper in mathematics teaching and learning (Knuth et al., 2005). It is defined as a generalized form of arithmetic that uses symbols, letters and signs for the purpose of generalization. Because of the usage of symbols, letters and signs, algebra becomes an abstract subject (Samo, 2009). Therefore, students consider that algebra is one of the most difficult areas of mathematics and consequently, many students have misconceptions and difficulties in learning algebra (Kieran, 1992). In order to understand students' misconceptions and difficulties, the reasons for students' difficulties in algebra and present strategies to overcome these difficulties, many studies paid attention to the students' understanding of algebra from different point of views (Dede, 2004; Dede & Argun, 2003; MacGregor & Stacey, 1993). As a result of these studies, researchers reported that students have difficulties in understanding algebraic concepts such as variable, algebraic expressions and unknown. Although these concepts are the reasons for students' difficulties in algebra, there is not any study in the literature which investigated how students define these terms. For this reason, the present study aimed to investigate students' understanding of algebraic concepts from the point of students' knowledge on the definitions of the algebraic concepts. In other words, the aim of this study was to reveal students' concept images related to algebraic concepts.

Definitions have a special place in the learning and teaching of mathematics since it is the starting point in order to understand the meaning of a concept (Long & DeTemple, 2003). Before students begin the school, they learn about the concept from their daily life experiences (Clements, Swaminathan, Hannibal, & Sarama, 1999) and they define mathematical concepts in an informal way (Engelbrecht, 2010) which may be considerably different from the formal definitions (Vinner, 1991). To understand how students define the concept, it is essential to identify students' understandings and beliefs concerning the concept. In order to do this, Tall and Vinner suggested to assess students' concept images that a learner relies on in order to solve a mathematical task. In relation to concept image and concept definition, Tall and Vinner (1981) generated a theory which the present study is grounded.

Tall and Vinner (1981) stated that "We shall use the term concept image to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. It is built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures" (p. 152). Further, concept definition is "a form of words used to specify that concept" (p. 152). A

student may memorize the concept definition of any concept from the textbooks or from the teacher. Surprisingly, when the students try to recall a concept or solve the problem, personally created concept image come to their mind, instead of concept definition (Gutierrez & Jaime, 1999; Heinze & Ossietzky, 2002). However, Dickerson and Pitman (2012) stated that students' concept image might be incomplete or faulty. Due to this, students have difficulties in learning mathematics subjects.

Despite the vast research on the students' understanding in algebra, the overall image that emerges from the literature is that students' concept images of algebraic concepts have not been investigated yet. Starting from this point of view, the aim of the study was to answer the following research questions, using the terminology of Tall and Vinner (1981).

What kinds of concept images of algebraic concepts do middle school students have?

What are the performances of middle school students on defining algebraic concepts correctly?

2. Method

2.1 Research Design

A phenomenological method, which is one of the qualitative researches, was used to support methodological perspective and to reveal the findings of the study. Creswell (1998) stated that phenomenological method is best suited for the research which aims to understand participants' common experiences and thoughts of a phenomenon. He emphasized that this is important to develop a deeper understanding about the characteristics of the phenomenon. Due to the fact that the purpose of the study was to gain deeper understanding about middle school students' concept images of the algebra concepts, phenomenological design was chosen.

2.2 Setting and Participants

In Turkey, the teaching of algebra is carried out in 6th grade level for the first time (Ministy of National Education [MoNE], 2013). Students have learnt the definitions of variable, term, unknown, constant term and coefficient when they are 6th graders. Based on the curricula (MoNE, 2013), it is assumed that algebraic concepts asked in data collection tool are learned by 7^{th} grade students. Therefore, the participants of the study were selected from the 7^{th} grade students in a private school in Istanbul, Turkey. Twenty seven middle school students were the participants of the study. Moreover, 12 students among 27 students were selected for semi-structured interview based on the purposeful sampling strategy. "The purpose of purposeful sampling is to select information-rich cases whose study will illuminate the questions under study" (Patton, 1990, p. 169). The students were selected based on their responses to the questionnaire administered before the semi-structured interviews. Six students were males and the rest of them were females. Moreover, the number of students whose mathematics performance was high, moderate and low was the same, which is four students. These students gave the researchers a strong impression that they are all suitable candidates for interviews capable of freely expressing themselves.

2.3 Questionnaire

An instrument on algebraic concepts was designed and used by the researcher to determine 7^{th} grade students' concept images of the algebraic concepts. The instrument consisted of 10 open-ended questions related to the algebraic concepts. For the purpose of the present study, only responses regarding the definition of 4 major algebraic concepts such as term, constant term, coefficient and variable were analyzed. Each participant wrote the definitions of the given terms without time limitation.

2.4 Semi-Structured Interviews

In this study, information gathered from the questionnaire was limited to portray a general description of 7^{th} grade students' concept images of the algebraic concepts. Thus, interviews were conducted with 12 students to clarify and expand on their responses to the questionnaire.

During the interview, it was expected to explain the definitions of the given algebraic concepts. If it was necessary, each of the participants was asked the questions "what is a variable?", "what is a constant term?", "what is a coefficient?" and "what is a term?". With the permission of the participants, all the interviews were video-taped using a digital camera. The duration of all interviews was approximately 40 minutes.

2.5 Data Analysis

The data was analyzed via the content analysis approach developed by Pilkington (2001). To begin with, all the videos of interviews were transcribed and the initial codes were formed. Then the categories were generated and they were integrated to create the themes. Moreover, students' performances on defining algebraic concepts

correctly were also investigated. While analyzing the correctness of the students' definitions, the definitions taken from Turkish Language Institution [TDI] (http://www.tdk.gov.tr), the middle school mathematics lesson curriculum (MoNE, 2016), textbooks and related literature (Baykul, 2000; Capraro & Joffrion, 2006) were used. Two experts in the field of mathematics education validated the analysis of students' images. Due to the fact that the students did not give complete formal definition of the concepts, the experts discussed whether or not students' definitions could be accepted as correct until a complete consensus was reached.

3. Findings

In this study, the aim was to examine 7^{th} grade students' concept images of the algebraic concepts and their performances on defining these concepts correctly. Within the scope of the study, it was asked the definitions of term, constant term, variable and coefficient. Throughout the finding section, students' concept images of the given terms were presented separately.

3.1 Students' Concept Images of the "Term"

Based on the analysis of the data obtained from interviews, it was found that 12 students made five different definitions of "term". Initially, 7 students defined the "term" as each element of algebraic expression which can be separated from each other by addition sign (+) and subtraction sign (-). For instance, Participant 4 stated that the terms of 2x+3 are 2x and 3. Moreover, Participant 8 gave another example regarding the term. She said that $4t^2$, 6z and 5 are the terms of $4t^2+6z+5$. Apart from this definition, 1 student defined the "term" in a different way. He explained that terms are the unknowns and the knowns. In other words, he noted that terms of 8xy+10 are 8xy and 10. Based on the definition taken from Turkish Language Institution, these students' definitions of the "term" could be regarded as correct.

On the other hand, 4 students among 12 students did not make the definition of the "term" correctly. Participant 1 and Participant 10 explained that the "term" is the element of algebraic expressions which contain coefficient and unknown. Based on their definition, Participant 10 stated that $4t^2+6z+5$ has 2 terms which are $4t^2$ and 6z. It can be specified that these students' definition of the "term" is not sufficient due to the fact that they did not consider the constant term as a term.

However, the "term" was defined by Participant 5 as the element of algebraic expression comprised of the unknown and the constant term. According to him, the coefficient is not a part of the term. For instance, he said that xy and 10 are the terms of 8xy+10. The concept image of Participant 5 could be stated as incorrect. Lastly, Participant 12 described the term as each number and unknown are the terms of the algebraic expression. In other words, the terms of $5rs^2$ are 5, r and s^2. As a result, it can be concluded that Participant 5 defined the "term" incorrectly.

3.2 Students' Concept Images of the "Constant Term"

The analysis of the data indicated that students had 4 different concept images of "constant term". Depending on these concept images, 5 students defined constant term as unchanging and the term which remains the same. Regarding this definition, an excerpt of Participant 2 is given below.

> *The constant term of 2x+3 is 3 since there is not an unknown like x. 3 is always 3 and its value does not change.*

Although 6 students explained that the term which does not have any unknown is constant term, 4 students stated that numbers without unknowns are the constant term of the algebraic expression. For example, Participant 10 specified that the constant term of 8xy+10 is 10. Additionally, one participant defined constant term as the numbers which are added to unknown or subtracted from unknown. The Participant 5's excerpt is given as an example.

> *In the algebraic expression, $4v^2–2v+7$, 7 is added to the $4v^2$ and –2v. Thus, it is constant term.*

When students' concepts images were analyzed in terms of correctness, it could be stated that all are correct.

3.3 Students' Concept Images of the "Variable"

The findings of the study showed that, all students defined "variable" as unknown. They expressed that the variable in the algebraic expression does not have exact value. Instead of the variable in the algebraic expression, all numbers can be written. For instance, Participant 6 explained that:

> *In the algebraic expression, 7p+9k, p can be 5 or 11. Also, k can be all numbers. In other words, their value is not known and constant.*

Regarding the data, it could be said that all students have correct concept images of the "variable".

3.4 Students' Concept Images of the "Coefficient"

Analysis of the data revealed that students had different concept images of coefficient. Six students defined coefficient as the number located in front of the unknown. For instance, they stated that 8 is the coefficient of the algebraic expression 8xy+10. Based on the definition taken from Turkish Language Institution, students' definition of the "coefficient" is correct.

On the other hand, 4 students stated that coefficient is the number which is located in front of the unknown. Moreover, these students added that constant term of the algebraic expression is also the coefficient. Regarding this concept image, the excerpts of Participant 9 is presented below.

> *In the algebraic term, 2x+3, the coefficients are 2 and 3. Since 2 is the in front of x and 3 is the constant term.*

Besides, only Participant 6 stated that the number multiplied by unknown is the constant term of the algebraic expression. For instance, the participant explained that:

> *In the algebraic expression, 8xy+10, 8 is the number multiplied by both x and y. Also, 10 is the coefficient since it is multiplied by 1. For this reason, 8 and 10 are the coefficients.*

Lastly, one participant expressed that if the term is the square of another term, then it is the coefficient. For instance, she stated that:

> *The algebraic expression, $4t^2 + 6z + 5$, has one coefficient which is $4t^2$ since it is the square of 2t.*

In regard to the definition of coefficient presented in mathematics curriculum (MoNE, 2016), these students' concept images of definition is not correct.

4. Discussion and Conclusion

Mathematical definition has an important role in teaching and learning mathematics (Ertekin, Yazici, & Delice, 2015). Because of the high importance of definitions, it is crucial to define the basic concepts correctly in order to learn algebra. Initially, students seem to have two different concept images of the "term" in algebraic expression. Most of the students focused on the addition and subtraction signs while defining "term" of the algebraic expression. They stated that the "term" can be multiplication of the letters and numbers, multiplication of only numbers or multiplication of only letters between two signs (addition or subtraction sign). Moreover, some students stated that terms are formed by the unknowns and the knowns. Although students know what the term is and they gave correct answer to the question, they could not define it in a formal way. However, based on the Turkish Language Institution [TDI], the "term" is defined formally as the multiplication of one number and one variable or more than one variable. From the definition of TDI, students did not define "term" using variables. Instead of this, they use letters, numbers, known and unknown. As Tall and Vinner (1981) assert, students can use mathematical concepts without knowing the formal definitions. On the other hand, some students could not define the term correctly. They only focused on the letters and the numbers separately.

With regard to students' concept images of constant term, it was found that students have 4 concept images. Similar to their concept images of "term", they did not use variable while defining the "constant term". Instead of this, they preferred to describe it with different concepts such as unchanging, unknown, numbers without unknowns, numbers which are added to unknown or subtracted from unknown. However, their concept images are correct since MoNE (2016) provides a definition as the terms which do not contain any variable.

The findings of the present study showed that all students defined variable as unknown. This is in parallel to other studies which found that students use variable in the schools as unknowns, constant, varying quantities (Philipp, 1992; Schoenfeld & Arcavi, 1988; Usiskin, 1988). Actually, it is not surprising that students conceptualize the variables as unknowns since the mathematics curriculum in Turkey defined variables as unknowns. Also, teachers do not emphasize the difference between unknown and variables (Ulusoy, 2013). For this reason, it is expected that students defined variables as unknown. Furthermore, students think that variables must stand for only one specific value as reported in the study of Asquith et al. (2007). This may be another reason why students have difficulties with variables, which was also stated in the previous studies.

Another finding of the study refers to the students' images of coefficient. Almost half of the students defined coefficient correctly. They defined it as in the mathematics curriculum (MoNE, 2016). An interesting finding of this study is that some students explained that the constant term is also coefficient since they thought that coefficient is the multiplier of the unknown and the numbers. To this respect, they considered that the constant term is multiplier of 1.

To sum up, the findings of this study showed that although students' performances on defining the algebraic concepts were above 50% for each concept, 7th grade students' concept images of the algebraic concepts is not so strong for learning algebra conceptually. They defined the terms in an informal way. The reasons for students' misconceptions and difficulties in algebra might be that their lack of definitions. As it is emphasized by the United States' Common Core State Standards for Mathematics [CCSSM] (Common Core State Standards Initiative, 2010), definitions in the mathematics education has a vital role since mathematically proficient students use definitions to conceptualize the concepts in mathematics. For this reason, the teachers may attach particular importance to make students learn the definitions of the concepts correctly. Moreover, the teachers let students to define the concepts before presenting their formal definitions. Also, another implication is related to curriculum developers and textbook writers. They might design different activities to give opportunities for students to define the concepts on their own.

Due to the fact that definitions are so important in learning mathematics, there is a need for future studies to explore concept definitions and concept images. Although this study was carried out with restricted number of middle school students, the existence of perceptions about algebraic concepts points out to the fact that this perception exists in other individuals. For this reason, a research with larger groups can be suggested to investigate students' concept images and concept definitions about the other important topics in mathematics.

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