Journal of Educational and Developmental Psychology; Vol. 4, No. 2; 2014
ISSN 1927-0526 E-ISSN 1927-0534
Published by Canadian Center of Science and Education

A Relational Thinking Process of Elementery School Students with High Capability

Baiduri[1]

[1] Mathematics Education Department of University of Muhammadiyah Malang, East Java, Indonesia

Correspondence: Baiduri, University of Muhammadiyah Malang, East Java, Indonesia. E-mail: baiduriumm@gmail.com

Received: April 20, 2014 Accepted: May 20, 2014 Online Published: June 13, 2014

doi:10.5539/jedp.v4n2p24 URL: http://dx.doi.org/10.5539/jedp.v4n2p24

Abstract

The objective of this paper is to analyze the relational thinking process of the fifth year students with high capability in mathematics when they face mathematical problems. Two students, girl and boy, were chosen as subjects with high-capability in mathematics. Data were obtained through in-depth interviews based on the task of problem solving. All data were video-recorded. To obtain credible data, consistency and perseverance (improving perseverance), time triangulation and member check were observed. The obtained data were analyzed using a flow chart consisting of three-flow activities occuring simultaniously namely: data reduction, data presentation and conclusion drawing. The results suggest that the relational thinking processes of male and female students when facing mathematical problems by constructing inter- and intra-relation between two core elements in solving mathematic problems are that they understood the problems and answered the questions in the problems. The relations made by the male students were richer than those by female students.

Keywords: relational thinking, mathematics problems, capability in mathematics

1. Introduction

Thinking and problem solving are two inseparable matters, since one of objectives of thinking is to solve problems (Solso, 1995). To solve problems certainly needs processes of thinking, and thinking abilities can be trained using problem solving in general, and especially mathematics problems. According to Gestalt school and also Ahmadi and Supriyono (2004), processes of problem solving are called a thinking process. Problem solving is a very important matter in learning mathematics at school because by such problem solving, students may have capabilities in how to think, habits to persevere, high curiosity, self-confidence in any situation, and also may able to apply their knowledge and skills in problem solving in their daily lives in general (NCTM, 2000; Depdiknas, 2006; Pimta, Tayruakham, & Nuangchalerm, 2009). So that, abilities and skills in solving mathematical problems always raises attention from mathematics teachers from elementary level at school.

Problem solving is not monotonous and uniform activities, since problems are not always the same, dependent on the contents, forms and their processes. Hejný, Jirotková and Kratochvilová (2006) state that an approach to problem solving may be done in two ways, namely *procedural meta-strategy* and *conceptual meta-strategy*. Students handling problems using the conceptual-meta strategy may also be called as *relational thinking,* namely thinking that may use relationships among elements in sentences and among arithmetical structures (Molina, Castro, & Mason, 2008; Stephens, 2006; Carpenter & Franke, 2001) or that may analyse expressions (Molina & Ambrose, 2008) that may use structural thinking or algebraic thinking (Stephens, 2004).

Relational thinking is important in mathematics because there are many basic ideas in mathematics containing relationships among different representations from numbers and operation among numbers and among other mathematical objects (Molina, Castro, & Ambrose, 2005) and also a a good foundation to learn formal algebra (Molina & Ambrose, 2008). As a result, it is important to develop relational thinking among students as early as possible, namely among elementary school students. Moreover, the students' thinking style is also influenced by their ability and gender (Albaili, 1997; Zhu, 2007).

Based on the facts, this paper will analyse the relatioal thinking process of elementary school students with high capability in mathematics when they face mathematical problems, before solving them. High capability students are choosen in order to become one of references by students with average-or low-capability and may serve as a model for teachers in facilitating their students to prepare the students to solve especially mathematical problems and to solve any problems in general.

Furthermore, the nature of mathematics problems and relational thinking to solve mathematics problems based on the review of some related literatures will also be presented.

1.1 Mathematic Problems

A problem is situation one (including students) faces that need resolutions, and the way of the solution is not immediately known (Posamentier & Krulik, 1998; Polya, 1973). This means that a problem arises because there is a gap between the present situation and the intended objective. A gap is problem if one does not own certain rules that may immediately be used to solve the gap. A problem depends on individual and times, meaning that a gap is a problem for someone, but not for others. For certain people a gap at present is a problem, but not at other time, because the person may immediately be able to solve it by learning from his previous experiences.

In mathematics education, most experts in mathematics education state that a problem is a question or a mathematic problem to answer or to respond. But, it is also stated that not all mathematical problems automatically will become problems. A question will be a problem if the question shows a challenge that may not be solved by a routine procedure the problem solver has known. It is because there is no certain rule/law that may immediately be used to answer or to solve it. Mathematical problems are problems if the problems not routine ones (Hudoyo, 2005) or not standardized ones (McNeil, Grandau, Knuth, Alibali, & Stephens, 2006). Then Polya (1973) proposes that there two types of problems in mathematics namely:

a) Problem to find, namely to find, to determine or to obtain certain objects or values that are not known in the problem and that fulfill conditions proper to the problem.

b) Problem to prove, namely procedures to determine whether a statement is right or not right. Problem to solve consists of hypothesis and conclusion. The solution is made by making a logical statement from the hypothesis to the conclusion, meanwhile to prove that the statement is not right, it is merely made proposing an example of refusal so that the statement becomes not right.

Problem to find is a type of problem given to the students in order to train them on how to process a concept or principle is found. Moreover, Polya states (1973) that problem to find is more important in elementary mathematics, meanwhile problem to prove is more important in advanced mathematics.

From the structure, problem to find and to prove may be grouped into *well-structure* (Jonassen & Tessmer, 1997). It is this type of problem that might often be found out in schools and universities. Usually this kind of problems may be found at the end of chapters, that they need applications of concepts, rule and principles that have been learned under limited situations of problems. These problems have been well-defined, the objectives are known, they are limited under logical operators and the answers are convergent.

Based on the descriptions above, mathematical problems in this paper are types of problem to find in the form of story problems dealing with arithmetics in elementary schools in daily life that should be solved.

1.2 Relational Thinking

Relational thinking plays a central role in human cognition. It is a system of thinking centered upon a relation surrounding next objects, and considers anything in in terms of objects and interactions with others. As an example, relational thinkers will "see a form of a head" when they see a pair of spectacles and they will "see a form of air wave when they see birds flying". Relational thinking deals with *affordances* (all possible actions latent in the environment) since this method of thinking enables one to become aware of an action (Doumas & Hummel, 2005). Relational thinking including our ability to understand an analogy between objects or events seems to be different and to apply abstract rules in a new situation (Doumas & Hummel, 2005). Relational thinking considers additional information simultaneously and looks for patterns of the next connections of identification. This is often related to intuition and characteristics of applied mathematics, object-oriented programming, and inter-disciplinary fields. Therefore, relational thinking is to think by constructing relationships of various objects/contexts so that they are related one another.

To understand relational thinking in mathematics, researchers have looked for approaches by giving mathematic problems. When doing right/wrong sentences on number such as $257 - 34 = 257 - 30 - 4$ or $27 + 48 - 48 = 25$ designed based on arithmetics characteristics, in general there are two approaches employed: (1) doing computation on two sides and comparing the two results, (2) considering the whole sentence, making meaning of its structure and using the relationship either its elements or knowldge on the arithmetical structure to solve the problem (Carpenter, Franke & Levi, 2003). The first approach is called *procedural meta-strategy* and the second, *conceptual meta-strategy* (Hejný, Jirotková & Kratochvilová, 2006).

When students solve problems using conceptual meta-strategy, they are said to use relational thinking namely to think that makes use of the relation among elements in the sentence and that of arithmetical structure (Molina, Castro & Mason, 2008; Stephens, 2006) and to analyse the expression (Molina and Ambrose, 2008) or to think structurallly or algebraicly (Stephens, 2004). Relational thinking is the opposite of procedural thnking (Carlo, Anne, Paul & Paola, 2010; Stephens 2006). Furthermore, Carlo, Anne, Paul, and Paola (2010) state that relational thinking is almost the same with what Skemp (1976) said about relational comprehension, namely knowing what to do and why to do that.

In arithmetics, relational thinking depends on whether students may see and use varried possible variations among numbers in number sentences or given problems (Stephens, 2006). It means that students are said to think relationally if they approach number sentences with the focus of aritmethical relationship to replace computation. It is in line with the statement Carpenter, Franke, Madison, Levi and Zeringue (2005) make that relational thinking involves the use of basic charecterictis of number and operation to change mathematical expressions than to answer following a set of procedures. This means that relational thinking is related to many different relations students make in recognizing and constructing among and in numbers, expressions and operations. The use of basic chafacterstic to result in the structure of objective and to change expresssions may be explicit or implicit in students' reasoning logic, for instance $47 + 73 = (40 + 7) + (70 + 3) = (40 + 70) + (7 + 3) = 110 + 10 = 120$.

Students are said to think relationally if they (1) see the sign of "equal" as a symbol of relation; (2) may focus on the structure of expression and (3) may be able to give rationality in using a strategy to solve number sentence problems involving operation (Carpenter, Franke & Levi, 2003; Stephens, 2006). This means that students that solve problems using relational thinking make use of their understanding of numbers to consider arithmetical expressions from structural instead of procedural perspectives. Moreover, one thinks relationally or uses relational thinking when he investigates two or more ideas of mathematics of things, he looks for alternatives of relation among them and analyses or uses of the relationship to solve problems, to make decisions, or to further learn about situations or concepts employed (Molina, Castro, & Ambrose, 2005).

On the basis of some explanations above, the term relational thinking in mathematics that has been proposed is always related to implementing a plan of solution, the third stage of problem solving (Polya, 1973; Posamentier, Jaye, & Krulik, 2007). Relational thinking in this present research is to think of constructing a relation by making use of informational elements given (context), knowledge that has been possessed before and knowledge of mathematical characteristics/ structures when facing mathematic problems.

1.3 The Present Study

The objective of this present paper is to analyse the relational thinking process of male and female elementary school students with high capability in mahematics when they are given mathematical problems. Based on this objective, research problems are formulated as follows:

1) How is the relational thinking process of male elementary school students with high capability in mathemathics when they are given mathematical problems?

2) How is the relational thinking process of female elementary school students with high capability in mathemathics when they are given mathematical problems?

3) Is there any difference in relational thinking process between male and female elementary scholl students with high capability in mathematics when they are given mathematical problems?

2. **Method**

2.1 Subjects

To determine elementary school students with high capability in mathematcis, 31 fifth year elementary school students with the age of between 10 – 11 years were asked to solve problems of the numeracy test. The results of the test were rated using scores from 0 – 100, and then were grouped into three categories. The test scores of < 55 are categorized into low capability, $55 \leq$ test scores < 80 into midle, and test scores ≥ 80 into high. The results of the test showed that there are 5 students (16.13%) with high capability, consisting of 2 male and 3 female students.

Then two students (one male and one female) with relatively the same scores and with good communication skills were chosen to be the subjects of the study.

2.2 Instruments

The instruments were main instrument, namely the researcher himself and supporting instruments covering audio visual recorder, numeracy test, tasks of mathematic problem solving and interview guide.

2.2.1 Numeracy Test

Numeracy test is constructed by adopting the test items of the National Final Exams for elementary schools in the form of multiple-choice that had been changed into description problems which are in line with the standard of content of the 2006 Mathematics curriculum for fifth year students, especially the materials of the odd semester. The answer sheets that had been arranged were validated by elementary school mathematics teachers (who have got certification), by experts in mathematics education and experts in evaluation to evaluate the content of the tests ad language use. Based on the results of validation, the answer sheets are ready to use.

2.2.2 Tasks of Problem Solving

As in the numeracy test, the Tasks of Problem Solving were started by studying the content standart of the 2006 curriculum for fifth year students and the test instruments used by previous researchers to explore students' relational thinking. This present study refers to instruments developed by Stephens (2008) and Stephens and Wang (2008). Then, the researcher asked for permission to Stephens to be allowed to use his tests, but the tests are in the form of story problems. He permitted the researcher and suggested to develop the tests, either in terms of the data or questions given. The produced numeracy test was then validated by certified teachers of mathematics in elementary school, experts in mathematics education and experts in evaluation. The validation is in terms of the content and language use. Based on the results of validation, the readibility of the test instrument was informally made to two fifth year students of elementary school, male and female student. The two students were able to mention of what they knew and what were asked in the test instruments, of which these two aspects were very important to find in a test (Polya, 1973). In this tudy, two types of equal numeracy tests were developed, and one of them is as follows:

> *In her stall, Dewi sells candies put in three containers, of which each color is red, green and blue. Each container contains two-taste candies with the same form and size. The red container contains 81 pieces of candies with strawberry taste and 27, orange taste. The green container contains 23 pieces of candies with coffee and milk tastes. The blue container contains 24 candies with pineapple and the rest melon tastes. The number of candies in the red container is two times more than the green container. The number of candies in the blue container is not higher than those in the green one. How many candies with milk taste in the green container? How many candies with melon taste might be in the blue container?*

2.2.3 Interview Guide

The interview guide was developed in order to help unreveal the process of relational thinking of the subjects when they faced mathematical problems.

2.3 Data and Data Credibility

The mechanism of data collection, either the numercy I or II began by asking the subjects to read tasks of problem solving, continued by in-dept interviews (semi-structured interview). The data were video-recorded. And then to assure the obtained data credibility, the researcher made observations continuously/consistently and perseverantly (to improve perseverance), time triangulation and member check (Moleong, 2011; Sugiyono, 2011). Based on the credible data, an analysis with three-flow model of activities were made simultaneously: data reduction , data presentation and conclusion drawing (Miles & Huberman, 1992).

3. Results

3.1 The Relational Thinking Process of Male Student (MS)

The Relational Thinking Process of Male Student (MS) when facing mathematical problems deals three main elements, namely looking at the problem, understanding the problem and answering the problem. This may be known from the quotation of the following interview:

R What do you think after reading the problem?

MS (kept silent and scratched his head and said): "*Looking at the problem*".

R And then?

MS *Understanding the problem.*

R Is there anything else in your mind?

MS (Kept silent while looking at the problem, and said): *"Answering"*

(Note: R – researcher, MS – Male student)

Then he looked at the problem and related it to the question and the content. The content is related to numbers in the test. It is shown from the quotation of results of interviews as follows:

R What did you see in the problem?

MS *The question,* (then he looked at the test sheet)

R And then?

MS (Raising his head so that his face is up and said): *"the content"*

R What is meant by the content?

MS *Numbers* (while looking at the problem)

Understanding problems concerns with what is known in the problem. While what is known deals with numbers and instruction to operate the calculation. The figures the MS meant are number. It may be known from the following interview:

R What do you understand from the problem?

MS (kept silent, then read the problem)*: Each container contains two-taste candies with the same form and size. The red container contains 81 pieces of candies with strawberry taste and 27, orange taste. The green container contains 23 pieces of candies with coffee and milk tastes. The blue container contains 24 candies with pineapple and the rest melon tastes. The number of candies in the red container is two times more than the green container. The number of candies in the blue container is not higher than those in the green one.*

R From what you have mentioned, what is known or what is asked?

MS (kept silent and said) *"known".*

R What information is in the problem that states what is known?

MS (kept silent, and said): *information on the instruction to add, to subtract, to multiply or to divide.*

R O, yeah. Is there any more information stating what is known?

MS (kept silent and examined the problem,then said): *there are numbers*

Now questions are given dealing with those of the problems and the ways to answer it. The question is related to question mark (?) and the question word (how many). The ways to answer deals with counting operations used and the information (content) of the problem. It is delineated from the quotation of the following interview results:

R You said, "answering", what do you mean?

MS (Kept silent and examined the problem and said): *answering the question, the ways.*

R What is meant by the ways?

MS *Yeah, adding, subtracting, multiplying or dividing.*

R What is added, subtract, multiplied or divided?

MS *Numbers in here* (pointing the problem)

R From where do you known it is a question?

MS *There is a question mark, and also a question on how many.*

Then MS also related what is known and the question that what is known is used to answer the question. It is shown from the quotation of the following interview:

R Is there any relationship between what is known and what is asked?

MS Yes. What is known is used to answer the question.

Based on this fact, the relation thinking process of male student (MS) when facing with mathematical problems by constructing relations in and among the three core elements, namely looking at, understanding and answering the problem. Looking at the problem relates to the question and content of the problem. The content deals with any numbers in the problem. Understanding the problem concerns with what is known in the problem, while what is known in the problem relates to numbers and instruction to operate the calculation. Answering the problem concerns with the existence of the question mark (?) and question word (how many). The ways to answer deal with counting operation used and the information (content) in the problem (numbers). What is known and what is asked are related one another, where what is known is used to answer questions. The relational thinking process of the MS is presented in Picture 3.1.

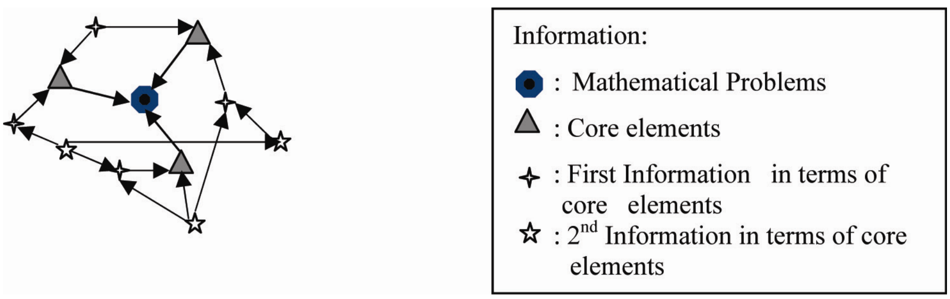

Figure 1. Relational thinking process of the MS

3.2 Relational Thinking Process of Female Students (FS)

The relational thinking process of female students (FS) when facing mathematical problems dealing with two core elements: the ways to answer the problem and the way to understand the problem. This is delineated in the quotation of the results of interviews below:

R What do think after reading the problem?

FS (examined the problem and said): *"ways to answer the problem"*

R Then what is the next?

FS (examined the problem and said): *"the problem is understood"*

The problem is understood relates to two things, namely numbers and instruction(s) in the problem. The two are called as what is known in the problem. Figures the FS mean numbers. It is delienated from the quotation of the interview results as follows:

R Ok. You have just said the problem is understood. what do you understand from the problem?

FS *The Number of candies in the containers* (looked at the problem and referring to the problem with the pencil)

R What else?

FS (Paid attention to the problem and referring to the problem with the pencil and then said): *"the instruction is the same"*

R What is instruction?

FS (Read the problem): *The number of candies in the red container is two times more than the green container. The number of candies in the blue container is not higher than those in the green one.*

R From what you have mentioned, what is known or asked?

FS *What is known*

The ways to answer the problem deal with question, counting operation used and information in the problem. The question deals with interrogative sentences and what is not yet known. Interrogative sentences related to question word and question mark. Information deals with numbers and instruction to answer the problem. This is shown from the quotation of the interview results below:

R You said ways to answer the problem, what do you mean?

FS *Yeah, ways to answer how many candies with milk taste in the green container? How many candies with melon taste might be in the blue container?*

R Oh, it is. How comes?

FS *First 81 + 47 then the result is divided into 2 and subtracted by 23* (while referring to the problem with the pencil).

R Ok. This is for which question?

FS *How many candies with melon taste are in the green container?*

R To answer how many candies with melon taste that might be in the blue container, how is the way?

FS *For question, the number of candies in the green container is subtracted by 46. Before that, the result is given "mark equal" (These must be candies with melon taste.* (while referring with the pencil to the writing that hasbeen made)

R Ok. How do you know the question you have made?

FS *From the sentence of the problem, asking.*

R From where do you know sentence ask?

FS *From the word how many and the question mark*

R Ok. Good. Is there something else stating something so that you know something to ask?

FS (kept silent and paid attention to the problem and said): *"from what has not been known".*

Then FS also relates between what is known and the question, that to answer the question needs what is known. It is known from the quotation of the interview results as follow:

R Ok, is there relation between what is known and what is asked?

FS *yes. To answer what is asked needs what is known.*

Based on the facts, the relational thinking process of the female students when facing mathematical problems is by relating in and between two core elements, namely the ways to answer and to understand the problem. Understanding the problem relates to what is known in the problem. The ways to answer the problem deals with the numbers and direction in the problem. What is known deals with the question, counting operation used and what is not known. The interrogative question is related to question words and question mark. Information in the problem deals with numbers and direction to answer the problem. What is known and what is asked is related one another, to answer a questions needs what is known. The relational thinking process of FS is shown in Picture 3.2.

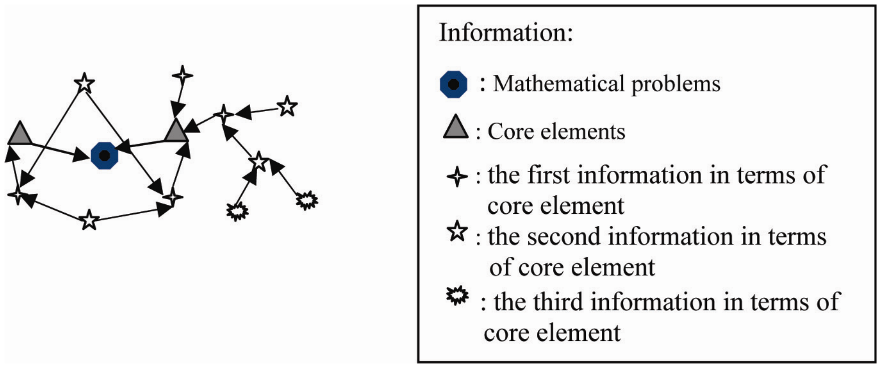

Figure 2. Relational thinking process of FS

4. Discussion

Relational thinking process of MS when facing mathematical problems is by making relationship in and among looking at, understanding and answering the problem. Looking at the problem in this case is a part of understanding the problem. So that the relational thinking process of MS when facing mathematical problems is the same as what is done by FS, namely constructing relationship in and among understanding and answering the problem. The relational thinking process of MS and FS when facing mathematical problems is on the first and third stage from the four-stage process of mathematical problem solving (Polya, 1973; Posamentier, Jaye, & Krulik, 2007). Understanding such a problem and doing a plan (to answer the problem) is very essential in solving mathematical problems. When MS and FS are making a relation in and among two important elements in problem solving, it means that he has made processes of understanding, opinions, and decisions as thinking processes (Sujanto, 2004). The relational thinking process MS and FS are mental representational of their mental experiences to solve problems. The mental representation is of one basic materials to construct thinking and determining one's thinking model (Sujanto, 2004).

Based on the obtained relations, the relation constructed by MS is richer than that by FS. It is in opposition with an opinion that female students performed relatively better than male students in the items of story problems (Bolger & Kellaghan, 1990). But, when answering the problem, MS relates it to the question in the problem and the ways, meanwhile FS relates it to the question, counting operation used and information in the problem. It means that relation constructed by FS in understanding the problem is richer than that is constructed by MS. It is in line with the opinion that female students perform relatively better than male students in terms of items of story problems (Bolger & Kellaghan, 1990). The reason that there might be some differences between the sexes is that the story test items need verbal competence (Murphy, 1982).

Then what is related between MS and FS in understanding problems is based on information in the problem. Such an understanding in the perspective of reading comprehension is included in the second level (text base), namely reading comprehension is merely based on what is in the reading text (Österholm, 2006; Van Dijk & Kintsch, 1983). Meanwhile the ways to answer the problem relates to the counting operation used and information in the problem (numbers and direction). It means that the ways to answer have been integrated between the knowledge that has been possessed (choosing or using counting operation) and information in the problem (numbers and direction). Such an understanding is included in the situation model, namely the third level/the highest in reading comprehension perspective (Österholm, 2006; Van Dijk & Kintsch, 1983).

Based on nature of the (binary) relation of set A to set B, it is an association of elements in A and those in B written as aRb or $R(a) = b$, with a an element in A and b element in B by the R relation, where the relational thinking process (R_t) of female and male students when facing problems may be represented as a relation using a domain in look at the problem (L_s), understanding (M_s) and answering problems (M_p). Mathematically it may be represented by the following formula:

$R_{tm} = R(L_s, M_s, M_p)$ dan $R_{tf} = R(M_s, M_p)$

where

R_{tm} represents male student's relational thinking processes. R_{tf} expresses female student's relational thinking process, $L_s = R\ (u_1, u_2)$, where u_1 = is a quesition in the problem, u_2 = content of the problem. $M_s = R(D_s)$, where

D_s = what is known in the problem (for male and female students). $D_s = R(x_1, x_2)$, with x_1 = numbers, x_2 = is an instruction to make a counting operation (for male student) or x_2 = an instruction in the problems (for female student). $M_p = R(q,w)$, where q = question, w = way to answer questions (for male student). $M_p = R(q,o,i)$ where q = quesitons, o = counting operatios used, i = information in the problems (for female student)

5. Conclusion and Future Research

Relational thinking process of MS and FS when facing mathematical problems is by construction relation in and among understanding and answering the problem, which is an important stage (the main element) in solving mathematical problems (Polya, 1973; Posamentier, Jaye & Krulik, 2007). It means that there is similarity in the relational thinking process of MS and FS when facing mathematical problems. But there is a different relationship constructed in and between the two elements. In general, the relation constructed by male students is richer than female students. The female and male students' understanding is related to what is known in a problem. Meanwhile in answering the problem, male students relate it with the question, and the way to count, the female students related it to the question and the counting operation employed an the information in the problem. In general, the relation constructed by male students is richer than that of female students. When constructing a relations, they relate it to their past experiences (schemes) the have possesed and the problem at hand. It is an important process in solving a problem (Mayer, 1983). Relational thinking gives an overview of what will be done by students when facing mathematical problems.

This research is limited to when students face mathematical problems in the form of problems to find in the form story among elementary school students with high capability in mathematics. Therefore, it is necessary to study the relational thinking process of elementary school students with low or average capability in mathematics or secondary high school or university students and at the stage of solving mathematical problems developed by Polya.

References

Ahmadi, A., & Dan Supriyono, W. (2004). *Psikologi Belajar*. Edisi Revisi. Jakarta: PT Rineka Cipta.

Albaili, M. A. (1997). Differences in Thinking Styles among Low, Average, and High-Achieving College Students. *Educational Psychology*, *17*(1), 171-177. http://dx.doi.org/10.1080/0144341970170112

Bolger, N., & Kellaghan, T. (1990). Methods of measurement and gender differences in scholastic achievement. *Journal of Educational Measurement*, *31*, 275-293.

Carlo, M., Anne, C., Paul, P. W., & Paola, V. (2010). Equality Relation and Structural Properties. P*roceedings of CERME 6, January 28th-February 1st 2009, Lyon France*. Retrieved from http://www.map.fr/publication/edition-electronique/cerme6/wg4-16.marchini-et-al. pdf

Carpenter, T. P., & Franke, M. L. (2001). Developing algebraic reasoning in the elementary school: Generalization and proof. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference. The future of the teaching and learning of.* Melbourne: University of Melbourne.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Posrtmouth: Heinemann.

Carpenter, T. P., Franke, M. L., Madison, Levi, L., & Zeringue, J. K. (2005). Algebra in Elementary Schooll: Developing Relational Thinking. *ZDM*, *37*(1), 53-59.

Depdiknas. (2006). *Kurikulum Tingkat Satuan Pendidikan. Kompetensi Dasar Pelajaran Matematika untuk Sekolah Dasar (SD)/Madrasah Ibtidaiyah (MI), Sekolah Menengah Pertama (SMP)/Madrasah Tsanawiyah (MTs), Sekolah Menengah Atas (SMA)/Madrasah Aliah (MA)*. Jakarta: Pusat Kurikulum, Balitbangdiknas, Jakarta.

Doumas, L. A. A., & Hummel, J. E. (2005). Approaches to Modeling Human Mental Representations: What Works, What Doesn't, and Why. In K. J. Holyoak, & R. G. Morrison (Eds.), *The Cambridge Handbook of Thinking and Reasoning.* Cambridge: University press.

Hejný, M., Jirotková, D., & Kratochvilová, D. (2006). Early conceptual thinking. In J. Novotná, H. Moraová, M. Krátká, & N. Stheliková (Eds.), *Proceedings 30th Conferences of the International Group for the Psychology of Mathematics Education* (3rd ed., pp. 289-296). Prague: PME.

Hudojo, H. (2005). *Kapita Selekta Pembelajaran Matematika*. Malang: Universitas Negeri Malang.

Jonassen, D. H., & Tessmer, M. (1996/1997). An outcomes-based taxonomy for instructional systems design, evaluation, and research. *Training Research Journal*, *2*, 11-46.

Mayer, R. (1983). *Thinking, problem solving, cognition*. New York: W.H. Freeman and Company.

McNeil, N. M., Grandau, L., Knuth, E. J., Alibali, M. W., & Stephens, A. C. (2006). Middle-School Students' Understanding of the Equal Sign: The Books They Read Can't Help. *Cognition and Instruction*, *24*(3), 367-385. http://dx.doi.org/10.1207/s1532690xci2403_3

Miles, M. B., & Huberman, A. M. (1992). *Analaisis Data Kualitatif: Buku Sumber Tentang Metode-metode Baru*. Terjemahan oleh: Tjetjep Rohendi Rohedi. Jakarta: UI Press.

Moleong, L. J. (2011). *Metodologi Penelitian Kualitatif* (Edisi Revisi). Bandung PT. Remaja Rosdakarya.

Molina, M., & Ambrose, R. (2008). From an operational to a relational conception of the equal sign: Thirds grades' developing algebraic thinking. *Focus on Learning Problems in Mathematics*, *30*(1), 61-80.

Molina, M., Castro, E., & Ambrose, R. (2005). Enriching Arithmetic Learning by Promoting Relational Thinking. *The International Journal of Learning*, *12*(5), 265-270**.**

Molina, M., Castro, E., & Mason, J. (2008). Elementary school students' approaches to solving true/false number sentences. *PNA*, *2*(2), 75-86. Retreived from http://www.pna.es/Numeros/pdf/Molina2008Elementary.pdf

Murphy, R. J. L. (1982). Sex differences in objective test performance. *British Journal of Psychology*, *52*, 213-219.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author

Österholm, M. (2006). Charactherizing reading comprehension of mathematical texts. *Educational Studies in Mathematics*, *63*(3), 325-346. http://dx.doi.org/10.1007/s10649-005-9016-y

Pimta, S., Tayruakham, S., & Nuangchalerm, P. (2009). Factor Influencing Mathematical problem Solving Ability of Sixth Grade Students. *Journal of Social Sciences*, *5*(4), 381-385. http://dx.doi.org/10.3844/jssp.2009.381.385

Polya, G. (1973). *How to Solve it* (2nd ed.). Princeton University Press.

Posamentier, A. S., & Krulik, S. (1998). *Problem-solving strategies for efficient and elegant solutions*: *A Resource for the Mathematics Teacher*. Corwin Press, Inc. California USA.

Posamentier, A. S., Jaye, D., & Krulik, S. (2007). *Exemplary Practices for Secondary Math Teachers*. Association for Supervision and Curiculum Development. Alexandria, Virgina USA.

Skemp, R. R. (1976). Relational understanding and Instrumental understanding. *Mathematics Teaching*.

Solso, R. L. (1995). *Cognitive Psychology*. Boston: Allyn and Bacon.

Stephens, C. A. (2006). Equivalence and relational thinking: Preservice elementary teachers' awareness of opportunities and misconceptions. *Journal of Mathemathics Teacher Education*, *9*, 249-278. http://dx.doi.org/10.1007/s10857-006-9000-1

Stephens, M. (2004). *Researching relational thinking*. Japan: University of Tsukuba.

Stephens, M. (2006). Describing and exploring the power of relational thinking. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, Cultures and Learning Spaces, Proceeding of the 29th annual conference of the Mathematics Education Research Group of Australasia* (pp. 479-486). Canberra: MERGA. Retrieved from http://www.merga.net.au/documents/ RP552006.pdf

Stephens, M. (2008). Some key junctures in relational thinking. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating current and charting directions* (Proceedings of the 31th annual conference of the Mathematics Education Group of Australia, pp. 491-498). Brisbane: MERGA.

Stephens, M., & Wang, X. (2008). Some key junctures in relational thinking. *Journal of Mathematics Education*, *17*(5), 36-40.

Sugiyono. (2011). *Metode Penelitian Kuantitatif, Kualitatif dan R&D*. Bandung: Alfabeta.

Sujanto, A. (2004). *Psikologi Umum*. Jakarta: PT Bumi Aksara.

Van Dijk, T. A., & Kintsch, W. (1983). *Strategies of discourse comprehension*. New York: Academic Press.

Zhu, Z. (2007). Gender differences in mathematical problem solving patterns: A review of literature. *International Education Journal*, *8*(2), 187-203.