International Journal of Statistics and Probability; Vol. 6, No. 1; January 2017
ISSN 1927-7032 E-ISSN 1927-7040
Published by Canadian Center of Science and Education

Installation and Dispatch of the Traffic Patrol Service Platform

Tang Jiahui[1], Zhang Yuanbiao[2], Peng Churu[3], & Huang Xinxin[3]

[1] Mathematical Modeling Innovative Practice Base, JiNan University, ZhuHai GuangDong, China

Correspondence: Mathematical Modeling Innovative Practice Base, JiNan University, ZhuHai GuangDong,China

Received: August 11, 2016 Accepted: December 5, 2016 Online Published: December 21, 2016

doi:10.5539/ijsp.v6n1p95 URL: https://doi.org/10.5539/ijsp.v6n1p95

Abstract

In this paper, we construct three mathematical models to install and dispatch the traffic patrol service platform properly based on the real data of a certain city. Firstly, we build the shortest path model based on the Floyd algorithm to determine the jurisdictional of each platform. Then, we designed the dispatch model combined with 0-1 integer programming and the Hungarian algorithm to find the dispatching schemes when coming across large-scale emergencies. Lastly, we build the multiple-objective location model to optimize the present distribution situation of the traffic patrol platforms considering the workload differences among these existing platforms and overlong response time in some places.

Keywords: Floyd algorithm, shortest path model, dispatch model, multiple-objective location model

1. Introduction

The police undertake four major functions, including the criminal enforcement, public security management, traffic management and servicing the masses. In order to implement these functions effectively, we need to set up traffic patrol service platforms in some city traffic arteries and important positions. The function and police resources of each traffic patrol service platform are basically the same. However, the police resources are not infinite. As a result, how to install and dispatch the traffic patrol service platform reasonably has been a practical task faced by the police department according to the actual situation such as the distribution intersection nodes and crime probability difference of an area (Yu, Song, Zhao & Li, 2012). The shortest path model based on the Floyd algorithm and dispatch model can solve such kinds of problems effectively and properly.

The shortest path problem is a basic but very important problem in network optimization, and the Floyd algorithm is a common method to solve the shortest path problem. The Floyd algorithm begins with the weighted adjacency matrix, which represents the distance between any two intersection nodes v_i and v_j. Then it inserts an extra vertex to be the transfer station, and compares the shortest path's distance before and after inserting the vertex v_k, choosing the minimum to get a new distance matrix. Keeping loop iteration till all the vertices are as transfer stations between $v_i$and v_j. Next, the final weighted adjacency matrix will be worked out, which reflects the shortest distance information between any two vertices, becoming a distance matrix of the original image. Lastly, we can get the shortest path between any two points and the shortest path distance from the final distance matrix (Zhu & Zhang, 2012)

The Vehicle Scheduling Problem (VSP) has been a hot topic since it was put forward by Dantzing and Ramser(1959) for the first time (Dantzing & Ramser, 1959), which attracted many experts to research, including experts in operational research, combinatorial mathematics, graph theory and network analysis, and so on. Savelsbergh(1985) proved VSP is a NP-Complete problem(Savelsbergh, 1985), so how to get the answer is the key point in VSP when in large scale. Laporte and Mercure(1992) came up with a kind of vehicle routing problem with stochastic travel time. They also developed the chance constrained model and solved the problem successfully by the branch cut algorithm(Laporte & Mercure, 1992). Recently, some intelligent algorithms are widely applied into VSP solving, such as genetic algorithm， ant colony algorithm, particle swarm optimization and so on, which achieved much progress in VSP(Baker 2003; Berger,2003; Blanton,1993; Bullnheimer,1999; Salman,2002; Ellabib, 2002). In China, there are also some scholars studied the vehicle scheduling problem. Huo and Wang(2005) solved the vehicle scheduling problem based on constraint programming(Huo & Wang,2005). Xie, Li and Guo (2000) has carried on the thorough research concerning the logistics vehicle scheduling problem, and put forward a variety of algorithm (Xie, Li & Guo,2000).

In this paper, we develop three mathematical models to install and dispatch the traffic patrol service platform properly based on the real date of a certain city. Firstly, we distribute the jurisdictional of each platform according to the shortest path model. Then, we designed the dispatch model to find the reasonable dispatching schemes when come across large-scale emergencies. We lastly proposed the multiple-objective location model to optimize present distribution situation of the traffic patrol service platforms considering the workload differences and the overlong response time in some places.

2. Problem-Descriptions

This paper develops several mathematical models to install and dispatch the traffic patrol service platform reasonably, based on the data and assumption of Problem B in China Undergraduate Mathematical Contest in Modeling (China Undergraduate Mathematical Contest in Modeling[CUMCM],2011). The specific backgrounds of this problem are as following:

(1) The traffic network and the distribution of the existing 20 traffic patrol service platforms in Area A are shown in Fig1. We are required to determine the jurisdictional scope of each platform, making sure that the police can arrive at the scene within 3 minutes when emergency happens in its jurisdictional scope. And the speed of police car is 60Km/h.

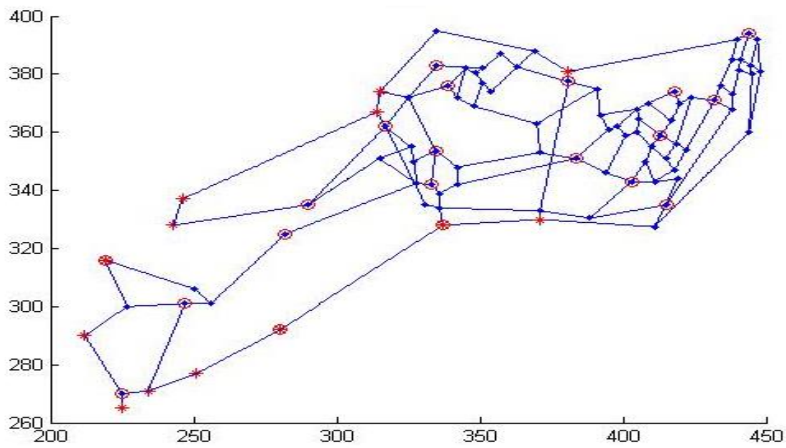

Figure 1. The traffic network and platforms location of city A

(2) When coming across large-scale emergencies, it is required to blockade all the 13 roads in city A as quickly as possible by dispatching the 20 traffic patrol service platforms. Actually, the police resource of each platform can only blockade one intersection node. What we are supposed to do is figuring out the proper methods to dispatch the traffic patrol service platforms.

(3) Considering the workload differences among these existing platforms and overlong response time in some places, the government is going to add 2 to 5 police platforms to improve the present situation. We are required to determine the specific quantity and position of these traffic patrol service platforms that are going to be set.

3. Development of Models

3.1 Model for Calculating the Shortest Path

In order to make sure that police can arrive at the scene as quickly as possible when there are emergencies in its jurisdictional scope, we find out the nearest traffic patrol service platform of each intersection and nominate the nearest one to be the responsible one for this intersection node. In other words, we determine the jurisdiction of each platform by finding the traffic patrol service platform that is responsible for each intersection node. When all the intersection nodes are allocated to the nearest platform, the jurisdiction is determined. Floyd algorithm is an ideal method to solve such a problem.

3.1.1 The Principle of Floyd Algorithm

Floyd algorithm was put up in 1962, which can be used to solve any kind of network shortest path problem when arc power is real, as well as figure out the shortest path distance between any two points. Floyd algorithm can be understood as a successive approximation approach, whose basic methods are as follows(Mao Yuan-jie,1942):

The Floyd algorithm begins with the weighted adjacency matrixD^0 and then generates a sequent of recursive matrixes $D^1,\cdots,D^k,\cdots,D^n$ through iteration. The formula of calculating $D^0$and D^k are as follows:

$$D^0 = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{nn} \end{bmatrix} \tag{1}$$

$$D^k = (d_{ij}^k)_{n\times n} \tag{2}$$

While d_{ij} is the distance between any two intersection nodes; k is the number of iterations; d_{ij}^k is the shortest path's distance between the two intersection nodes v_i and v_j when the number of intermediate nodes isn't greater than k. We can calculate d_{ij}^k by the following iterative formula:

$$d_{ij}^k = min\{d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1}\} \tag{3}$$

When $k=n$, the value of d_{ij}^n is the shortest distance between v_i and v_j, which is $d(v_i,v_j)$.

3.1.2 Result— the Jurisdictional Scope

In order to confirm the jurisdictional scope of each traffic patrol service platform, we firstly figure out the length of each path, which is the distance between every two intersection nodes. Secondly, we arrange these numbers into a weighted adjacency matrix D_0 of 20×72 dimensions. Then, based on the Floyd algorithm and MATLAB, we find out the nearest traffic patrol service platform of each intersection and nominate the nearest one to be the responsible one for the corresponding intersection. That is to say, when all the intersection nodes are assigned, the jurisdiction is determined. The specific results are shown in Table 1.

Table 1. jurisdictional scope of the traffic patrol service platform in city A

Platforms	Jurisdictional scope	Platforms	Jurisdictional scope
1	1,67,68,69,71,73,74,75,76,78	11	11, 26, 27
2	2, 39, 40, 43, 44, 70, 72	12	12, 25
3	3, 54, 55, 65, 66	13	13, 21, 22, 23, 24
4	4, 57, 60, 62, 63, 64	14	14
5	5, 49, 50, 51, 52, 53, 56, 58, 59	15	15, 28, 29
6	6	16	16, 36, 37, 38
7	7, 30, 32, 47, 48, 61	17	17, 41, 42
8	8, 33, 46	18	18, 80, 81, 82, 83
9	9, 31, 34, 35, 45	19	19, 77, 79
10	10	20	20, 84, 85, 86, 87, 88, 89, 90, 91, 92

We can see from table 1.that the workload differences among traffic patrol service platforms in Area A is obvious. For example, the 6th、the 10th and the 14th platforms, they only need to be responsible for the intersection node where they are, because they are so far away from other road crosses that they can't rush to the scene in 3 minutes according to the shortest distance calculated by the Floyd algorithm. Oppositely, the 1th、the 5th、and the 20th platforms, their jurisdictional scope are wide and the workload are especially more. Considering the existing problems, the government is going to add 2 to 5 police platforms, and we will discuss the specific number and location of them later.

3.2 Model for Dispatching Traffic Patrol Service Platforms

We know that there are total 20 traffic patrol service platforms in Area A, and if it comes across large-scale emergencies, we need blockade all the 13 roads in Area A as quickly as possible by dispatching the 20 traffic patrol service platforms. Now, we are going to build a dispatch model based on 0-1 integer programming and the Hungarian algorithm to solve such a dispatching problem reasonably. Firstly, we introduce the 0-1 variables x_{ij}（$i=1,2,...,13; j=1,2,...,20$）:

When the j^{th} traffic patrol service platform was assigned to blockade the i^{th} traffic artery, $x_{ij}=1$. Otherwise, $x_{ij}=0$.

3.2.1 The Objective Functions

We know that there are total 20 traffic patrol service platforms in Area A, and when large-scale emergencies happens, we need dispatch all the 20 traffic patrol service platforms reasonably to all the 13 roads. Reasonably dispatching means that blockade all the 13 roads base on the principal of time priority. Considering the speeds of police car are same in all conditions, we transfer the time priority to distance priority, which is:

$$\min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} l_{ij} \tag{4}$$

Where l_{ij} is the shortest path's length between the j^{th} traffic patrol service platform and the i^{th} traffic artery; x_{ij} represents whether the j^{th} traffic patrol service platform was assigned to blockade the i^{th} traffic artery, if yes, then $x_{ij}=1$, or $x_{ij}=0$.

3.2.2 The Constraints

(1) In order to guarantee that the whole Area A is blockaded, every traffic artery requires at least one traffic patrol service platform, which means:

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1,2,\cdots,n; n = 20 \tag{5}$$

(2) Each traffic patrol service platform can only be assigned to manage a traffic artery, which means:

$$\sum_{j=1}^{m} x_{ij} = 1, \qquad i = 1,2,\cdots,m; m = 13 \tag{6}$$

3.2.3 The Methods

Firstly, we figure out the shortest path from the j^{th} traffic patrol service platform to the i^{th} traffic artery and its length l_{ij}. And we arrange the date of distance into the coefficient matrix of x_{ij} ,matrix X_0 :

$$X_0=\begin{bmatrix} l_{11} & l_{12} & \cdots & l_{1n} \\ l_{21} & l_{22} & \cdots & l_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ l_{m1} & l_{m2} & \cdots & l_{mn} \end{bmatrix} \tag{7}$$

Then, in order to meet the requirements of the Hungarian algorithm to matrix, we add *n*-*m* imaginary missions to convert the original matrix X_0 of $m \times n$ dimensions into *n* phalanxX_1:

$$X_1=\begin{bmatrix} l_{11} & l_{12} & \cdots & l_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ l_{m1} & l_{m2} & \cdots & l_{mn} \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{8}$$

Next, we use the Hungarian algorithm to transfer phalanxX_1 into the identity matrixX_2:

$$X_2=\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & \cdots & 0 \end{bmatrix} \tag{9}$$

Finally, we can figure out the proper Scheduling Scheme of the traffic patrol service platforms when large-scale emergencies happen according to the matrixX_2.

3.2.4 Result— Scheduling Scheme to Blockade Area A

The specific results are shown in Table 2.

Table 2. Scheduling Scheme of the traffic patrol service platform

Platforms	A12	A14	A16	A9	A10	A13	A11	A15	A8	A7	A2	A5	A4
Traffic arteries	A12	A14	A16	A21	A22	A23	A24	A28	A29	A30	A38	A48	A62

We can see from the table 2 that the specific Scheduling Scheme is: the 12th, 14th, and 16th platforms stay at their original positions to stand by; the 9th platform is appointed to block the 21th traffic artery; the 10th platform is appointed to block the 22th traffic artery; the 13th platform is appointed to block the 23th traffic artery; the 11th platform is appointed to block the 24th traffic artery; the 15th platform is appointed to blockade the 28th traffic artery; the 8th platform is appointed to block the 29th traffic artery; the 17th platform is appointed to block the 30th traffic artery; the 2th platform is appointed to block the 38th traffic artery; the 5th platform is appointed to block the 48th traffic artery; the 4th platform is appointed to block the 62th traffic artery.

3.3 Model of Multiple-Objective Location

3.3.1 The Preliminary Location of New Traffic Patrol Service Platforms

Concerning that the added traffic patrol service platforms are mainly used to optimize the imbalanced workload situation and decrease the response time, and the existing 20 traffic patrol service platforms are not to make any changes, we consider decreasing the response time firstly. The specific methods are as follows:

We firstly find out the intersection nodes, whose guardian platforms can’t arrive in 3 minutes when accident happens. Then we find out the traffic patrol service platforms that can arrive in these non-effective guarded platforms within 3 minutes, making them the alternative new platforms’ location. We can see the preliminary location of these new platforms in table 3:

Table 3. The preliminary location of these new platforms in table 3

Non-effective guarded platforms	**Preliminary location of new platforms**
A28	A26，A27，A28，A29
A29	A28，A29
A38	A38，A39，A40
A39	A38，A39；A40
A61	A48；A61
A92	A87；A88；A89；A90；A91；A92

3.3.2 Location Decision of New Platforms

Objective1: Minimize the total response time, which means decreasing the response time of platforms as much as possible. Thus we obtain the first objective function V_1:

$$V_1 = \min \sum_{i=1}^{n} t_i \qquad (i = 1, 2, \cdots, 92) \tag{10}$$

With V_1 representing the cumulative time spent by each traffic patrol service platform to arrive at the designated area; t_i represents response time, which is the amount of time spent by the j^{th} traffic patrol service platform to the i^{th} intersection node.

According to the equation ‘time = distance/ speed’, we can generate the value of t_i

$$t_i = s_i / v \qquad (i = 1, 2, \cdots, 92) \tag{11}$$

With s_i representing the distance between the j^{th} traffic patrol service platform and the i^{th} intersection node; v represents the value of police car speed, and we assume v is 60Km/h in this paper.

At the same time, in order to make sure that the police can timely tackle the accidents in its jurisdictional area, the response time is supposed to less than 3minutes:

$$t_i < 3\min = 0.05\text{h} \quad (i = 1, 2, \cdots, 92) \tag{12}$$

Objective2: Maximize the coverage of crime probability, which can increase the efficiency of police resources usage. Thus we obtain the second objective function V_2:

$$V_2 = \max \sum_{i=1}^{n} a_i A_i \quad (i = 1, 2, \cdots, 92) \tag{13}$$

With V_2 representing the crime probability in areas managed by all the traffic patrol service platforms; a_i representing the crime probability in the i^{th} intersection node; A_i representing whether the i^{th} intersection node is properly guarded by a certain traffic patrol service platform, if yes, then A_i =1, or A_i =0.

Objective3: Minimize the number of total traffic patrol service platforms, which can decrease the cost of setting platforms. Thus we obtain the third objective function V_3:

$$V_3 = \min Z = \min \sum_{j=1}^{n} X_j \tag{14}$$

With Z representing the number of total traffic patrol service platforms in Area A; X_j representing whether the j^{th} traffic artery is equipped with traffic patrol service platform, if yes, then X_j =1, or X_j =0.

Considering that changing the location of these existing 20 traffic patrol service platforms can cost a lot, which goes against the cost-benefit principle, so the government will only set new traffic patrol service platforms in the remaining intersection nodes and the existing 20 platforms remain their present positions. And these can be concluded through the following two equations:

$$\text{When } j = 1, 2, \cdots, 20, \quad X_j = 1; \tag{15}$$

$$\text{When } j = 21, 22, \cdots, 92, \quad X_j \in \{0, 1\}; \tag{16}$$

After adding 2 to 5 new platforms, the number of total traffic patrol service platforms in Area A should be from 22 to 25:

$$22 \le Z \le 25 \tag{17}$$

Objective4: Minimize the workload difference among traffic patrol service platforms. In this paper, we use the mean difference of workload to measure the degree of workload imbalance, and the smaller of the mean difference, the more balanced of the workload distribution. Thus we obtain the forth objective function V_4:

$$V_4 = \min \left(\frac{\sum_{j=1}^{Z} \left| \sum_{i=1}^{n} a_i A_{i,j} - \bar{X} \right|}{Z} \right) \tag{18}$$

With V_4 representing the mean difference of workload; a_i representing the case happening probability in the i^{th} intersection node; A_{ij} representing whether the j^{th} traffic patrol service platform is arranged to the i^{th} intersection node. If yes, then $A_{i,j}$ =1, or $A_{i,j}$ =0.

3.3.3 Result—the final location of new traffic patrol service platforms

On the basis of the preliminary scheme, we combine the multi-objective location decision model, which aims at minimizing the total response time, maximizing the coverage of the crime probability, minimizing the number of total traffic patrol service platforms, and minimizing the workload difference, to figure out the number of traffic patrol service platform and their location eventually. And we find that adding 4 traffic patrol service platforms is OK, which located in A28, A39, A61, A92 intersection nodes.

4. Concluding Remarks

Firstly, we use the Floyd algorithm to confirm the jurisdictional scope of each traffic patrol service platform, making sure that police can arrive at the scene as soon as possible when there are emergencies in its jurisdictional scope, and the specific results can be seen in table1. However, we are surprised to find that the workload differences among some traffic patrol service platforms in Area A is obvious, for example, the 6th only needs to be responsible for the intersection where it is and the 1th platform needs manage 10 platforms.

Then, we develop a dispatch model based on 0-1 integer programming and the Hungarian algorithm to dispatch the 20 traffic patrol service platforms in Area A, guaranteeing that the police can blockade all the 13 traffic arteries as quickly as possible when large-scale emergencies occur. The specific scheduling scheme is shown in Table 2.

Lastly, on the basis of multi-objective location decision model, we find that adding 4 traffic patrol service platforms in

A28, A39, A61, A92 intersection nodes can decrease the workload disequilibrium phenomenon and cut down the long response time.

In this paper, we find that the mathematical models combined with Floyd algorithm can solve the problem about the installation and dispatch of the patrol service platform effectively and feasibly, including the shortest path model, the dispatch model and the multiple-objective location model, which provides a good reference for the practical situations.

References

Baker, B. M., & Ayechew, M. A. (2003). A genetic algorithm for the vehicle routing problem. *Computers and operations research*, *30*(5), 787-800. https://doi.org/10.1016/S0305-0548(02)00051-5

Berger, J., & Barkaoui, M. (2003). A hybrid genetic algorithm for the capacitated vehicle routing problem. *Genetic and evolutionary computation conference*. Springer Berlin Heidelberg, 646-656. http://dx.doi.org/10.1007/3-540-45105-6_80

Blanton, J. L., & Wainwright, R. L. Multiple vehicle routing with time and capacity constraints using geneticalgorithms. International conference on genetic algorithms. 1993,452-459.

Bullnheimer, B., Hartl, R. F., & Strauss, C. (1999). Applying the ANT system to the vehicle routing problem. *Meta-heuristics*. Springer US, 285-296. http://dx.doi.org/10.1007/978-1-4615-5775-3

China Undergraduate Mathematical Contest in Modeling, The competition problems in 2011. http://www.mcm.edu.cn.

Dantzing, G., & Ramser, J. (1959). The truck dispatching problem. *Management Science*, *10*(6), 80-91. https://doi.org/10.1287/mnsc.6.1.80

Ellabib, I., Basir, O. A., & Calamai, P. (2002). An experimental study of a simple ant colony system for the vehicle routing problem with time windows. *Lecture notes in computer science*, *2463*, 53-64. http://dx.doi.org/10.1007/3-540-45724-0_5

Huo, J. Z., & Wang, X. H. (2005). Solving vehicle scheduling problem based on constraint programming. *Logistics technology*, *9*, 110-112.

Laporte, G., & Mercure, H. (1992). The vehicle routing problem with stochastic travel times. *Transportation Science*, *26*(3), 161-170. http://dx.doi.org/10.1287/trsc.26.3.161

Mao Yuan-jie. (2013). Floyd Algorithm and matlab program realization of shortest path problem. *Journal of Hebei north university(natural science edition)*, *29*(5), 1673-1942. http://dx.doi.org/10.3969/j.issn.1673-1492.2013.05.004

Salman, A., Ahmad, I., & Al-Madani S. (2002). Particle swarm optimization for task assignment problem. *Microprocessors & Microsystems*, *26*(8), 363-371. http://dx.doi.org/10.1016/S0141-9331(02)00053-4

Savelsbergh, M. W. P. (1985). Local search in routing problems with time windows. *Annals of Operations Research*, *4*(1), 285-305. http://dx.doi.org/10.2498/cit.1002276

Xie, B. L., Li, J., & Guo, Y. H. (2000). Genetic algorithm for vehicle scheduling problem of non-full loads with time windows. *Journal of Systems Engineering*, *9(*03), 235-239.

Yu, J. X., Song, D. C., Zhao, X. Y., & Li, J. Q. (2012). Research on the reasonable dispatch of service platform of traffic and patrol police. *Science Technology and Engineering*, *12*(1), 1671-1815.

Zhu, H., & Zhang, Y. (2011). Finding shortest path between nodes based on improved floyd algorithm. *Network and Multimedia*, *12*(3), 1002-8684. http://dx.doi.org/10.16311/j.audioe.2011.12.025

Copyrights

Copyright for this article is retained by the author(s), with first publication rights granted to the journal.

This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (http://creativecommons.org/licenses/by/4.0/).