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Problem Solving at the Middle School Level: A Comparison of Different Strategies

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Abstract

This article sheds light and reflects on how students in grades seven and eight read and understand implicit data when solving a story problem. Problem solving experiences help in adding up to the child's mathematical knowledge and promote a higher level of critical thinking abilities. Seventh and eighth grade students were selected from two private schools. Both schools are of the same socio-economic status. All the students in seventh and eighth grades from these two schools participated in the study, regardless of their school grades or their English proficiency. The results show that very few students understand the implicit data given in the text and use it in the resolution of their problem, even when this data is crucial in solving the problem. The majority of the students try to solve the problem without searching for the implicit data or understanding it. They also have difficulties choosing a strategy and following it until they get an answer, and never seem to check if the answers they found are correct or even logical.

Problem solving has been and still is the basis for learning mathematics. This research is a reflection of what our students think and do once they encounter a story problem. Thus, it sheds some light on the importance of developing teaching strategies that enhance students' understanding of implicit data and that encourage them to follow their chosen strategy and verify their answers at the end.

Keywords: mathematics teaching, problem solving, implicit data, story-problem, middle school

1. Introduction

Solving a problem situation in mathematics discipline is one of the most important competencies taught to students in the middle school level. This competency requires the mastering of many other competencies such as reading the problem, listing and grouping the given data, identifying the useful data, and building a strategy of resolution (Baraké & Rouadi, 2014). One of the difficulties the students face is the identification of the useful data especially if the given data is large (Baraké & Rouadi, 2014). One of the other difficulties that we noticed in our earlier studies is reading the problem and understanding it. The students face difficulties in reading and understanding the problem, which affects their resolution strategy. Their choosing and following a strategy is affected, and very few follow the chosen strategy until the problem is solved.

The main purpose of this study is to check the methods used by students to solve two story problems that contain implicit data. A brief introduction on the importance of solving story problems at the middle school level will be presented, followed by the importance of a good reading of a problem. We will then present two problems we used with learners as well as the possible strategies that could be used to solve them. Finally, we will analyze the students' answers and conclude about the use of different strategies in solving the two problem situations we presented with and the impact of the implicit data on the problem resolution.

2. Problem Solving in Middle School

Mathematics educators have been called to teach mathematics through problem solving (National Council of Teachers of Mathematics [NCTM], 2000). As stated in Principles and Standards for School Mathematics (NCTM, 2000, p. 52): "Solving problems is not only a goal of learning mathematics but also a major means of doing so. By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations…" Thus, problem solving experiences help in adding up to the student's mathematical knowledge and promote a higher level of critical thinking abilities. According to the NCTM (2000), problem solving in math has four main objectives: building new mathematical knowledge, solving problems that arise in other contexts than mathematics, adapting and applying appropriate strategies and reflecting on the process of mathematical problem solving.

Problem solving has been viewed from varying perspectives such as means–end analyses (Newell & Simon, 1972), text processing (Kintsch, 1994) and schema theory (Marshall, 1995).

In the means-end analysis, developed by Newell and Simon (1972), the student divides the main goal of the problem into subgoals or obstacles; he then tries to eliminate these obstacles by solving them one after the other, always keeping in mind the main purpose or goal of the problem. Once all the obstacles have been eliminated and subgoals achieved, the main goal has also been achieved. This method allows the student to concentrate on smaller subtasks, which are easier to solve than the main purpose of the problem, or the main task. On the other hand, the text processing developed by Kintsch (1994), explains how difficult it is for readers to understand and use the information given in a text. The reader's prior knowledge is a primary factor that helps the reader make use of the information. Therefore, even though students might understand the general idea of the text, especially in mathematics problem-solving, they might not be able to properly use the information presented in the text. The text of the problem should therefore take into account the background knowledge of the students, and should be as coherent and explicit as possible to facilitate understanding and use of information for the student. Finally, Marshall's (1995) schema theory suggests that planning and execution in problem-solving is a part of a schema that contains both declarative and procedural knowledge. In the problem-solving process, students should use identification, elaboration, planning and execution in order to reach their goal.

Moreover, students who are provided with opportunities to communicate verbally and through writing and listening will review dual benefit of communication to learning mathematics and learning to communicate mathematically (Wahyuningrum & Suryadi, 2014).

3. Reading the Problem and Implicit Data

Mathematical problem solving relies on many factors, among which reading and understanding the problem. It is based on the knowledge of language, mathematical terminology, and the ability of visualizing the problem in a drawn format which is the concrete representation of the problem. Evidence indicates that students' errors are frequently based upon the miscomprehension of the word problem (Cummins, Kintsch, Reusser, & Weimer, 1988). These miscomprehensions result from several possibilities including language learners (Mestre, 1988), inadequate reading strategies, insufficient conceptual or procedural mathematical knowledge (Mayer, 1992) and inability to coordinate knowledge structures necessary to solve the problem (Pape, 2004).

The comprehension of the problem can be divided into three categories (Peletier-Leculée & Sayac, 2004): Literal comprehension, integral comprehension and fine comprehension. In the literal comprehension, the student understands the successive words and explicit ideas of the text. This comprehension requires the automatic understanding of the words and the particular attention to the anaphors, the connectors and the sequencing of the sentences. The integral comprehension is a full comprehension of the text and its details. It adds to the literal comprehension the representation of the whole text. The student should know how to divide the text in different coherent units and withdraw the important information from each unit, keeping in mind the general meaning and orientation of the text. Finally, the fine comprehension includes the exploration of the implicit data of the text. The implicit data are information that are not clearly stated in the text and that the student should find using other information and clues from the text. Every text contains implicit data. Finding this data in a word problem requires a full and deep understanding of the text (Meurice & Degallaix, 2008). Any superficial understanding will lead to erroneous solving since a part of the required information will be missing.

4. The Experiment

The implicit data being very important to problem solving, we studied its effect in two word problems, the first for seventh and the second for eighth grade. Two private schools in northern Lebanon accepted to be part of our study. Both schools are of the same socio-economic status. All the students of grades 7 and 8 of these two schools participated in this study, regardless of their school grades or their English proficiency. As a total, 213 students participated in the study, of which 107 are in grade 7, and 106 in grade 8.

The study was conducted in the students' usual classroom, during their math session and in the presence of their math teacher. The work was individual, presented to the students as a test. Students were asked to solve story problems using any strategy they find suitable. They can answer through text writing, drawn schema, an equation, guessing and checking, or working backwards. Students of grade 7 were asked to solve problem 1 (the problem of Mrs. Koenig), and students of grade 8 were asked to solve problem 2 (the problem of Newtonville).

4.1 Problem 1: Mrs Koenig's Class

The students in Mrs. Koenig's class are in three groups working in teams.

• 20% are in room A;

• 4 students are in room B; and

• The remaining students, ½ are in the front of room C and the other 10 students are at the back of the room.

How many students are in Mrs. Koenig's class?

To solve this problem, students must be able to understand an implicit data, given in the third point of the problem: *½ are in front of room C and the other 10 students are at the back of the room.* Students must understand that if half of the students are in front of the room, then the other half are those 10 that are at back of the room. Understanding this implicit data allows the students to find the total number of students in room C and therefore solve the problem.

Students can represent the problem as shown in Figure 1.

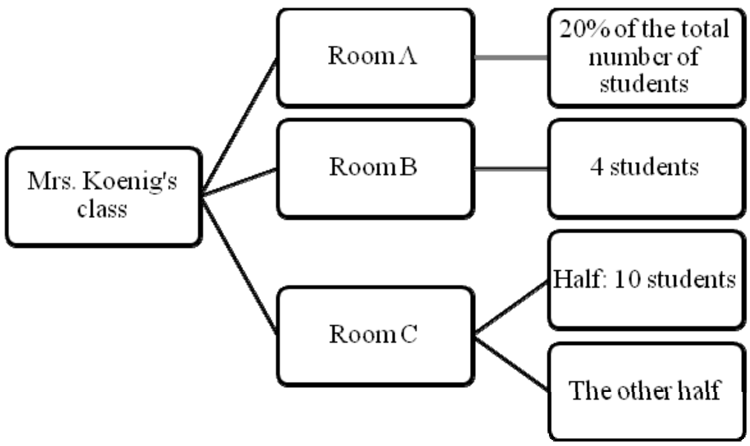

Figure 1. Representation of the problem

This representation will facilitate the resolution of the problem. Once the students get the number of students in class C, they must understand the meaning of the 20% that are in room A.

4.1.1 Procedures Used

The story problem of Mrs. Koenig's class was distributed to students; the allocated time was 30 minutes. Different problem solving behaviors were detected. Each solution was coded as correct if the student recorded an appropriate numerical answer. Next, the solution paths that a student has followed to reach an answer–whether correct or not–were examined and analyzed in detail to determine the type of error. We coded the answers according to the way the students understood the two major difficulties in this problem: ½ of class C, and 20% in class A.

4.1.2 Half of Class C

As we mentioned in the previous section, one of the difficulties of this problem is getting the total number of students in class C. The students must understand that if half of the students are at the front of the room and the remaining 10 at the back, then the room contains 10+10=20 students.

The answers of the students are divided in 5 categories:

1. Students who understood this implicit data and got the correct number of students in class C;
2. Students who considered that the main purpose of the problem is to get the number of students in class C;
3. Students who misunderstood and therefore misinterpreted the term "half" used in the problem;

4. Students who gave an incomprehensible analysis of this term;
5. Students who didn't give any answer concerning this part of the problem.

Table 1. Distribution of the answers of the students according to the understanding of the word "half"

Understanding of "half"	Number of students who used the strategy	Percentage
1. Good understanding of "half"	25	23%
2. Searched for the "half"	22	21%
3. Misinterpreted "half"	32	30%
4. Incomprehensible strategy	21	20%
5. No answer	7	6%
TOTAL	107	100%

Table 1 shows that more than 75% of the students were unable to understand the implicit data given in the problem. 21% of the students (22 students) considered that the purpose of the problem is to search for the number of students that constitute the "half" of class C. Mainly, these students considered that *half of class C* means *half of the rest of class C*, and therefore *half of 10*.

4.1.3 20% in Class A

When we proposed this story problem to students of grade 7, we considered the main difficulty being in understanding the implicit data given to find the number of students in class C. However, the worksheets of the students showed us that another main difficulty was working with the data given at the beginning of the problem: 20% of the students are in class A. In order to solve the problem, students must understand the meaning of 20% and must know how to get the number of students in class A from the rest (80%) that are in class B and C. Students had eight different understandings of this 20%:

1). Students considered that 20% means 80 students. This understanding didn't seem to bother the students although it gave them an illogical final answer: the total number of students of Mrs. Koenig's class is more than 80 (the exact number depends on the strategy used to get the number of students in class C).

2). Students considered that 20% means simply 20 students.

3). Students calculated 20% as 20/100 and used it as equal to 0.2.

4). Students considered that 20% is equal to 20/100. Although this fraction gives the same answer as the previous strategy (0.2), students who used this strategy didn't do the calculation and used it as 20/100 throughout solving the problem.

5). Students considered that 20% is equal to 100/20=5.

6). Students used 20% as such through solving the problem.

7). Students used an incomprehensible interpretation of 20%.

8). Students didn't use the 20% in their answers.

Table 2. Distribution of the answers according to the understanding of "20%"

Understanding of 20%	Number of students who used the strategy	Percentage
1. 20% = 80	1	1%
2. 20% = 20	18	17%
3. 20% = 0.2	19	18%
4. 20% = 20/100	32	30%
5. 20% = 100/5 = 5	1	1%

6. 20% = 20%	6	5%
7. Incomprehensible	27	25%
8. No answer	3	3%
TOTAL	107	100%

Table 2 shows that none of the 107 students were able to use the 20% correctly in solving the problem. 28% of them (30 students) either gave an incomprehensible answer or didn't use this 20% at all. 71 students (67%) have an erroneous understanding of the percentage (they used strategies 1 to 5).

4.1.4 Conclusion for Problem 1

The answers of the students to problem 1 show the difficulties they are having in solving a basic word problem. In fact, among the 107 students who solved the problem, none was able to follow a good strategy and give a correct answer. These difficulties are mainly due to three reasons. First, the students face difficulties in understanding the problem itself and the terms used in the problem. This is mainly due to the language used in the problem, which is not the students' mother tongue (all students are native Lebanese speakers). Second, the students have difficulties understanding the implicit data given in the problem. This is mainly due to the nature of the story problems given usually to the students, where all the data is clear and ready to be used (Baraké & Rouadi, 2014). Third, although teachers assured us that the students were familiar with the percentages, it seems that they still have major difficulties understanding the meaning of a percentage and using it in the resolution of the problem.

4.2 Problem 2: Newtonville

Seven middle schools are in the town of Newtonville. Each school has a different number of students.

School A has 3 fewer students than school B;

School B has 3 fewer students than school C;

School C has 3 fewer students than school D;

School D has 3 fewer students than school E;

School E has 3 fewer students than school F;

School F has 3 fewer students than school G.

If 2037 students attend Newtonville middle schools, how many students are in the school with the smallest number of students?

The problem of Newtonville is considered relatively easy to students. In fact, although the easiest way to solve this problem is to use an algebraic method (assigning x as the number of students in one of the schools), students who are not familiar with this method can use an arithmetic method to solve it.

4.2.1 Algebraic Method

To solve this problem algebraically, students must assign x as the number of students in one of the schools. There are two obvious assignations: x being the number of students in school A, or x being the number of students in school G. Both methods imply the same reasoning and the same calculations, as shown in the table below. The students using one of these methods will get a correct answer, but might have a lot of calculation errors in the process.

A third method used to solve this problem implies the use of a particular characteristic of this problem: the number of schools in the problem is an even number. The student can therefore assign x as the number of students in the median school, which will allows them to have far less calculations then with the two previous methods, as shown in the table below. This implicit data given in the problem enables the students to be more at ease when solving it.

Table 3. Three methods to solving the problem algebraically

	Method 1: x is the number of students in school A	Method 2: x is the number of students in school G	Method 3: x is the number of students in school D
School A	x	x-3-3-3-3-3-3	x-3-3-3
School B	x+3	x-3-3-3-3-3	x-3-3
School C	x+3+3	x-3-3-3-3	x-3
School D	x+3+3+3	x-3-3-3	x
School E	x+3+3+3+3	x-3-3	x+3
School F	x+3+3+3+3+3	x-3	x+3+3
School G	x+3+3+3+3+3+3	x	x+3+3+3
TOTAL	7x+63 = 2037	7x-63 = 2037	7x = 2037

As we can see in Table 3, it is easier to solve the problem using method 3 then it is using methods 1 and 2. The calculations to be done are minor calculations, and the errors in calculation are therefore less possible.

4.2.2 Arithmetic Method

Students can solve this problem in an arithmetic way, without having to use the unknown factor x. The benefits of this method is that the student does not have to use x, and therefore is not lost about which school to assign as x. The student has to simply know which school has the less students, and work with the data he has according to following:

If we eliminate the excess, all schools would have the same number of students. Therefore, the total number of students would be:

$2037 - (3 + 6 + 9 + 12 + 15 + 18) = 2037 - 63 = 1974$.

The number of students in each school would be: $1974/7 = 282$.

The class with the least students (school A) is 282.

School B has: $282 + 3 = 285$ students.

School C has: $282 + 6 = 288$ students. Etc.

4.2.3 Procedures Used

The story problem of Newtonville was distributed to students in grade 8; the allocated time was 30 minutes. Different problem solving behaviors were detected. Each solution was coded as correct if the student recorded an appropriate numerical answer. Next, the solution paths that a student has followed to reach an answer–whether correct or not–were examined and analyzed in detail to determine the type of error. We coded the answers according to their understanding of the problem and the difficulties they faced, and according to the method used to solve it.

4.2.4 Assignation of x

As we mentioned in the previous section, students can use one of four methods to solve the problem correctly: x is the number of students in school A, x is the number of students in school G, x is the number of students in school D, and without the use of x. However, some students used one additional method: the use of x in an ambiguous way and x is not assigned to any school. The distribution of the results is shown in Table 4.

Table 4. Distribution of the results according to the use of x in the solution

Method used	Number of students who used the strategy	Percentage
x: school A	26	24%
x: school G	2	2%
x: school D	0	0%

No x	59	56%
x: not any school	19	18%
TOTAL	106	100%

As we can see in this table, none of the students were able to use the implicit data given in the problem and assign x as the number of students in school D. However, 59 students (56%) used the arithmetic method and didn't use x while solving the problem. 19 students (18%) used x in an ambiguous way, not assigning it to any school.

4.2.5 Faced Difficulties

While solving this problem, students had many difficulties. In addition to the calculation errors, the problems left with no answer, and the correct answers, some students gave incomplete answers, starting with a strategy but not following through until they got an answer. Other students gave incomprehensible answers, where we didn't know what answer the student gave or which strategy he/she used. Finally, some students followed an incorrect logic to solve this problem: they either considered the number of students in school G as equal to 2037/7=291; or they considered the number of students in G as equal to 2037; or considered the number of students in A as equal to 2037; or considered that G has the less students. All these answers show that the students did not fully understand the word problem and therefore could not solve it correctly. A few students also started by following a correct strategy but gave a wrong answer and wrong conclusion at the end, showing that they understood part of the problem, but not enough to be able to answer correctly. The distribution of the students' answers over these strategies is shown in Table 5.

Table 5. Distribution of the answers according to the comprehension of the problem

	Comprehension of the problem	Number of students	Percentage
	1. Correct answer	15	14%
	2. Calculation error	3	3%
	3. Incomprehensible strategy	34	32%
	4. Incomplete strategy	16	15%
	5. No answer	2	2%
Wrong logic and understanding of the problem	6. G=291	7	6%
	7. G=2037	6	6%
	8. A=2037	5	5%
	9. G contains the smallest number of students	17	16%
	10. Correct calculations and strategy but wrong conclusion	1	1%
	TOTAL	106	100%

As we can see in this table, very few students (14%) were able to solve the problem and give a correct answer. 36 students (34%) were unable to solve the problem because of a lack of understanding of the problem. This shows that one of the main difficulties that students face is related to the understanding of the words and terms and the story told in the problem. Many students (32%) gave answers that were difficult to analyze, and that we therefore considered incomprehensible. In fact, the answer sheets of these students were full of numbers without any specifications, many of which we couldn't understand where they came from. Even though the answers of many students were illogical (schools with too few or too many students), none of their answer sheets show that they verified their answer or considered using another strategy to correct themselves.

4.2.6 Conclusion of Problem 2

Although problem 2 is considered relatively easy, very few students were able to use a correct strategy and give a correct answer. In fact, most of the students' answers show the major difficulties they face regarding word problem solving. Their worksheets show that most of them are unable to choose a correct strategy and follow it until they get an answer. Many students were even unable to recognize the illogical answers and therefore change their strategies to get more logical answers. A lot of students were also unable to understand the problem correctly and therefore were unable to choose a correct strategy to solve it. Students were also unable to recognize the implicit data of the problem and use it to their advantage.

5. Data Analysis and Conclusion

The results of our study show that students face many difficulties when solving a word problem: understanding the problem, recognizing the implicit data and using it in the resolution, choosing a strategy and following it through, analyzing their answer and making sure it's logical.

In fact, understanding a word problem is not easy for students. Some of them face language problems, and therefore cannot understand the words they are reading and the general meaning of the problem. This difficulty constitutes the first obstacle that students with low language proficiency face when solving a word problem. Although the words we used in both problems were easy and the students were certainly familiar with them, some of them were unable to understand the general meaning of the problem. These results are consistent with findings of other studies indicating that students with low language proficiency usually perform less well in word problems in math (Beal et al., 2010; Kempert et al., 2011).

One of the major difficulties the students face when solving a problem is recognizing the implicit data and using it in their strategies and their problem resolution. Whether this data is relevant to the resolution or not, students seem to manage to find a way around it and solve the problem–most of the time incorrectly–without using it. The importance of the implicit data in understanding and solving the problem are also discussed by Meurice and Degallaix (2008).

Students also face difficulties in choosing a strategy—whether correct or not—and following it through until they get an answer. This gives way to incomplete answers that show that the students either didn't know how to continue the resolution of the problem or didn't trust their strategy enough to continue through with it.

Finally, once the students get their final answers, very few of them go back to the main problem and verify whether their answers are correct or not. In addition to that, many students obtain illogical answers but are unable to recognize them as such. Looking back at the strategy and the solution is in fact what Polya (1957) considers as the fourth principle in solving a math problem.

As a conclusion to our study and to the major difficulties we recognized in the problem solving, we can offer many recommendations to the math teachers. First, to help the understanding of the problem, students can largely benefit from two major strategies: underlining the key words of the problem, and drawing a diagram or any other visual support. These two strategies can help students enhance their understanding of the problem by actively transforming the elements of the problem into a mental model (Mayer, 1992). Second, teachers must encourage students to follow their strategies until they get an answer. Whether these strategies are correct or not is irrelevant. Once the students get an answer, teachers must encourage them and motivate them to verify it using two methods: they first verify whether their answers are logical according to the problem or not. Many students get answers that are too far from logic but never think of verifying them (a class with 80 students or a school with 18 students for example). Once they find their answer to be logical, students must verify if it's correct or not by following the inverse pathway of the problem: *if school A has 18 students, how many students do schools B, C, D, E, F, and G have? And what is the total number of students in Newtonville?* This verification give the students the opportunity to autocorrect their strategies and to figure out what went wrong. Finally, the students can also benefit from reading between the lines of the problems, finding the implicit data and using it in the solving of the problem. The only way they can do that is by practicing on problems that can only be solved with this data. Unfortunately, many teachers find such problems difficult to students and hence never give them the chance to solve them.

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