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Detection and Modeling of Asymmetric GARCH Effects in a Discrete-Time Series

Emmanuel Alphonsus Akpan[1] & Imoh Udo Moffat[2]

[1] Department of Mathematical Science, Abubakar Tafawa Balewa University, Bauchi

[2] Department of Mathematics and Statistics, University of Uyo, Uyo

Correspondence: Emmanuel Alphonsus Akpan, Department of Mathematical Science, Abubakar Tafawa Balewa University, Bauchi. E-mail: eubong44@gmail.com

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Abstract

This study traced the patterns of discrete time series over time with respect to GARCH effect and asymmetric GARCH effect. Particularly, we paid attention to the weakness of the GARCH model in modeling the asymmetry of GARCH effect. In order to handle this weakness, we applied the sign and size bias test which comprises sign bias test, negative size bias test, positive size bias test, and Lagrange Multiplier test in order to identify the asymmetric effect in the residual series of the GARCH model. Where the asymmetric effect is present and significant, we fit the asymmetric GARCH models. Exploring the share price returns of Zenith bank plc obtained from the Nigerian Stock Exchange from January 4, 2006 to May 26, 2015, our findings indicated the presence of GARCH effect and was adequately captured by GARCH(0,1) model. Also, the sign and size bias test for asymmetric GARCH effect on the residual series of GARCH(0,1) model showed a joint significance as indicated by the Lagrange Multiplier test. Moreover, the asymmetric GARCH effect was adequately captured by EGARCH(0,1) and TGARCH(0,1) models. In addition, the significance of the size bias test indicated that the size of negative and positive returns has an impact on the predicted heteroscedasticity. Hence, we concluded that GARCH(0,1) model adequately predicted the GARCH effect but failed to capture the asymmetric effect in the share price returns of the discrete series. However, this was complemented by both EGARCH(0,1) and TGARCH(0,1) models with the size of both the negative and positive effects taken into consideration.

Keywords: Discrete-time series, EGARCH,GARCH effect, Heteroscedasticity, Sign and size test, TGARCH, Volatility.

1. Introduction

Statistically, returns are the natural logarithm transformed share prices whose characterizations have more attractive statistical properties, and easier to handle than the share prices. The fact that large absolute returns tend to appear in clusters, indicating a possible presence of heteroscedasticity, is hardly compatible with the assumption of constant variance(Franses and Dijk, 2003). Thus there is need for appropriate models that can capture the time-varying heteroscedasticity as neglecting its presence in linear models results in inefficient ordinary least squares estimates of ARMA parameters though still consistent and asymptotically normally distributed, their variance-covariance matrix is no longer the usual one. Thus, making the t-statistics invalid and cannot be used to examine the significance of the individual explanatory variables in the model. Also, over-parameterization of an ARMA model and low statistical power are identified as part of the consequences for neglecting heteroscedasticity. Lastly, neglecting heteroscedasticity can lead to spurious nonlineality in the conditional mean and difficulty in computing the confidence interval for forecasts (see for example, Deshon and Alexander, 1996; Franses and Dijk, 2003; Fan and Yao, 2005; Astriou and Hall, 2007). Moreover, the formal tests for the presence of heteroscedasticity (ARCH/GARCH effects) areusually Lagrange Multiplier and Ljung-Box on the squares of the residual series obtained from ARIMA modeling of the return series. Once these ARCH/GARCH effects are identified, then GARCH models could be applied (Akpan, Moffat and Ekpo, 2016;Akpan and Moffat, 2015;Ogum, Beer and Nouyrigat, 2005; Mgbame and Ikhatua, 2014; Atoi, 2014; Onwukwe, Samson and Lipcsey, 2014;Yaya, 2013; Emenike, 2010). Meanwhile, in an attempt to model the asymmetric GARCH, previous studies in Nigeria only extended the fitted GARCH models to the asymmetric ones and thereafter, access the significance of coefficient of leverage effect without prior formal test for the presence of asymmetric GARCH effect and thus created a gap in knowledge by not exploring a formal test in detecting the asymmetric GARCH effect. Hence, we seek to fill this gap by using sign and size bias test (which are categorized into; sign bias test, negative size bias test and positive size bias test) and Lagrange Multiplier (which checks the joint significance of sign and size test) to detect the asymmetric GARCH effect in the share price returns of Zenith bank plc. In this regard, the fitted GARCH model would be evaluated based on its ability to model the changing conditional variance and its failure to capture the asymmetries in the series while extensions to EGARCH and TGARCH models are made in order to overcome the weaknesses of the GARCH model. Furthermore, the remaining content of this paper is organized as follows: section 2 which treats the methods to be implored; the results and discussion of the findings are presented in section 3 while section 4 accommodates the conclusion of the overall findings of this work.

2. Methods

2.1 Return

The return series R_t can be obtained given that P_t is the price of a unit share at time, t and P_{t-1} is the share price at time t−1.

$$R_t = \nabla lnP_t = (1-B)lnP_t = ln\,P_t - ln\,P_{t-1} \tag{1}$$

The R_t in equation (1) is regarded as a transformed series of the share price, P_t meant to attain stationarity, that is, both mean and variance of the series are stable (Akpan and Moffat, 2015). The letter $\boldsymbol{B}$is the backshift operator.

2.2 Model Selection Criteria

For a given dataset, when there are multiple adequate models, the selection criterion is normally based on summary statistics from residuals of a fitted model (for more details see Wei, 2006). For the purpose of this study, we consider the well-known Akaike's information criterion (AIC), (Akaike, 1973) defined as

$$\text{AIC} = -2\,ln(\text{likelihood}) + 2(\text{number of parameters}) \tag{2}$$

where the likelihood function is evaluated at the maximum likelihood estimates. The optimal order of the model is chosen by the value of the number of parameters, so that AIC is minimum (Wei, 2006).

2.3 Diagnostic Checking of the Model

Ljung and Box Test (1978) is given as

$$\text{Q(m)} = \text{T(T + 2)} \sum_{l=1}^{m} \frac{\hat{\rho}_l^2}{T-l} \tag{3}$$

where T is the number of observations. This test checks the joint significance of the first m lags of the ACFs of the residual series. The joint null hypothesis is stated as follows: H_0: $\rho_1 = \rho_2 = \cdots = \rho_m = 0$ against H_A: $\rho_1 \neq \rho_2 \neq \cdots \neq \rho_m \neq 0$. The decision rule is to reject H_0 if Q(m) $>\chi^2_\alpha$, where χ^2_α denotes the 100 (1 – α)th percentile of a Chi-squared distribution with m – (p + q) degree of freedom (see for example Akpan, Moffat and Ekpo, 2016).

2.4 Lagrange Multiplier Test

Another approach for testing the ARCH/GARCH effect (otherwise called heteroscedasticity is the changing conditional variance) is to apply the Lagrange Multiplier (LM) test of ARCH(q) against the hypothesis of no ARCH effects to $\{a_t^2\}$ series. The LM test is carried out by computing, $\chi^2 = \text{T}R^2$ in the regression of a_t^2 on a constant and q lagged values. T is the sample size and R^2 is the coefficient of determination. Under the null hypothesis of no ARCH effects, the statistic has a Chi-square distribution with q degrees of freedom. If the LM test statistic is larger than the critical value, then, there is evidence of the presence of ARCH effects (Greene, 2002).

2.5 Test for Asymmetries in Heteroscedasticity

2.5.1 Sign Bias (SB) Test

This test is used to verify whether previous positive and negative shocks have a different impact on heteroscedasticity. The test can be carried out as follows:

Obtain the residual series from GARCH model.

Test for sign bias in the following regression of the squares of residual series:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 N_{t-1}^- + e_t, \tag{4}$$

$$N_{t-1}^- = \begin{cases} 1 & if\ \hat{\varepsilon}_{t-1}^2 < 0 \\ 0, & otherwise \end{cases}$$

If positive and negative shocks have different impacts on heteroscedasticity then α_1 will be statistically significant.

2.5.2 Sign and Size Bias Test

This test as proposed by Engle and Ng (1993)verifies whether heteroscedasticity depends on both the sign and size of previous shocks. This test is based on the following regression:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 N_{t-1}^- + \alpha_2 N_{t-1}^- \hat{\varepsilon}_{t-1} + \alpha_3 N_{t-1}^+ \hat{\varepsilon}_{t-1} + e_{t,} \tag{5}$$
$$N_{t-1}^+ = 1 - N_{t-1}^-$$

where

N_{t-1}^- is the sign bias variable

$N_{t-1}^- \hat{\varepsilon}_{t-1}$ is the negative size bias variable

$N_{t-1}^+ \hat{\varepsilon}_{t-1}$ is the positive size bias variable

The null hypothesis of no sign and size bias corresponds to: $H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$. This can be tested with a Lagrange Multiplier test(see also, David, Dikko and Gulumbe, 2016;Lundbergh and Terasvirta, 2002; Hagerud, 1997).

2.6 Heteroscedastic Model

The statistical methods for modeling the volatility of a return are referred to as heteroscedastic models. Let R_t be the return of a share price at time index t. The basic idea behind volatility study is that the series $\{R_t\}$ is either serially uncorrelated or with minor lower-order serial correlations, but it is a dependent series. For the purpose of this study, we consider the GARCH model to account for the ARCH effect (volatility clustering) and the EGARCH and TGARCH models to account for the asymmetric (leverage) effect. To successfully, fit the heteroscedastic models, one starts with modeling ARIMA process to remove the linear dependence in the data (see Box, Jenkins and Reinsel, 2008; Cryer and Chen, 2008: Wei, 2006; Brockwell and Davis, 2002 ; Fuller, 1996, for more details on ARIMA modeling). The residual series of the fitted ARIMA model is used to model the GARCH process and if the asymmetric effect is detected in the residuals series of the fitted GARCH model, then the EGARCH and TGARCH models are entertained.

2.7 Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

Although the ARCH model is simple it often requires many parameters to adequately describe the volatility process of a share price return. Some alternative models must be sought. Bollerslev (1986) proposed a useful extension known as the generalized ARCH (GARCH) model. For a return series, R_t, let $a_t = R_t - \mu_t$ be the innovation at time t. Then, a_t follows a GARCH (p, q) model if

$$a_t = \sigma_t e_t,$$

$$\sigma_t^2 = \alpha_o + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{6}$$

where again e_t is a sequence of i.i.d. random variance with mean, 0, and variance, 1, $\alpha_o > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$,

$and \sum_{i=1}^{max(p,q)} (\alpha_i + \beta_i) < 1$(Tsay, 2010).

Here, it is understood that $\alpha_i = 0$, for $i > p$, and $\beta_i = 0$, for $i > q$. The latter constraint on $\alpha_i + \beta_i$ implies that the unconditional variance of a_t is finite, whereas its conditional variance σ_t^2, evolves over time.

2.8 EGARCH Model

The Exponential GARCH (EGARCH) model represents a major shift from ARCH and GARCH models (Nelson, 1991). Rather than modeling the variance directly, EGARCH models the natural logarithm of the variance, and as such, no parameter restrictions are required to ensure that the conditional variance is positive. The EGARCH (p, q) is defined as,

$$R_t = \mu_t + a_t, \quad a_t = \sigma_t e_t,$$

$$ln\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \frac{|a_{t-i}| + \gamma_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \, ln\sigma_{t-j}^2 \tag{7}$$

where again, e_t is a sequence of i.i.d. random variance with mean, 0, and variance, 1, and γ_k is the asymmetric coefficient. The process is covariance stationary if and only if$\sum_{j=1}^{q} \beta_j < 1$.

2.9 TGARCH Model

According to Francq and Zakoian (2010), a natural way to introduce asymmetry is to specify the conditional variance as a function of the positive and negative parts of the past innovations. The Threshold GARCH (TGARCH) class of models introduces a threshold effect into the volatility. Moreover, this model takes the asymmetry into account while keeping the linear function form of conditional variance.

By definition, (a_t) is called a TGARCH (p,q) process if it satisfies an equation of the form

$$a_t = \sigma_t e_t$$

$$\sigma_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(|a_{t-i}| - \gamma_i a_{t-i}\right) + \sum_{j=1}^{q} \beta_j \sigma_{t-j} \quad (8)$$

where α_0, α_i and β_j are real numbers. Under the constraints $\alpha_0 > 0, \alpha_{i,+} \geq 0, \alpha_{i,-} \geq 0, \beta_j \geq 0$ the variable σ_t is always strictly positive and represents the conditional standard deviation of a_t. In general, the conditional standard deviation of a_t is $|\sigma_t|$, therefore, imposing the positivity of σ_t is not required (contrary to the classical GARCH models based on the specification of σ_t^2) (Francq and Zakoian, 2010).

3. Result and Discussion

Considering the daily closing share prices of Zenith Bank plc from 04/01/2006 to 26/05/2015 obtained from the Nigerian Stock Exchange with the series made up of 2451 observations. The motivation for the choice of share price series of Zenith Bank Plc is that Zenith Bank Plc has emerged the most active and capitalized stock of the Nigerian Stock Exchange. From the plot assessment, the series presented in figure 1 indicates that the series is not stationary. Also, the fact that ACF of the share price series persists and decays slowly is an indication that the series is not stationary (see Figure 2).

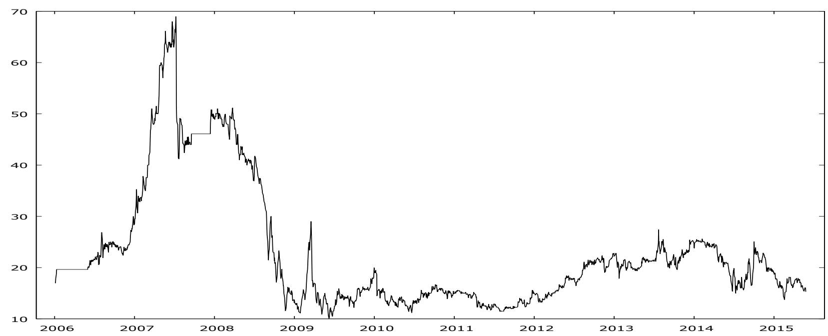


Figure 1. Plot of Share price series of Zenith Bank

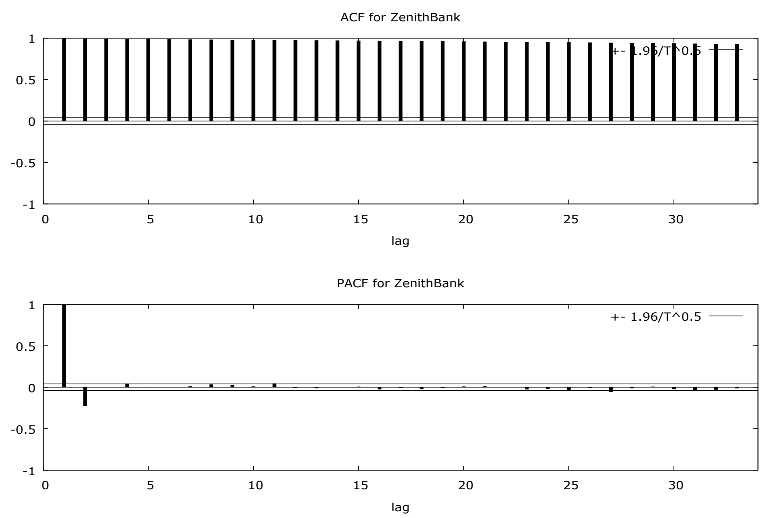


Figure 2. ACF and PACF of the Share Price Series

However, we take the log difference of the share price series (returns) to ensure stationarity. In addition, volatility clustering (heteroscedasticity) is evident in the returns and the fact that return series clusters around zero mean entails stationarity (see Figure 3).

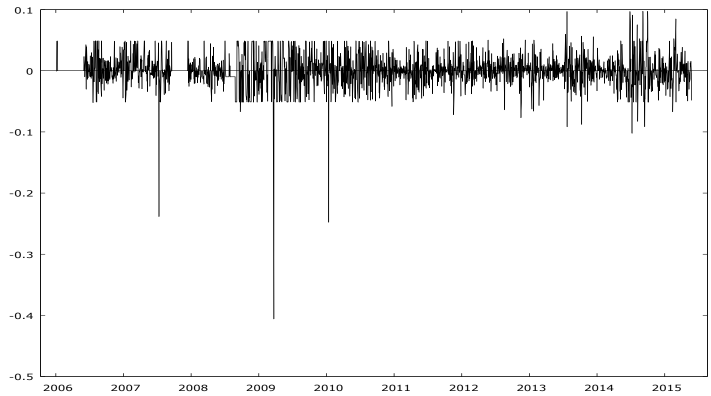


Figure 3. Plot of the Return Series of Zenith Bank

3.1 ARIMA Modeling

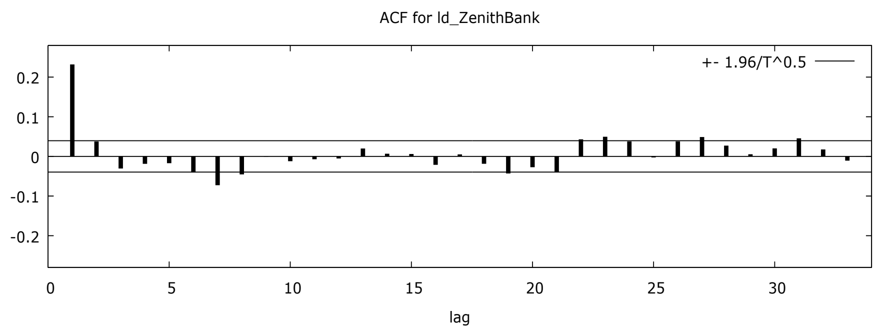
In order to model the linear dependence in the return series, ARIMA (1, 1, 3) model was identified since the ACF of the share price return series cuts off after lag 3 and the PACF cuts off after lag 1 (see Figure 4)

The estimated ARIMA(1,1,3) model is presented below

$$(1-0.8289B)\nabla logP_t \; = \; (1-0.5885B \; -0.1734B^2 \; -0.0633B^3)\varepsilon_t \tag{9}$$

s.e:	(0.065054)	(0.067549)	(0.025665)	(0.020043)
z-value:	(12.7420)	(-8.7127)	(-6.7569)	(-3.1584)
p-value:	$(< 2.2e^{-16})$	$(< 2.2e^{-16})$	$(1.41e^{-11})$	(0.001586)

The diagnostic checking on the ARIMA(1,1,3) model using Ljung – Box test indicates that the model is a good fit since the null hypothesis that the first eight (8) lags of the residuals of the model are not autocorrelated is not rejected given that the p-value of 0.3062 associated with Chi square of 9.4444 with degree of freedom, 8 is less than 0.05 significance level.



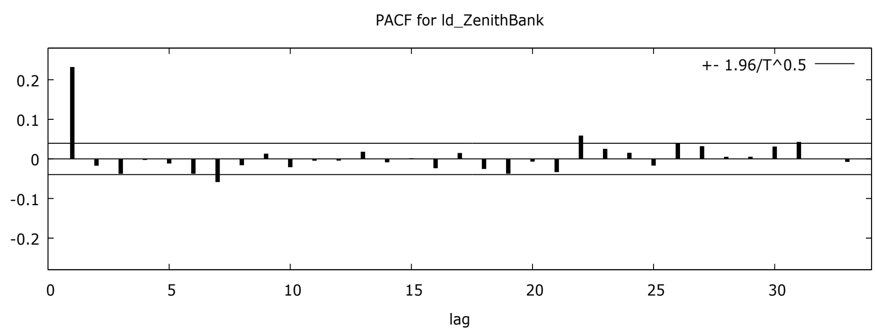


Figure 4. ACF and PACF of the Returns of Zenith Bank

3.2 Symmetric GARCH Modeling

3.2.1 Detection of Heteroscedasticity

To detect the presence of heteroscedasticity (ARCH effect), we assess the ACF of ε_t^2 of ARIMA(1,1,3) model. If at least one lag term in the squares of residual series (ε_t^2) is found to be statistically significant, then heteroscedasticity is said to exist. From figure 5, the first lag term of ACF of the squares of the residual series is significant and heteroscedasticity is said to exist.

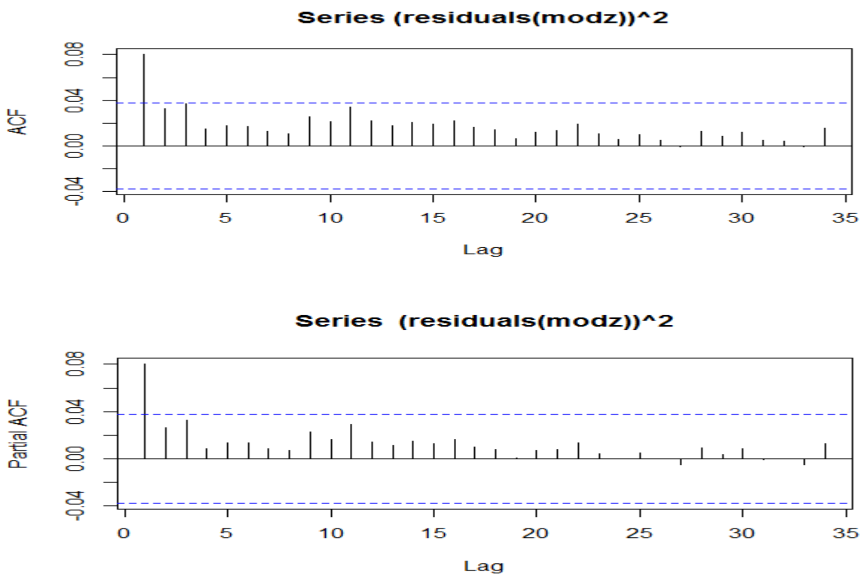


Figure 5. ACF and PACF of Squares of Residual Series

Also, according to the Lagrange Multiplier test, the hypothesis of no ARCH effects up to lag 24 is rejected since the Lagrange Multiplier test value of 2095 with corresponding probability (of Chi Square with 24 degree of freedom), $0.000 < 0.05$ further confirms the presence of heteroscedasticity.

3.2.2 Estimation of GARCH Model

Having detected the presence of heteroscedasticity, we move on to estimate the GARCH model. We fit tentatively, GARCH(0,1), GARCH(0,2) and GARCH(0,3) models to the residual series of ARIMA(1,1,3) model. Their information criteria are almost the same (see Table 4) but based on the principle of parsimony and the fact that present conditional variance depends only on the immediate past conditional variance, we select GARCH(0,1) model.

Table 4: Information Criteria for GARCH Model

	GARCH(0,1)	GARCH(0,2)	GARCH(0,3)
AIC	−11417.96	-11525.57	−11625.91

The estimated GARCH(0,1) model is presented in equation (10) below

$$R_t = 8.91031 \times 10^{-4} + \varepsilon_t$$

s.e: (0.00009)

z-value: (2.2344)

p-value: (0.0255)

$\varepsilon_t = \sigma_t e_t, e_t \sim N(0,1)$

$$\sigma_t^2 = 3.38191e^{-4} + 0.631176\varepsilon_{t-1}^2 \quad (10)$$

s.e: $(1.4758e^{-05})$ (0.05227)

z-value: (22.9165) (12.0745)

p-value: (<0.0001) (<0.0001)

The diagnostic checking on the GARCH(0,1) model indicates that the model is adequate since the Lagrange Multiplier test statistics of 4.372158 with corresponding probability of Chi- Square with lag 8, 0.8221 > 0.05 level of significance.

3.3 Asymmetric GARCH Modeling

3.3.1 Test for Asymmetries in Heteroscedasticity

With the symmetric GARCH being successfully represented by GARCH(0,1) model, then we move to test for the asymmetric effect in the residuals of the fitted GARCH(0,1) model based on sign and size bias test. The estimated test is presented is presented below

$$\hat{\varepsilon}_t^2 = 3.19\text{e}^{-05} + 4.04\text{e}^{-05} N_{t-1}^- - 0.009639 N_{t-1}^- \hat{\varepsilon}_{t-1} + 0.010276 N_{t-1}^+ \hat{\varepsilon}_{t-1} \quad (11)$$

s.e:	(2.89e^{-05})	(3.84e^{-05})	(0.002139)	(0.002753)
t-value:	(1.106081)	(1.051076)	(-4.505697)	(3.733321)
p-value:	(0.2688)	(0.2933)	(0.0000)	(0.0002)

From equation (11), the coefficient of N_{t-1}^- is not significant at 0.05 level of significance. The implication is that the effect of the sign of negative and positive shocks on heteroscedasticity is not different from the one predicted by GARCH(0,1) model. Also, the negative and positive size bias test statistics are highly significant at 0.05 level of significance. The implication is that the impact of large/small (negative/positive) returns on heteroscedasticity were not predicted by GARCH(0,1) model. Moreover, the joint significance of the sign and size bias test for asymmetric GARCH effect indicates the presence of leverage effect since the Lagrange Multiplier test statistic of 35.19073 with the corresponding probability value, 1.11e^{-07}< 0.05 level of significance, rejecting the null hypothesis of no sign and size bias correspond to: $H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$.

3.3.2 Estimation of Asymmetric GARCH

With the asymmetric GARCH effect detected, we extend symmetric GARCH(0,1) model to the asymmetric type; EGARCH(0,1) model and TGARCH(0,1) model.

The estimated EGARCH(0,1) model is presented in equation (12)

$$R_t = 6.83823 \times 10^{-5} + \varepsilon_t$$

s.e:	(0.000329079)
z-value:	(0.2078)
p-value:	(0.8354)

$\varepsilon_t = \sigma_t e_t, e_t \sim N(0,1)$

$$ln\sigma_t^2 = -7.84212 + 0.570692 \left(|\varepsilon_{t-1}| - 0.0808165\varepsilon_{t-1}\right)(\sigma_{t-1})^{-1} \quad (12)$$

s.e:	(0.0389357)	(0.0403487)	(0.0280480)
z-value:	(-201.4)	(14.14)	(-2.881)
p-value:	(0.0000)	(2.03e-045)	(0.0040)

The statistical significance of the leverage effect parameter of the fitted EGARCH(0,1) model indicates that the leverage effect would impact negatively on the conditional variance and that the model is able to predict both large and small negative impacts on the returns. The diagnostic checking on the EGARCH(0,1) model shows that the model is adequate, since the Lagrange Multiplier test statistic of 5.927093 with the p-value of Chi-Square of first 8 lags, 0.6554 > 0.05 level of significance.

The estimated TGARCH(0,1) model is presented in equation (13)

$$R_t = 4.65978 \times 10^{-6} + \varepsilon_t$$

s.e:	(4.97470e-05)
z-value:	(0.09367)
p-value:	(0.9254)

$\varepsilon_t = \sigma_t e_t, e_t \sim N(0,1)$

$$\sigma_t = 4.15618e^{-4} + 0.485568\left(|\varepsilon_{t-1}| - 0.231281\varepsilon_{t-1}\right) \quad (13)$$

s.e:	(1.04875e-05)	(0.0281060)	(0.0426081)
z-value:	(39.63)	(17.28)	(5.428)
p-value:	(0.0000)	(7.09e-067)	(5.70e-08)

The statistical significance of the leverage effect parameter of the fitted TGARCH(0,1) model indicates that the leverage effect would impact negatively on the conditional variance and that the model is able to predict both large and small negative impacts on the returns. The diagnostic checking on the TGARCH(0,1) model shows that the model is adequate, since the Lagrange Multiplier test statistic of 1.227263 with the p-value of Chi-Square of first 8 lags,0.9964> 0.05 level of significance does not reject the null hypothesis of no heteroscedasticity.

Furthermore, to choose between EGARCH(0,1) and TGARCH(0,1) models, we consider their information criteria (AIC), -11304.56822 and -11474.16299, respectively. Since their information criteria are almost the same, it follows that both models can be entertained in modeling the asymmetric effect of the return series.

The findings of this paper confirmed the presence of asymmetric (leverage) effect in the return series of Zenith Bank Plc which is in tandem with the findings of Onwukwe, Samson and Lipcsey (2014) whose study fitted ARCH(1), ARCH(2), GARCH(1,1), EGARCH(1,1) and TGARCH(1,1) models to the return series of Zenith Bank Plc from 4th January 2004 to 31st August 2012; EGARCH(1,1) model was selected as the best model and concluded that asymmetric conditional heteroscedastic model is more suitable than the symmetric conditional heteroscedastic model. Though the order of model specification are slightly different which could be as a result of the differences in time span considered, this particular work differs from the work of Onwukwe, Samson and Lipcsey (2014) in that it combines GARCH(0,1), EGARCH(0,1) and TGARCH(0,1) models to express the full characterizations in the series. The evidence of volatility clustering and leverage effect provided so far is in tune with both Nigerian and International evidence of financial data exhibiting the phenomenon of volatility clustering and leverage effect.

4. Conclusion

Following the inability of the symmetric GARCH in capturing and modeling the leverage effect of a discrete-time series, this study has been able to take into consideration asymmetric effect in modeling such a series. The sign and size bias tests were explored to detect the asymmetric effect the series. The findings revealed that the asymmetric effect is present in the share price returns series since the joint negative and positive size bias tests were significant. However, this asymmetric salient feature in the series could not be predicted by GARCH (0,1) model, but was adequately captured by both EGARCH(0,1) and TGARCH(0,1) models. The implication of the study is that, in contrast to GARCH, the EGARCH provided a natural way of avoiding the positivity constraint on the coefficients since the logarithm can be of any sign. Also, TGARCH avoided the imposition of conditional standard deviation, σ_t by modeling the modulus form of the conditional standard deviation, $|\sigma_t|$. Hence, it can be concluded that the current conditional variance depends on the modulus and sign of the past return series. Furthermore, subsequent studies will consider other forms of asymmetric GARCH models not captured in this study.

References

Akaike, H. (1973). A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control, 6*(19), 716 – 723.

Akpan, E. A., & Moffat, I. U. (2015). ARCH Modeling of the Returns of First Bank of Nigeria. *American Journal of Science and Industrial Research, 6*(6), 131-140.doi:10.5251/ajsir.2015.6.6131.140

Akpan, E. A., Moffat, I. U., & Ekpo, N. B. (2016). ARMA – ARCH Modeling of the Returns of First Bank of Nigeria. *EuropeanScientific Journal. 12*(8), 257 – 266. https://doi.org/10.19044/esj.2016.v12n18p257

Asteriou, D., & Hall, S. G. (2007). Applied Econometrics. A Modern Approach. Publisher: Palgrave Macmillam.

Atoi, N. V. (2014). Testing Volatility in Nigerian Stock Market Using GARCH Models. *CBN Journal of Applied Statistics, 5* (2).

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics, 31*(3), 307-327. https://doi.org/10.1016/0304-4076(86)90063-1

Box, G. E. P., Jenkins, G. M., & Reinsel, G. C. (2008). Time Series Analysis: Forecasting and Control. (3rd ed.). New Jersey: Wiley and Sons, 47- 89. https://doi.org/10.1002/9781118619193

Brockwell, P. J., & Davis, R. A. (2002). Introduction to Time Series and Forecasting. (2nd ed.).Springer, 83 – 219. https://doi.org/10.1007/0-387-21657-X_3

Cryer, J. D., & Chan, K. (2008). Time Series Analysis with Application in R. (2nd ed.). New York: Springer, 55 – 190.

David, R. O., Dikko, H. G., & Gulumbe, S. U. (2016). Modeling Volatility of the Exchange Rate of the Naira to major Currencies. *Central Bank of Nigeria Journal of Applied Statistics*, *7*(2), 159-187.

Deshon, R. P., & Alexander, R. A. (1996). Alternative Procedures for Testing Regression Slope Homogeneity when Group Error Variances are unequal. *Psychological Methods, 1*, 261-277. https://doi.org/10.1037/1082-989X.1.3.261

Emenike, K. O. (2010). Modeling Stock Returns Volatility in Nigeria using GARCH Models. Available at mpra.ub.uni-muenchen.de/23432/. Extracted 20 May, 2017.

Engle, R. F., & Ng, V. K. (1993). Measuring and Testing the Impact of News on Volatility. *Journal of Finance, 48*(5), 1749 - 1778. https://doi.org/10.1111/j.1540-6261.1993.tb05127.x

Fan, J., & Yao, Q. (2003).Nonlinear time series: Nonparametric and Parametric methods. (2nd ed.). New York: Springer(pp 143 -171). https://doi.org/10.1007/b97702

Francq, C., & Zakoian, J. (2010). GARCH Models: Structure, Statistical Inference and Financial Applications.(1st ed.). Chichester, John Wiley & Sons Ltd.(pp 19 – 220). https://doi.org/10.1002/9780470670057

Franses, P. H., & Van Dijk, D. (1999). Outlier Detection in the GARCH(1,1) Model. Available at feweco1999070809440.pdf. Extracted on 6 July, 2017.

Fuller, W. A. (1996). Introduction to Statistical Time Series.(2nd ed.). New York, John Wiley and Sons, 21 – 94.

Greene, W. H. (2002). Econometric Analysis. (5th ed.). New York, Prentice Hall, 269.

Hagerud, G. E. (1997). Specification Test for Asymmetric GARCH. Available at Swopec. hhs.se/hastef/papers/hastef0163.pdf. Extracted on 4 July, 2017.

Ljung, G., & Box, G. C. (1978). On a Measure of Lack of Fit in Time series models. *Biometrica, 2*(66), 265-270. https://doi.org/10.1093/biomet/65.2.297

Lundbergh, S., & Terasvirta, T. (2002). Evaluating GARCH Models. *Journal of Econometrics, 110*,417 – 435. https://doi.org/10.1016/S0304-4076(02)00096-9

Mgbame, C. O., & Ikhatua, O. J. (2014). Accounting Information and Stock Volatility in the Nigerian Capital Market: A GARCH Analysis Approach.*International Review of Management and BusinessResearch, 2* (1).

Nelson, D. B. (1991). Conditional Heteroscedasticity of Asset Returns. A new Approach. *Econometrica, 59*, 347 - 370. https://doi.org/10.2307/2938260

Ogum, G., Bee, R. F., & Nouyrigat, G. (2005). Emerging Equity Market Volatility: An Empirical Investigation of Markets in Kenya and Nigeria, *Journal of Africa Business, 6*, 139-154. https://doi.org/10.1300/J156v06n01_08

Onwukwe, C. E., Samson, T. K., & Lipcsey Z. (2014). Modeling and Forecasting daily Returns Volatility of Nigerian Banks Stocks. Available at eujournal.org/index.php/esj/article/347. Extracted 9 May, 2017.

Tsay, R. S. (2010). *Analysis of Financial Time Series.* (3rd ed.). New York: John Wiley & Sons Inc. 97 – 140. https://doi.org/10.1002/9780470644560

Wei, W. W. S. (2006). Time Series Analysis Univariate and Multivariate Methods. (2nd ed.). New York:Adison Westley, 33 – 66.

Yaya, O. S. (2013). Nigeria Stock Exchange: A Search for Optimal GARCH Modeling using High Frequency Data. CBN *Journal of Applied Statistics, 4*(2).