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Exploring Mathematics Teachers' Noticing as Pedagogical Discourse Through an Adapted Lesson Study

Emine Gül Çelebi[1]

[1] Mathematics and Science Education, TED University, Ankara, Turkey

Correspondence: Emine Gül Çelebi, Mathematics and Science Education, TED University, Ankara, Turkey.

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Abstract

Although positive effects of lesson study on teachers learning are reported, only some studies have investigated teacher noticing as an analytical tool for supporting teachers with an explicit focus, as in LS, and more empirical evidence is needed. This qualitative interpretive case study design aims to investigate the noticing processes of a group of mathematics teachers conducting a lesson study cycle focused on teaching algebraic expressions using manipulatives in middle school. Data is collected through the audio recordings of the participants' lesson study meetings. Participants were six elementary mathematics teachers who attended a graduate course selected based on voluntariness. This study aims to incorporate Lee and Choy's (2019) teacher noticing framework with Sfard's (2008) commognitive theory, which views learning *as changes in discourse* and noticing as a *discourse structure* covering observation, interpretation, and reasoning processes (van Es, 2011). Results showed that teachers focused more on aspects of students' learning than issues of their instructional practice. However, their noticing was mostly related to future decisions and actions regarding issues of teaching methods and sequencing of the lesson, whereas teachers' dominant noticing form related to students learning was interpretive. Results illustrate the applicability of these noticing frameworks as an analytic tool where noticing is conceptualized as a pedagogical discourse for the analysis of a lesson study review discussion by a group of mathematics teachers who focus on teaching algebraic expressions.

Keywords: lesson study, mathematics teacher noticing, pedagogical discourse, student thinking

1. Introduction

Effective teaching requires various types of knowledge and skills, as well as noticing how and what to teach. Teachers should understand and collect evidence of student thinking and use this understanding and evidence to support instruction (NCTM, 2014). Mathematics teacher noticing has been a focus of research for more than two decades, appearing as a critical element of teaching (e.g., Mason, 2002; Van Es & Sherin, 2002). Noticing is crucial in teachers' professional practice, as teachers have been called to be faced with "blooming buzzing confusion of sensory data" within the classroom (Sherin & Star, 2011, p. 69), where they must select what to pay attention to and what to ignore. Mason (2002) posited a discipline-specific perspective of noticing that "every act of teaching depends on noticing—what students do or say, evaluating these against criteria, and deciding on what might be said or done in the future (p. 7).

However, the definitions and perspectives regarding noticing vary, and there are a few different theoretical roots and conceptualizations of teacher noticing in the literature. Noticing is characterized as a cognitive process in which teachers mainly observe and interpret worthwhile classroom events, based on the work of Van Es and Sherin (2002). Similarly, scholars developed an expertise-related perspective of noticing as an ability to recognize, interpret, and generate instructional decisions based on these interpretations of the critical events of teaching. (e.g., Bastian et al., 2022) focus on the differences between novice and expert teachers' noticing. A more social and situated perspective builds on Goodwin's (1994) notion of professional vision, stressing teacher noticing as a social, practice-based way of seeing and understanding events of particular interest to teaching and classroom practice. From a sociocultural perspective, there are specific ways that members of a community of practice engage in activities and learn through their participation in these activities (Lave, 2011; Lave & Wenger, 1991; Wenger, 1998). Noticing is also a discursive practice where teachers produce identifications, interpretations, and plans complementarily (Sherin et al., 2011). Since communication and meanings are produced within classroom discourse, those meanings are realized mainly by teachers' pedagogical choices. As teachers selectively pay attention, take action, and respond to these events in the classroom, their noticing is structured within the social practice of teaching and learning as a pedagogical discourse. Teachers' pedagogical discourse distinguishes reform-oriented practice from traditional practice as it shapes and guides the what, how, and why of teaching and who learns (Heyd-Metzuyanim & Shabtay, 2019). To enhance the quality of mathematics instruction and student learning, it is essential to notice content-specific characteristics of instruction (Çopur-Gençtürk & Rodrigues, 2021; Sherin & van Es, 2009). However, research suggests that teachers may have difficulty recognizing the mathematical characteristics of a task (Star et al., 2011), and they may recognize more general pedagogical features than student thinking (Ball, 2001; Schifter, 2001). Teachers' level of noticing can shift from witnessing to making sense of pedagogical events through student thinking (Sherin & Han, 2004; van Es et al., 2017). In addition, teachers' noticing may remain at a more basic level when experiencing LS for the first time (Vermunt et al., 2019). The Lesson Study (LS) can provide a framework for the teachers to engage with a co-learning environment by attending critical instances during a lesson, enabling identification, making sense, and reasoning based on these events.

LS provides a convenient form and space for teachers to understand student learning and their instructional practices (Fernandez & Yoshida, 2004). What makes LS different from other professional development activities are research lessons and post-lesson discussions (Groth, 2011; Murata, 2011). Through the collaborative nature of planning, enacting, and reviewing research lessons, the LS approach helps teachers improve their lesson planning, teaching, and skills for collecting evidence of student learning (Lee, 2019; Lee & Tan, 2020). However, it also helps teachers develop pedagogical content knowledge (Leavy & Horigan, 2016) and constructive dispositions and beliefs (Lewis et al., 2009). Those benefits in LS are tightly related to the teacher's noticing, incorporating, attending, interpreting, and responding to critical instructional events (Lee, 2019). However, only some studies have investigated teacher noticing as an analytical tool supporting teachers with an explicit focus, as in LS, and more empirical evidence is needed. Some of the problems that have been found in studies about teacher learning in LS are the need for time for many cycles (Fang et al., 2009), the lack of institutional or administrative support (González & Vargas, 2020), and the lack of theoretical underpinnings (Huang & Shimizu, 2016). When reviewing the findings of research on lesson study and teacher learning, Xu and Pedder (2015) also asserted the need for clear theoretical frameworks that can be used to explain teacher learning in LS. Moreover, in mathematics education, it is significant to consider the applicability of the existing noticing frameworks as an analytic tool for different contexts and environments (Amador, 2020).

In this study, an explicit focus is given to the mathematics teachers to notice the teaching and learning processes clear of algebraic expressions. Among all learning areas of secondary mathematics, algebra is identified as a gatekeeper to higher-level mathematics courses and careers; namely, students who do poorly in algebra risk their professional opportunities to be successful in the future workforce (Moses & Cobb, 2001). Since algebra topics start first with mastering arithmetic operations and gradually move towards abstract algebraic operations or expressions, this might also be challenging for students (Baroudi, 2006; Jupri & Drijvers, 2016). Quadratic equations are a core topic that is reported to be one of the most challenging topics that started to be learned in 8^{th} grade in the Turkish National Curriculum (MoNE, 2008). Although factorization is a useful method for solving quadratic equations, students frequently fail to pay attention to the method's structure and conceptual meaning (Sönnerhed, 2009). Moreover, there are few studies focusing on the teachers' knowledge of quadratic equations that is necessary for addressing students' learning difficulties (Hu et al., 2021).

To address the abovementioned issues regarding the theoretical foundations of teacher learning during LS, I aimed to incorporate Lee and Choy's (2019) teacher noticing framework for mathematics teachers and researchers to learn during lesson study with two discursive lenses of teacher learning and noticing. The learning lens refers to Sfard's (2008) commognitive theory that views learning *as changes in discourse,* and the noticing lens draws on the view that noticing itself is a *discourse structure* covering observation, interpretation, and reasoning processes (van Es, 2011). Based on these theoretical foundations, I investigated the noticing processes of a group of mathematics teachers who have participated in an eight-week LS focused on teaching algebraic expressions using manipulatives in middle school. The current study aims to highlight teachers' noticing during LS review discussions and draw on research on how teachers' actions and instructional responses are related to their noticing. The following question guided this research: What and how do teachers notice pedagogical discourse while reviewing their research lesson during Lesson Study?

2. Theoretical Backgrounds

2.1 Lesson Study

The lesson study cycle has five essential steps. (1) Determining a research theme and a goal; (2) planning a lesson to address the research theme; (3) conducting the research lesson; (4) the post-lesson discussion; and (5) reflecting on the whole process and learnings from it (Fujii, 2016; Lewis & Hurd, 2011). Although many variations and adaptations exist worldwide (e.g., Ming Cheung & Yee Wong, 2014; Regan et al., 2016; Takashi & McDougal, 2016), LS is a continuous, inquiry-oriented professional learning model for teachers based in Japan. As a professional learning model, lesson study aims to improve teaching quality and emphasizes student thinking while studying curriculum resources (Adler & Alswaikh, 2019). Interest in LS has grown since the Third International Mathematics and Science Study (TIMMS) video study, where it has been referenced as a strategy for improvement in educational practice (Stigler & Hiebert, 1999). LS in different contexts LS should have several key characteristics to be effective: (1) It should be school- or district-wide, not individual; (2) the study of teaching materials is essential; (3) one cycle, including the research lesson and the post-lesson discussion, is conducted within several weeks, not hours; and (4) it aims to improve pedagogical expertise (Takashi & McDougal, 2019).

Recent research has shown that LS has positive effects on teacher learning (Murata, 2011; Vrikki et al., 2017; Warwick et al., 2016) since it assists in the "development of teacher knowledge of content, pedagogy, and children's thinking" (Hart, 2011, p. 290). LS allows teachers to focus more on students' thinking, learning difficulties, and misconceptions during the design of the lesson (González & Vargas, 2020). It is also demonstrated that experienced teachers are more engaged in their students' thought processes through their use of the LS (Yang et al., 2021). However, practitioners need time to comprehend the key elements of this professional development strategy (Gunnarsdóttir & Pálsdóttir, 2019), and the visible effects of LS may not be apparent until teachers have participated in multiple LS cycles (Dudley, 2013).

It is known that collaborative debriefing research lessons in LS give teachers access to evidence of student thinking and pedagogical content knowledge that would otherwise remain hidden (Dudley, 2013). Reviewing their research lesson, teachers collaboratively notice classroom events as "socially organized ways of seeing and understanding" classroom events. In that sense, I consider a social and situated perspective of teacher noticing in the LS context as an effective pedagogical practice supporting teachers learning from lesson study. This paper will focus on one of the aspects that bring about the benefits of LS: the improvement of teachers' pedagogical expertise during the review of their research lesson.

2.2 Teacher Noticing as a Pedagogical Discourse

The social and situated perspective of learning set forth by members of a particular community engages in specific ways of practising and learning from their participation (Lave, 2011; Lave & Wenger, 1991). Similarly, Goodwin (1994) conceptualizes the notion of professional vision; as a capability to attend to events encountered in a profession that might be shaped by and developed by the discursive practices of the professionals. This process develops a new object of knowledge about a professional craft—expertise—that distinguishes teaching from other professions. This notion also follows Mason's (2002) characterization of noticing as a discipline-specific collection of practices where teachers live and learn from their practice, which informs their future practices. Although noticing has been defined in many ways, scholars recently adopted a view of noticing that stands for skills and knowledge necessary for expertise in mathematics teaching. Sherin et al. (2011) explain that teachers' selective attention and knowledge-based reasoning are the two main components of noticing as part of an interrelated and cyclical process. The main areas of noticing that teachers focus on in their practice as they observe, analyze, and reason are features of interpretive discourse (van Es, 2011).

In that sense, Lee and Choy (2017) provide a framework for mathematics teachers and researchers to learn during lesson study based on van Es (2011). The framework comprises three foci for noticing: The content, confusion or difficulties regarding the content, and the teacher's actions targeted to overcome the challenges and teach the content. The first is the key point, referring to the lesson's mathematical concepts or big ideas. The second is the difficult point, which relates to errors, confusion, or common misconceptions while students learn the content. The third one is the critical point, referring to the teachers' approach, considerations, and actions targeting the problematic points of the students while they are learning the task. Teachers begin to use a critical stance in their professional practice while anticipating students' difficult points and selecting or designing their tasks for the specific content.

LS, as a form of collaborative teacher inquiry, provides a context for teacher noticing (Amador & Carter, 2018) to improve teaching and learning by giving convenient means to understand students' developmental progress of thinking and their pedagogical practice (Warwick et al., 2016). A recent study by Tyskerud and colleagues (2023) investigated teacher's pedagogical discourse patterns from the reflection meetings during a multi-cycle lesson study, revealing the shift in the form and structure of pedagogical discourse from delivery to explorative, which indicates teacher learning as a result of participating in LS.

In addition, LS helps to reveal teachers' thinking and practice and provides opportunities to learn from one another about what is important to observe and how it should be used to make future decisions and take action to improve student learning (van Es et al., 2017). However, noticing is not always productive for mathematics teachers because the mathematical features of the tasks can be hard to notice (Choy, 2016), and other elements of classroom practice that are not mathematical are more likely to be attended by them (Star et al., 2011).

Although the interaction among teachers and sharing of ideas while reviewing their research lessons contribute to collaborative learning outcomes for the teachers during LS, there is a need for more knowledge regarding the underlying nature of their interactions during LS (Warwick et al., 2016). Based on van Es' characterization of noticing as a discourse structure and building on Lee and Choy's (2017) three points for productive noticing, I analyze the mechanism of LS teacher discussions during the post-lesson review.

In this regard, Lee and Choy (2017) highlighted noticing as a key to teacher learning in LS. On the other hand, by providing three explicit foci in the LS context, their research focused on how teachers may develop their discipline-specific noticing based on van Es' (2011) framework but not as a discourse for attending and interpreting important events and planning for the future. Following Lee and Choy's view that LS supports the development of teacher noticing, I investigated a group of mathematics teachers' noticing as a pedagogical discourse. In doing that, I needed to make some adjustments (Figure 1). First, to gain more detailed and context-aware empirical evidence of the teacher's noticing, I aimed to identify the core topics of what teachers notice as "Focus" instead of "Three points", covering the content, difficulties, and teacher decisions to these difficulties. Secondly, based on Lee and Choy's theoretical framework, I used teacher noticing levels as forms of pedagogical discourse by investigating the discussions of teachers' actions and responses to instances that they consider critical in their LS post-lesson review meeting. In this study, I defined pedagogical discourse as a tool for teachers' noticing and meaning-making of critical events in the teaching and learning processes realized by teachers' actions and responses to these events at the LS review meeting.

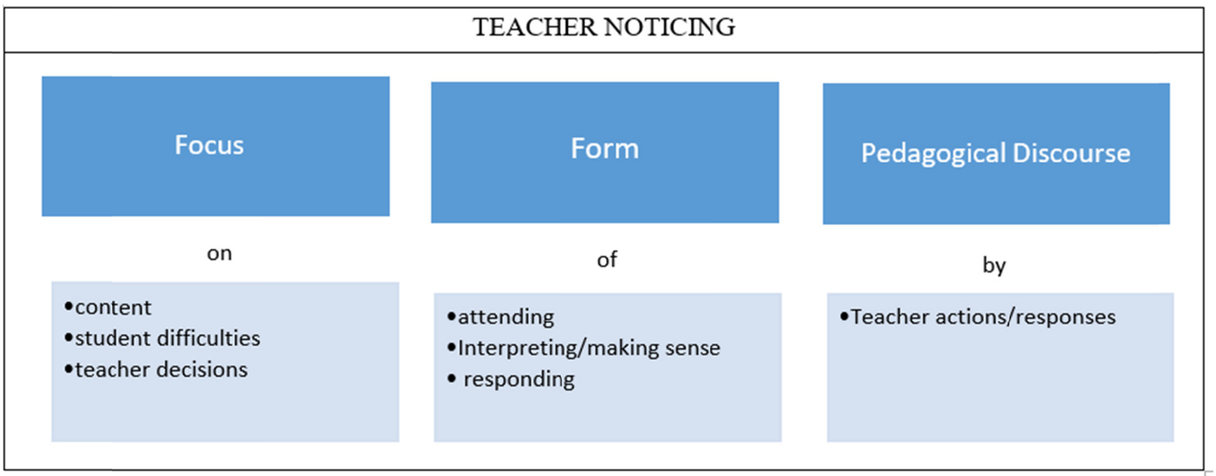

Figure 1. Noticing as a pedagogical discourse

3. Method

This research was carried out at a government university throughout one semester with middle school mathematics teachers who enrolled in a graduate course and conducted a LS as part of a requirement of the course. I used a qualitative and interpretive case study design to explore what a group of mathematics teachers noticed as pedagogical discourse during the post-lesson review meeting of their LS.

3.1 The Participants and Setting

The case of this study involves six elementary mathematics teachers. The participants' demographic information represented a range of novice to veteran teachers, as given in Table 1. Participants were also graduate students specializing in mathematics education at a large public university in Turkey. They were teaching middle school mathematics and pursuing a master's degree in mathematics education during the study. They were from different schools across districts; their names were pseudonyms; they voluntarily participated in the study; and they had not previously participated in an intervention related to a lesson study. The participants were taking a graduate course on “Math Teaching Resources”, which required effectively engaging in conversation about using resources in mathematics teaching and collaboratively working with teaching resources comprising curriculum materials, lesson plans, and digital resources. Voluntary participants were selected from the teachers who attended the course. I conducted this study with these voluntary participants. The participant teachers were to conduct a LS as part of the course requirement, whereas I will be a participant in the LS process as a knowledgeable other. Teachers have not participated in an intervention related to LS before. In the study, T1, T2, T3, T4, T5, and T6 were used instead of the actual names of the teachers.

Table 1. Demographic information of the teachers in the LS group

Teacher	Years of service	Workplace
T1	5	Public middle school
T2	6	Public middle school
T3	15	Public middle school
T4	1,5	Private school
T5	3	Not working then
T6	1	Public middle school

3.2 Data Collection

The data were the audio recordings of the six participants and the author, who participated in all Lesson Study meetings. The data for this study were collected during a graduate course of 14 weeks, *Math Teaching Resources*. In the first two weeks, they were introduced to the approaches to lesson study for connecting the lesson to goals and creating lesson plans. By the third week, the participants were asked to develop a lesson plan related to middle school algebra involving manipulatives and teach the research lesson in a classroom. They met for lesson planning once a week for four weeks. The primary learning objective of this lesson was to factor algebraic expressions using manipulatives. One of the participants, T3, who had 15 years of teaching experience, was a volunteer teaching the research lesson in one of her classes. Before the teaching session, they met as a group once again to review their lesson plan. In the eighth week, T3 taught the lesson in one of her 8^{th}-grade classes. Following the lesson, they met right away and made an initial review of the teaching practice. The following week, they met for the evaluation of the lesson plan. This post-meeting allowed an in-depth examination of the teachers’ sharing and refining of their knowledge of the lesson. Participants reflected on the task by evaluating the teaching and student learning at four-lesson study meetings that lasted an average of 1–1.5 hours each. They focused on learning objectives, using manipulatives, developing activities, and planning the lesson. In this study, I have just focused on post-lesson meetings.

The data for this study consist of video recordings of the one session held a week after the research lesson. The session lasted about 60 minutes. The instructor of the lesson was in an observer role at that meeting, whereas I was in a dual role of knowledgeable others and researcher. I audio-recorded the session and then transcribed it.

3.3 Development of the Coding Scheme and Data Analysis

Development of the coding scheme, in which I contextualized the use of these components**.** I primarily used the framework created by Lee and Choy (2017), including three forms of noticing (i.e., attending to, making sense of, and deciding to respond) about the Three-Point framework (i.e., key, difficulty, and critical points). Moreover, I analyzed the post-lesson LS meeting to present findings and focused on discussions about learning, noticing instructional practice, and student thinking. So, I also relied on a list of considerations for aspects of teaching (i.e., instructional process, learners, and learning) developed by Davis (2006) and the instructional triangle of Cohen et al. (2003). Codes were assigned to segments of the post-meeting transcripts (e.g., paragraphs and expressions) to uncover the emerging themes. The code development process involved a) a literature review and the development of the codes based on teacher noticing, aspects of teaching and learning; b) the addition of emergent codes through open coding (Glaser & Strauss, 1967); and c) the application of the codes to the data. I first worked on developing codes, then coded the segments of the transcripts. To provide trustworthiness, I asked another researcher with expertise in Lesson study and noticing the processes of mathematics teachers. The other researcher also coded the transcripts individually. To achieve consensus, we discussed the application and adequacy of codes to the transcripts. In the initial stages of coding, we adopted a qualitative approach to develop a coding system where we looked for patterns and regularities in the transcript segments and assigned preliminary possible coding categories to the data segments. Then we reread the transcript as we modified and added some other coding categories with the help of a list of codes available from the literature. In light of the literature and using the code development process described above, an analysis of the author's and another researcher's transcripts led to the characterization of two elements in the discussions during the LS post-lesson review:

1) Learning and Learners

2) The instructional process

Table 2 specifies the 'child' codes within each parent code category to describe each sub-category. Moreover, the quantitative measures of the child codes across two main categories of parent codes were presented to provide an overview of how these codes changed in the level of noticing. (a: Attention, b: Interpretive, c: Responding)

Table 2. List and description of parent and child codes

Instructional	Code 1	Description
Practice	IP1	Teachers describe/interpret/ adapt the **mathematical content** of the lesson. This could be related to constructing a mathematical model for a concept (e.g., constructing a rectangular area using zero pairs)
	IP2	Teachers identify/decide the **methods and strategies used** to support students in learning a mathematical concept, including asking further questions, probing, redirecting, and collaborative work.
	IP3	Teachers sequence the lesson or activities. This could be about using **different questions /problems** or introducing a concept or strategy before or during the task.
	Code 2	Description
Learners and Learning	LL1	Teachers recognize/ interpret students' **strategies for a specific mathematical task**. This could be related to different ways used by the students to model a trinomial (e.g., constructing "x" with units or trying the difference if two squares)
	LL2	Teachers recognize/interpret how their **students think** regarding a mathematical concept, procedure, situation, or model, including students who need special education. This could explain how a student thinks while learning to factorize or how students connect critical concepts of the lesson.
	LL3	Teachers recognize/interpret how students **invented strategies or alternative conceptions** for modeling a mathematical procedure.

4. Results

Findings related to what teachers noticed came from our coding scheme derived from teachers' lesson analysis. Codes emerging from the study of a group of middle school teachers' discussions during their lesson study review meeting are illustrated. In this regard, I first present the quantitative measures of the sub-codes across three main coding categories to provide an overview of how these codes are defined. Then, I will provide more detailed findings of how teachers noticed what they noticed regarding the *Instructional Process* (IP), *Learners, and Learning* (LL) to demonstrate teachers' noticing within and between each category. Analyses include the qualitative nature of the teachers noticing, in particular the sub-codes of noticing, and provide a trajectory of changes in the level of noticing, if any, within the discussion of lesson analysis.

Figure 2 provides the frequency of sub-codes due to the noticing levels: (a) attending, (b) interpreting, and (c) responding to, across the two principal codes of teacher noticing for the lesson analysis session. In the following sections, a detailed qualitative analysis of teacher discussions regarding these primary teacher noticing categories, IP, and LL, is presented with the quantitative frequencies of their sub-codes to provide a detailed analysis of teacher discussions. These results indicate that overall, teacher discussions are focused primarily on learners and learning, followed by an instructional process in their study of research lessons. Since the research lesson aimed to teach and learn algebraic expressions with manipulatives, there was a particular emphasis on manipulative use.

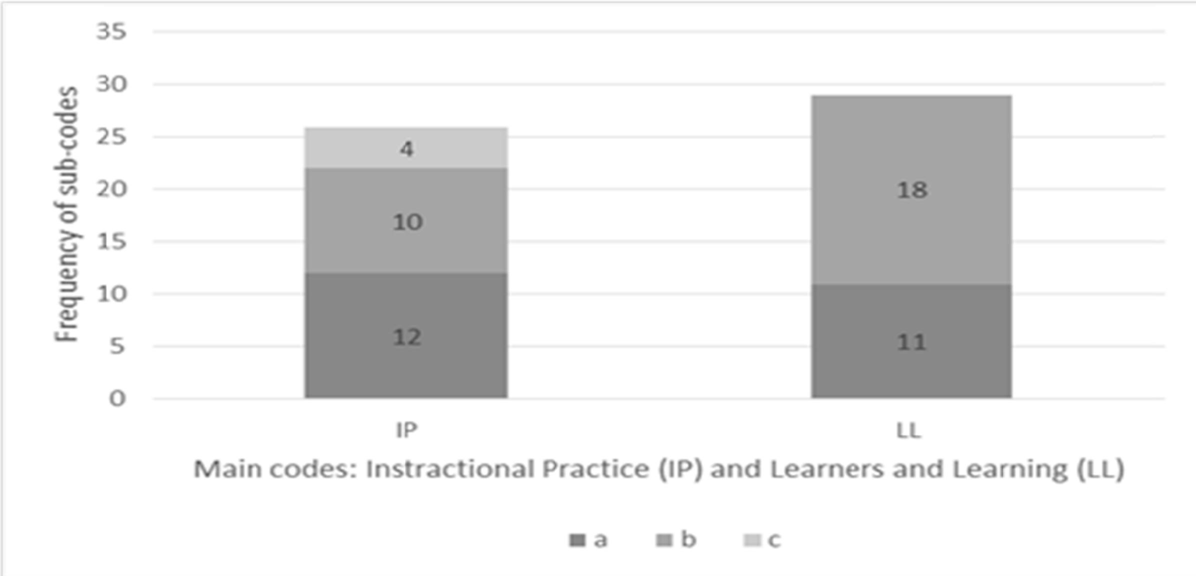

Figure 2. Frequency of sub-codes due to the noticing levels

4.1 Learners and Learning

One of the critical features of teachers' reflection on the lesson study was what they recognized as noteworthy in their students' learning. To capture what teachers found noteworthy in lesson study, I derived codes from the teachers' post-lesson discussion meetings. Overall, learners and learning (LL) were noted as the most commented-on topic among the main topics of the lesson study. Teacher comments covered *student strategies* (LLA), *student thinking* (LLB), and *students' alternative ideas* (LLC). Based on the analysis of the frequencies of the sub-codes, student thinking (LLB) was the most noted (58%) sub-category, followed by students' alternative ideas (LLC) (21%), and student strategies (LLA) (21%).

4.1.1 Student Thinking

Student thinking was the most noted category in the learners and learning theme. This category was defined as teachers making sense of their students' thinking regarding a mathematical concept, procedure, situation, or model, including adaptations for students with special needs. Most comments regarding student thinking focused on student difficulties, except one focused on the mathematical topic, and one commented on the critical situations regarding that thinking. Moreover, 63% of the comments were at the base-attention level; for the rest, the teachers made sense of the student's thinking. Teachers first noted that students have difficulty understanding rectangular area models and adding zero pairs to overcome that difficulty.

T2: They first struggled with constructing an area model for $2x + 6$. Some added zero pairs: positive x square and negative x square. Moreover, it was not wrong! It might be so because we told them to construct a rectangle as they wished, so they thought they could do that.

T3: One student said, "We can also construct a square with these pieces!"

T2: This means they naturally think about expressions like the difference between two squares.

In addition, there were issues around how students had difficulty learning to factorize. These were associated with the connection between understanding the area concept and the factors of these trinomials.

T3: 60% of the students would do the correct factoring.

T4: It is a complex topic to learn.

Researcher: Why do you think they have difficulty with negative terms or learning factoring in general?

T1: I think multiplication is easy for the students. But thinking the other way around, factoring is hard for them.

T2: I would expect them to factorize algebraically and then try to find the area model accordingly.

T3: But they did not use their knowledge.

T2: I was nervous about that. They treated it like a completely new task.

T4: They acted as if they knew nothing about factoring before!

T2: I could do it like that as well.

T1: Could it be that the product is an area, and they could not precisely relate to the fact that the lengths of its sides are the factors?

Through these comments, teachers attempted to make sense of their students' thinking when factoring trinomials. Moreover, the teachers generated possible explanations for this, as students might have difficulties connecting the lengths of the rectangle's sides as the trinomials' factors.

4.1.2 Student Strategies

Comments regarding different methods used by the students were primarily interpretive (75%). Comments categorized under this category referred to those that focused on making sense of and recognizing the strategies used by the students to deal with a particular mathematical task. Students might use different ways to model a trinomial; one might start with the difference between two squares or construct an x using unit tiles. The key strategy noted by the teachers was that most students begin with an x square and then continue winding the units around that square. However, their strategy was pointed out as a difficulty for students to form an appropriate area model for a trinomial:

T3: A different strategy was used at first when modelling.

T2: They start with an x square and wrap the others around like a flower. Most of them began like that.

T1: But it is only working for some. Not for -7x+18, for instance. It is not working because of the negative term. Although it works for some, it is wrong in general.

Researcher: So they generated a strategy.

T3: But this needed clarification since it is only sometimes working.

T4: They might have written the factors algebraically but needed modelling help.

T3: It worked with the positive terms but not with the negatives.

Although the teachers were making sense of a strategy that students in the classroom commonly use, they did not reason about the event for responding; there was no comment at the responding level in this sub-category.

4.1.3 Alternative Student-Generated Conceptions

The final category was related to teachers' recognition of how students invented their strategies and alternative conceptions for modelling an algebraic expression. There were four comments regarding student-generated strategies at the making sense/interpretation level, except one at the attention-base level. There were no comments at the responding level either. The critical alternative strategy invented by the students was noted as "twirling" or "wrapping around an x square (x^2)," where most students start with an x square and then continue winding the units around that square. Teachers commented on this strategy as they tried to make sense of these student-generated strategies as a learning and teaching opportunity:

T3: There is lots of wrapping around!

T5: But it works well with the (x^2+ 4x + 4)

All: Yes, it is surprising.

T3: I would not think like that, anyway.

T5: Me neither.

T2: They do it like a flower with the x square (x^2) at the centre and wrap the rest around it. Most of the students started to do that.

T3: I was thinking ofthe same type at all times.

T4: Yes, starting from the beginning and moving on both sides.

T3: They were doing it correctly. First, I told them not to do that, but I was curious. Let me check it, I said. I saw the correct answers and told them to do as they wished.

T2: Why did they think like that?

T3: Maybe they thought about tessellations. That was the only topic that they used, something like using crayons.

These comments indicated that teachers had noticed student-generated conceptions and how they think alternatively. They tried to make sense of this strategy and evaluate whether it was helpful. They also tried to generate a new understanding by identifying and analyzing this new strategy as a critical lesson incident to help students reach the lesson objective.

4.2 Instructional Practice

The other central theme from the teachers' post-lesson discussion was the practice of instruction. Instructional Practice (IP) was also the second-most-noticed topic among the main topics. Teacher comments about the instructional practice ranged from comments on mathematical Content (IPA) to discussions about the instructional strategies (IPB) and instructional sequencing (IPC) of the lesson. Analysis of the sub-code frequencies showed that comments on mathematical Content (IPA) were the most noted (45%). Comparatively, discussions about instructional sequencing (IPC) were less frequent (20%) in teachers' post-lesson discussion meetings.

4.2.1 Mathematical Content

In the post-discussion meeting, teachers commented on many features of the mathematical content of the research lesson. These comments included key mathematical topics, student difficulties as they learn them, and teachers' decisions and considerations shaping the instruction to support students in learning the content and reaching the objectives. In most of the comments, teachers were attending to aspects of mathematical content. They were about student difficulties, key mathematical topics, and critical teacher decisions to overcome student difficulties in shaping instruction.

These results indicate that teachers' talk about key mathematical topics of the lesson was mainly focused on attending to the need for constructing a rectangular area model and understanding zero pairs while working with this model. The teacher who taught the research lesson commented that she was unaware of the rectangular area model as a critical aspect in teaching factorization of the trinomials. She also pointed out the possible confusion that some students might experience about the different colours of the negative and positive terms in the area representation of the trinomials.

> T3: "For the beginning of the lesson, I would not change things so much. However, the procedure of constructing a rectangle and addressing what the long and short sides represent was essential, which I overlooked beforehand. Next time I teach this topic, I will emphasize the connections between the rectangle's area and the factors of the trinomials. Furthermore, I would also change some of the exercises in the handout since students got confused about the colours. The negative terms create that confusion. I would allow more time early to explain them on the board".

In the post-lesson discussion meeting, the teachers also pointed out the students difficulties in constructing an area model and using zero pairs. They recognized possible confusion among students when they were first introduced to the model. T3: We assumed they understood the "rectangle" and gave them handouts. However, they did not.

> T2: Yes, we have mentioned how the area can be found while constructing a rectangle. We asked, "How would you do it with the manipulatives you have?" It was not evident to them.

Moreover, later on, teacher comments become interpretive, generating a new understanding of how students think about zero pairs. Soon after carrying this understanding to the next level, they refine their decisions for the future.

> T6: "First, we shall start with favourable terms, then introduce the zero pairs. It is critical to introduce that it is possible to construct with zero pairs. Then it would be much easier to understand how to factorize full square expressions

4.2.2 Instructional Strategies

Teachers commented on the instructional strategies and methods used, and they decided to use them in the future to support students' understanding, such as questioning, giving a hint, and redirecting. Moreover, in most of these comments (5/7), teachers respond to the situations (critical points) of instruction to support students in learning the content. The rest of the comments were at the attention and making sense. When faced with possible student confusion about constructing rectangular area models and zero pairs, questioning was used to support students in overcoming the difficulty. Here, the teachers are talking about refining the task based on their new understanding of difficult points for their students. They think they should allow more discussion time and allow students to practice modelling different algebraic expressions.

Researcher: Why do you think they did not understand the negative ones?

T3: Zero pairs should be introduced first, which needs to be clarified for students.

T2: They should be introduced to these tiles before the main session, or they should do trials. We asked them, "What happens when you put these two tiles together?" " When they said it did not work, I told them, "Could it be done in other ways? Try repositioning the tiles." As if it were irrelevant, I told them, "What if you put one x squared and one negative x squared on your table? Then they said, "Then it must be zero!" A-ha! Then they asked, Can we use this?"

T3:" But we did not teach it in the lesson? How could they know?

Moreover, they also discussed how using manipulatives in their plan supported students in learning factorization. Although their learning objectives did not involve factorizing trinomials, which require addition or subtraction of a term, some students also explored that option. In the following dialogue, teachers discuss how they used virtual and concrete manipulatives in their lessons and how they could be used in their future lesson plans.

T2: A few students showed me samples of adding and subtracting term strategies in factoring. Perhaps they did not know the strategy, but they used it. For example, they added and subtracted fours.

T3: At the end of the activity, we gave the models and asked them to find the algebraic expression and their factors, and they used the virtual manipulatives a lot.

T1: We might use more virtual manipulatives in the future with better planning. It might be better to introduce the concrete manipulatives for one lesson and then make more room for virtuals at the computer lab.

T3: We only wanted to try out the virtual manipulatives for a small section of the lesson, but it went pretty well.

T4: It might be more favourable in terms of timing.

T2: It would work better if we could construct our examples using virtual manipulatives.

4.2.3 Instructional Sequencing

Teachers' comments were mainly on the Sequencing of the tasks, questions, and exercises related to the mathematical concepts of the lesson. These comments were also associated with using different questions, tasks, or activities before or during the implementation. All comments under the responding level sub-category were used to understand their students' thinking about the mathematical concept. During the post-lesson discussion, the researcher asked what they would do differently if they were to plan this lesson individually. Teachers were likely to choose other examples if they did the plan separately. They commented that they would include more difficult questions but start with more straightforward exercises or use specific forms or total squares for initial work:

T5: They would understand better with total squares, so I start with them.

T1: I would choose a couple of more different questions. They would be difficult ones. I would prefer, for instance, terms.

T4: Sometimes, we give the model first, and factors are required. Most of our examples use negative terms. I could have changed the order of these questions. We could have started with more straightforward examples and then moved on to more difficult ones.

T6: On the other hand, the teacher who taught the research lesson noticed a possible confusion that she would not have noticed if she were to plan this lesson individually. However, she realized that working with the manipulatives prevented students from falling into that mistake.

T3: I would introduce the units first. Then, students would think that X's are composed of units. That would be an unexpected situation for me.

T2: The possibility that some students might think that six units would equal an "x" was why we thought it was better not to do it, which is what they thought.

T3: They tried that option but saw that these were unequal, so they quit.

T1: But there are many drawings.

T2: That is too much of the required units for an X in the figures, but sometimes they do it as if they were equal. They realize these are not equal while doing manipulatives, but they draw as they like when drawing the model.

This conversation has set the stage for the benefits of using manipulatives, of which we will present the related results further. In addition, there were also comments on what teachers found critical in helping students learn the content and their decisions regarding future practices:

T1: Instead of starting with an example in the first place, a whole-class discussion would work well after they try their own.

T3: It sounds reasonable.

T1: We could say, "Your way of wrapping around works well with the positively termed trinomials, but this is another way of doing it, which you could come across often." And then to show that is what we expected.

These comments indicate uncertainties around how to deal with confusion regarding modelling trinomials using manipulatives. They discussed identifying whether to provide instructions or leave them completely free to discover a way to model working for each trinomial. Finally, they came to terms with the fact that it was better not to provide them with a way to model, followed by a discussion explaining the method commonly used to model using manipulatives.

5. Discussion

With teacher noticing as a pedagogical discourse, this study has investigated the noticing processes of a group of mathematics teachers' post-lesson discussion as part of an eight-week LS. This research was carried out to contribute to the literature regarding teacher noticing during LS and how noticing during LS post-lesson meetings contributes to teacher learning and change in practice. The scope of this research study has been limited to the noticing processes of a team of mathematics teachers who work at different middle schools and have enacted a cycle of an eight-week Lesson Study. This LS also focused on algebraic expressions using manipulatives in an eighth-grade class that one of the teachers taught during the LS review discussions.

Table 3. Teacher noticing pedagogical discourse: Focus, forms, and sample comments of actions and responses

FOCUS		FORM	%	ACTIONS -RESPONSES	SAMPLE COMMENTS
Instructional Practice	Math content	**Attention** Interpretive Responding	77 23 -	Identification, interpretation, and adaptation of the critical mathematical topics	"*Rectangular area model for teaching*."
	Methods and strategies	Attention Interpretive **Responding**	15 15 70	Identification and deciding on the methods and strategies to support students learning	"*Factorization with manipulatives, using rectangular area models and "zero pairs."*
	Sequencing of the lesson	**Responding**	100	Deciding on the structure of the lesson such as tasks, questions, activities	"*Focus on modeling trinomials by using manipulatives*."
Learners and Learning	Student Strategies	Attention **Interpretive** Responding	25 75 -	Recognition and interpretation of a critical strategy and difficulties such as beginning with x^2	*"Using x^2 in modeling."*
	Student thinking	**Attention** Interpretive Responding	63 37 -	Recognition and interpretation of difficulties in understanding the area model such as twirling or wrapping around	*"They used a negative and positive x^2 in modeling for constructing an area model."*
	Alternative conceptions	Attention **Interpretive** Responding	20 80 -	Recognition and interpretation of the student-generated idea of	*"wrapping around an X square"*

Table 3 shows teachers noticing as pedagogical discourse at post-lesson discussion encompassing focus, the form of noticing associated with teacher actions, and responses discussing critical instances from the research lesson. Findings revealed that mathematics teachers who participated in an LS review discussion were noticing elements of *Instructional Practice* (i.e., mathematical content, methods, and strategies used for teaching, sequencing of the lesson), as well as the features of *Learners and Learning* (student strategies, student thinking, and student-generated alternative conceptions). These two main themes coincide with key aspects of teaching practice from the literature, such as *teachers, students, and content* (Cohen et al., 2003) or *instruction, learners, and learning, subject matter knowledge, and assessment* (Davis, 2006).

In this study, the *Learners and Learning* theme was identified as the most noted theme. As the number of comments on this theme is higher than the other main category, this result suggests that teachers have noticed how students learn and think, which is an expected shift that occurs as a positive effect of LS on teachers' learning (Hart, 2011). Another reason might be that there are relatively experienced teachers in the LS team with experience levels between 1 and 15 years of teaching, which would direct the team to focus more on students' thought processes through the LS process (González & Vargas, 2020). Overall, this result might be interpreted as teachers noticing not being at a base level, aligning with the literature expecting a shift of teachers' noticing from general-pedagogical to content-specific noticing, specifically on *student thinking* (Sherin & Han, 2004; van Es et al., 2017). This result is consistent with the dominant form when teachers notice the sub-themes. Table 3 shows that the teachers noticed that they were at their most interpretive while focusing on *student strategies and alternative conceptions and ideas* but remained attentive to the essence of *student thinking*. Nevertheless, there is still a dominant interpretive form when discussing *student strategies and alternative conceptions,* and this might be due to the support given to the teachers as an explicit focus (Lee & Choy, 2017) in teaching algebraic expressions and factorization of trinomials within LS. However, there are no teacher comments regarding *student thinking*, *strategies*, or *conceptions* at a responding-to level or proposing a way of action. This result aligned with the literature that teachers' noticing may remain at more base levels at the initial cycles of LS (Vermunt et al., 2009) and that teachers might need time to understand the dynamics of LS (Gunnarsdóttir & Pálsdóttir, 2019).

The other theme that was identified was *Instructional practice*. Since the sub-themes are related to mathematical content and the related pedagogy, this theme covers content-specific features of teachers' noticing as well. The key topic teachers attended or identified for this review meeting was rectangular area models while teaching factorization of trinomials. Overall, teachers have remained mostly attentive to the *mathematical content* of the lesson. This result is consistent with the literature reporting that teachers might have difficulty noticing the mathematical features of a task (Choy, 2016; Star et al., 2011). However, teachers became more interpretive and dominantly responsive regarding *methods and strategies* used in the lesson that supported student learning; they became fully responsive to the *sequencing of the lesson* as features of the sequence and the lesson's structure. Although these sub-themes include content-specific features such as future decisions to use rectangular area models, starting with introducing what a zero pair was, or modelling trinomials with manipulatives (Table 3), the teaching *methods and strategies* and the *sequencing of the lesson* themes are common general pedagogical features that teachers are noticing as well. Teachers becoming more responsive to these sub-themes is consistent with the research reporting that teachers tend to notice general pedagogical features rather than student thinking (Ball, 2001; Schifter, 2011).

6. Conclusion

This study was based on a team of middle school mathematics teachers working at different institutions who conducted an LS within the scope of a graduate course to learn about and teach algebraic expressions and factorization of trinomials with manipulatives. Hence, this study's findings are considered empirical evidence for Lee and Choy's (2017) framework of noticing in the LS context working as an analytical tool in different environments. Moreover, this study provided evidence on noticing as a discourse structure (Van Es, 2011) that different levels of noticing (i.e., attending, making sense/interpreting, and responding) are defined as pedagogical discourse by the analysis of the discussion of teacher's and learners' actions and responses within the classroom practice during the post-lesson review meeting. The theoretical underpinnings of this study define teacher noticing as a pedagogical discourse presenting explicit foci for noticing (Lee & Choy, 2017), where the focus, form, actions, and responses of teacher noticing are seen as discursive structures (van Es, 2011). Since the discursive perspective this study draws on uses the commognition theory (Sfard, 2008), where learning occurs as a shift in discourses, this might be an indicator that the pedagogical discourse of teachers in the LS team is more content-specific than general pedagogical. However, since this LS was one cycle, unlike Tyskerud et al. (2023), there is a need for more empirical evidence to conclude whether or not this change happened during the LS process or specifically during the post-lesson review meeting. While our findings provide information about teachers' noticing as pedagogical discourse when reviewing jointly planned LS research lessons, future research might explore teacher noticing at other phases of LS conducted in different contexts and under different conditions. For example, future research might investigate teacher noticing as discourse during the research lesson and lesson planning phases, which might be conducted by a team of teachers working at institutions. Moreover, the team of teachers might prepare to teach other learning strands of middle school mathematics for more than one cycle. Findings from future work might inform mathematics education researchers and education leaders to learn more about teacher noticing as a discourse construct and ways to integrate teacher noticing as a developmental tool for learning about pedagogical practice (Amador & Carter, 2018) within LS.

Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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