Temporal Disaggregation Methods in Flow Variables of Economic Data: Comparison Study

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Abstract

Annual Gross Domestic Product (GDP) for Nigeria using observed annual time-series data for the period 1981-2012 was studied. Five different econometric disaggregation techniques, namely the Denton, Denton-Cholette, Chow-Lin-maxlog, Fernandez, and Litterman-maxlog, are used for quarterisation. We made use of quarterly Export and Import as the indicator variables while disaggregating annual into quarterly data. The time series properties of estimated quarterly series were examined using various methods for measuring the accuracy of prediction such as, Theil's Inequality Coefficient, Root Mean Squared Error (RMSE), Absolute Mean Difference (MAD), and Correlation Coefficients. Results obtained showed that export and import are not good indicators for predicting GDP for Nigeria is concerned for the period covered. Denton method proved to be the worst using Mean Absolute Difference (MAD) and Theil's Inequality Coefficient. However, RSME% and Pearson's correlation coefficient gave robust values for Litterman-maxlog, thereby making it the best method of temporal disaggregation of Nigeria GDP.

Keywords: temporal disaggregation, Denton, Chow-Lin-maxlog, Fernandez, Litterman-maxlog, GDP, export, import

1. Introduction

A traditional problem faces economic researchers is the interpolation or distribution of economic time series observed at low frequency into compatible higher frequency data. Interpolation refers to estimation of missing observations of stock variables, a distribution (or temporal disaggregation) problem occurs for flow aggregates and time averages of stock variables. Temporal disaggregation is the process of deriving high frequency data from low frequency data, and is closely related to benchmarking and interpolation.

Temporal disaggregation has been extensively studied by previous econometric and statistical literature and many different solutions have been proposed (Di Fonzo (1994), (2002), Quilis (2004)). Broadly speaking, two alternative approaches have been followed: methods which do not involve the use of related series but rely upon purely mathematical criteria or time series models to derive a smooth path for the unobserved series; methods which make use of the information obtained from related indicators observed at the desired higher frequency.

All disaggregation methods ensure that the sum, the average, the first or the last value of the resulting high frequency series is consistent with the low frequency series. They can deal with situations where the high frequency is an integer multiple of the low frequency (e.g. years to quarters, weeks to days), but not with irregular frequencies (e.g. weeks to months). Temporal disaggregation methods are widely used in official statistics. For example, in France, Italy and other European countries, quarterly figures of Gross Domestic Product (GDP) are computed using disaggregation methods. Outside of R, there are several software packages to perform temporal disaggregation: Ecotrim by Barcellan et al. (2003); a Matlab extension by Quilis (2012); and a RATS extension by Doan (2008).

Temporal disaggregation is used to disaggregate or interpolate a low frequency to a higher frequency time series, while the sum, the average, the first or the last value of the resulting high-frequency series is consistent with the low frequency series. Disaggregation can be performed with or without the help of one or more indicator series. It can deal with all situations where the high frequency is an integer multiple of the low frequency (e.g. weeks to days), but not with irregular frequencies (e.g. weeks to months). The selection of a temporal disaggregation model is similar to the selection of a linear regression model. Despite the number of empirical studies conducted to evaluate the merits of various methods of temporal disaggregation, there is no consensus that one method is consistently superior in all situations. Rather, a common conclusion is that the choice of method depends on the desired application. However, few empirical results have attempted to establish the conditions under which some of these methods may have an advantage over competing models. Most empirical studies have focused on applying these methods to relatively well-behaved series; for example, constructing quarterly estimates of GDP (Abeysinghe and Lee, 1998; Di Fonzo and Marini, 2005a; Trabelsi and Hedhili, 2005) manufacturing (Brown, 2012), or retail and wholesale trade data (Brown, 2012; Dagum and Cholette, 2006; Di Fonzo and Marini, 2005b) from observed annual levels.

2. Denton Process

The first method used for interpolation is the proportional Denton procedure. This method also computes the interpolation of a time series observed at low frequency by using a related high-frequency indicator time series. The Denton process imposes the condition that the sum of the interpolated series within each year equals the annual sum of the underlying series for that particular year. The Denton process may be useful in cases where the higher frequency indicators do not considerably associated with the low-frequency time series of the interest (Denton, 1971). Specifically, this method minimizes the distance between the two time series as much as possible using quadratic minimization framework.

The Denton process (1970, 1971) is stated as follows:

Let *G* be an integer, and assume that our concern is *G* per year intra-annual time periods (in our case quarters). Let *T* be a number of years and the time series of interest spans over *T* years, consisting $n = G \times T$ observations. The original figures are given in column-vector form as follows:

$$t = [t_1, t_2, \dots, t_n]'$$
(1)

Further, assume that a column-vector of T annual sums is available from another data source, which is represented by

$$b = \begin{bmatrix} b_1, b_2, \dots, b_n \end{bmatrix}$$
(2)

Denton (1970, 1971) proposed a method in order to make adjustment in the preliminary vector t to derive a new column vector

$$\lambda = [\lambda_1, \lambda_2, \dots, \lambda_n]$$
⁽³⁾

The Denton method satisfies the two conditions: (i) minimization of the distortion of the primary series (ii) equalization of the sum of the *G* observations of the derived series in a specific year to the given annual sum for that year. A penalty function given by $p(\lambda, b)$, and select the λ so as to minimize the penalty function given the following constraint

$$\sum_{(N-1)G+1}^{NG} \lambda_i = b_N \qquad for \qquad N = 1, \ 2, \dots, T$$
(4)

3. Chow-Lin Method

This procedure is known as the best linear unbiased estimator (BLUE) approach, which was developed by Chow and Lin (1971, 1976). According to Chow-Lin Method a regression model relates the unknown disaggregated series and a set of known high frequency indicators. Suppose that annual series of N years are available which is to be disaggregated into quarterly series, which is related to the k indicator (related) series and then relationship between the disaggregated series (to be estimated) and indicators series is

$$y = X\beta + e \tag{5}$$

where y is $(n \times 1)$ vector (n = 4N) of the quarterly series to be estimated, X is the matrix $(n \times k)$ of the k indicator variables which are observed quarterly, β is a $(k \times 1)$ vector of coefficients, and *e* is the $(n \times 1)$ vector of stochastic disturbances with mean, E(e) = 0 and variance, E(ee') = V, where V is a $(n \times n)$ matrix.

It has to be mentioned that the disaggregated model at the high frequency level (here quarterly) is subject to the usual aggregation constraints

$$Y = B' y \tag{6}$$

Substituting (5) into (6) gives a regression equation for the observed annual series in relation to the quarterly indicator series:

(7)

$$Y = B' x \beta + B' e$$

The regression coefficients β then can be calculated by using the Generalized Least Squares (GLS) estimator as

$$\hat{\boldsymbol{\beta}} = \left[\boldsymbol{X}' \boldsymbol{B} (\boldsymbol{B}' \boldsymbol{V} \boldsymbol{B})^{-1} \boldsymbol{B}' \boldsymbol{X} \right]^{-1} \boldsymbol{X}' \boldsymbol{B} (\boldsymbol{B}' \boldsymbol{V} \boldsymbol{B})^{-1} \boldsymbol{Y}$$
(8)

The estimated sub-period of the quarterly time series data is derived as

$$\hat{Y} = X\hat{\beta} + VB(B'VB)^{-1} \left[Y - B'X\hat{\beta} \right]$$
(9)

4. Fernandez Random Walk Process

Fernandez (1981) proposed the usual regression model of Chow-Lin; and estimates $\hat{\beta}$ and \hat{y} but assuming that the disturbances (residuals) in the disaggregated model follow a random walk process as:

$$u_{i} = u_{t-1} + \varepsilon_{t} \qquad t = 1, 2, ..., n$$

$$\varepsilon_{t} \sim N(0, \sigma^{2}), \qquad u_{0} = 0$$

5. Litterman Random Walk Markov Process

The other variant of Chow-Lin is the random walk Markov model derived by Litterman (1983 and also Di Fonzo, 1987) as

$$u_i = u_{t-1} + \varepsilon_t$$
$$\varepsilon_i = \alpha \varepsilon_{t-1} + e_t$$

6. Estimating the Autoregressive Parameter

There are several ways to estimate the autoregressive parameter r in the Chow-Lin and Litterman methods. An iterative procedure has been proposed by Chow and Lin (1971). It infers the parameter from the observed autocorrelation of the low frequency residuals, u. In a different approach, Bournay and Laroque (1979) suggest the maximization of the likelihood of the GLS-regression:

$$L(\rho,\sigma^{2}_{\varepsilon}\beta) = \frac{\exp\left[-\frac{1}{2}u'(C\Sigma C')^{-1}u\right]}{(2\pi)^{\frac{N}{2}} \cdot \left[\det(C\Sigma C')\right]^{\frac{N}{2}}}$$
(10)

Where

 $C = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ is the conversion matrix

and

 $\rho = \hat{\beta} X$

The maximum likelihood estimator of the autoregressive parameter, $\hat{\rho}$, is a consistent estimator of the true value, thus it has been chosen as the working estimator. However, in some cases, $\hat{\rho}$ turns out to be negative even if the true $\hat{\rho}$ is positive.

A final approach is the minimization of the weighted residual sum of squares (RSS), as it has been suggested by Barbone et al. (1981):

$$RSS(\rho, \sigma^2_{\varepsilon}, \beta) = u'(C\Sigma C')^{-1}u$$
(11)

Contrary to the maximum likelihood approach, σ_{ε}^2 does not cancel out. The results are thus sensitive to the specification of Σ , with different implementations leading to different but inconsistent estimations of ρ .

7. Data and Methods

The data used for this research obtained from the Central Bank of Nigeria (CBN) is on Gross Domestic Product (GDP), the original series were in annual which was disaggregated into quarterly series in this work using five methods Denton, Denton-Cholette, Chow-lin-maxlog, Fernandez, and Litterman-maxlog methods.

All analyses were carried out using *R* version 3.1.3 (R Development Core Team, 2015)

	Min. of	Median of	Max. of					Low	High
Method	Residual	Residual	Residual	Estimate	Std. error	t-value	P-value	Freq.	Freq.
Denton	227251	372520	888889	NA	NA	NA	NA	32	128
Denton-Cholette	227251	372520	888889	NA	NA	NA	NA	32	128
Chow-Lin-Maxlog	-338794	193524	322845	141512	77133	1.835	0.0762	32	128
Fernandez	-24017	121252	637621	62818	4805	13.07	3.73E-14	32	128
Litterman	-23640	121629	637998	62724	1208	51.93	2.00E-16	32	128

Table 1. Results of Temporal Disaggregation without Indicator

Where NA means "not applicable" since Denton and Denton-Cholette do not use regression.

8. Metrics for Measuring the Accuracy of Prediction

We begin by examining the performance of each method based on statistics that measure the accuracy of each method with respect to the levels of the original series. Theil's (1961) inequality coefficient, U, which is a measure of accuracy used in forecasting (Leuthold, 1975) used by Trabelsi and Hedhili (2005), is given by equation

$$U = \frac{\frac{1}{N} \sum_{n=1}^{N} (p_n - a_n)^2}{\sqrt{\frac{1}{N} \sum_{n=1}^{N} (p_n^2)} + \sqrt{\frac{1}{N} \sum_{n=1}^{N} (a_n^2)}}$$

where p_n is the predicted value and an is the actual value in quarter *n*. The U statistic takes on a value between 0 and 1, where U = 0 indicates that the method used is a perfect predictor of the actual series (Leuthold, 1975; Trabelsi and Hedhili, 2005). Consistent with Trabelsi and Hedhili (2005), we also calculate the mean of the absolute differences between actual, a, and predicted values, p.

Abeysinghe and Lee (1998) employed a different criterion, the Root Mean Squared Error as a percent of the mean (RMSE%) of the observed series, in which a lower value implies a more accurate prediction. The Root Mean Square Error (also called the root mean square deviation, RMSD) is a frequently used measure of the difference between values predicted by a model and the values actually observed from the environment that is being modelled. These individual differences are also called residuals, and the RMSE serves to aggregate them into a single measure of predictive power. The RMSE of a model prediction with respect to the estimated variable X_{model} is defined as the square root of the mean squared error:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (X_{obsi} - X_{model,i})^2}{n}}$$

Table 2. Summary Statistics of the True Quarterly GDP and Estimated

	True values	Denton	Denton-Cholette	Chow-Lin	Fernandez	Litterman
Minimum	56260	36422	55745	55744	55745	55591
Ist Qtr	72522	73946	71059	71059	71059	70925
Median	93173	93206	93206	93206	93206	93180
Mean	108244	108244	108244	108244	108244	108244
3rd Qtr	128624	135518	135518	135518	135518	135427
Maximum	263679	224855	224855	224790	224855	226958
Std. Dev.	47713.61	46172.42	46073.53	46073.50	46073.53	46076.39
Kurtosis	0.96	-0.01	-0.02	-0.02	-0.02	-0.01
Skewness	1.25	0.98	1.00	1.00	1.00	1.00

It can be observed from table 2 that all the five methods used have the same mean of 108244 as the true GDP values. Denton-cholette and Fernandez share the similar properties. Denton, Denton-Cholette, Chow-Lin and Fernandez have a uniform median of 93206, while Litterman has a different value.

Table 3. Absolute Mean Difference (MAD), Inequality Coefficient (U) and Root Mean Squared Error % (RMSE)

	Denton	Denton-Cholette	Chow-Lin	Fernandez	Litterman
MAD	6618.2200	6117.1500	6117.5400	6117.1500	6128.83
Theil's U	0.0241	0.0197	0.0197	0.0197	0.0199
RMSE%	10.5600	10.1700	10.1700	10.1700	10.0900

Table 3 indicates that the three methods Denton-Cholette, Chow-Lin, and Fernandez perform similarly with Mean Absolute Difference (MAD) of 6117.3, this means that any of the three methods is better that either Denton or Litterman methods as far as the data used is concerned. Considering the Inequality coefficient (U), the minimum value 0.0197 is common with the Denton-Cholette, Chow-Lin, and Fernandez, the worst method is Denton with coefficient 0.02. Using the RSME% as a metric, Litterman method is the best method with the least value 10.09%, while Denton is least efficient with the highest percentage of 10.56 %.

Table 4. Correlation matrix of the various methods

	Denton	DentonCH	Chow-Lin	Fernandez	Litterman
GDPreal	0.97067	0.97283	0.97282	0.97283	0.97324

Disaggregated values from Denton, Denton-Cholette, Chow-Lin, and Fernandez methods related favorably with the real GDP, but the series produced using Litterman method was more efficient with a correlation coefficient of 0.97324.



Figure 1. Temporal disaggregation using various methods

From fig. 1 above, it can be seen that all other four methods of disaggregation are similar in shape with the real GDP annual series (frame 1) expect Denton (frame 2) having departures at the beginning of the series.



Figure 2. Comparing the true values with the disaggregated

Comparing the true values with the disaggregated using the five methods, it is discovered that it is only the Denton method that is having a slight change at the beginning of the series.

Table 5. Using some indicator series

Method	Estimate	Std. error	t-value	P-value	low freq.	high freq.
Intercept	4.73E+05	2.80E+05	1.691	0.102	32	128
exports	6.89E+00	5.75E+00	1.200	0.240	32	128
imports	1.21E+01	1.11E+01	1.090	0.284	32	128

The result from using exports and imports as indicator series indicates that Nigeria GDP is not significantly influenced by her exports and imports, that is, they are not good indicators for predicting the GDP. The R^2 of 0.08 is too low when talking about accuracy and reliability of any model.



Plot of Quarterly GDP against Time

Figure 3. Graphical comparison of the five methods with the real GDP series

Using the data in the appendix and the plotted graph in fig. 3, it can be summarized that Litterman method is much closer to the original series of GDP while the other methods seem to be away from it.

9. Conclusion and Recommendation

In this paper, we disaggregated annual Gross Domestic Product (GDP) of Nigeria to quarterly series using Denton, Denton-Cholette, Chow-Lin, Fernandez, and Litterman methods. Further test was carried out to verify the possibility of using a related indicator series (export) to make forecast of GDP, but the result obtained showed that export is not a good indicator for predicting GDP as far as Nigeria is concerned for the period covered. Later, comparison is made among the quarterly GDP series obtained by these methods (Appendix). It was found out that Denton method is the worst using the metrics such as Mean Absolute Difference (MAD) and Theil's Inequality coefficient. However, RSME% and Pearson's correlation coefficient gave robust values for Litterman, thereby making it the best method of temporal disaggregation of Nigeria GDP. It can be recommended that Litterman method of temporal disaggregation could be used by users in need of higher frequency data, such as the GDP, import, export and so on. Future researchers who wish to do comparison of methods should explore the Cubic Spline Interpolation method.

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QTRS	GDPreal	Denton	Denton-Cholette	Chow-Lin-maxlog	Fernandez	Litterman-maxlog
1981Q1	63433.0800	36422.0424	62818.0061	62880.1077	62818.0061	62739.1053
Q2	62446.9700	61526.8704	62796.0317	62799.0177	62796.0317	62769.8860
Q3	61818.9800	75315.4840	62752.0828	62722.5250	62752.0828	62791.0293
Q4	63353.2500	77787.8832	62686.1594	62650.6296	62686.1594	62752.2595
1982Q1	61555.1700	68944.0679	62598.2616	62583.3313	62598.2616	62577.8853
Q2	61383.7900	62501.5716	62196.4549	62195.7368	62196.4549	62166.5007
Q3	60930.5000	58460.3944	61480.7391	61487.8454	61480.7391	61473.1307
Q4	62857.1000	56820.5361	60451.1144	60459.6565	60451.1144	60509.0534
1983Q1	58056.4900	57581.9968	59107.5807	59111.1687	59107.5807	59341.8448
Q2	57335.0200	57891.8624	57965.2150	57965.3868	57965.2150	58095.6454
Q3	57041.7000	57750.1328	57024.0171	57022.3093	57024.0171	56925.4734
Q4	57947.5900	57156.8080	56283.9872	56281.9352	56283.9872	56017.8365

APPENDIX: Disaggregated Quarterly GDP using the five Methods

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1984Q1	57654.9100	56111.8880	55745.1251	55744.2635	55745.1251	55591.4676
Q2	56515.5800	55974.6378	55957.0032	55956.9622	55957.0032	55898.1874
03	56260.0400	56745.0574	56919.6216	56920.0316	56919.6216	56999.2900
04	56824 2000	58423 1468	58632 9801	58633 4727	58632 9801	58765 7850
100501	62202 2000	(1002.0000	(1007.0799	(1007.2976	(1007.0799	(0977.9992
1985Q1	63303.3000	61008.9060	61097.0788	61097.2876	61097.0788	008/7.8882
Q2	63021.6900	62940.9743	62945.2138	62945.2245	62945.2138	62823.7604
Q3	63095.2000	64219.3516	64177.3850	64177.2856	64177.3850	64266.8844
Q4	63593.0800	64844.0381	64793.5925	64793.4723	64793.5925	65044.7371
198601	64371.7400	64815 0337	64793 8362	64793,7853	64793 8362	65168 1487
02	64245 6400	64655 8635	64654 8442	64654 8415	64654 8442	64821 3455
	64426 5100	(4266 5274	64034.6442	64276 6407	64034.0442	64012.0164
Q3	04420.5100	04300.5274	043/0.0105	043/0.040/	043/0.0105	04213.8104
Q4	64740.5600	63947.0255	63959.1530	63959.1825	63959.1530	63581.1395
1987Q1	64039.0700	63397.3577	63402.4538	63402.4666	63402.4538	63185.9115
Q2	63716.3800	63418.2436	63418.4886	63418.4895	63418.4886	63318.7830
03	63815.5900	64009.6829	64007.2574	64007.2513	64007.2574	64085,4075
04	64425 9300	65171 6758	65168 7602	65168 7527	65168 7602	65406 8580
109901	68564.0700	66004 2222	66002.0071	66002.0051	66002.0071	67010 2156
1900Q1	08504.0700	00904.2222	00902.9971	00902.9931	00902.9971	07019.3130
Q_2	685//.1300	683/6.5069	683/6.4480	683/6.4482	683/6.4480	684/3.0289
Q3	68743.3300	69588.5298	69589.1130	69589.1137	69589.1130	69567.3354
Q4	69525.0300	70540.2910	70540.9920	70540.9930	70540.9920	70349.8701
1989Q1	73786.1000	71231.7905	71232.0850	71232.0874	71232.0850	71116.7536
02	73521.1800	72524,7540	72524.7682	72524,7693	72524,7682	72413.7597
03	73548 9200	7//19 1818	74419 0416	74419 0404	74/19 0/16	7//51 9218
Q3 04	73340.9200	74415.1010	74417.0410	76014 0020	74412.0410	77109 2650
Q4	74254.0000	/0913.0/3/	/0914.9032	70914.9029	70914.9032	77108.3030
1990Q1	821/2.5200	80012.4298	80012.3590	80012.3597	80012.3590	79925.8218
Q2	81916.0200	82107.0063	82107.0029	82107.0034	82107.0029	82110.8280
Q3	81811.6600	83198.8033	83198.8370	83198.8365	83198.8370	83251.8332
Q4	82705.8600	83287.8207	83287.8612	83287.8603	83287.8612	83317.5771
199101	82187.6000	82374.0585	82374.0756	82374.0749	82374.0756	82656.9666
02	81965 6600	81923 3843	81923 3851	81923 3850	81923 3851	82000 4517
Q2 03	81720 8200	01925.50 4 5	01725.5051 01025 7000	81025 7000	01925.5051 01025 7000	02000.4317
Q3	81/39.8300	81955.7979	01955.7090	81955.7900	01955.7090	01/92./110
Q4	82/51.4500	82411.2993	82411.2896	82411.2900	82411.2896	82194.4100
1992Q1	84525.0600	83349.8887	83349.8846	83349.8855	83349.8846	83082.8264
Q2	84118.2000	84114.2330	84114.2328	84114.2331	84114.2328	84051.3744
Q3	83966.5000	84704.3321	84704.3341	84704.3336	84704.3341	84836.7161
04	84678.8700	85120.1862	85120.1886	85120,1878	85120,1886	85317.7232
199301	85920 2000	85361 7952	85361 7962	85361 7961	85361 7962	85514 9994
02	85420.0100	85567 7677	85567 7678	85567 7670	85567 7678	85500.0610
	05420.9100	85507.7077	05507.7070	05507.7079	05507.7070	85590.9019
Q3	85354.6100	85/38.1038	85/38.1033	85/38.1033	85/38.1033	8365/.3985
Q4	85844.7500	85872.8033	85872.8027	85872.8028	85872.8027	85776.9102
1994Q1	86357.4700	85971.8663	85971.8661	85971.8662	85971.8661	85961.1550
Q2	86151.0100	86145.5910	86145.5910	86145.5910	86145.5910	86172.8967
03	86188.4400	86393.9773	86393.9775	86393.9773	86393.9775	86406.4666
04	86531.5400	86717.0253	86717.0255	86717 0254	86717 0255	86687.9417
199501	883/1 7300	8711/ 73/9	8711/ 7350	87114 7357	87114 7350	87075 2467
02	88061 6400	87602 5468	87602 5468	87602 5472	97602 5469	97659 2922
	00001.0400	07092.3400	07092.3400	07092.3472	87092.3408	87038.3833
Q3	88105.0500	88450.4609	88450.4609	88450.4605	88450.4609	88462.8495
Q4	88137.8000	89388.4773	89388.4773	89388.4766	89388.4773	89449.7405
1996Q1	92123.0100	90506.5960	90506.5960	90506.5966	90506.5960	90515.5949
Q2	91729.3900	91473.0141	91473.0141	91473.0144	91473.0141	91491.9866
03	91731.7400	92287.7315	92287.7315	92287.7312	92287.7315	92293.4360
04	91633,9500	92950.7484	92950 7484	92950 7478	92950 7484	92917.0726
100701	94716 5500	93462 0647	93462 0647	93462 0652	93462 0647	03442 6234
0	04440.0400	04064.9452	04064 8452	04064.9457	04064 8452	04022 7290
Q2	94440.0400	94004.8453	94004.8453	94004.845/	94004.8433	94032.7280
Q3	94452.0200	94759.0903	94759.0903	94759.0900	94759.0903	94758.0238
Q4	94222.1900	95544.7997	95544.7997	95544.7990	95544.7997	95597.4248
1998Q1	97531.5300	96421.9733	96421.9733	96421.9735	96421.9733	96438.0385
02	97115.5500	97050.8655	97050.8655	97050.8656	97050.8655	97074.7206
03	97133 0200	97431 4761	97431 4761	97431 4759	97431 4761	97425 2677
04	96688 0100	07563 2051	07562 2051	07563 2050	07563 2051	07530 0021
100001	00000.0100	97505.0051	07447.0506	07/47 0520	07447.0506	97550.0951
1999QI	98099.4800	9/44/.8520	9/44/.8526	9/44/.8530	9/44/.8526	9/552.5864
Q2	98394.1200	97733.0476	97733.0476	97733.0478	97733.0476	97778.7569
Q3	98546.7300	98419.3900	98419.3900	98419.3897	98419.3900	98377.3777
Q4	98066.8400	99506.8799	99506.8799	99506.8795	99506.8799	99398.6489

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2000Q1	103201.1600	100995.5172	100995.5172	100995.5186	100995.5172	100775.3965
Q2	103182.9500	102425.9662	102425.9662	102425.9670	102425.9662	102322.6073
Q3	103234.4300	103798.2270	103798.2270	103798.2263	103798.2270	103887.0976
Q4	102713.4600	105112.2996	105112.2996	105112.2981	105112.2996	105346.9086
2001Q1	108099.7600	106368.1838	106368.1838	106368.1841	106368.1838	106610.8253
Q2	108093.1900	107501.5820	107501.5820	107501.5822	107501.5820	107618.0171
Q3	108083.7100	108512.4941	108512.4941	108512.4937	108512.4941	108412.2547
Q4	107506.5300	109400.9201	109400.9201	109400.9199	109400.9201	109142.0828
2002Q1	112632.9800	110166.8600	110166.8600	110166.8618	110166.8600	110061.4061
Q2	113328.2100	111585.0590	111585.0590	111585.0599	111585.0590	111530.4907
Q3	113096.1000	113655.5170	113655.5170	113655.5159	113655.5170	113697.0328
Q4	112728.3800	116378.2340	116378.2340	116378.2323	116378.2340	116496.7405
2003Q1	124036.7900	119753.2101	119753.2101	119753.2122	119753.2101	119653.0807
Q2	123928.7500	122702.6335	122702.6335	122702.6346	122702.6335	122676.1885
Q3	123782.6300	125226.5042	125226.5042	125226.5029	125226.5042	125282.7942
Q4	123258.9900	127324.8222	127324.8222	127324.8203	127324.8222	127395.1066
2004Q1	114617.6200	128997.5875	128997.5875	128997.5890	128997.5875	129140.5056
Q2	123702.9200	130825.2616	130825.2616	130825.2628	130825.2616	130852.0454
Q3	142373.6100	132807.8446	132807.8446	132807.8438	132807.8446	132731.8030
Q4	146881.8700	134945.3363	134945.3363	134945.3343	134945.3363	134851.6759
2005Q1	120048.9200	137237.7369	137237.7369	137237.7369	137237.7369	137153.6656
Q2	128755.4600	139452.7424	139452.7424	139452.7419	139452.7424	139449.6485
Q3	153933.5900	141590.3527	141590.3527	141590.3518	141590.3527	141635.8170
Q4	159193.4200	143650.5679	143650.5679	143650.5694	143650.5679	143692.2689
2006Q1	128579.7900	145633.3879	145633.3879	145633.3969	145633.3879	145682.9302
Q2	135438.6300	147755.3218	147755.3218	147755.3288	147755.3218	147755.8060
Q3	162498.7700	150016.3694	150016.3694	150016.3678	150016.3694	149989.4750
Q4	169304.4300	152416.5309	152416.5309	152416.5165	152416.5309	152393.3988
2007Q1	135774.7400	154955.8061	154955.8061	154955.7777	154955.8061	154907.9597
Q2	142790.4600	157414.3208	157414.3208	157414.2967	157414.3208	157404.2250
Q3	173067.4800	159792.0748	159792.0748	159792.0763	159792.0748	159813.2549
Q4	182618.5900	162089.0683	162089.0683	162089.1193	162089.0683	162125.8304
2008Q1	142071.4000	164305.3012	164305.3012	164305.4286	164305.3012	164392.4170
Q2	150862.2000	166689.9288	166689.9288	166690.0344	166689.9288	166723.3634
Q3	183678.8200	169242.9513	169242.9513	169242.9397	169242.9513	169201.9671
Q4	195590.1400	171964.3687	171964.3687	171964.1473	171964.3687	171884.8025
2009Q1	149191.4700	174854.1808	174854.1808	174853.6607	174854.1808	174801.9443
Q2	162101.1600	177966.1663	177966.1663	177965.7329	177966.1663	177957.0859
Q3	197084.3300	181300.3253	181300.3253	181300.3676	181300.3253	181331.9722
Q4	210600.3800	184856.6576	184856.6576	184857.5688	184856.6576	184886.3277
2010Q1	160117.0500	188635.1634	188635.1634	188637.3408	188635.1634	188557.6973
Q2	174733.9700	192325.7213	192325.7213	192327.5321	192325.7213	192261.2001
Q3	212771.6800	195928.3315	195928.3315	195928.1471	195928.3315	195941.3711
Q4	228709.5200	199442.9938	199442.9938	199439.1900	199442.9938	199571.9415
2011Q1	171265.8600	202869.7083	202869.7083	202860.6651	202869.7083	203155.7363
Q2	187833.0600	206492.5937	206492.5937	206485.0699	206492.5937	206724.6844
Q3	228454.8200	210311.6502	210311.6502	210312.4088	210311.6502	210288.0317
Q4	246447.1000	214326.8778	214326.8778	214342.6862	214326.8778	213832.3776
2012Q1	182119.4400	218538.2763	218538.2763	218575.9069	218538.2763	217321.6217
Q2	199831.5600	221696.8252	221696.8252	221728.1307	221696.8252	220696.8220
Q3	243263.1000	223802.5244	223802.5244	223799.3613	223802.5244	223916.7630
Q4	263678.9100	224855.3741	224855.3741	224789.6011	224855.3741	226957.7932

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