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Nonlinear Models for Plant Height of Rye Cultivars at Sowing Times

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Abstract

Adjusting nonlinear Gompertz and Logistic models will help in the understanding of the growth pattern of the rye crop and also in the height response of the plant, when planted in different environmental conditions. The the aims of this study were to adjust the nonlinear Gompertz and Logistic models for plant height and indicate the one that best describes growth of two rye cultivars in five sowing times. Ten uniformity trials were conducted with the rye crop in the 2016 harvest. In each trial, ten randomly selected plants were evaluated from the first expanded leaf weekly. In each plant height was measured. The adjustment of the Gompertz and Logistic models as a function of the accumulated thermal sum was performed with the average plant height at each evaluation. The parameters *a*, *b*, and *c* were estimated for each model. The confidence interval for each parameter and the inflection points, maximum acceleration, maximum deceleration and asymptotic deceleration were calculated. The quality of fit of the models was verified by the coefficient of determination, Akaike's information criterion and residual standard deviation. Intrinsic non-linearity and non-linearity of the parameter effect were quantified. Both models describe satisfactorily the plant height. The model that best describes the growth of rye cultivars is Logistic.

Keywords: *Secale cereale* L., growth models, soil cover plant, winter cereals

1. Introduction

Rye (*Secale cereale* L.) is a winter cereal from the family *Poaceae*. The crop is efficient both as soil cover and grain production. In Brazil, the rye cultivated area is 3.6 thousand hectares, with a grain productivity of 2,222 kg ha^{-1} (CONAB, 2017), being also a potential alternative for crop rotation during winter. It stands out by its hardiness and for playing an important role as cover plant (Doneda et al., 2012). Its initial growth is vigorous, which enables rye to supply early fodder, being a good option as animal forage (Meinerz et al., 2012).

It is important to define cultivars and sowing times that enable the adequate plant growth and development in order to maximize production gains. Therefore, crop growth might be characterized through mathematical modeling, assisting in decision-making along the cycle of a crop (Rosa, Moreira, Rudorff, & Adami, 2010). Mathematical models are, basically, a simplified description of a mathematical system, elaborated to better understand the functioning of a real system. In this way, the nonlinear models describe growth curves that enable the interpretation of the processes involved in plant growth, since their parameters allow practical interpretation (Sorato, Prado, & Morais, 2014).

Among the mathematical models that describe plant growth, the nonlinear Gompertz and Logistic models stand out, for being one of the most employed to describe plant behavior based in the observation of the crop itself. The adjustment of these models has been made to evaluate the triticale plants (Karadavut, 2009), the height of maize (Mangueira, Savian, Muniz, Sermarini, & Crosariol Neto, 2016), the development of cashew (Muianga, Muniz, Nascimento, Fernandes, & Savian, 2016), the morphological characters of *Crotalaria juncea* (Bem et al., 2017), the growth of cocoa fruits (Muniz, Nascimento, & Fernandes, 2017), and the productive features of *Crotalaria juncea* (Bem et al., 2018).

The adjustment of growth models for rye plant height is critical for comparing cultivars in different sowing times. The adjustment of the nonlinear Gompertz and Logistic models will help in the understanding of the growth pattern of the rye crop and also in the height response of the plant, when planted in different environmental conditions (sowing times). Therefore, it is expect that the Gompertz and Logistic models will be fitted to plant height and describe successfully the growth of two rye cultivars in five sowing times, allowing the choice of the most adequate model. So, the aims of this study were to adjust the nonlinear Gompertz and Logistic models for plant height and indicate the one that best describes growth of two rye cultivars in five sowing times.

2. Material and Methods

Ten uniformity trials were conducted (blank experiments) in a rye crop (*Secale cereale* L.), in an experimental area in the Department of Crop Science of the Federal University of Santa Maria, Rio Grande do Sul (located at 29º42′ S, 53º49′ W, with 95 m altitude), in 2016. The climate in the region, according to the Köppen climate classification, is Cfa humid subtropical, with hot summers and without a defined dry season (Heldwein, Buriol, & Streck, 2009) and the soil is classified as dystrophic sandy Red Argisol (Santos et al., 2013).

In these uniformity trials, all the procedures (sowing, fertilization, crop care, and evaluation) were homogeneously performed along the whole experimental area. The seeds of both rye cultivars were sown in five times. Each cultivar in each sowing times represents a uniformity trial. Sowing was conducted in May 3, 2016 (time 1), May 25, 2016 (time 2), June 7, 2016 (time 3), June 22, 2016 (time 4), and July 4, 2016 (time 5), in order to represent the sowing times indicated or not for the crop. In each sowing, the soil was prepared in the conventional way, with light disking, and base fertilization of 25 kg ha^{-1} of N, 100 kg ha^{-1} of P_2O_5, and 100 kg ha^{-1} of K_2O.

The BRS Progresso and Temprano cultivars were broadcast sown, with a density of 455 seeds m^{-2}. These cultivars were chosen due to their distinct features, namely, the BRS Progresso is intended to grain production and the Temprano is recommended as soil cover and pastureland. In the first sowing time, each cultivar was sown in an area of 320 m² (20 m × 16 m). In the other sowing times, each cultivar occupied 375 m² (25 m × 15 m). When the plants were between stage 3 and 4 (Large, 1954), an application of 25 kg of N ha^{-1} was made.

In each uniformity trial, the plants were randomly chosen and the plant height (PH, in cm) was measured with a millimeter ruler (Figure 1). Plant height is the distance from the plant base to the insertion of the flag leaf.

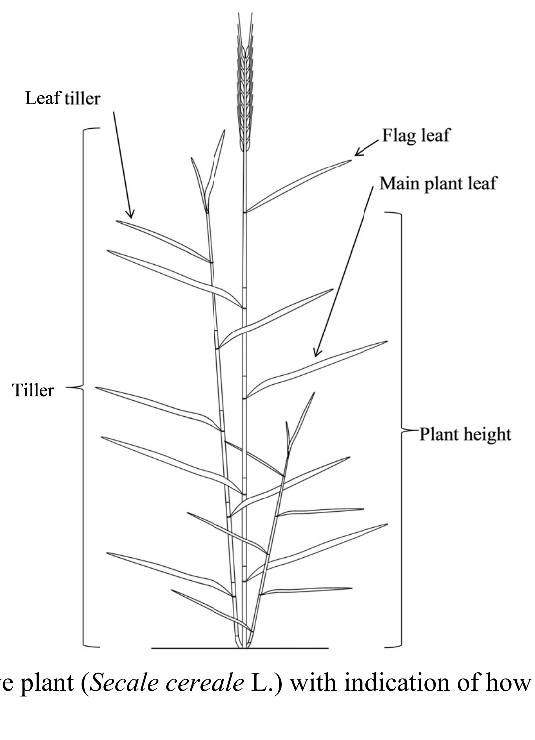

Figure 1. Representation of a rye plant (*Secale cereale* L.) with indication of how the plant height was measured

Source: Authors.

The evaluations were made in the times between the appearance of the first expanded leaf, stage 1 (Large, 1954) until November 3, 2016 (time 1), October 27, 2016 (time 2), November 10, 2016 (time 3), and November 18, 2016 (times 4 and 5). Altogether, 16, 15, 17, 17, and 15 evaluations were made with the BRS Progresso cultivar in times 1, 2, 3, 4, and 5, respectively. For the Temprano cultivar 18, 17, 18, 19, and 17 evaluations were made in times 1, 2, 3, 4 e 5, respectively.

The minimum and maximum air temperatures in ºC were recorded from the sowing time until the end of the evaluation by the Meteorological Station of the Federal University of Santa Maria, located 50 m from the experimental area. The daily thermal sum was calculated from the minimum and maximum air temperatures (Equation 1) by the Gilmore and Rogers (1958), and Arnold (1960) method,

$$dTS = \frac{Tmax + Tmin}{2} - Tb \quad (1)$$

where, dTS is the daily thermal sum (ºC), Tmax é is the daily maximum temperature (ºC), Tmin is the minimum daily temperature (ºC), and Tb is the basal temperature for rye, of 0 ºC (Bruckner & Raymer, 1990). Then, the accumulated thermal sum was calculated (Equation 2),

$$aTS = \sum dTS \quad (2)$$

where, aTS is the accumulated thermal sum and $\sum dTS$ is the summation of the daily thermal sum.

For plant height (dependent variable), the nonlinear Gomperz and Logistic models were adjusted according to the accumulated thermal sum (independent variable) in each uniformity trial. The input data for the dependent variable were the averages of the ten plants of each evaluation.

The assumptions of the mathematical models were checked, based on the residues, through the Shapiro-Wilk test for the normality of the residues, the Durbin-Watson test for the presence of autocorrelation of the residues, and the Breusch-Pagan for the homoscedasticity of the residues.

The Gompertz (Windsor, 1932) (Equation 3),

$$y = a \cdot exp[-exp(b - cx)] \quad (3)$$

and Logistic models were adjusted (Nelder, 1961) (Equation 4),

$$y = a/[1 + exp(-b - cx)] \quad (4)$$

where, y is the dependent variable, x is the independent variable, a is the asymptotic value, b is the ratio between the initial growth value and the final value, and c is the maximum rate of relative growth.

Were have calculated for the Gompertz model the point of inflection (PI) (Equations 5 and 6),

$$xi = \frac{b}{c} \quad (5)$$

$$yi = \frac{a}{e} \quad (6)$$

the point of maximum acceleration (PMA) (Equations 7 and 8),

$$xi = \frac{b - \ln\left(\frac{3+\sqrt{5}}{2}\right)}{c} \quad (7)$$

$$yi = a \cdot exp\left(-\frac{3+\sqrt{5}}{2}\right) \quad (8)$$

the point of maximum deceleration (PMD) (Equations 9 and 10),

$$xi = \frac{b - \ln\left(\frac{3-\sqrt{5}}{2}\right)}{c} \quad (9)$$

$$yi = a \cdot exp\left(-\frac{3-\sqrt{5}}{2}\right) \quad (10)$$

and the point of asymptotic deceleration (PAD) (Equations 11 and 12),

$$xi = \frac{b - \ln(36.8 - 9.77\sqrt{14.06})}{c} \quad (11)$$

$$yi = a \cdot exp\left[-\left(36.8 - 9.77\sqrt{14.06}\right)\right] \quad (12)$$

where, a b and c are the parameters of the model and e is the base of the Napierian logarithm (Mischan & Pinho, 2014).

For the Logistic model were calculated the point of inflection (PI) (Equations 13 and 14),

$$\text{xi} = \frac{\text{-b}}{\text{c}} \quad (13)$$

$$\text{yi} = \frac{\text{a}}{2} \quad (14)$$

the point of maximum acceleration (PMA) (Equations 15 and 16),

$$\text{xi} = \frac{1}{\text{c}}\left[\text{-b} - \ln\left(2 + \sqrt{3}\right)\right] \quad (15)$$

$$\text{yi} = \frac{\text{a}}{3 + \sqrt{3}} \quad (16)$$

the point of maximum deceleration (PMD) (Equations 17 and 18)

$$\text{xi} = \frac{1}{\text{c}}\left[\text{-b} - \ln\left(2 - \sqrt{3}\right)\right] \quad (17)$$

$$\text{yi} = \frac{\text{a}}{3 - \sqrt{3}} \quad (18)$$

and the point of asymptotic deceleration (PAD) (Equations 19 and 20),

$$\text{xi} = \frac{1}{\text{c}}\left[\text{-b} - \ln\left(5 - 2\sqrt{6}\right)\right] \quad (19)$$

$$\text{yi} = \frac{\text{a}}{2\left(3 - \sqrt{6}\right)} \quad (20)$$

where, *a*, *b* and *c* are the parameters of the model (Mischan & Pinho, 2014).

For each model (Gompertz and Logistic), the estimation of their parameters (*a*, *b*, or *c*) were compared between the cultivars (BRS Progresso *versu*s Temprano) and each sowing time (times 1, 2, 3, 4, and 5), with a total of 30 comparisons (two models × three estimates of each model × five sowing times). Then, for each model (Gompertz and Logistic), the estimation of their parameters (*a*, *b*, or *c*) were compared between their sowing times (1 *versus* 2, 1 *versus* 3, 1 *versus* 4, 1 *versus* 5, 2 *versus* 3, 2 *versus* 4, 2 *versus* 5, 3 *versus* 4, 3 *versus* 5, and 4 *versus* 5) in each cultivar (BRS Progresso and Temprano), totaling 120 comparisons (two models × three estimates of each model × two cultivars × 10 combinations of sowing times).

For these comparisons, were adopted the criterion of confidence interval overlap of the parameters of each model (Gompertz and Logistic). For this, the lower and upper limits of the 95% confidence interval were calculated. Therefore, for each model (Gompertz and Logistic), were adopted the following criterion to compare the cultivars in each sowing time: if the parameter estimate (*a*, *b*, or *c*) for a certain cultivar is between the lower and upper limits of the confidence interval of the parameter of another cultivar, both of the estimates do not differ. The parameter estimates differ, between the cultivars, if no estimate is placed between the lower and upper limits of the confidence interval of the parameter of another cultivar. In the same way, comparing the sowing times in each cultivar, were considered that: if a parameter estimate (*a*, *b*, or *c*) for a certain season is within the lower and upper limits of the confidence interval of the parameter of another season, these estimates are not different. The parameter estimates between the sowing times differ when none of the estimates is within the lower and upper limits of the confidence interval of the parameter of another sowing time.

In order to choose the adequate models for plant height, the evaluators of the adjustment quality were determined: coefficient of determination (R^2) (Equation 21); Akaike information criterion (AIC) (Equation 22); and residual standard deviation (RSD) (Equation 23),

$$R^2 = 1 - \frac{\text{SQR}}{\text{SQT}} \quad (21)$$

$$\text{AIC} = \ln(\sigma^2) + 2(\text{p} + 1)/\text{n} \quad (22)$$

$$\text{RSD} = \sqrt{\frac{\text{SQR}}{\text{n} - \text{p}}} \quad (23)$$

where, SQR is the sum of the squared residues and SQT is the sum of the total squares, $\ln(\sigma^2)$ is the logarithm of the variance of the errors, p is the number of parameters of the model and n is the number of evaluations. A high R^2 value is intended for these evaluators of the adjustment quality, since this will represent a better adjustment of the model. In contrast, for AIC and RSD, the lower the value, the better is the adjusted model.

A non-linear model is considered the best, compared to others, when it presents a pattern close to the linear one. In order to analyze the pattern of the models, Ratkowsky (1983) recommends checking the nonlinearity measures of the curves using Bates and Watts (1988), which quantified the nonlinearity found in the models based on the geometrical concept of curvature, and showed that the nonlinearity might be decomposed in intrinsic nonlinearity (IN) and the nonlinearity caused by the effect of the parameter (EP). A model should be chosen among others if it presents the lowest values of nonlinearity due both to its intrinsic nonlinearity and to the effect of the parameter.

The statistical analyses were performed using Microsoft Office Excel® and the statistical software R (R Development Core Team, 2018).

3. Results and Discussion

The cultivar BRS Progresso presented its maximum plant height values in times 2 and 3, while the Temprano cultivar has shown its maximum values in times 3 and 4 (Table 1). These values were obtained from the average of ten plants in each evaluation. Thus, the cultivars have shown a great performance in the field in these sowing times. That is, the BRS Progresso cultivar had a good development outside (time 2) the time indicated for sowing (time 3), with June and July being the months indicated for cultivars indicated for grain production (Embrapa, 2014). Moreover, the Temprano cultivar, with a sowing time starting in March (Embrapa, 2014), showed its best development in June. This development is related to the lack of big rainfalls, with a predominance of low intensity rain, although in times not so staggered during the period of raising the cultivar, which enabled the most adequate development for the crop (Figure 2).

Table 1. Minimum, mean and maximum plant height (PH, cm), based on the mean of 10 plants in each evaluation, of cultivars BRS Progresso e Temprano in five sowing times of rye (*Secale cereale* L.)

Time	Minimum	Mean	Maximum
BRS Progresso			
Time 1 (May 3, 2016)	2.30	59.67	126.30
Time 2 (May 25, 2016)	1.80	66.21	164.20
Time 3 (June 7, 2016)	1.80	70.58	169.20
Time 4 (June 22, 2016)	1.40	68.86	158.60
Time 5 (July 4, 2016)	2.30	72.47	142.60
Temprano			
Time 1 (May 3, 2016)	2.40	37.00	139.20
Time 2 (May 25, 2016)	1.70	31.37	139.60
Time 3 (June 7, 2016)	1.10	48.20	156.30
Time 4 (June 22, 2016)	0.90	48.59	157.80
Time 5 (July 4, 2016)	1.20	54.25	143.40

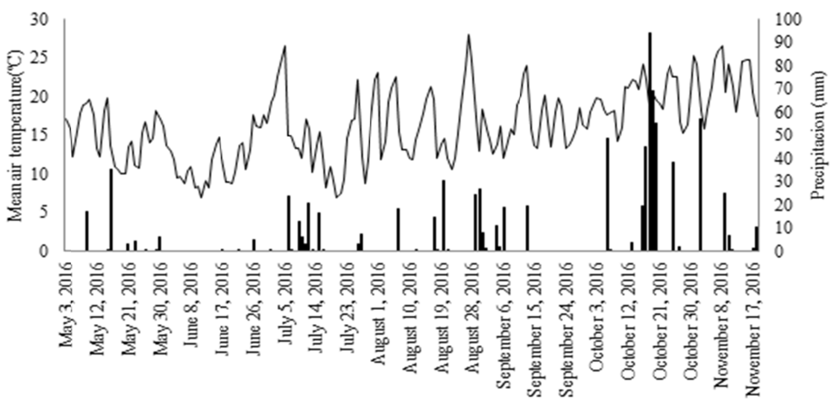

Figure 2. Precipitation (mm) and mean air temperature (ºC), of the rye experiment (*Secale cereale* L.)

For the plant height of the cultivars, in the sowing times, the p-value was higher than 0.05 in the Shapiro-Wilk (SW) normality test, in the Gompertz and Logistic models, indicating that the residues of each model follow the normal distribution. The independence test of Durbin-Watson (DW), for both models, presented a p-value above 0.05, showing that the residues are independent, that is, are not autocorrelated at a significance level of 5%. And in the homogeneity test of the residues of Breush-Pagan (BP), for both models, the p-value was also higher than

0.05, which shows that the residues present a constant variance at a 5% significance level, for the plant height of the cultivars in the sowing times. So, the assumptions of normality, independence, and homogeneity of the residues were met (Table 2).

Table 2. P-value of the Shapiro-Wilk (SW), Durbin-Watson (DW) and Breusch-Pagan (BP) tests applied on the Gompertz and Logistic model residues for plant height as a function of accumulated thermal sum (°C) of cultivars BRS Progress and Temprano in five sowing times of rye (*Secale cereale* L.)

Time	Estatistic	BRS Progresso	Temprano	BRS Progresso	Temprano
		Gompertz		Logistic	
Time 1	SW	0.140	0.058	0.540	0.050
May 3, 2016	DW	0.278	0.504	0.486	0.616
	BP	0.225	0.521	0.140	0.490
Time 2	SW	0.066	0.994	0.366	0.152
May 25, 2016	DW	0.144	0.138	0.780	0.276
	BP	0.539	0.335	0.402	0.122
Time 3	SW	0.205	0.457	0.075	0.050
June 7, 2016	DW	0.930	0.172	0.638	0.362
	BP	0.090	0.116	0.050	0.050
Time 4	SW	0.271	0.271	0.052	0.744
June 22, 2016	DW	0.192	0.192	0.442	0.050
	BP	0.300	0.300	0.234	0.489
Time 5	SW	0.086	0.086	0.331	0.089
July 4, 2016	DW	0.050	0.050	0.360	0.326
	BP	0.497	0.497	0.209	0.112

In the work of Muniz et al. (2017), the assumptions of normal distribution, independence, and homogeneity of residues were met for the volume of cocoa fruits in the Gompertz and Logistic models, in the same way that in em Fernandes, Pereira, Muniz, and Savian (2014), the assumptions were met for the accumulation of the fresh pulp of coffee fruits. Therefore, the assumptions for the models of this study were met and the Gompertz and Logistic models are adequate for the adjustment for plant height of the rye cultivars BRS Progresso and Temprano, evaluated in five sowing times.

For each model (Gompertz and Logistic), the parameter estimates (*a*, *b*, or *c*) were compared between the cultivars (BRS Progresso *versu*s Temprano) in each sowing time (times 1, 2, 3, 4, and 5) (Table 3) and between the sowing times (1 *versus* 2, 1 *versus* 3, 1 *versus* 4, 1 *versus* 5, 2 *versus* 3, 2 *versus* 4, 2 *versus* 5, 3 *versus* 4, 3 *versus* 5, and 4 *versus* 5) in each cultivar (BRS Progresso and Temprano) (Table 4), through the criterion of confidence interval overlap.

Table 3. Estimates of the parameters (*a*, *b* and *c*), lower limit ($LL_{2,5\%}$) and upper limit ($UL_{97,5\%}$) of the confidence interval ($CI_{95\%}$) of the Gompertz and Logistic models for plant height as a function of accumulated thermal sum °C) of cultivars[(1)] BRS Progress and Temprano in five sowing times of rye (*Secale cereale* L.)

Time	Parameter	Estimate	$LL_{2.5\%}$	$UL_{97.5\%}$	Estimate	$LL_{2.5\%}$	$UL_{97.5\%}$
		BRS Progresso			Temprano		
Gompertz							
Time 1	a^{ns}	116.3112	110.9945	121.6280	124.1218	109.1728	139.0707
May 3, 2016	b^{*}	3.4657	2.8908	4.0406	7.6396	4.8778	10.4013
	c^{ns}	0.0038	0.0032	0.0044	0.0050	0.0032	0.0069
Time 2	a^{*}	150.3814	135.1986	165.5641	102.4608	85.2911	119.6305
May 25, 2016	b^{*}	2.7986	2.1357	3.4616	10.3026	3.7672	16.8381
	c^{ns}	0.0031	0.0023	0.0039	0.0073	0.0027	0.0120
Time 3	a^{ns}	131.9025	122.4404	141.3647	135.3721	124.8593	145.8849
June 7, 2016	b^{*}	3.4758	2.4438	4.5079	5.3429	3.7751	6.9108
	c^{ns}	0.0040	0.0028	0.0052	0.0040	0.0028	0.0052
Time 4	a^{ns}	124.5188	114.2665	134.7711	128.2198	118.0222	138.4174
June 22, 2016	b^{*}	4.4471	2.4920	6.4023	9.0893	5.3013	12.8772
	c^{ns}	0.0053	0.0031	0.0076	0.0066	0.0038	0.0093
Time 5	a^{*}	125.2007	118.7027	131.6987	115.0514	106.2586	123.8442
July 4, 2016	b^{*}	3.4076	2.6244	4.1907	7.9221	4.6416	11.2026
	c^{ns}	0.0042	0.0033	0.0052	0.0067	0.0039	0.0094
Logistic							
Time 1	a^{ns}	113.5268	108.9470	118.1067	123.7441	111.7260	135.7622
May 3, 2016	b^{*}	-5.7793	-6.5887	-4.9699	-10.6636	-13.3512	-7.9759
	c^{ns}	0.0057	0.0048	0.0065	0.0066	0.0049	0.0084
Time 2	a^{*}	143.7418	134.4917	152.9918	104.8720	89.7693	119.9747
May 25, 2016	b^{*}	-5.1513	-5.9597	-4.3429	-10.4469	-14.5073	-6.3864
	c^{ns}	0.0051	0.0042	0.0060	0.0072	0.0043	0.0101
Time 3	a^{ns}	129.7035	121.4863	137.9207	133.1832	125.3150	141.0515
June 7, 2016	b^{*}	-5.6549	-7.1023	-4.2076	-8.0024	-9.6676	-6.3371
	c^{ns}	0.0058	0.0043	0.0074	0.0056	0.0044	0.0069
Time 4	a^{ns}	124.2929	115.5518	133.0339	128.1100	119.9451	136.2748
June 22, 2016	b^{*}	-6.6908	-8.9602	-4.4214	-11.7823	-15.2583	-8.3064
	c^{ns}	0.0073	0.0047	0.0098	0.0081	0.0057	0.0106
Time 5	a^{*}	123.6520	120.0644	127.2395	114.3118	107.2020	121.4216
July 4, 2016	b^{*}	-5.8360	-6.5921	-5.0800	-10.7092	-14.0612	-7.3572
	c^{ns}	0.0064	0.0056	0.0073	0.0086	0.0058	0.0113

Note. [(1)] Comparison of the estimates of the parameters (*a*, *b* and *c*) among the cultivars: * Estimates differ to 5% of significance. [ns] Not significant.

Table 4. Comparison of the estimates of the parameters (*a*, *b* and *c*) between sowing times[(1)] based on the confidence interval ($CI_{95\%}$) of the Gompertz and Logistic models for plant height as a function of the cumulative thermal sum (°C) of cultivars BRS Progress and Temprano in five sowing times of rye (*Secale cereale* L.)

Time	Time	Gompertz BRS Progresso	Gompertz Temprano	Logistic BRS Progresso	Logistic Temprano
a					
Time 1 (May 3, 2016)	Time 2 (May 25, 2016)	*	*	*	*
Time 1 (May 3, 2016)	Time 3 (June 7, 2016)	*	ns	*	ns
Time 1 (May 3, 2016)	Time 4 (June 22, 2016)	ns	ns	*	ns
Time 1 (May 3, 2016)	Time 5 (July 4, 2016)	*	ns	*	ns
Time 2 (May 25, 2016)	Time 3 (June 7, 2016)	*	*	*	*
Time 2 (May 25, 2016)	Time 4 (June 22, 2016)	*	*	*	*
Time 2 (May 25, 2016)	Time 5 (July 4, 2016)	*	ns	*	ns
Time 3 (June 7, 2016)	Time 4 (June 22, 2016)	ns	ns	ns	ns
Time 3 (June 7, 2016)	Time 5 (July 4, 2016)	ns	*	ns	*
Time 4 (June 22, 2016)	Time 5 (July 4, 2016)	ns	*	ns	*
b					
Time 1 (May 3, 2016)	Time 2 (May 25, 2016)	*	ns	ns	ns
Time 1 (May 3, 2016)	Time 3 (June 7, 2016)	ns	ns	ns	ns
Time 1 (May 3, 2016)	Time 4 (June 22, 2016)	ns	ns	ns	ns
Time 1 (May 3, 2016)	Time 5 (July 4, 2016)	ns	ns	ns	ns
Time 2 (May 25, 2016)	Time 3 (June 7, 2016)	ns	ns	ns	ns
Time 2 (May 25, 2016)	Time 4 (June 22, 2016)	ns	ns	ns	ns
Time 2 (May 25, 2016)	Time 5 (July 4, 2016)	ns	ns	ns	ns
Time 3 (June 7, 2016)	Time 4 (June 22, 2016)	ns	ns	ns	*
Time 3 (June 7, 2016)	Time 5 (July 4, 2016)	ns	ns	ns	ns
Time 4 (June 22, 2016)	Time 5 (July 4, 2016)	ns	ns	ns	ns
c					
Time 1 (May 3, 2016)	Time 2 (May 25, 2016)	ns	ns	ns	ns
Time 1 (May 3, 2016)	Time 3 (June 7, 2016)	ns	ns	ns	ns
Time 1 (May 3, 2016)	Time 4 (June 22, 2016)	ns	ns	ns	ns
Time 1 (May 3, 2016)	Time 5 (July 4, 2016)	ns	ns	ns	ns
Time 2 (May 25, 2016)	Time 3 (June 7, 2016)	ns	ns	ns	ns
Time 2 (May 25, 2016)	Time 4 (June 22, 2016)	ns	ns	ns	ns
Time 2 (May 25, 2016)	Time 5 (July 4, 2016)	*	ns	*	ns
Time 3 (June 7, 2016)	Time 4 (June 22, 2016)	ns	ns	ns	*
Time 3 (June 7, 2016)	Time 5 (July 4, 2016)	ns	ns	ns	*
Time 4 (June 22, 2016)	Time 5 (July 4, 2016)	ns	ns	ns	ns

Note. [(1)] Comparison of the estimates of the parameters (*a*, *b* and *c*) between sowing times: * Estimates differ to 5% of significance. [ns] Not significant.

For example, comparing the estimate of parameter *a* of the Gompertz model between the BRS Progresso and Temprano cultivars, in time 1 (May 3, 2016), the following was verified: the estimate of parameter *a* (116.3112) for the BRS Progresso cultivar is within the confidence interval of the estimate of this parameter (124.1218) for the Temprano cultivar, that is, between its lower (109.1728) and upper (139.0707) limits. It was also verified that the estimate of parameter *a* (124.1218) for the Temprano cultivar is outside the confidence interval of the estimate of parameter *a* (116.3112) for the BRS Progresso cultivar, that is, is outside its lower (110.9945) and (121.6280) upper limits. In this example, since at least one estimate of parameter *a* for a cultivar is within the confidence interval of the other cultivar, it was concluded that the estimate of parameter *a* (116.3112) for the BRS Progresso cultivar does not differ from the estimate of this parameter (124.1218) for the Temprano cultivar (Table 3).

In another example, comparing the estimate of parameter *b* for the Gompertz model between the BRS Progresso and Temprano cultivars, in time 1 (May 3, 2016), the following was verified: the estimate of parameter *b* (3.4657) for the BRS Progresso cultivar is outside the confidence interval of this parameter (7.6396) for the Temprano cultivar, that is, outside its lower (4.8778) and upper (10.4013) limits. It was also verified that the estimate of parameter *b* (7.6396) for the Temprano cultivar is outside the confidence interval of this parameter (3.4657) for the BRS Progresso cultivar, that is, is outside its lower (2.8908) and upper (4.0406) limits. Therefore, in this example, since no estimate of parameter *b* is within the lower and upper limits of the confidence interval of this parameter for the other cultivar, it was concluded that the estimate of parameter *b* (3.4657) for the BRS Progresso cultivar differs, at a 5% level of significance, from the estimate of this parameter (7.6396) for the Temprano cultivar (Table 3).

For each model (Gompertz and Logistic), the comparisons of the estimates of their parameters (*a, b*, or *c*) between the sowing times in each cultivar (Table 4) are interpreted according to the following example: comparing the estimate of parameter *a* of the Gompertz model between time 1 (May 3, 2016) and time 2 (May 25, 2016), for the BRS Progresso cultivar, the following was verified: the estimate of parameter *a* (116.3112) for time 1 is outside the confidence interval of this parameter (150.3814) for time 2, that is, outside its lower (135.1986) and upper (165.5641) limits. It was also verified that the estimate of parameter *a* (150.3814) for time 2 is outside the confidence interval of the estimate of this parameter (116.3112) for time 1, that is, is outside its lower (110.9945) and upper (121.6280) limits. Therefore, in this example, since no estimate of parameter *a* is within the lower and upper limits of the confidence interval of parameter *a* from the other period, it was concluded that the estimate of this parameter (116.3112) for time 1 differs, at a 5% significance level, from the estimate of parameter *a* (150.3814) for time 2 (Tables 3 and 4).

Comparing the cultivars by the method of confidence interval overlap, the results have shown the same pattern between the estimates of the parameters of the Gompertz and Logistic models (Table 3). That is, comparing the BRS Progresso and Temprano cultivars with the Gompertz and Logistic models in each sowing time can be inferred that their behavior is the same as the asymptote, the ratio between their initial and final growth values, and the maximum rate of relative growth.

Analyzing the estimate of parameter *a* in the Gompertz and Logistic (Table 3) models there was a difference at a 5% level of significance between the cultivars for plant height, in times 2 and 5. This pattern shows that the asymptote or the maximum plant height is different between the rye cultivars in these sowing times, that is, in times 2 and 5, there is a different growing pattern between the BRS Progresso and Temprano cultivars until the maximum height for the culture. For the Gompertz and Logistic models, the estimates of parameter *b* differ at a 5% level of significance for plant height between the cultivars in all sowing times. These results show that the growth of BRS Progresso e Temprano cultivars is different, exhibiting different transition moments in their growth rates. It can also be said that the BRS Progresso cultivar presents a higher growth rate in relation to the Temprano cultivar.

Analyzing parameter *c* for the Gompertz and Logistic models, it can be seen that in all the sowing times the maximum growth rate exhibits the same pattern between the BRS Progresso and Temprano cultivars in the sowing times (Table 3). Therefore, by analyzing the estimates of the parameters can be inferred that both models might be indicated to represent the growth of BRS Progresso and Temprano cultivars since they both present the same pattern of significance for the parameters.

Comparing the estimates of the parameters of the nonlinear models for plant height, between the sowing times in each cultivar, it was verified was no significant effect in the estimates of the parameters in 75.83% of the comparisons (Table 4). This comparison between the estimates of the parameters was found in a study comparing the sowing times of *Crotalaria juncea* in the Gompertz and Logistic models (Bem et al., 2018). However, in this study a smaller non-significant effect was observed in the estimates of the parameters in the comparisons between the sowing times. Thus, it can be inferred that for the models that present parameter estimates with a non-significant effect, the use of the model from any sowing time might be adequate since it presents the same growth pattern.

In order to make inferences about the growth of a certain agricultural crop, the points of inflection, maximum acceleration, maximum deceleration, and asymptotic deceleration might be used. So, were might realize that, through the point of inflection that for the Gompertz and Logistic models, the BRS Progresso cultivar needs a smaller thermal sum until the crop reaches 50% of its growth than the Temprano cultivar in each sowing time (Table 5). Further, comparing both models, in the Gompertz model were might infer that the BRS Progresso and Temprano cultivars reach their maximum growth rates with the least thermal sum, when compared to the growth rates of the cultivars in the Logistic model. However, in the Logistic model, since the plants present a higher thermal sum they reach the point of inflection with higher plant heights. Different values for the point of inflection between the cultivars and the progenies might also be observed in the work of Deprá, Lopes, Noal, Reiniger, and Cocco (2016) when they analyzed features from maize cultivars and progenies based on the thermal sum.

Table 5. Coefficient of determination (R^2), akaike information criterion (AIC) and residual standard deviation (RSD), intrinsic nonlinearity (IN) and the nonlinearity caused by the effect of the parameter (EP), point of inflection (PI), point of maximum acceleration (PMA), point of maximum deceleration (PMD), point of asymptotic deceleration (PAD) of the Gompertz and Logistic models for plant height as a function of the cumulative thermal sum (°C) of cultivars BRS Progress and Temprano in five sowing times of rye (*Secale cereale* L.)

		Time 1 May 3, 2016	Time 2 May 25, 2016	Time 3 June 6, 2016	Time 4 June 22, 2016	Time 5 July 4, 2016	Time 1 May 3, 2016	Time 2 May 25, 2016	Time 3 June 6, 2016	Time 4 June 22, 2016	Time 5 July 4, 2016
		BRS Progresso					Temprano				
Gompertz											
R²		0.992	0.981	0.975	0.961	0.985	0.967	0.931	0.981	0.969	0.972
AIC		3.262	4.525	4.735	5.195	4.084	4.203	4.457	4.185	4.534	4.453
RSD		4.413	8.132	9.123	11.477	6.531	8.459	10.159	7.722	9.864	8.885
IN		0.184	0.225	0.295	0.447	0.215	0.215	0.665	0.301	0.408	0.433
EP		0.354	0.732	0.596	0.777	0.408	0.408	1.304	0.576	0.705	0.780
PI	x	914.183	892.737	864.842	832.171	804.269	1525.640	1403.057	1321.942	1382.116	1186.640
	y	42.788	55.322	48.524	45.808	46.059	45.662	37.693	49.801	47.169	42.325
PMA	x	660.315	585.733	625.377	652.077	577.115	1333.442	1271.990	1083.820	1235.770	1042.479
	y	8.484	10.970	9.622	9.083	9.133	9.054	7.474	9.875	9.353	8.393
PMD	x	1168.052	1199.741	1104.308	1012.264	1031.424	1717.837	1534.124	1560.064	1528.462	1330.800
	y	79.385	102.638	90.026	84.986	85.452	84.715	69.931	92.394	87.512	78.525
PAD	x	1388.258	1466.037	1312.022	1168.478	1228.458	1884.550	1647.812	1766.611	1655.404	1455.846
	y	98.545	127.411	111.755	105.499	106.077	105.162	86.810	114.694	108.635	97.478
Logistic											
R²		0.992	0.989	0.977	0.969	0.995	0.976	0.950	0.986	0.979	0.979
AIC		3.254	3.982	4.681	4.990	3.114	4.022	4.376	4.026	4.265	4.226
RSD		4.258	6.088	8.776	10.268	3.967	7.191	8.657	6.630	8.215	7.681
IN		0.138	0.169	0.247	0.326	0.114	0.253	0.370	0.177	0.245	0.284
EP		0.298	0.443	0.516	0.621	0.222	0.633	0.899	0.357	0.490	0.568
PI	x	1018.485	1009.659	968.861	922.479	909.067	1613.214	1454.386	1417.413	1445.957	1246.477
	y	56.763	71.871	64.852	62.146	61.826	61.872	52.436	66.592	64.055	57.156
PMA	x	786.398	751.534	743.227	740.907	703.926	1413.980	1271.042	1184.147	1284.337	1093.192
	y	23.991	30.376	27.410	26.266	26.131	26.150	22.162	28.145	27.073	24.157
PMD	x	1250.572	1267.783	1194.496	1104.052	1114.208	1812.447	1637.730	1650.678	1607.577	1399.761
	y	89.536	113.366	102.294	98.027	97.521	97.594	82.710	105.038	101.037	90.155
PAD	x	1422.479	1458.976	1361.623	1238.543	1266.156	1960.020	1773.533	1823.458	1727.290	1513.299
	y	103.111	130.553	117.803	112.889	112.307	112.390	95.250	120.963	116.356	103.823

With the other points shown (Table 5) there is a pattern in the Gompertz and Logistic models similar to the pattern previously presented. This is due to the fact that the cultivars exhibit different characteristics for production. In this way, since the BRS Progresso cultivar is intended to grain production, it presents a faster development than the Temprano cultivar, which needs to develop mass in order to be promising to be used as soil cover and pastureland, demanding more thermal sum and having a longer crop cycle. So, based on these points, the investigator might choose the best sowing time, opting or not for faster initial growths. He might also predict the moment in which the crop will start to decline in growth, until it reaches the maximum peak, resembling the asymptote of the chosen model. Therefore, according to Bem et al. (2018), the period between the maximum point of acceleration and the end of the point of inflection is the best time to infer about management procedures such as fertilization, pest and disease control, and herbicide application, since in this interval the plant will respond efficiently. However, crop management is necessary along the whole period in which the crop is in the field.

In order to appoint an adequate model, it should analyze the quality of their adjustments. So, the choice of the best model for each cultivar and sowing time should take into account the R^2, AIC, and RSD values. Regarding R^2, the higher its value, the better will be the adjustment of the model. In contrast, a better adjustment is found when the AIC and RSD values are smaller. Thus, comparing the models in each combination of cultivar and sowing time the indicators are always favorable to the Logistic model (higher R^2 and lower AIC e RSD) (Table 5). In the same way, Karadavut (2009), Muianga et al. (2016), and Muniz, Nascimento, and Fernandes (2017), analyzing the quality of the adjustment of the models, have also found a better quality for the Logistic model.

In order to indicate the nonlinear model that presents the better pattern of the crop, one should appoint the model with the pattern closest to the linear model. In this way, through the intrinsic nonlinearity and the nonlinearity caused by the effect of the parameter it was verified that the Logistic model is indicated for the BRS Progresso and Temprano cultivars since it presents the smallest values, showing that this nonlinear model is closest to the linear one. Analyzing these parameters in the Logistic model, it can be verified that in the BRS Progresso cultivar, time 5 (Table 5, Figure 3) followed by time 1 exhibit the model closest to the linear, while for the Temprano cultivar time 3 (Table 5, Figure 3) presents the best model. In this context, in order to indicate the best models in their studies, Fernandes et al. (2014) evaluated the coffee tree, testing the linear pattern of the models using the same evaluators employed in the present study.

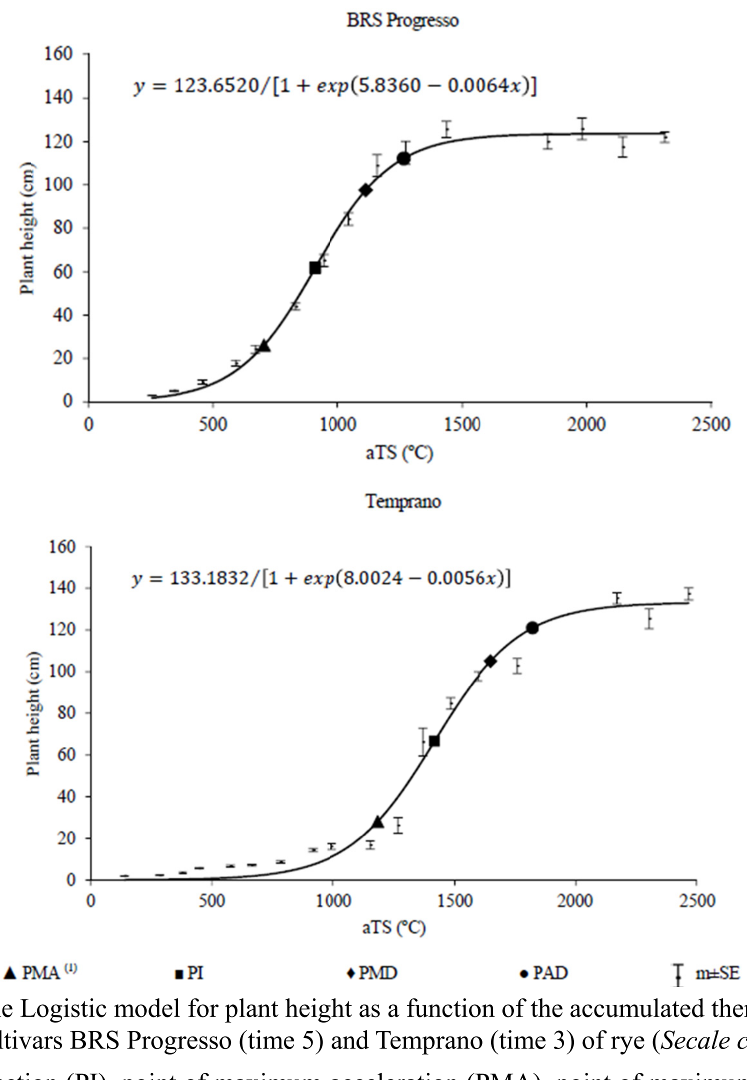

Figure 3. Graph of the Logistic model for plant height as a function of the accumulated thermal sum (aTS, in °C) of the cultivars BRS Progresso (time 5) and Temprano (time 3) of rye (*Secale cereale* L.)

Note. [1] Point of inflection (PI), point of maximum acceleration (PMA), point of maximum deceleration (PMD), point of asymptotic deceleration (PAD), mean ± standard error (m ± SE), estimated based on 10 rye plants, at each evaluation time of sowing times.

In order to exemplify the growth of the crop, the equation y = 123.6520/[1 + exp(5.8360 – 0.0064x)] in the Logistic model was obtained for plant height, based on the accumulated thermal sum (aTS) of the BRS Progresso cultivar, during time 5. Taking for instance a notional number of 750 ºC for the thermal sum, the estimated value for the plant height is 32.71 cm, a value found between the point of maximum acceleration and the point of inflection. For this case, the crop procedures that enable the initial growth of the plant should have been executed. In the case growth is not satisfactory for the crop; the investigator should check for the reason for this crop pattern and then apply the proper crop management practices, obtaining therefore a crop with a promising pattern for soil cover or grain production. So, for the rye crop, in subsequent investigations, the option should be for the equation obtained in the present study, using a new thermal sum from the harvest and analyzing if the feature is similar condition for the sowing time and cultivar, in order to check if crop management might already be employed.

Growth models are important to choose the better time to introduce the crop, in order to obtain better quality plant agronomic features. The option for the Logistic model might be prioritized by the investigator to describe the height increase of the rye, as well as in Berger (1981), where the Logistic model was the most appropriate to describe the progress of plant diseases, but the chosen model should be the one that best meets the perspectives of the research, taking into account the sowing time. In practice, enables the investigator to choose the growth model according to the characteristics of the crop, considering the sowing time for each cultivar. The information generated in this study is valid for the BRS Progresso and Temprano in the studied environment and might serve as a reference for the crop in future investigations. For other genotypes and environments, other studies should be conducted.

4. Conclusions

The Gompertz and Logistic models describe successfully the plant height pattern in the BRS Progresso and Temprano rye cultivars at sowing time.

The model that best describes the growth of the BRS Progresso and Temprano rye cultivars regarding plant height is the Logistic model.

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