The Development of Mathematical Problem-Solving and Reasoning Abilities of Sixth Graders by Organizing Learning Activities Using Open Approach

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Abstract

The researchers found that sixth-grade students at Traimit Pattana Suksa School had limited problem-solving and mathematical reasoning skills, which was attributed to the way their learning activities were organized by their teachers. The traditional approach did not allow students the freedom to think and practice solving a variety of problems in unconventional ways that mirror everyday life. To address this, the researchers applied an open approach to organizing activities and developed learning activities that fostered problem-solving and mathematical reasoning skills of the students. The goal of the study was to achieve an average score of not less than 70% using action research based on the concepts of Kemmis and McTaggart. Data were collected using a Sub-test at the end of the operating spiral, Mathematical Problem-Solving Ability Test, Math Reasoning Ability Test, and student behavior observation form. The data were analyzed using descriptive statistics, including percentage, mean, and standard deviation. The findings showed that the open approach to organizing activities can effectively develop the mathematical problem-solving and reasoning capabilities of students. The students were able to create work pieces, explain different types of 3D geometric shapes, show how to find the volume of a rectangular shape from given problem situations, and provide reasons to verify their ideas.

According to the test results, 13 students (81.25% of the total number of students) had the ability to solve mathematics problems at 70% or higher, and 15 students (93.75% of the total number of students) had mathematical reasoning ability that met the criteria of 70% or higher.

Keywords: open approach, mathematical problem-solving ability, mathematical reasoning ability

1. Introduction

To cultivate proficient math skills in a country, it is necessary to prioritize the development of problem-solving and reasoning abilities, as these are the primary goals of education. However, a broad examination of math education reveals that many students lack proficiency in these skills. Learners often hesitate to share their own ideas or think beyond what has been taught in class. Responses to class questions tend to be uniform, and problem-solving methods are often simply imitations of examples previously presented by teachers. Additionally, the teaching methods and approaches frequently prioritize the teacher as the center of the learning process, failing to promote students’ problem-solving and reasoning skills in math (Inprasitha, 2014).

An analysis of student math performance and national basic education test results (O-NET) at the researcher’s school has found that sixth-grade students consistently score below the national average in math. These tests require students to analyze problems through multiple steps, integrating and applying knowledge from multiple content areas. The results indicate that students lack the skills to solve complex, multi-step problems and are unable to analyze problems requiring them to demonstrate how to find the solution. They also have difficulty explaining their thought processes when arriving at the answer as a result of teacher-led activities. Consequently, the researcher examined concepts, forms, and teaching techniques to enhance students’ problem-solving and reasoning abilities in math more effectively. It was found that effective learning activities should allow students to practice solving diverse problems, giving them the freedom to think according to their abilities and experiences. Students need to solve problems from unfamiliar scenarios, fostering creativity and analytical abilities (Nowyenphon, 2001). An Open Approach, which focuses on using open-ended problems to teach
students problem-solving skills and reasoning in math, is an effective teaching method. It encourages students to share problem-solving methods with their peers (Inprasitha, 2003).

The analysis indicates that math education still prioritizes content and exercises over the learning process. As a result, students often struggle when presented with unfamiliar problems. The researcher emphasizes the importance of promoting and developing problem-solving and reasoning abilities in math education. Thus, the researcher is interested in utilizing an open teaching method to foster these skills, providing guidance for enhancing the quality of math education.

2. Literature Review

The aim of this study is to investigate the development of mathematical problem-solving and reasoning abilities in sixth-grade students by utilizing an open approach to learning activities. The researcher conducted a thorough review of relevant literature to support this study.

2.1 Organizing Learning Activities Using an Open Approach Method

One teaching method that has been advocated for promoting independent thinking and creativity in students is the open approach to learning activities. The National Council of Teachers of Mathematics (2000) recommends that students must be provided with opportunities to create knowledge based on their prior knowledge and experiences. Nohda (1998) describes the open approach as a teaching method that encourages interaction between mathematics and learners by providing them with a variety of problem-solving processes to stimulate creativity and problem-solving thinking. Inprasitha Maitree (2004) further emphasizes that the open approach aims to enable all students to learn mathematics in a way that is responsive to their abilities while making their own decisions. Teachers who utilize this method should understand their students’ concepts as much as possible.

The open approach to teaching consists of four steps. First, the teacher presents an open-ended problem in class and explains the relationships, rules, and conditions of the problem. Second, the students engage in self-directed problem-solving, where they are free to use their own mathematical thinking to solve the problem. Third, the students discuss and compare their problem-solving ideas in class, and these ideas are recorded in an activity sheet or notebook for assessment. Finally, the teacher or students compare concepts to see the similarities and differences of those concepts and positively reinforce diverse ideas. Research has shown that organizing learning activities through an open approach leads to increased student engagement, enthusiasm, and the ability to express their opinions freely. It also provides low proficiency learners with the opportunity to share their knowledge or solutions with other learners. Additionally, learners gain experience by listening to other people’s opinions and accepting new and unfamiliar concepts. Further, organizing learning activities through an open approach enables learners to develop mathematical language that they can use to explain and reason, as well as convey mathematical concepts. This language is not formal terminology but is the language learners use in everyday life, including mathematical concepts, visual conceptual writing, and gestures (Premprayoon et al., 2011)

After examining the use of open approach learning activities, it can be concluded that this teaching method is designed to support students of all levels to learn mathematics in a way that is tailored to their individual needs. This approach encourages students to use various methods to solve problems and express their ideas and concepts. Additionally, it allows students to use their creativity to create unique works while also building their problem-solving skills. The teacher’s role is to facilitate the learning process by providing problem situations, learning goals, and materials, as well as creating a positive learning environment. This approach creates an exciting learning atmosphere by using a range of materials and innovative problem situations. Students have the opportunity to systematically and rationally practice problem-solving skills while also exchanging ideas and listening to others’ opinions within a group and classroom setting.

2.2 Mathematical Problem-Solving Ability

Based on the sources cited, it can be inferred that mathematical problem-solving ability involves using existing knowledge and experience to find solutions to mathematical problems, often through a process that involves planning, gathering data, forming opinions, suggesting various solutions, and testing appropriate solutions to reach conclusions. The problem-solving process is considered the ultimate goal of mathematics teaching, with Polya’s (1980) concept of four important steps—understanding the problem, problem-solving plan, implementing the plan, and checking the results—being widely accepted and used. The ability to solve mathematical problems requires learners to rely on their knowledge and understanding of problem-solving steps and strategies, as well as their previous experiences and basic skills, to apply to new situations that are consistent with daily life. To help learners develop their problem-solving abilities, teachers should choose activities that encourage gradual learning, provide ample opportunities for independent thinking and problem-solving, and define a variety of problem situations or propositions for students to practice analyzing and solving problems. Measuring and evaluating the ability to solve
mathematical problems can be done through assessments that require students to demonstrate their understanding of problem-solving steps, strategies, and solutions, either through showing how to solve problems or writing independent answers, and can use rubric evaluation criteria for clearer ratings.

2.3 Mathematical Reasoning Ability

The concept of mathematical reasoning is defined by various organizations and researchers. According to the Ministry of Education (2017), mathematical reasoning is the ability to listen, reason, and use mathematical facts to support or argue in order to lead to conclusions. The Institute for the Promotion of Teaching Science and Technology (IPST) defines mathematical reasoning as a process that involves analyzing facts, concepts, and mathematical situations to generate new ideas or situations. Another definition by Kammanee Tissana (2002) defines rational thinking as a thought process that aims to understand logically explained ideas by distinguishing factual information and evaluating them based on evidence. The importance of mathematical reasoning is emphasized by the Department of Curriculum and Instruction Development (2008), which divides it into two types: inductive and deductive reasoning. Mathematical reasoning is essential in helping learners develop confidence, logical thinking, and the ability to explain ideas. Teachers play a critical role in developing students’ mathematical reasoning abilities by setting clear goals, adjusting teaching concepts, arranging additional activities, and creating a classroom environment where students can freely express their ideas and exchange thoughts.

Based on these definitions, it can be concluded that mathematical reasoning is the ability to explain and reason to make decisions, supporting or opposing an idea that is being explained in words and actions during problem-solving. Mathematical reasoning is vital in analyzing and planning solutions, and its development requires the active involvement of teachers in organizing activities that encourage learners to speak, explain, and express their ideas freely. In assessing students’ mathematical reasoning skills, teachers should use tests that encourage learners to express their ideas, write and explain their reasons, and ensure that the questions are open-ended.

2.4 Action Research

Kemmis and McTaggart (1988) proposed a four-step approach for implementing action research to improve the teaching and learning environment in schools.

Step 1: Plan—In this step, the problem that needs to be solved is identified and explored. The researcher needs to investigate the situation of the problem, what the problem is, who is involved, and what are the ways to solve it.

Step 2: Act—In this step, the concepts that are defined as activities in the planning stage are put into action. Relevant people are consulted, and the plan is analyzed to determine if it is reasonable and practical. Unexpected obstacles may arise, so the program may need to be flexible. The researcher must use appropriate judgment and decision-making to implement changes in the specified procedures.

Step 3: Observe—As research activities are carried out, obstacles and disruptions may occur. Therefore, it is necessary to use observation carefully and generously note any changes that occur in real-world conditions.

Step 4: Reflect—In this step, an assessment is made to examine the process, problems, or limitations that hinder the practice. Researchers and those involved need to examine the problems that arise in terms of social, environmental, and the school’s educational system. Through discussion of the problems, information can be gathered that will lead to further improvement and action planning.

![Figure 1. Action research spiral based on the concept of Kemmis and McTaggart (1988)](image-url)
Based on the process of action research discussed by Kemmis and McTaggart (1988), it can be concluded that action research is a cyclical process that consists of four important steps: planning, action, observation, and reflection. The first step, planning, involves identifying and exploring the problem, defining objectives, and determining the appropriate methods and tools to be used. The second step, action, involves implementing the plan and making any necessary adjustments or modifications as unforeseen obstacles arise. The third step, observation, involves monitoring and documenting the results of the implemented plan in order to identify changes and any issues that may have arisen. The final step, reflection, involves assessing the results of the research and examining the limitations and challenges that arose during the process. This information is then used to develop a new plan for the next cycle of the action research process until the desired objectives are achieved. In summary, the process of action research is a continuous cycle of planning, implementation, observation, and reflection, with each cycle informing the next until the desired outcome is achieved.

3. Method

The methodology used in this research is action research, based on Kemmis and McTaggart’s (1988) concept, to develop the problem-solving and mathematical reasoning abilities of sixteen sixth graders through open approach learning activities. The research tools included 15 plans for learning activities, sub-tests, and student behavior observation tests. The methodology consisted of four steps. In the planning step, the researcher gathered information from learners’ academic performance and relevant literature, created a research tool consisting of learning activity plans, and having experts evaluate it. In the acting step, the researcher collected data through quantitative (sub-tests and ability tests) and qualitative (behavior observation form and video recording) data collection methods. The collected data was analyzed into quantitative and qualitative data in the data analysis sub-step. The researcher analyzed the data using basic statistics and observational recordings of the learners’ learning behaviors. In the observation step, the researcher observed the process of organizing learning activities using an open approach and recorded the students’ problem-solving and reasoning abilities. In the reflection step, the researcher used the data obtained from the previous steps to analyze, evaluate, discuss, and reflect together with co-researchers and advisors to improve learning activities and plan operations for the next cycle.

4. Data Collection

In this study, data was collected through four main steps. First, before the implementation of the learning activities, the researcher orientated the students and established an agreement on the procedures for organizing learning activities using an open approach. Second, the learning management plan was implemented in the second semester of the academic year 2022. There were 15 plans in total, divided into four operational spirals. Spiral 1 included learning management plan 1–4, Spiral 2 included learning management plan 5–8, Spiral 3 included learning management plan 9–12, and Spiral 4 included learning management plan 13–15. Third, after the end of each operational spiral, students were given a sub-test to evaluate their mathematical problem-solving and reasoning behavior. This step aimed to obtain information to reflect on the results of practice and improve teaching in the next operational spiral. Finally, at the end of all four learning activities in the operational spiral, students took a mathematical problem-solving ability test and a mathematical reasoning ability test. The goal was for students to achieve an average score of at least 70 percent. The scores and results were then analyzed and summarized.

5. Data Analysis

In this research, the collected data was analyzed both quantitatively and qualitatively. The quantitative data was analyzed using basic statistics such as percentage, mean, and standard deviation. The sub-tests at the end of all four spirals, as well as the mathematical problem-solving ability test and mathematical reasoning ability test, were used to obtain this data. The data was then compared to the criteria of having an average score of not less than 70 percent. The qualitative data, on the other hand, was obtained during the learning activities using an open approach. Observational recordings of learners’ learning behaviors and videos that record student behavior during learning activities were used to obtain this data. The researcher analyzed this data and interpreted and summarized the approaches to develop learners’ problem-solving and mathematical reasoning abilities. The findings were then presented in the form of a lecture to analyze how organizing learning activities using an open approach encourages learners to develop problem-solving and mathematical reasoning abilities. The results were also used as a guideline for considering solutions to improve and develop effective learning activities.
6. Results and Discussion

This study aimed to improve the mathematical problem-solving and reasoning abilities of sixth graders by implementing learning activities that followed an open approach. The goal was to achieve an average score of not less than 70 percent in both problem-solving and reasoning abilities. The research followed an action research pattern according to Kemmis and McTeggart (1988), with 16 sixth graders as the target group. The researcher developed and designed 15 learning management activities, which were divided into 4 spirals, by synthesizing processes from studying actual problems in mathematics class, research documents, concepts, and learning theories.

After completing the learning activities in each spiral, the researcher administered sub-tests to assess the students’ progress. Data obtained from the sub-tests were used to compare mean scores, standard deviation, number of students who met the criteria, and the percentage of students who met the criteria. The results of the study are presented in Table 1, which compares the percentages of mean scores of the sub-tests at the end of the 4 operating spirals.

Table 1. Comparison of percentages of mean scores of the sub-tests at the end of the 4-operating spiral

<table>
<thead>
<tr>
<th>Comparison List</th>
<th>Spiral 1</th>
<th>Spiral 2</th>
<th>Spiral 3</th>
<th>Spiral 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of average score</td>
<td>77.19</td>
<td>72.81</td>
<td>68.75</td>
<td>70.21</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.15</td>
<td>3.52</td>
<td>3.20</td>
<td>5.77</td>
</tr>
<tr>
<td>The number of students who passed the criteria</td>
<td>13</td>
<td>10</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>Percentage of students who passed the criteria</td>
<td>81.25</td>
<td>62.50</td>
<td>62.50</td>
<td>81.25</td>
</tr>
</tbody>
</table>

Based on the data presented in Table 1, it was observed that the sub-test at the end of the first operating spiral had the highest percentage of the mean score (77.19%), with a standard deviation of 2.15. Also, 13 out of 16 students (81.25%) passed the criteria. However, the sub-tests at the end of the second and third operating spirals showed a decline in the students’ average scores and the number of students who passed the criteria. Specifically, the sub-test at the end of the second operating spiral had an average score of 72.81% with a standard deviation of 3.52, and only 10 out of 16 students (62.50%) passed the criteria. The sub-test at the end of the third operating spiral had an average score of 68.75% with a standard deviation of 3.20, and only 10 out of 16 students (62.50%) passed the criteria.

The decrease in the students’ scores and passing rate in the second and third operating spirals may be due to the subjective nature of the tests, which required students to draw pictures, write down ideas or methods for finding answers to problem situations. Students who lack reading, writing, and numeracy skills may find it difficult to express their ideas accurately, leading to incorrect answers and failing to meet the criteria. However, in the sub-tests at the end of the fourth operating spiral, the students’ average scores increased compared to the third test, despite having similar characteristics. Thirteen out of 16 students (81.25%) passed the criteria, which was the same as the first operating spiral. However, the average score of the students in the fourth operating spiral was still less than that of the first operating spiral. This may be because the sub-tests at the end of the first operating spiral required students to write and illustrate the concept of reasoning, which was not included in the fourth operating spiral’s written test that required students to demonstrate their ideas for solving problems, including calculations to find the answer.

6.1 Mathematics Problem-Solving Ability Test Results

Regarding the mathematical problem-solving ability test results, the researcher administered a test to the students to evaluate their proficiency in solving mathematical problems. The test was designed by the researcher and the results were then analyzed to compare them with the set criteria. The criteria stated that the students should score an average of 70 percent or more of the full score in order to be considered proficient. The data obtained from the analysis of the results is presented in Table 2.

Table 2. The results of the mathematics problem-solving ability test

<table>
<thead>
<tr>
<th>Total number of students</th>
<th>Score</th>
<th>( \bar{X} )</th>
<th>S.D.</th>
<th>Number of students who passed the criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full</td>
<td>Pass the criteria</td>
<td>Highest</td>
<td>Lowest</td>
</tr>
<tr>
<td>-------------------------</td>
<td>------</td>
<td>------------------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>16</td>
<td>30</td>
<td>21</td>
<td>27</td>
<td>12</td>
</tr>
</tbody>
</table>
The results suggest that most students demonstrated knowledge and understanding of the problem-solving process and were able to apply concepts or principles to find solutions to problems. The majority of students achieved passing scores, including 13 students who were in the good and middle groups. However, three students did not pass the criteria, and they were in the group that was quite late in reading and writing fluency, which may have hindered their understanding of the problem situation and their ability to write down ideas. Despite this, the average math problem-solving ability score of all students in the class was 74.17, which was higher than the specified criterion of 70%, indicating that the research achieved its objective.

6.2 Mathematics Reasoning Ability Test Results

Following the completion of all four research phases, the researcher administered a mathematics reasoning ability test to the students. The test was designed by the researcher and the resulting data was analyzed to determine how well the students performed. The analysis was conducted by comparing the test scores against a predetermined criterion, which stated that the students should achieve an average score of at least 70% of the maximum possible score. The data analysis results are presented in Table 3.

<table>
<thead>
<tr>
<th>Total number of students</th>
<th>Score</th>
<th>X</th>
<th>S.D.</th>
<th>Number of students who passed the criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full</td>
<td>Pass the criteria</td>
<td>Highest</td>
<td>Lowest</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>10</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

Upon examining Table 3, it was observed that the sixth-grade students had varying levels of mathematical reasoning ability, with the highest score being 9 points, the lowest score being 6 points, and the mean score being 7.88 points, indicating a success rate of 78.75%. The standard deviation was calculated to be 0.99, and a total of 15 students achieved a passing score, accounting for 93.75% of all students.

The results of the mathematical reasoning ability test indicate that the students possessed a strong grasp of the relevant concepts and were able to use their knowledge to reason through problems presented in the test. This was due to their ability to apply the basic knowledge and skills acquired through classroom activities. While most students were able to perform well on the test, one student in the group of slow learners did not meet the required score. This was attributed to the student’s difficulty in expressing their ideas in writing due to limited reading and writing fluency. However, the student’s ability to answer questions and provide detailed explanations during a question-answer test demonstrated their proficiency in mathematical reasoning. Despite this single instance, the class as a whole achieved an average score of 78.75%, exceeding the specified criterion of 70%. This indicates that the research objectives were successfully achieved.

7. Conclusion

In this study, the researcher aimed to develop problem-solving and mathematical reasoning abilities of sixth graders by organizing learning activities using an open approach. After completing four learning activities, the students were tested on their mathematical problem-solving and reasoning abilities. The results showed that the students’ average score for math problem-solving ability was 22.25 out of 30, representing 74.17% with a standard deviation of 3.89. Thirteen students passed the criteria of 70%, representing 81.25% of the total number of students who met the specified criteria. The open approach allowed students to research and practice on their own according to their potential, while the teacher gave advice and support. The self-learning stage enabled students to express ideas and methods to find answers together, analyzing the relationship of given information and then showing how to think using mathematical knowledge step by step. The open approach is consistent with the research findings of teaching using an open approach as an opportunity for students to learn on their own, think independently, and develop problem-solving thinking into a body of mathematical knowledge.

The students’ average score for mathematical reasoning ability was 7.88 out of 10, representing 78.75% with a standard deviation of 0.99. Fifteen students passed the criteria, representing 93.75% of the total number of students who met the specified criteria. The open approach allowed students to think and analyze ways to express various concepts and ways of thinking on their own, able to think independently. The self-learning stage enabled students to express ideas in order to obtain answers and summarize the results according to the specified problem situation. Students developed their own mathematical reasoning by explaining their answers to group members. In the discussion stage, students were given the opportunity to express themselves and explain their
way of thinking to others, allowing them to try to find reasons to support their own answers. The open approach is consistent with the guidelines for organizing teaching and learning activities for students to have the ability to reason, as well as the reasoning development guidelines of the Institute for the Promotion of Teaching Science and Technology.

Overall, the use of an action research model to develop problem-solving abilities and mathematical reasoning abilities of sixth graders through learning activities by using an open approach found that the open approach can help develop problem-solving and mathematical reasoning skills. The self-learning stage encourages students to use mathematical knowledge to find concepts or methods for obtaining answers to problem situations correctly and stepwise, while the discussion stage encourages students to get confident to speak, express different ideas, ask questions on doubtful issues, and accept other people’s opinions or reasons.

8. Recommendation

The researcher recommends that teachers who use the open approach in organizing learning activities should consider the unique characteristics of each student, including their physical, mental, emotional, social, and cognitive abilities, as well as their prior knowledge. It is crucial for teachers to pay attention to all students and create appropriate student groupings, as open approaches involve group work. In some cases, there may be students who are not cooperative or may struggle to contribute to group activities. Teachers should closely observe and motivate these students to participate and become interested in learning. By doing so, teachers can create an inclusive and supportive learning environment that fosters problem-solving and mathematical reasoning skills.

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References


Students. Doctoral thesis. Department of Mathematics Education. Faculty of Education. Srinakharinwirot University.


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