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New Bounds on Poisson Approximation for Random Sums of Independent Binomial Random Variables

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Abstract

In this paper, we use the Stein-Chen method to obtain new bounds on Poisson approximation for random sums of independent binomial random variables. Some results related to sums of independent binomial distributed random variables are also investigated. The received results in the present study are more general and sharper than some known results.

Keywords: Binomial random variable, Poisson approximation, Random sums, Stein-Chen method

1. Introduction

In recent times, Poisson approximation problem for random sums of discrete random variables has attracted the attention of mathematicians. Readers who are interested in this problem can refer to (Hung & Giang, 2016b), (Kongudomthrap & Chaidee, 2012), (Teerapabolarn, 2013), (Teerapabolarn, 2014b), (Vellaisamy & Upadhye, 2009) and (Yannaros, 1991) for more details. We need to recall some results concerning the bounds in Poisson approximation for random sums of discrete random variables.

Let $Z_1, Z_2, \dots$ be a sequence of independent Bernoulli random variables, each with probability of success $P(Z_i = 1) = p_i = 1 - P(Z_i = 0), i = 1, 2, \dots$, and let N be a positive integer-valued random variable and independent of Z_i's. Let U_{λ^*} be a Poisson random variable with mean λ^*, $V_N = \sum_{i=1}^{N} Z_i$, $\lambda^* = E\left(\lambda_N^*\right)$ and $\lambda_N^* = \sum_{i=1}^{N} p_i$. In 1991, Yannaros gave a uniform bound for the total variation distance between the distributions of V_N and U_{λ^*} as follows, see (Yannaros, 1991):

$$d_{TV}(V_N, U_{\lambda^*}) \leq E\left|\lambda_N^* - \lambda^*\right| + E\left(\frac{1 - e^{-\lambda_N^*}}{\lambda_N^*} \sum_{i=1}^{N} p_i^2\right). \quad (1)$$

Let $X_1, X_2, \dots, X_n$ be n independently distributed binomial random variables, each with probabilities

$$P(X_i = k) = C_{r_i}^k p_i^k (1 - p_i)^{r_i - k},$$

where $p_i \in (0, 1); r_i = 1, 2, \dots; i = 1, 2, \dots n; k = 0, 1, \dots r_i; C_{r_i}^k = \dfrac{r_i!}{k!\,(r_i - k)!}$.

Suppose that N is a positive integer-valued random variable and independent of X_i's. Let U_λ be a Poisson random variable with mean λ, $W_N = \sum_{i=1}^{N} X_i$, $\lambda_N = \sum_{i=1}^{N} r_i p_i$ and $\lambda = E(\lambda_N)$. In 2014, Teerapabolarn used the Stein-Chen method to obtain a uniform bound for the total variation distance between the distribution functions of W_N and U_λ as follows, see (Teerapabolarn, 2014a):

$$d_{TV}(W_N, U_\lambda) \leq E\left(\frac{1 - e^{-\lambda_N}}{\lambda_N} \sum_{i=1}^{N} r_i p_i^2\right) + \min\left\{1, \sqrt{\frac{2}{\lambda e}}\right\} E\left|\lambda_N - \lambda\right|. \quad (2)$$

This paper is organized as follows. The second section is a brief introduction to Stein-Chen method. In section 3, we give main results of this paper, and conclusions of this study are presented in the last section.

In addition, throughout this paper, d_{TV} is denoted the total variation distance, defined by

$$d_{TV}(X, Y) = \sup_A \left|P(X \in A) - P(Y \in A)\right|,$$

where $A \subseteq \mathbb{Z}_+ := \{0, 1, 2, \dots\}$.

2. Preliminaries

The Stein-Chen method has been dealt with in detail in many articles (the reader is referred to (Chen, 1975) and (Barbour, Holst & Janson, 1992) for fuller development). The Stein-Chen method can be summarized as follows.

Let us denote by $F_{W_n}(A)$ the probability distribution function of a discrete random variable $W_n \in A$ and we will be denoted by $P_{\lambda_n}(A) = \sum_{k\in A} e^{-\lambda_n}\frac{{\lambda_n}^k}{k!}$ the Poisson distribution function ($\lambda_n > 0$), defined on the set $A \subseteq \mathbb{Z}_+$. The best known method for estimating

$$\Delta = \sup_x \left|F_{W_n}(A) - P_{\lambda_n}(A)\right|$$

is basing on the following arguments (see (Chen, 1975) for more details).

Assume that h is a bounded real-valued function defined on $\mathbb{Z}_+$ and

$$P_{\lambda_n}(h) = e^{-\lambda_n}\sum_{k=0}^{\infty} h(k)\frac{{\lambda_n}^k}{k!}.$$

Consider the function $f(.)$ which is a solution of the Stein's equation

$$\lambda_n f(w+1) - wf(w) = h(w) - P_{\lambda_n}(h). \tag{3}$$

Setting

$$h(w) = h_A(w) = \begin{cases} 1, & \text{if} \quad w \in A, \\ 0, & \text{if} \quad w \notin A. \end{cases}$$

Give $h = h_A$ and take the expectation of both sides of the equation (3), we have

$$F_{W_n}(A) - P_{\lambda_n}(A) = E\left[\lambda_n f(W_n+1) - W_n f(W_n)\right]. \tag{4}$$

Thus, the problem of estimating Δ can be reduced to that of estimating the difference of the expectations

$$|E\lambda_n f(W_n+1) - EW_n f(W_n)|.$$

According to Barbour et al. (see (Barbour, Holst & Janson, 1992), for $C_{w-1} = \{0, 1, ..., w-1\}$, the solution f_A of (3) is of the form

$$f_A(w) = \begin{cases} (w-1)!{\lambda_n}^{-w}e^{\lambda_n}\left[P_{\lambda_n}(h_{A\cap C_{w-1}}) - P_{\lambda_n}(h_A)P_{\lambda_n}(h_{C_{w-1}})\right], & \text{if}\, w \geq 1, \\ 0, & \text{if}\, w = 0. \end{cases} \tag{5}$$

Before starting the main results in next section, we also need the following lemmas, which is directly obtained from (Barbour, Holst & Janson, 1992) and (Teerapabolarn & Wongkasem, 2007).

Lemma 1 *Let $Vf_A(w) = f_A(w+1) - f_A(w)$. Then, for $A \subseteq \mathbb{Z}_+$ and $k \in \mathbb{Z}_+ \setminus \{0\}$,*

$$\sup_{w\geq k}|Vf_A(w)| \leq \min\left\{{\lambda_n}^{-1}\left(1 - e^{-\lambda_n}\right), \frac{1}{k}\right\}.$$

Lemma 2 *Let $w_0 \in \mathbb{Z}_+$ and $k \in \mathbb{Z}_+ \setminus \{0\}$, we have*

$$\sup_{w\geq k}\left|Vf_{C_{w_0}}(w)\right| \leq {\lambda_n}^{-1}\left(e^{\lambda_n} - 1\right)\min\left\{\frac{1}{w_0+1}, \frac{1}{k}\right\}.$$

Lemma 3 *Let U_{λ_N} and U_λ denote a Poisson random variable with mean λ_N and λ, respectively. Then, for $A \subseteq \mathbb{Z}_+$, the total variation distance between the distributions of U_{λ_N} and U_λ satisfies the following inequality:*

$$d_{TV}(U_{\lambda_N}, U_\lambda) \leq \min\left\{1, \sqrt{\frac{2}{e\lambda}}\right\}E|\lambda_N - \lambda|. \tag{6}$$

3. Main Results

The following lemma is established for proving the main results.

Lemma 4 *Let $X_1, X_2, \ldots$ be a sequence of independent binomial distributed random variables. Setting $W_n = \sum_{i=1}^{n} X_i$ and $\lambda_n = E(W_n)$. Then,*

$$
\begin{aligned}
&E\left[\lambda_n f(W_n+1) - W_n f(W_n)\right] \\
&\qquad = \sum_{i=1}^{n} \sum_{k\ge 1} k C_{r_i}^{k} p_i^{k+1} (1-p_i)^{r_i-k} E\left[f(W_i+k+1) - f(W_i+k)\right],
\end{aligned}
$$

where f is a bounded real-valued function defined on $\mathbb{Z}_+$.

Proof. We have

$$E\left[\lambda_n f(W_n+1) - W_n f(W_n)\right] = \sum_{i=1}^{n} E\left[r_i p_i f(W_n+1) - X_i f(W_n)\right].$$

Setting $W_i = W_n - X_i$,

$$
\begin{aligned}
&E\left[r_i p_i f(W_i+X_i+1) - X_i f(W_i+X_i)\right] \\
&= E\left[E\left[(r_i p_i f(W_i+X_i+1) - X_i f(W_i+X_i)) / X_i\right]\right] \\
&= E\left[r_i p_i f(W_i+1)\right] P(X_i=0) \\
&\quad + E\left[r_i p_i f(W_i+2) - f(W_i+1)\right] P(X_i=1) \\
&\quad + \sum_{k\ge 2} E\left[r_i p_i f(W_i+k+1) - k f(W_i+k)\right] P(X_i=k) \\
&= E\left[(r_i p_i P(X_i=0) - P(X_i=1)) f(W_i+1)\right] \\
&\quad + \sum_{k\ge 2} E\left[(r_i p_i P(X_i=k-1) - kP(X_i=k)) f(W_i+k)\right] \\
&= E\left[\left(r_i p_i (1-p_i)^{r_i} - r_i p_i (1-p_i)^{r_i-1}\right) f(W_i+1)\right] \\
&\quad + \sum_{k\ge 2} E\left[\left(r_i p_i C_{r_i}^{k-1} p_i^{k-1} (1-p_i)^{r_i-k+1} - k C_{r_i}^{k} p_i^{k} (1-p_i)^{r_i-k}\right) f(W_i+k)\right] \\
&= -E\left[r_i p_i^2 (1-p_i)^{r_i-1} f(W_i+1)\right] \\
&\quad + \sum_{k\ge 2} E\left[\left(r_i p_i C_{r_i}^{k-1} p_i^{k-1} (1-p_i)^{r_i-k+1} - (r_i-k+1) C_{r_i}^{k-1} p_i^{k} (1-p_i)^{r_i-k}\right) f(W_i+k)\right] \\
&= -E\left[r_i p_i^2 (1-p_i)^{r_i-1} f(W_i+1)\right] \\
&\quad + \sum_{k\ge 2} E\left[\left(\frac{r_i-k+1}{r_i} r_i p_i C_{r_i}^{k-1} p_i^{k-1} (1-p_i)^{r_i-k+1} - (r_i-k+1) C_{r_i}^{k-1} p_i^{k} (1-p_i)^{r_i-k}\right) f(W_i+k)\right] \\
&\quad - \sum_{k\ge 2} E\left[\left(\frac{r_i-k+1}{r_i} - 1\right) r_i p_i C_{r_i}^{k-1} p_i^{k-1} (1-p_i)^{r_i-k+1} f(W_i+k)\right] \\
&= -E\left[r_i p_i^2 (1-p_i)^{r_i-1} f(W_i+1)\right] \\
&\quad - \sum_{k\ge 2} E\left[(r_i-k+1) C_{r_i}^{k-1} p_i^{k+1} (1-p_i)^{r_i-k} f(W_i+k)\right] \\
&\quad - \sum_{k\ge 2} E\left[\left(\frac{r_i-k}{r_i} - 1\right) r_i C_{r_i}^{k} p_i^{k+1} (1-p_i)^{r_i-k} f(W_i+k+1)\right] \\
&\quad + E\left[r_i p_i^2 (1-p_i)^{r_i-1} f(W_i+2)\right] \\
&= r_i p_i^2 (1-p_i)^{r_i-1} E\left[f(W_i+2) - f(W_i+1)\right] \\
&\quad + \sum_{k\ge 2} k C_{r_i}^{k} p_i^{k+1} (1-p_i)^{r_i-k} E\left[f(W_i+k+1) - f(W_i+k)\right] \\
&= \sum_{k\ge 1} k C_{r_i}^{k} p_i^{k+1} (1-p_i)^{r_i-k} E\left[f(W_i+k+1) - f(W_i+k)\right].
\end{aligned}
$$

This finishes the proof. □

The following theorems present non-uniform and uniform bounds for the distance between the distribution functions of W_N and U_λ, which are the expected results.

3.1 A Uniform Bound on Poisson Approximation for Random Sums of Independent Binomial Random Variables

Theorem 1 *For $A \subseteq \mathbb{Z}_+$, we have*

$$\begin{aligned} d_{TV}(W_N, U_\lambda) \leq E\left(\sum_{i=1}^{N} \min\left\{\lambda_N^{-1}\left(1-e^{-\lambda_N}\right) r_i, \frac{1-(1-p_i)^{r_i}}{p_i}\right\} p_i^2\right) \\ + \min\left\{1, \sqrt{\frac{2}{\lambda e}}\right\} E\left|\lambda_N - \lambda\right|. \end{aligned} \tag{7}$$

Proof. Let $f = f_A$ be defined as in (5) and applying (4), we have

$$\left|P(W_n \in A) - \sum_{k\in A} \frac{\lambda_n^k e^{-\lambda_n}}{k!}\right| = \left|E[\lambda_n f(W_n+1) - W_n f(W_n)]\right|. \tag{8}$$

Taking account of Lemma 4 and Lemma 1, it follows that

$$\begin{aligned} &\left|E\left[r_i p_i f(W_n+1) - X_i f(W_n)\right]\right| \\ &\leq \sum_{k\geq 1} k C_{r_i}^k p_i^{k+1}(1-p_i)^{r_i-k} E\left|f(W_i+k+1) - f(W_i+k)\right| \\ &\leq \sum_{k\geq 1} k C_{r_i}^k p_i^{k+1}(1-p_i)^{r_i-k} \sup_{w\geq k}\left|Vf(w)\right| \\ &\leq \sum_{k\geq 1} k C_{r_i}^k p_i^{k+1}(1-p_i)^{r_i-k} \min\left\{\frac{1-e^{-\lambda_n}}{\lambda_n}, \frac{1}{k}\right\} \\ &= \min\left\{\frac{1-e^{-\lambda_n}}{\lambda_n} p_i \sum_{k\geq 1} k C_{r_i}^k p_i^k (1-p_i)^{r_i-k}, p_i \sum_{k\geq 1} C_{r_i}^k p_i^k (1-p_i)^{r_i-k}\right\} \\ &= \min\left\{\frac{1-e^{-\lambda_n}}{\lambda_n} p_i \sum_{k\geq 1} k P(X_i = k), p_i\left(\sum_{k\geq 0} P(X_i = k) - (1-p_i)^{r_i}\right)\right\} \\ &= \min\left\{\frac{1-e^{-\lambda_n}}{\lambda_n} p_i E(X_i), p_i\left(1-(1-p_i)^{r_i}\right)\right\}. \end{aligned}$$

Thus,

$$\left|E\left[r_i p_i f(W_n+1) - X_i f(W_n)\right]\right| \leq \min\left\{\lambda_n^{-1}\left(1-e^{-\lambda_n}\right) r_i, \frac{1-(1-p_i)^{r_i}}{p_i}\right\} p_i^2. \tag{9}$$

Combining (8) with (9), gives

$$d_{TV}(W_n, U_{\lambda_n}) \leq \sum_{i=1}^{n} \min\left\{\lambda_n^{-1}\left(1-e^{-\lambda_n}\right) r_i, \frac{1-(1-p_i)^{r_i}}{p_i}\right\} p_i^2. \tag{10}$$

From Lemma 3 and (10), it follows the fact that

$$\begin{aligned} d_{TV}(W_N, U_\lambda) &= \sum_{n=1}^{\infty} P(N=n)\, d_{TV}(W_n, U_\lambda) \\ &\leq \sum_{n=1}^{\infty} P(N=n)\left[d_{TV}(W_n, U_{\lambda_n}) + d_{TV}(U_{\lambda_n}, U_\lambda)\right] \\ &= \sum_{n=1}^{\infty} P(N=n)\, d_{TV}(W_n, U_{\lambda_n}) + d_{TV}(U_{\lambda_N}, U_\lambda) \\ &\leq \sum_{n=1}^{\infty} P(N=n) \sum_{i=1}^{n} \min\left\{\lambda_n^{-1}\left(1-e^{-\lambda_n}\right) r_i, \frac{1-(1-p_i)^{r_i}}{p_i}\right\} p_i^2 \\ &\quad + \min\left\{1, \sqrt{\frac{2}{\lambda e}}\right\} E\left|\lambda_N - \lambda\right| \end{aligned}$$

$$\leq E\left(\sum_{i=1}^{N}\min\left\{\lambda_N^{-1}\left(1-e^{-\lambda_N}\right)r_i,\frac{1-(1-p_i)^{r_i}}{p_i}\right\}p_i^2\right)$$
$$+\min\left\{1,\sqrt{\frac{2}{\lambda e}}\right\}E\left|\lambda_N-\lambda\right|.$$

This finishes the proof. □

Remark 1 For $r_1 = r_2 = ... = r_n = 1$, we have a uniform bound on Poisson approximation for the random sums of independent Bernoulli random variables:

$$d_{TV}\left(V_N,U_{\lambda^*}\right)\leq E\left(\lambda^{*-1}_N\left(1-e^{-\lambda^*_N}\right)\sum_{i=1}^{N}p_i^2\right)+\min\left\{1,\sqrt{\frac{2}{\lambda^* e}}\right\}E\left|\lambda^*_N-\lambda^*\right|. \quad (11)$$

Remark 2 Let us consider:

$$\min\left\{1,\sqrt{\frac{2}{\lambda^* e}}\right\}\leq 1$$

and

$$\min\left\{\frac{1-e^{\lambda_N}}{\lambda_N}r_i,\frac{1-(1-p_i)^{r_i}}{p_i}\right\}p_i^2\leq\frac{1-e^{\lambda_N}}{\lambda_N}r_ip_i^2.$$

Thus, the bounds in (7) and (11) are sharper than the bounds in (2) and (1), respectively.

Corollary 1 *For $N = n \in \mathbb{Z}_+$ is fixed, then $\lambda = \lambda_n = \sum_{i=1}^{n} r_ip_i$ and*

$$d_{TV}\left(W_n,U_{\lambda_n}\right)\leq\sum_{i=1}^{n}\min\left\{\lambda_n^{-1}\left(1-e^{-\lambda_n}\right)r_i,\frac{1-(1-p_i)^{r_i}}{p_i}\right\}p_i^2. \quad (12)$$

Remark 3 The result (12) is a uniform bound on Poisson approximation for sums of independent binomial random variables. This bound is sharper than those reported in (Teerapabolarn, 2014a).

3.2 A Non-uniform Bound on Poisson Approximation for Random Sums of Independent Binomial Random Variables

Theorem 2 *For $w_0 \in \mathbb{Z}_+$, we have*

$$\begin{aligned}\left|P(W_N\leq w_0)-P\left(U_\lambda\leq w_0\right)\right|&\leq\min\left\{\frac{2\lambda}{w_0+1},\min\left\{1,\sqrt{\frac{2}{e\lambda}}\right\}E\left|\lambda_N-\lambda\right|\right\}\\&+E\left(\sum_{i=1}^{N}\lambda_N{}^{-1}\left(1-e^{-\lambda_N}\right)\min\left\{\frac{e^{\lambda_N}r_i}{(w_0+1)},\frac{\left(1-(1-p_i)^{r_i}\right)e^{\lambda_N}}{p_i}\right\}p_i^2\right).\end{aligned} \quad (13)$$

Proof. For $C_w = \{0, ..., w\}$ and $w_0 \in \mathbb{Z}_+$, let $h_{w_0} : \mathbb{Z}_+ \to \mathbb{R}$ such that

$$h_{C_{w_0}}(w)=\begin{cases}1 & \text{if } w\leq w_0,\\ 0 & \text{if } w>w_0.\end{cases}$$

According to Barbour et al. (see (Barbour, Holst & Janson, 1992) on p.7), the solution $f_{C_{w_0}}(w)$ of (3) is expressed in the form of

$$f_{C_{w_0}}(w)=\begin{cases}(w-1)!\lambda_n{}^{-w}e^{\lambda_n}\left[P_{\lambda_n}\left(h_{C_{w_0}}\right)P_{\lambda_n}\left(1-h_{C_{w-1}}\right)\right] & \text{, if } w_0<w,\\ (w-1)!\lambda_n{}^{-w}e^{\lambda_n}\left[P_{\lambda_n}\left(h_{C_{w-1}}\right)P_{\lambda_n}\left(1-h_{C_{w_0}}\right)\right] & \text{, if } w_0\geq w,\\ 0 & \text{, if } w=0.\end{cases}$$

Given $f = f_{C_{w_0}}$ and $h = h_{C_{w_0}}$, the Stein's equation

$$h_{C_{w_0}}(w)-\sum_{k\leq w_0}e^{-\lambda_n}\frac{\lambda_n^k}{k!}=\lambda_nf(w+1)-wf(w).$$

Taking expectations of both sides, and applying Lemma 2 and Lemma 4, we have

$$
\begin{aligned}
&\left|P(W_n \le w_0) - P\left(U_{\lambda_n} \le w_0\right)\right| \\
&\le \sum_{i=1}^{n}\left(\sum_{k\ge 1} kC_r^k p_i^{k+1}(1-p_i)^{r_i-k} E\left|f\left(W_i+k+1\right)-f\left(W_i+k\right)\right|\right) \\
&\le \sum_{i=1}^{n}\left(\sum_{k\ge 1} kC_r^k p_i^{k+1}(1-p_i)^{r_i-k} \lambda_n^{-1}\left(e^{\lambda_n}-1\right)\min\left\{\frac{1}{w_0+1},\frac{1}{k}\right\}\right) \\
&= \sum_{i=1}^{n}\lambda_n^{-1}\left(e^{\lambda_n}-1\right)\min\left\{\frac{p_i\sum_{k\ge 1} kP\left(X_i=k\right)}{w_0+1}, p_i\sum_{k\ge 1}P\left(X_i=k\right)\right\} \\
&= \lambda_n^{-1}\left(e^{\lambda_n}-1\right)\sum_{i=1}^{n}\min\left\{\frac{r_i}{w_0+1},\frac{1-(1-p_i)^{r_i}}{p_i}\right\}p_i^2.
\end{aligned}
$$

Thus,

$$
\left|P(W_n \le w_0) - P\left(U_{\lambda_n} \le w_0\right)\right| \le \lambda_n^{-1}\left(e^{\lambda_n}-1\right)\sum_{i=1}^{n}\min\left\{\frac{r_i}{w_0+1},\frac{1-(1-p_i)^{r_i}}{p_i}\right\}p_i^2. \tag{14}
$$

In addition, by using Lemma 3, Teerapabolarn showed that (see (Teerapabolarn, 2013) for more details):

$$
\left|P\left(U_{\lambda_N} \le w_0\right) - P\left(U_{\lambda} \le w_0\right)\right| \le \min\left\{\frac{2\lambda}{w_0+1}, \min\left\{1, \sqrt{\frac{2}{e\lambda}}\right\} E\left|\lambda_N-\lambda\right|\right\}. \tag{15}
$$

Combining (14) and (15) gives

$$
\begin{aligned}
&\left|P\left(W_N \le w_0\right) - P\left(U_{\lambda} \le w_0\right)\right| \\
&\le \sum_{n=0}^{\infty} P\left(N=n\right)\left|P\left(W_n \le w_0\right) - P\left(U_{\lambda} \le w_0\right)\right| \\
&\le \sum_{n=0}^{\infty} P\left(N=n\right)\left|P\left(W_n \le w_0\right) - P\left(U_{\lambda_n} \le w_0\right)\right| \\
&\quad + \left|P\left(U_{\lambda_N} \le w_0\right) - P\left(U_{\lambda} \le w_0\right)\right| \\
&\le \sum_{n=0}^{\infty} P\left(N=n\right)\frac{1-e^{-\lambda_n}}{\lambda_n}\sum_{i=1}^{n}\min\left\{\frac{r_i e^{\lambda_n}}{w_0+1},\frac{\left(1-(1-p_i)^{r_i}\right)e^{\lambda_n}}{p_i}\right\}p_i^2 \\
&\quad + \min\left\{\frac{2\lambda}{w_0+1}, \min\left\{1, \sqrt{\frac{2}{e\lambda}}\right\} E\left|\lambda_N-\lambda\right|\right\} \\
&\le E\left(\frac{1-e^{-\lambda_N}}{\lambda_N}\sum_{i=1}^{N}\min\left\{\frac{r_i e^{\lambda_N}}{w_0+1},\frac{\left(1-(1-p_i)^{r_i}\right)e^{\lambda_N}}{p_i}\right\}p_i^2\right) \\
&\quad + \min\left\{\frac{2\lambda}{w_0+1}, \min\left\{1, \sqrt{\frac{2}{e\lambda}}\right\} E\left|\lambda_N-\lambda\right|\right\}.
\end{aligned}
$$

This finishes the proof. □

Remark 4 For $r_1 = r_2 = ... = r_n = 1$, we have a non-uniform bound on Poisson approximation for the random sums of independent Bernoulli random variables:

$$
\begin{aligned}
\left|P(V_N \le w_0) - P\left(U_{\lambda^*} \le w_0\right)\right| \le{}& \min\left\{\frac{2\lambda^*}{w_0+1}, \min\left\{1, \sqrt{\frac{2}{e\lambda^*}}\right\} E\left|\lambda_N^*-\lambda^*\right|\right\} \\
&+ E\left(\frac{\left(e^{\lambda_N^*}-1\right)}{\left(w_0+1\right)\lambda_N^*}\sum_{i=1}^{N}p_i^2\right).
\end{aligned} \tag{16}
$$

Corollary 2 *For $N = n \in \mathbb{Z}_+$ is fixed, then $\lambda = \lambda_n = \sum_{i=1}^{n} r_i p_i$ and*

$$\left|P(W_n \leq w_0) - P\left(U_{\lambda_n} \leq w_0\right)\right| \leq \lambda_n^{-1}\left(e^{\lambda_n} - 1\right)\sum_{i=1}^{n} \min\left\{\frac{r_i}{w_0 + 1}, \frac{1 - (1 - p_i)^{r_i}}{p_i}\right\} p_i^2. \quad (17)$$

Remark 5 The result (17) is a non-uniform bound on Poisson approximation for sums of independent binomial random variables.

4. Conclusions

We conclude this paper with the following comments. Bounds for the distance between the distribution function of random sums of independent binomial random variables and an appropriate Poisson distribution function were obtained. The received results in this paper are sharper than those reported in (Teerapabolarn, 2014a), (Teerapabolarn, 2014b), and (Yannaros, 1991). Moreover, non-uniform bounds on Poisson approximation for sums (and random sums) of independent binomial random variables are given. The results will be more interesting and valuable if we discuss Poisson approximation for random sums of dependent binomial random variables. We shall continue studying this matter in our future research.

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