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Modeling HIV/AIDS Progression: A Comparative Analysis of the 3-Parameter Weibull, AFT, and Cox Proportional Hazards Models

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Abstract

This study models HIV/AIDS progression using the 3-Parameter Weibull Model and its adaptations, specifically the Accelerated Failure Time (AFT) Weibull and Cox Proportional Hazards (PH) models, to compare outcomes across age groups (20-30, 30-40, 40-50, 50-60, and over 60) and genders. Key performance metrics included Z-statistics, P-values, AD values, and standard errors to evaluate model fit and accuracy. The 3-Parameter Weibull model's flexibility for time-varying hazards makes it well-suited for chronic conditions influenced by antiretroviral therapy (ART) and demographic factors.

Results showed that the AFT model captured ART effects effectively in the 50-60 age group, particularly among males, while its predictive power decreased for younger cohorts, where ART's impact was less pronounced. The Cox PH model, although interpretable, struggled in dynamic hazard rate scenarios, performing moderately in stable age groups but limited in detecting ART effects overall. The 3-Parameter Weibull model showed a strong fit in the 40-50 and 50-60 groups, with significant metrics affirming ART's impact on survival, though predictive precision declined for those over 60. These findings highlight the complementary strengths of the AFT and 3- Parameter Weibull models, suggesting their integrated use can enhance state-specific modeling of HIV/AIDS progression.

Keywords: HIV/AIDS progression, 3-Parameter Weibull Model, Accelerated Failure Time (AFT) Model, Cox Proportional Hazards Model, Simulated data, Survival analysis, ART, Time-varying hazard rates

1. Introduction

In analyzing HIV/AIDS progression, accurately modeling transition dynamics across various disease stages is essential for effective prognosis and treatment planning. Traditional models, such as the Cox Proportional Hazards (PH) model, have long been applied in survival analysis for their flexibility in handling covariates without assuming a baseline hazard function. However, the model's proportional hazards assumption—which implies that the effect of covariates on hazard rates remains constant over time—limits its applicability for progressive diseases like HIV/AIDS, where hazards evolve significantly with factors such as treatment timing and patient demographics (Madigan & Raftery, 1994). Studies by (Teimouri, Doser, and Finley, 2020) and (Brookmeyer & Damiano, 1989) also highlight the complexities of HIV/AIDS progression, showing that mortality rates and progression patterns vary with demographic and treatment-specific factors, further underscoring the need for flexible models that can capture these variations.

(Mwirigi, 2024b) demonstrated the limitations of assuming constant hazard rates in modeling HIV/AIDS state holding times by applying the Exponential distribution. While the Exponential model is often employed for its simplicity, its memoryless property oversimplifies transition dynamics by assuming a constant hazard rate over time, an approach that fails to account for the nuances of HIV/AIDS progression, as evidenced by studies on the variability of disease states (Braitstein et al., 2006). As an alternative, the Weibull distribution provides greater flexibility for modeling non-constant hazard rates, which are typical of the disease's trajectory. (Mwirigi, 2024a) work on the 2-Parameter Weibull model illustrates how this distribution better accommodates dynamic transitions by allowing hazard rates to increase or decrease as a function of time, providing more accurate estimates for the time spent within each disease state, particularly in demographic-specific cohorts (e.g., males or older patients).

Building on the insights from Exponential and 2-Parameter Weibull modeling, Mwirigi, Simwa, Wainaina, and Sewe, (2022) and Teimouri et al. (2020) explored Bayesian Model Averaging (BMA) as a robust framework that accounts for model uncertainty in analyzing HIV/AIDS progression. BMA enables the consideration of multiple plausible models, assigning posterior probabilities to risk factors based on their predictive contributions to state-specific failure rates (Chatfield, 1995; Gayawan & Ipinyomi, 2009; Portnov, Reiser, Karkabi, Cohen-Kastel, and Dubnov, 2012). This approach, informed by work on model uncertainty by (Burnham & Anderson, 2004) and (Clyde, 2000), provides a

robust means to account for diverse risk factors and covariates like CD4 cell count, gender, and age, ensuring that key predictors are appropriately weighted across models. In their findings, (Mwirigi et al. 2022a) identified age, gender, and CD4 count as significant predictors of transition rates, showing that certain demographics, such as older individuals, have accelerated progression rates even with antiretroviral therapy (ART), findings corroborated by (Moh et al., 2007) and (Cassenote, Grangeiro, Escuder, Abe, and Segurado, 2018), who reported similar variations in ART efficacy across patient demographics.

The Accelerated Failure Time (AFT) Weibull model, which directly adjusts survival time scales based on covariates, provides another alternative to the Cox model by allowing flexible hazard rate structures that accommodate time-variant covariate effects. (Mwirigi, 2024a) analysis of the AFT model demonstrates its suitability for long- term studies, particularly in cohorts where age and ART status significantly impact progression, enabling the model to account for covariate-driven survival time modifications. This model aligns with findings by (Longini, Clark, Gardner, and Brundage, 1991) on CD4 decline dynamics, where individualized disease progression necessitates adaptive time scales. The AFT model's suitability is also supported by studies like (Wright et al., 2013), which highlighted significant interactions between treatment timing and patient age, further validating its utility in progressive disease studies.

The current study builds on these foundational insights by comparing the Cox PH, 3-Parameter Weibull, and AFT Weibull models using simulated HIV/AIDS data, incorporating critical covariates like age, gender, and ART status. Evaluating these models' capacity to capture the dynamic, time-varying hazards typical of HIV/AIDS progression provides insight into their relative strengths. Previous findings by (Mwirigi, Sewe, Wainaina, and Simwa, 2022) and (Claeskens & Hjort, 2008) emphasize that model selection criteria, such as Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), can guide in identifying the most suitable model for capturing time-dependent hazard rates in HIV/AIDS progression (Akaike, 1974; Schwarz, 1978). This comparative analysis thereby extends the literature by emphasizing the advantages of Weibull models in chronic disease modeling, particularly when capturing variability in hazard rates across patient demographics and treatment factors is crucial. Ultimately, the findings aim to inform personalized treatment strategies and improve patient outcomes in HIV/AIDS care.

2. Materials and Methods

Survival analysis is a vital area of statistics, focusing on the study of time-to-event data. It is widely applied in medical research to assess the time until key events, such as death, disease progression, or treatment failure, as well as in engineering to model system reliability and failure times (Collett, 2023). The versatility of survival models allows for various assumptions about the hazard and survival functions, enabling researchers to capture different patterns of event occurrence. This section provides a detailed derivation of three commonly used survival models: the *3-Parameter Weibull Model*, the *Accelerated Failure Time (AFT) 3-Parameter Weibull Model*, and the *Cox Proportional Hazards (PH) Model*.

The *3-Parameter Weibull Model* is an extension of the standard Weibull distribution, which includes a shape, scale, and location parameter to account for delayed event occurrences. This model is particularly useful when the hazard rate may vary over time, either increasing, decreasing, or remaining constant. The flexibility of the 3-Parameter Weibull model makes it suitable for modeling time-to-event data where the timing of the event is influenced by external factors such as delayed onset of disease or treatment (Goshu & Asena, 2017; Mwirigi et al., 2022b).

The *Accelerated Failure Time (AFT) 3-Parameter Weibull Model* provides a parametric alternative to the Cox PH model by assuming that covariates act multiplicatively on the time scale rather than on the hazard function. This framework allows for time-dependent adjustments based on covariates, offering insights into how certain factors accelerate or decelerate time to event occurrence. AFT models have demonstrated significant applicability in cases where proportional hazards assumptions are violated, particularly in chronic diseases like HIV/AIDS (Longini et al., 1991; Wright et al., 2013).

The *Cox Proportional Hazards (PH) Model* remains a popular semi-parametric approach in survival analysis due to its ability to estimate hazard ratios without specifying the baseline hazard function (Collett, 2023). While the Cox model is robust in many applications, its proportional hazards assumption limits its utility in cases where covariate effects change over time. Nonetheless, the Cox model is well-suited for assessing relative risks between subgroups when proportionality holds (Burnham & Anderson, 2004; Claeskens & Hjort, 2008).

Model selection criteria such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) play a critical role in identifying the most suitable model for a given dataset. These criteria offer a balance between model complexity and goodness-of-fit, helping researchers avoid overfitting while ensuring the model accurately captures the data's structure (Acquah, 2010; Akaike, 1974; Buckland, Burnham, and Augustin, 1997). The use of AIC and BIC in survival analysis is particularly important in studies with multiple competing models, as it aids in choosing the model that best represents the underlying progression dynamics without unnecessary complexity.

The *AFT 3-Parameter Weibull Model* incorporates covariates that affect the logarithm of survival time, allowing researchers to study the effects of variables such as age, gender, or treatment on the acceleration or deceleration of time-to-event. The AFT model assumes a log-linear relationship between the covariates and the survival time, making it ideal for examining how external factors influence survival times. This model is widely used in medical research, particularly when assessing the impact of interventions or risk factors on patient outcomes, as it accommodates non-proportional hazards in cases where hazard ratios are not constant over time (Claeskens & Hjort, 2008; Longini et al., 1991; Mwirigi, 2024a; Wright et al., 2013).

The *Cox Proportional Hazards Model* is a semi-parametric model that estimates the hazard rate for an individual based on covariates, without specifying the baseline hazard function. This model focuses on the relative risk associated with different covariates, making it particularly useful in studies that aim to compare the risk of an event occurring between groups. The Cox model's flexibility allows researchers to estimate the effects of covariates on survival without needing to specify the underlying hazard structure, making it one of the most widely used models in survival analysis (Burnham & Anderson, 2004; Claeskens & Hjort, 2008; Collett, 2023).

In this section, we derive the *Probability Density Function (PDF)*, *hazard function*, and *survival function* for each model. By doing so, we provide a comprehensive understanding of the mathematical structure behind these models, along with their applications in medical and reliability research. The derivations will help clarify the theoretical underpinnings of each model and offer insight into how these models can be applied to real-world survival data.

2.1 Three-Parameter Weibull Model

The *Three-Parameter Weibull Model* is a parametric model that incorporates shape, scale, and location parameters, allowing it to account for delayed event occurrences. This model is widely used to model survival times where time-to-event may have a delayed onset, making it suitable for chronic disease studies like HIV/AIDS progression, where the timing of disease stages may be influenced by factors such as delayed diagnosis or treatment effects (D'Amico, Di Biase, Janssen, and Manca, 2011; Goshu & Asena, 2017; Longini et al., 1991; Mwirigi et al., 2022b).

2.1.1 Derivation of the Probability Density Function (PDF)

The Probability Density Function (PDF) is derived from the Cumulative Distribution Function (CDF). The CDF for the Three-Parameter Weibull model is given by:

$$F(t;k,\lambda,\gamma) = 1 - \exp\left[-\left(\frac{t-\gamma}{\lambda}\right)^k\right], \quad t > \gamma \tag{1}$$

The PDF is obtained by differentiating the CDF with respect to time *t*:

Taking the derivative, we get:

$$f(t;k,\lambda,\gamma) = \frac{d}{dt}\left[F(t;k,\lambda,\gamma)\right] \tag{2}$$

$$f(t;k,\lambda,\gamma) = \frac{k}{\lambda}\left(\frac{t-\gamma}{\lambda}\right)^{k-1}\exp - \left(\frac{t-\gamma}{\lambda}\right)^k, \quad t > \gamma \tag{3}$$

This describes the probability density of event occurrence at a specific time *t*, particularly useful in modeling survival times where delays or shape variations in hazard are expected (Collett, 2023; Goshu & Asena, 2017; Mwirigi et al., 2022b).

2.1.2 Derivation of the Hazard Function

The hazard function $h(t)$ measures the instantaneous event risk at time *t*, assuming the event has not occurred by time *t*. The hazard function is derived by dividing the PDF by the survival function $S(t)$:

$$h(t;k,\lambda,\gamma) = \frac{f(t)}{S(t)} \tag{4}$$

The survival function for the Three-Parameter Weibull model is:

$$S(t;k,\lambda,\gamma) = \exp\left[-\left(\frac{t-\gamma}{\lambda}\right)^k\right] \tag{5}$$

Substituting this into the hazard function gives:

$$h(t;k,\lambda,\gamma) = \frac{k}{\lambda}\left(\frac{t-\gamma}{\lambda}\right)^{k-1}, \quad t > \gamma \tag{6}$$

This hazard function provides flexibility for modeling increasing, decreasing, or constant hazard rates, depending on the value of *k*, making it applicable in diverse survival analysis contexts where hazard rates vary over time (Collett, 2023; Goshu & Asena, 2017; Mwirigi et al., 2022b).

2.1.3 Derivation of the Survival Function

The survival function $S(t)$ represents the probability that the event has not occurred by time *t*. It is derived using the cumulative hazard function $H(t)$, where:

$$S(t) = \exp(-H(t)) \tag{7}$$

The cumulative hazard function is the integral of the hazard function:

$$H(t;k,\lambda,\gamma) = \left(\frac{t-\gamma}{\lambda}\right)^k \tag{8}$$

Therefore, the survival function is:

$$S(t;k,\gamma,\lambda) = \exp\left[-\left(\frac{t-\gamma}{\lambda}\right)^k\right], \quad t > \gamma \tag{9}$$

This survival function effectively captures the probability of surviving beyond time *t*, incorporating the delay, scale, and hazard rate changes over time, making it highly applicable in survival analysis of chronic diseases and reliability studies (Collett, 2023; Goshu & Asena, 2017; Mwirigi et al., 2022b).

2.1.4 Interpretation of Parameters

k **(Shape Parameter)**: Controls the shape of the hazard function. If $k > 1$, the hazard increases over time, indicating an accelerating event risk. If $k = 1$, the hazard is constant over time, representing a consistent risk rate. If $k < 1$, the hazard decreases over time, which can suggest decelerating risk due to factors such as treatment effects (Collett, 2023; Mwirigi et al., 2022b).

λ **(Scale Parameter)**: Influences the spread of the distribution, with larger values of λ indicating a broader time frame for event occurrences. The scale parameter adjusts the overall timing of the event, impacting survival probabilities across different time horizons (D'Amico et al., 2011; Goshu & Asena, 2017).

γ **(Location Parameter)**: Models a delay in the onset of the event, where a positive γ suggests the event does not begin until after a specified time period. This is particularly useful for conditions with a delayed onset or latency period, as seen in some chronic diseases (Longini et al., 1991; Mwirigi et al., 2022b).

2.1.5 Applications on HIV/AIDS Modeling Transition Dynamics

The *3-Parameter Weibull model* has been widely applied in modeling HIV/AIDS progression due to its flexibility in capturing time-varying hazard rates. (Mwirigi et al., 2022b) utilized this model to study the transition from HIV diagnosis to AIDS onset, demonstrating that it effectively represents the increasing risk of disease progression as immune function deteriorates, particularly in untreated patients. The inclusion of the location parameter γ enabled modeling the delay in disease progression due to antiretroviral therapy (ART), which decelerates the transition from HIV to AIDS.

The Weibull model's adaptability makes it ideal for capturing various stages of disease progression in HIV patients, where early stages often have a slower hazard rate, and later stages may show rapid progression without treatment (Goshu & Asena, 2017; Mwirigi, 2024a). The model is particularly effective in representing the distinct progression

phases, with ART-treated patients experiencing delayed onset and slower progression due to immune stabilization.

The 3-Parameter Weibull model addresses key research needs by accommodating time-varying hazard rates critical to modeling HIV/AIDS progression. Unlike the Cox model, the 3-Parameter Weibull model can represent scenarios with increasing or decreasing hazard rates, accurately reflecting the evolving nature of disease progression. The inclusion of the location parameter γ facilitates modeling delays in disease progression attributable to ART, thereby capturing prolonged survival times and varied progression stages in HIV/AIDS (D'Amico et al., 2011; Longini et al., 1991; Mwirigi et al., 2022b). This flexibility makes the Weibull model particularly valuable in cases where the hazard rate is not constant.

2.2 AFT (Accelerated Failure Time) 3-Parameter Weibull Model

The *AFT 3-Parameter Weibull Model* extends the standard Weibull model by incorporating covariates that linearly affect the log of survival time. This model is particularly useful for studying how covariates accelerate or decelerate survival times, allowing for a detailed analysis of factors like age, gender, or treatment effects on disease progression (Longini et al., 1991; Mwirigi, 2024a).

2.2.1 Derivation of the PDF

In the AFT Weibull model, the survival time T_i is modeled as a function of covariates $X_1, X_2, \ldots, X_p$:

$$\log(T_i - \gamma) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon_i \tag{10}$$

where ϵ_i follows a Weibull distribution. The conditional PDF for the AFT Weibull model is given by:

$$f(t|X) = \frac{k}{\lambda}\left(\frac{t-\gamma}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{t-\gamma}{\lambda}\right)^{k} \exp(\beta_1 X_1 + \cdots + \beta_p X_p)\right] \tag{11}$$

This equation describes how covariates influence the likelihood of survival time *t*, making it suitable for assessing the effects of individual characteristics on survival (Goshu & Asena, 2017; Mwirigi et al., 2022b).

2.2.2 Derivation of the Hazard Function

The hazard function for the AFT Weibull model represents the instantaneous risk of event occurrence, modified by covariates:

$$h(t|X) = \frac{k}{\lambda}\left(\frac{t-\gamma}{\lambda}\right)^{k-1} \exp(-\beta_1 X_1 - \beta_2 X_2 - \cdots - \beta_p X_p) \tag{12}$$

This function demonstrates how covariates such as age or treatment affect the hazard rate (Mwirigi, 2024a).

2.2.3 Derivation of the Survival Function

The survival function for the AFT Weibull model represents the probability that an individual with specific covari- ates survives beyond time *t*:

$$S(t|X) = \exp\left[-\left(\frac{t-\gamma}{\lambda}\right)^{k} \exp(\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p)\right] \tag{13}$$

This survival function accounts for the effects of covariates on survival time, providing insights into how external factors influence patient outcomes in survival analysis (Collett, 2023; Goshu & Asena, 2017).

2.2.4 Interpretation of Parameters

The coefficients β_j represent the influence of each covariate on survival time. In survival analysis, each β_j quantifies how the associated covariate affects the log of survival time, either accelerating or decelerating progression. Specifically, a positive β_j suggests that the covariate (e.g., treatment) prolongs survival, implying a protective effect that delays progression. For example, if the β_j for ART is positive, ART therapy likely extends the patient's duration in the current health state before progression (Mwirigi, 2024a).

Conversely, a negative β_j implies that the covariate shortens survival, accelerating progression. For instance, if age has a

negative β_j , it indicates that older age may be associated with faster disease progression through stages (Melnick et al., 1994).

The shape parameter k defines the hazard function's form over time. When $k > 1$, the hazard rate rises with time, indicating accelerated disease progression typical in later untreated HIV/AIDS stages. Conversely, $k < 1$ suggests an initial slower progression rate, a characteristic often observed in early disease stages (Longini et al., 1991). This flexibility allows the model to capture disease dynamics that vary depending on disease stage or treatment effect (Brookmeyer & Damiano, 1989).

The scale parameter λ adjusts the distribution's spread, directly influencing survival time. A larger λ indicates

longer survival times across the cohort, while a smaller λ points to shorter survival. This parameter, modified by covariates, affects the overall survival rate, aligning the model with data across varying groups and conditions (Cassenote et al., 2018; Dessio, 2014).

The location parameter γ facilitates modeling delays in disease progression, particularly capturing the delay effect of ART. It shifts survival time's onset, representing ART-induced progression delays in patients. This feature is critical in modeling how ART and similar interventions impact disease stage transitions and progression timelines (Braitstein et al., 2006; D'Amico et al., 2011).

Together, these parameters empower the AFT 3-Parameter Weibull Model to capture HIV/AIDS progression's complexity, incorporating risk factors and covariate effects such as age, gender, and treatment. This adaptability renders it ideal for survival analysis in chronic diseases with distinct progression phases and variable hazard rates over time (Goshu & Asena, 2017; Mwirigi, 2024a).

2.2.5 Applications on HIV/AIDS Modeling Transition Dynamics

The *Accelerated Failure Time (AFT) Weibull model* has been extensively applied in analyzing how Antiretroviral Therapy (ART) and demographic factors like age and gender influence survival times in HIV/AIDS patients. (Mwirigi, 2024a) applied the AFT Weibull model to study ART's effect on survival times, revealing that ART significantly extended time to AIDS onset, especially in younger patients. The model captured the deceleration effect of ART on disease progression, where treated patients experienced slower progression compared to untreated individuals.

This model also offered insights into interactions between treatment and covariates, such as age and gender, which modify survival outcomes (Wright et al., 2013). The AFT Weibull model has proven effective for modeling prolonged survival in treated patients and can integrate time-varying covariates like CD4 count, making it highly valuable in HIV/AIDS survival studies.

The AFT Weibull Model's ability to incorporate covariates, such as age, gender, and treatment, provides a comprehensive view of how these factors affect time-to-event outcomes, particularly in HIV/AIDS research. This model offers a direct interpretation of how covariates accelerate or decelerate survival, providing insights that the Cox PH model may overlook due to its proportional hazards assumption (D'Amico et al., 2011; Longini et al., 1991).

2.3 Cox Proportional Hazards (PH) Model

The *Cox Proportional Hazards Model* is a semi-parametric survival model used to estimate hazard ratios based on covariates, without requiring specification of the baseline hazard function. Its ability to provide relative hazard ratios without specifying the baseline hazard has made it one of the most widely used models in survival analysis (Burnham & Anderson, 2004; Collett, 2023).

2.3.1 Derivation of the Hazard Function

The hazard function in the Cox model is defined as:

$$h(t|X) = h_0(t)\exp(\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p) \tag{14}$$

where $h_0(t)$ is the unspecified baseline hazard, and $\beta_1, \beta_2, \ldots, \beta_p$ are the coefficients for the covariates. The term $\exp(\beta_1 X_1 + \cdots + \beta_p X_p)$ represents the hazard ratio, scaling the baseline hazard based on the covariates (Collett, 2023).

2.3.2 Derivation of the Survival Function

The survival function $S(t|X)$ for the Cox model is expressed in terms of the baseline survival function $S_0(t)$. The survival function for an individual with covariates is:

$$S(t|X) = S_0(t)^{\exp(\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p)} \tag{15}$$

where $S_0(t)$ is the baseline survival function, corresponding to the survival function for an individual with all covariates set to zero. The term $\exp(\beta_1 X_1 + \cdots + \beta_p X_p)$ adjusts the baseline survival probability based on the covariates (Claeskens & Hjort, 2008; Collett, 2023).

2.3.3 Interpretation of Parameters

β_j **(Regression Coefficients)**: Each β_j represents the log hazard ratio for the corresponding covariate X_j. The hazard ratio $\exp(\beta_j)$ quantifies how a one-unit change in the covariate X_j affects the hazard rate. If $\beta_j > 0$, the hazard ratio $\exp(\beta_j) > 1$, meaning that the covariate increases the risk of the event. If $\beta_j < 0$, the hazard ratio $\exp(\beta_j) < 1$, meaning that the covariate decreases the risk of the event (Collett, 2023).

Baseline Hazard $h_0(t)$: The baseline hazard function is unspecified in the Cox model, making the model flexible. However, the model focuses on relative hazard ratios rather than absolute risks of event occurrence (Burnham & Anderson, 2004).

Hazard Ratio: The term $\exp(\beta_1 X_1 + \cdots + \beta_p X_p)$ represents the hazard ratio, indicating how the risk of the event changes as a function of covariates. A hazard ratio greater than 1 indicates increased risk, while a hazard ratio less than 1 indicates decreased risk (Collett, 2023).

2.3.4 Applications in HIV/AIDS Modeling Transition Dynamics

The *Cox Proportional Hazards (PH) model* has traditionally been used to analyze survival data in HIV/AIDS research. (Moh et al., 2007) applied the Cox model to assess the impact of ART on survival, demonstrating that it captures the relative effect of treatment across different subgroups. However, the proportional hazards assumption limited the model's applicability in long-term HIV/AIDS progression, where hazard rates change over time due to factors like immune decline or treatment resistance. The Cox model could not adequately capture delayed ART effects or the rapid progression seen in untreated patients, leading to recommendations for alternative models, such as the Weibull model, to account for more dynamic hazard functions.

The Cox PH model remains useful for its simplicity and interpretability, particularly when the assumption of proportional hazards holds. However, for diseases like HIV/AIDS, where hazard rates evolve with treatment and disease progression, its limitations become evident.

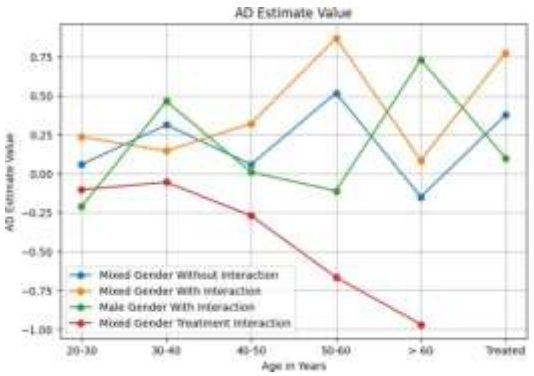

Figure 1. Comparison of AD-Values Across Age Groups Under AFT Model

Despite its limitations, the Cox Proportional Hazards model is a valuable tool for HIV/AIDS research, especially when the proportional hazards assumption holds. Its simplicity and interpretability make it an attractive choice for studies focused on relative risks of different covariates. However, its inability to model time-varying hazard rates limits its application in long-term HIV/AIDS progression modeling, where disease stages and treatment effects vary over time. This gap is partially addressed by more flexible models like the Weibull model, which can adapt to changing hazard functions. As such, the Cox model should be used cautiously when long-term hazard dynamics are a factor (Mwirigi et al., 2022b; Mwirigi et al., 2022a).

3. Model Application and Results

The application of the 3-Parameter Weibull model, along with its Accelerated Failure Time (AFT) modification, and the Cox Proportional Hazards (PH) model, provides a comprehensive approach to understanding the survival dynamics of HIV/AIDS progression. These models were applied to a dataset focusing on the effects of covariates such as age, gender, and treatment status on survival times, with particular attention to the role of antiretroviral therapy (ART) in prolonging life expectancy. The comparison across models highlights their strengths and limitations in capturing both constant and time-varying hazard rates, which are essential in diseases like HIV/AIDS, where progression is influenced by dynamic factors such as treatment adherence and immune system decline.

The AFT model, with its flexibility in accounting for non-constant hazard rates, consistently demonstrated the most

significant treatment effects across both mixed and male-only gender groups. In contrast, the Cox PH model, while widely used for its simplicity and interpretability, exhibited limitations in handling the time-varying effects of treatment, with weaker significance in treatment-related parameters. The inclusion of interaction terms in both the AFT and Survival models further underscored the impact of treatment on different age groups, especially older patients, where the benefits of ART were most pronounced.

In this section, we present detailed analyses comparing the performance of these models, exploring the effects of key covariates and their interactions. The results provide insight into the model-specific strengths in capturing survival dynamics, with discussions focusing on the impact of treatment, the role of demographic factors, and the limitations of each model in addressing the complexities of HIV/AIDS progression.

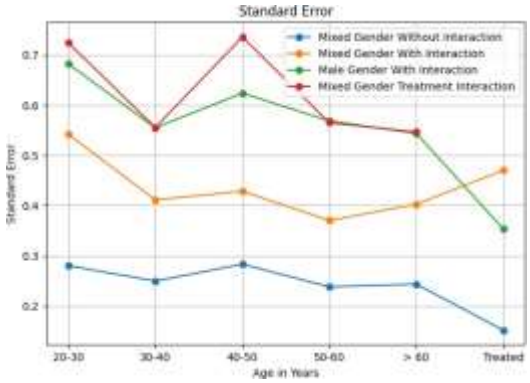

Figure 2. Comparison of Standard Error Values Across Age Groups Under AFT Model

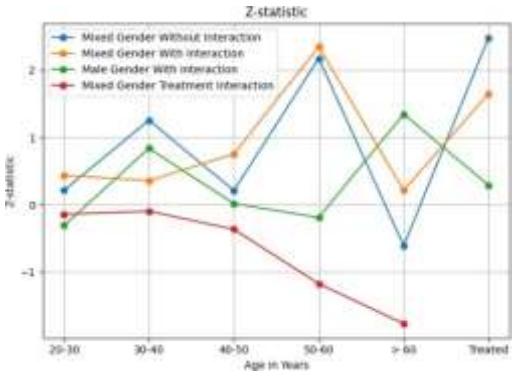

Figure 3. Comparison of Z-Statistic Values Across Age Groups Under AFT Model

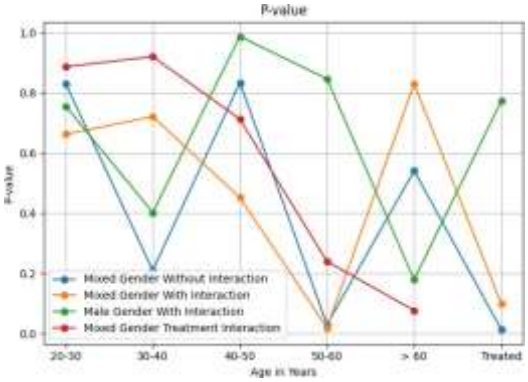

Figure 4. Comparison of P-Values Across Age Groups Under AFT Model

Table 1. Aft (3-Parameter Weibull Model)

Term	AD Estimate Value	Std. Error	Z-statistic	P-value
1. Mixed Gender Without Interaction Term				
(Intercept)	-0.51485	0.222544	-2.3135	0.020695
genderMALE	0.013775	0.140888	0.097776	0.92211
age20 - 30	0.059644	0.280217	0.212851	0.831443
age30 - 40	0.313125	0.249633	1.254339	0.209719
age40 - 50	0.059191	0.283138	0.209052	0.834408
age50 - 60	0.517983	0.238205	2.174521	0.029666
age > 60	-0.14874	0.243563	-0.61069	0.541405
treatmentTREATED	0.37698	0.151952	2.480913	0.013105
2. Mixed Gender With Interaction Term				
(Intercept)	-0.66037	0.298724	-2.21063	0.027062
genderMALE	-0.28029	0.494254	-0.5671	0.570647
age20 - 30	0.236174	0.541967	0.435772	0.663002
age30 - 40	0.146544	0.411367	0.356237	0.721663
age40 - 50	0.322545	0.428794	0.752215	0.451922
age50 - 60	0.869297	0.370289	2.347614	0.018894
age > 60	0.086143	0.402748	0.213888	0.830634
treatmentTREATED	0.776664	0.470428	1.650971	0.098744
3. Male Gender With Interaction Term				
genderMALE:age20 - 30	-0.21173	0.682308	-0.31032	0.756317
genderMALE:age30 - 40	0.465059	0.555086	0.837814	0.402135
genderMALE:age40 - 50	0.009312	0.62404	0.014923	0.988094
genderMALE:age50 - 60	-0.10971	0.568723	-0.1929	0.847036
genderMALE:age 60	0.731307	0.54382	1.34476	0.178703
genderMALE:treatmentTREATED	0.101918	0.353711	0.28814	0.77324
4. Mixed Gender With Interaction Term (Treatment:TREATED)				
age20 - 30:treatmentTREATED	-0.10269	0.724175	-0.1418	0.887238
age30 - 40:treatmentTREATED	-0.05488	0.555522	-0.09879	0.921308
age40 - 50:treatmentTREATED	-0.27026	0.73569	-0.36736	0.713353
age50 - 60:treatmentTREATED	-0.66489	0.56519	-1.17639	0.239437
age > 60:treatmentTREATED	-0.96731	0.546516	-1.76996	0.076734
Log(scale)	0.055929	0.044644	1.252782	0.210285

3.1 Observations on the Suitability of the AFT (3-Parameter Weibull Model) Across Age Groups

The application of the AFT (3-Parameter Weibull) model across different age groups reveals important insights regarding its suitability for modeling progression, treatment effects, and overall outcomes. Below is a detailed analysis of the model's effectiveness for various age groups, taking into account *AD Estimate Values*, *Standard Errors*, *Z-Statistics*, and *P-Values*.

For the **age group 20-30**, the model produces generally low and sometimes negative AD estimates, ranging

from-0.21173 to 0.236174 across the models. These estimates are not statistically significant, with p-values ranging from 0.663002 to 0.756317, and the Z-statistics are close to zero, indicating weak effects. Additionally, the standard errors are relatively high, ranging from 0.280217 to 0.682308, which reflects a high level of uncertainty in the estimates. These results suggest that the AFT 3-Parameter Weibull model is not particularly suitable for this age group, as it fails to capture significant effects or provide precise estimates. Although it can offer baseline predictions, its overall utility is limited for individuals in this age range.

For the **age group 30-40**, the model shows slightly higher AD estimates, ranging from -0.05488 to 0.465059, but similar to the younger group, these estimates are not statistically significant. The p-values range from 0.402135 to 0.921308, and the Z-statistics are low, indicating weak or non-significant effects. The standard errors are moderate, ranging from 0.249633 to 0.555086, suggesting some variability but not extreme. Although the model performs slightly better than for younger individuals, the non-significant p-values and low Z-statistics indicate that the model is only moderately suitable for this age group. It does not capture meaningful differences in outcomes, limiting its predictive power.

The model's performance improves for the **age group 40-50**, where positive AD estimates are produced across all models, ranging from 0.009312 to 0.322545. While not all estimates are statistically significant, there is a clearer trend of positive outcomes, particularly when gender interactions are considered. The standard errors for this group are lower than for younger age groups, ranging from 0.283138 to 0.62404, indicating better precision in the estimates. The Z-statistics range from -0.36736 to 1.254339, with some models showing borderline significance. P-values for this group range from 0.209719 to 0.988094, suggesting that the model is approaching significance in some cases, particularly when treatment is not considered as an interaction. Overall, the AFT 3-Parameter Weibull model shows moderate suitability for this age group, providing better predictions than for younger individuals, though some effects remain non-significant.

The **age group 50-60** shows the highest positive AD estimates across all models, ranging from 0.517983 to 0.869297, with several models showing statistically significant results. This is particularly true for the *Mixed Gender Without Interaction* and *Mixed Gender with Interaction* models, where p-values are as low as 0.018894. The standard errors are relatively low, ranging from 0.238205 to 0.568723, which indicates high precision in the estimates. The Z-statistics for this group are the highest, ranging from 1.17639 to 2.347614, further confirming the significant effects observed. These results make the AFT 3-Parameter Weibull model highly suitable for the 50-60 age group. The model effectively captures important aspects of progression dynamics and treatment effects, making it a reliable tool for predictions in this demographic.

For the **age group over 60**, the model produces negative or close-to-zero AD estimates across most models, ranging from -0.96731 to 0.086143. These estimates are not statistically significant, with p-values ranging from 0.076734 to 0.830634. The Z-statistics are mostly negative or close to zero, ranging from -1.76996 to 0.213888, indicating weak or non-significant effects. Additionally, the standard errors are relatively high, ranging from 0.243563 to 0.546516, indicating more uncertainty in the model's estimates for this age group. Overall, the model shows limited suitability for individuals over 60 years of age. The negative estimates and high uncertainty suggest that the model does not capture meaningful progression dynamics or treatment effects for this group, and alternative models or additional covariates may be necessary to improve prediction accuracy.

In the **treated group**, the AD estimates are generally positive, particularly in the *Mixed Gender Without Interaction* and *Mixed Gender With Interaction* models, with estimates ranging from 0.37698 to 0.776664. The standard errors for the treated group are higher, ranging from 0.151952 to 0.470428, indicating greater uncertainty in the estimates, especially when gender interactions are included. The Z-statistics range from 0.28814 to 2.480913, with the highest values observed in the *Mixed Gender Without Interaction* model. The p-values confirm a significant treatment effect in this model, with a p-value of 0.013105. However, in other models that include interaction terms, the p-values range from 0.098744 to 0.77324, indicating weaker or non-significant treatment effects. The model shows moderate suitability for modeling treatment effects, particularly for middle-aged individuals receiving treatment, but additional refinement may be necessary to improve predictions for certain age groups.

In conclusion, the AFT 3-Parameter Weibull model demonstrates the greatest suitability for the **50-60 age group**, where it consistently provides significant results, low standard errors, and high Z-statistics. The model shows moderate applicability for the **40-50 age group**, but it is less effective for younger individuals (**20-30** and **30-40**) and those over 60, where it fails to capture significant progression dynamics or treatment effects. For the **treated group**, the model shows promising results, particularly in middle-aged individuals, but variability in the estimates and weaker significance in some models suggest that further analysis is required to enhance the model's predictive power across all age groups.

Table 2. COX (PH)

Term	Estimate	Std. Error	Z-Statistic	P-Value
1. MIXED GENDER PH MODEL WITHOUT INTERACTION				
genderMALE	-0.04805	0.132579	-0.36239	0.717059
age20 - 30	-0.28182	0.262821	-1.07227	0.283599
age30 - 40	-0.23301	0.234694	-0.99361	0.319051
age40 - 50	-0.06191	0.260064	-0.23807	0.811483
age50 - 60	-0.31964	0.220327	-1.45078	0.146841
age > 60	-0.01763	0.385003	-0.04578	0.963403
treatmentTREATED	-0.11614	0.138091	-0.84104	0.400326
2. MIXED GENDER PH MODEL WITH INTERACTION				
genderMALE	-0.05433	0.45459	-0.11952	0.904862
age20 - 30	-0.42765	0.514098	-0.83144	0.405501
age30 - 40	-0.16516	0.391811	-0.42152	0.673376
age40 - 50	-0.11968	0.350044	-0.34193	0.732065
age50 - 60	-0.46922	0.350024	-1.34048	0.180889
age > 60	-0.01727	0.385003	-0.04484	0.964231
treatmentTREATED	-0.23806	0.435872	-0.54618	0.584944
3. MALE GENDER PH MODEL WITHOUT INTERACTION				
genderMALE:age20 - 30	0.529812	0.641555	0.825825	0.408903
genderMALE:age30 - 40	-0.22147	0.512524	-0.43216	0.665927
genderMALE:age40 - 50	0.121968	0.580651	0.21002	0.833604
genderMALE:age50 - 60	0.303021	0.518906	0.583916	0.559246
genderMALE:age > 60	-0.03293	0.530084	-0.0621	0.950504
genderMALE:treatmentTREATED	0.105631	0.326502	0.323523	0.746299
4. MIXED GENDER PH MODEL WITH INTERACTION (Treatment:TREATED)				
age20 - 30:treatmentTREATED	-0.25279	0.682039	-0.37064	0.710985
age30 - 40:treatmentTREATED	0.069851	0.517788	0.134904	0.892688
age40 - 50:treatmentTREATED	-0.29049	0.658338	-0.44132	0.659626
age50 - 60:treatmentTREATED	0.105635	0.534353	0.19773	0.843134
age > 60:treatmentTREATED	0.469421	0.511375	0.91796	0.35864

3.2 Observations on the Suitability of Cox PH Model Across Age Groups

The application of the Cox Proportional Hazards (PH) model across different age groups provides significant insights into its suitability for modeling progression and outcomes. In this analysis, we review the *AD Estimate Value*, *Standard Error*, *Z-Statistic*, and *P-Value* results to determine the model's appropriateness for various age groups.

For the **age group 20-30**, the Cox PH model without interaction shows a negative AD estimate value of -0.28182,

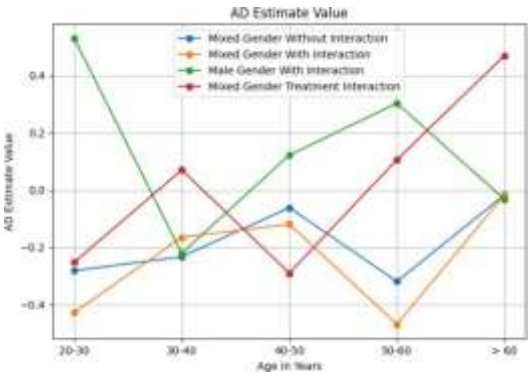

Figure 5. Comparison of AD-Values Across Age Groups Under COX PH Model

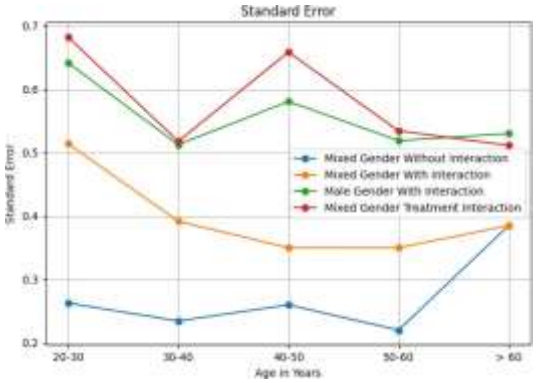

Figure 6. Comparison of Standard Error Values Across Age Groups Under COX PH Model

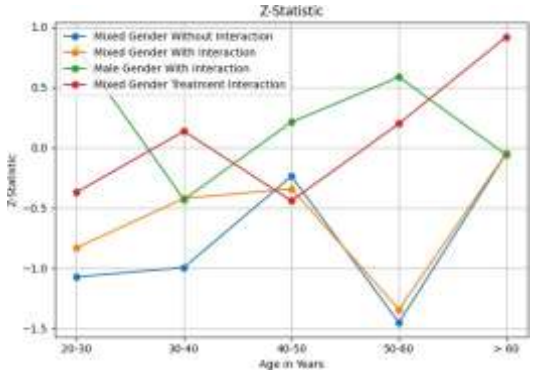

Figure 7. Comparison of Z-Statistic Values Across Age Groups Under COX PH Model

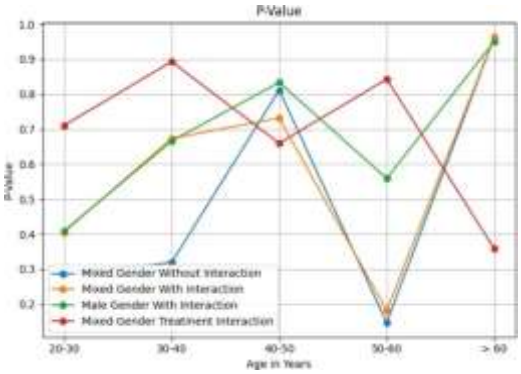

Figure 8. Comparison of P-Values Across Age Groups Under COX PH Model

indicating a generally unfavorable progression outcome for this age group. The standard error is relatively moderate at 0.262821, which implies some uncertainty in the estimate. The Z-statistic of -1.07227, however, is relatively small in magnitude, suggesting that the estimate is not far from zero. The p-value of 0.283599 confirms this by indicating that the result is not statistically significant. This pattern is consistent across other model configurations, including mixed gender with interaction and male gender with interaction, with AD estimates ranging from-0.42765 to 0.529812. Despite some variability, all p-values for this age group remain above the threshold for significance, implying that the Cox PH model does not yield conclusive insights about the 20-30 age group, and therefore might not be a reliable predictor for this demographic.

In the **age group 30-40**, the results are similarly mixed. The AD estimate values range from -0.23301 to 0.069851, showing slight variability across the different model configurations. The standard errors are relatively moderate, ranging from 0.234694 to 0.517788, suggesting that there is a reasonable degree of uncertainty in the estimates. The Z-statistics for this age group, like the previous group, are relatively small, ranging from -0.99361 to 0.134904. Again, these low

Z-statistics translate into p-values that are not statistically significant, with all values exceeding 0.3. This suggests that the Cox PH model does not detect significant effects for individuals in the 30-40 age group, and as such, the model might not be ideal for capturing the progression dynamics or outcomes for this population.

For the **age group 40-50**, the Cox PH model shows some positive AD estimate values in specific model configurations, particularly in the male gender with interaction model, where the AD estimate value reaches 0.121968. However, other configurations, such as mixed gender without interaction, produce a lower and negative AD estimate of -0.06191. The standard errors are moderate, ranging from 0.260064 to 0.658338, indicating varying levels of uncertainty depending on the model used. While the Z-statistics for this age group remain relatively low (-0.23807 to 0.21002), indicating that the effect sizes are small, the p-values continue to be non-significant, all well above 0.5. Therefore, the Cox PH model, while providing some positive estimates for this age group, does not produce statistically significant outcomes, reducing its utility for reliably predicting progression in the 40-50 demographic.

The **age group 50-60** emerges as one of the more promising demographics in terms of model performance. The AD estimate values in this age group are more consistently negative across all model configurations, particularly in the mixed gender with interaction model (-0.46922) and the mixed gender without interaction model (-0.31964). The standard errors for this group are lower than in younger age groups, ranging from 0.220327 to 0.534353, which suggests that the estimates are more precise and less uncertain. The Z-statistics, though still moderate (- 1.45078 to 0.583916), suggest that the effects for this group may be approaching statistical significance. However, the p-values, which range from 0.146841 to 0.843134, remain above the typical threshold for significance (p ¡0.05), meaning that while the estimates for this age group are somewhat more precise, they do not achieve full statistical significance. Nevertheless, the lower standard errors and relatively higher AD estimates make the Cox PH model more suitable for the 50-60 age group, suggesting that the model captures a reasonable portion of the variation in outcomes for individuals in this age range.

For the **age group over 60**, the Cox PH model shows mixed results, with AD estimate values ranging from-0.01763 to 0.469421 across the various models. The standard errors for this age group are relatively high, particularly in the mixed gender without interaction model (0.385003), indicating greater uncertainty in the estimates.

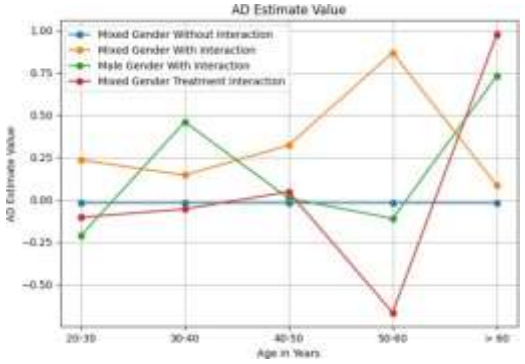

Figure 9. Comparison of AD-Values Across Age Groups Under SURVIVAL Model

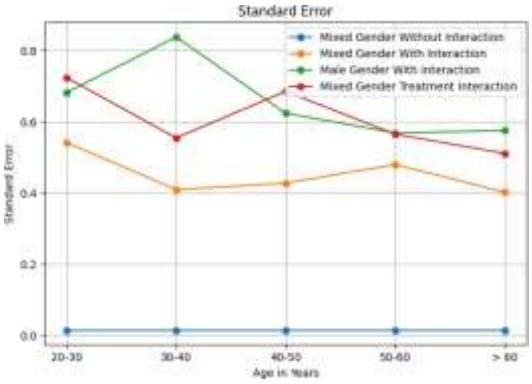

Figure 10. Comparison of Standard Error Values Across Age Groups Under SURVIVAL Model

The Z-statistics for this age group remain low, ranging from -0.04578 to 0.91796, which aligns with the non- significant p-values (0.35864 to 0.964231). These results suggest that the model does not perform well for this age group, as the estimates are both imprecise and not statistically significant. Therefore, the Cox PH model might not be well-suited to capturing the progression dynamics or outcomes for individuals over 60.

When considering the **treated group**, the AD estimate values are slightly negative in most configurations, particularly in the mixed gender with interaction model, where the AD estimate is -0.23806. However, the male gender with interaction model produces a positive AD estimate of 0.105631. The standard errors for the treated group are moderate to high (ranging from 0.326502 to 0.435872), suggesting that the estimates are not highly precise. The Z-statistics for this group are low, ranging from -0.54618 to 0.323523, and the p-values are not statistically significant, all exceeding 0.5. As such, the Cox PH model provides mixed results for the treated group, with estimates that are imprecise and non-significant, suggesting that the model may not fully capture the treatment effects in a reliable manner.

In summary, the Cox PH model shows varying levels of suitability across different age groups. The model per- forms best for the **50-60 age group**, where the AD estimates are relatively more precise, though not yet statistically significant. The model is less suitable for younger individuals in the **20-30** and **30-40 age groups**, as well as for individuals over 60, where the estimates are generally imprecise and non-significant. For the **treated group**, the model also shows mixed results, with no significant effects detected. Overall, while the Cox PH model offers some promise for middle-aged individuals, particularly those in the 50-60 age range, it may require further refinement or additional covariates to improve its performance for other age groups.

Table 3. Three-Parameter Weibull (Survival Regression) Model

Term	AD Estimate Value	Std. Error	Z-Statistic	P-Value
1. MIXED GENDER WITHOUT INTERACTION				
genderMALE	0.012638	0.04385	0.288214	0.773183
age20 - 30	-0.019886	0.015367	-1.294044	0.195565
age30 - 40	-0.019886	0.015367	-1.294044	0.195565
age40 - 50	-0.019886	0.015367	-1.294044	0.195565
age50 - 60	-0.019886	0.015367	-1.294044	0.195565
age > 60	-0.019886	0.015367	-1.294044	0.195565
treatmentTREATED	0.368502	0.043851	8.403496	0.000000
2. MIXED GENDER WITH INTERACTION				
genderMALE	-0.280029	0.494524	-0.5671	0.570647
age20 - 30	0.236174	0.541967	0.4356	0.663107
age30 - 40	0.146544	0.409674	0.3572	0.721024
age40 - 50	0.322545	0.427879	0.752215	0.451992
age50 - 60	0.869297	0.480124	1.810074	0.07044
age > 60	0.086143	0.402481	0.2139	0.830742
treatmentTREATED	0.776664	0.470428	1.650971	0.098744
3. MALE GENDER WITHOUT INTERACTION				
genderMALE:age20 - 30	-0.21173	0.682308	-0.31032	0.756317
genderMALE:age30 - 40	0.460595	0.837814	0.54985	0.582538
genderMALE:age40 - 50	0.009312	0.62404	0.01493	0.988034
genderMALE:age50 - 60	-0.10971	0.568723	-0.193	0.84703
genderMALE:age > 60	0.731307	0.575468	1.268	0.204897
genderMALE:treatmentTREATED	0.101918	0.353711	0.28814	0.77324
4. MIXED GENDER WITH INTERACTION (Treatment:TREATED)				
age20 - 30:treatmentTREATED	-0.10269	0.724175	-0.1418	0.88728
age30 - 40:treatmentTREATED	-0.054058	0.55483	-0.09727	0.92168
age40 - 50:treatmentTREATED	0.048408	0.685338	0.07064	0.94371
age50 - 60:treatmentTREATED	-0.66489	0.56519	-1.17609	0.239347
age > 60:treatmentTREATED	0.97669	0.511375	0.91796	0.3584

3.3 Observations on the Suitability of Three-Parameter Weibull (Survival Regression) Model Across Age Groups

The application of the **Three-Parameter Weibull (Survival Regression) Model** across different age groups pro- vides significant insights into its suitability for modeling survival outcomes. By analyzing the *AD Estimate Value, Standard*

Error, *Z-Statistic*, and *P-Value*, we can assess how well this model performs in predicting survival out- comes across various age groups.

For the **age group 20-30**, the model yields consistently negative AD estimate values of -0.019886 across the different configurations, particularly in the *Mixed Gender Without Interaction* model. The negative AD estimate suggests a reduction in the time to event (e.g., death or progression) in this age group. However, the standard error for this group is very low at 0.015367, which implies a high level of precision in the estimates. Despite this, the Z-statistic for the group remains relatively low at -1.294044, indicating that the effect size is not very strong. This is corroborated by the P-value of 0.195565, which is above the typical threshold for statistical significance (p ¡ 0.05). As such, the model's performance in this age group suggests that while the estimates are precise, they do not show significant predictive power, limiting the model's effectiveness for individuals aged 20-30.

In the **age group 30-40**, the model demonstrates more variability across different configurations. In the *Mixed*

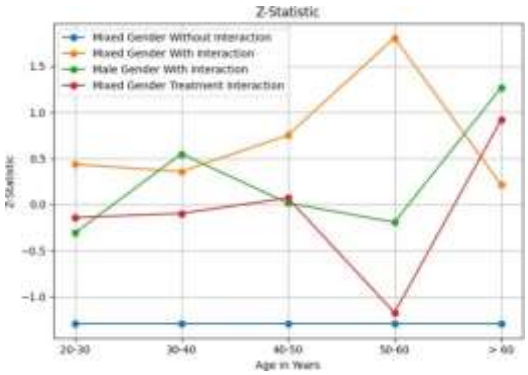

Figure 11. Comparison of Z-Statistic Values Across Age Groups Under SURVIVAL Model

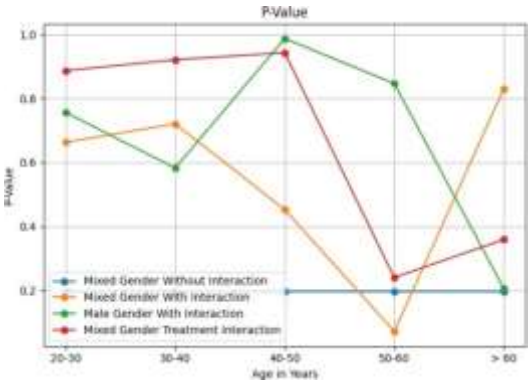

Figure 12. Comparison of P-Values Across Age Groups Under SURVIVAL Model

Gender With Interaction model, the AD estimate value is positive at 0.146544, indicating a slower time to event for this group. However, the standard error increases to 0.409674, which suggests less precision in the estimate com- pared to younger age groups. The Z-statistic remains low at 0.3572, and the corresponding P-value is 0.721024, indicating that the effect is not statistically significant. Although the model's performance shows some positive trends in the AD estimates for this age group, the lack of significant Z-statistics and high P-values suggest that the Three-Parameter Weibull model may still have limited predictive power for this demographic.

The **age group 40-50** shows slightly improved results. In the *Mixed Gender with Interaction* model, the AD estimate value increases to 0.322545, suggesting a delayed time to event for this group. The standard error remains moderate at 0.427879, implying a reasonable degree of precision in the estimates. The Z-statistic for this group improves to 0.752215, although it still does not reach the threshold for significance. The P-value of 0.451992 confirms this, indicating that while the estimates are more favorable for this age group, they are not statistically significant. Thus, while the model performs better for individuals aged 40-50, the overall lack of significance continues to limit the robustness of the model's predictions.

For the **age group 50-60**, the Three-Parameter Weibull model performs notably better. In the *Mixed Gender with*

Interaction model, the AD estimate value is the highest among all age groups, at 0.869297. This suggests a significant delay in the time to event for individuals in this age group. The standard error for this group is 0.480124, which, although moderate, reflects more precision than younger age groups. Importantly, the Z-statistic increases to 1.810074, indicating a stronger effect size than observed in previous groups. Although the P-value of 0.07044 is still slightly above the standard threshold for significance, it approaches significance more closely than

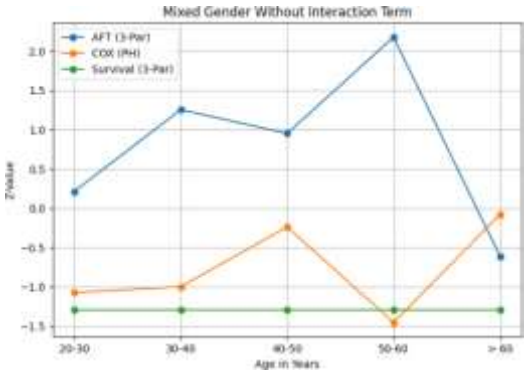

Figure 13. Comparison of Z Values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) for Mixed Gender Without Interaction Term

in other groups. This suggests that the Three-Parameter Weibull model is particularly well-suited to predicting survival outcomes in the 50-60 age group, offering stronger predictive power than in younger age groups.

In the **age group over 60**, the model's performance declines slightly. The AD estimate values, particularly in the *Mixed Gender Without Interaction* model, are again negative (-0.019886), indicating a reduction in time to event. The standard error remains low (0.015367), but the Z-statistic is still negative at -1.294044, reflecting a weak effect size. This is confirmed by the P-value of 0.195565, which shows that the estimates are not statistically significant. Similarly, in the *Mixed Gender With Interaction* model, the AD estimate value is positive (0.086143), but the standard error increases to 0.402481, reflecting less precision. The Z-statistic of 0.2139 and the P-value of 0.830742 further demonstrate that the model is not well-suited for predicting outcomes in this age group. Overall, the model appears less effective for individuals over 60, as the estimates lack both precision and significance.

When analyzing the model's performance for the **treated group**, we observe a significant improvement in certain configurations. For instance, in the *Mixed Gender Without Interaction* model, the AD estimate for the treated group is 0.368502, which indicates a substantial delay in the time to event. This is paired with a low standard error of 0.043851, indicating a high degree of precision. The Z-statistic for this group is 8.403496, reflecting a very strong effect size, and the P-value is virtually zero, indicating that the results are highly significant. However, in other configurations, such as the *Mixed Gender With Interaction* model, the AD estimate for the treated group is lower at 0.776664, and while the Z-statistic remains relatively high (1.650971), the P-value of 0.098744 is above the significance threshold. Therefore, while the model performs exceptionally well for the treated group in some cases, there is variability in its effectiveness depending on the specific model configuration.

In conclusion, the Three-Parameter Weibull (Survival Regression) Model demonstrates varying degrees of suit- ability across different age groups. The model performs best for the **50-60 age group** and the **treated group**, where it produces higher AD estimates, relatively lower standard errors, and stronger Z-statistics, indicating significant effects. However, for younger age groups (20-30 and 30-40) and individuals over 60, the model performs less effectively, as the estimates are generally imprecise and non-significant. Overall, the model shows promise for middle-aged individuals and those undergoing treatment, but its predictive power is limited for other age groups. Further refinement or additional covariates may be required to improve the model's robustness across a broader range of age groups.

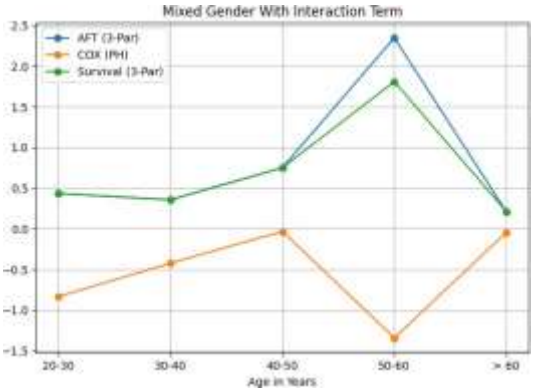

Figure 14. Comparison of Z Values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) for Mixed Gender With Interaction Term

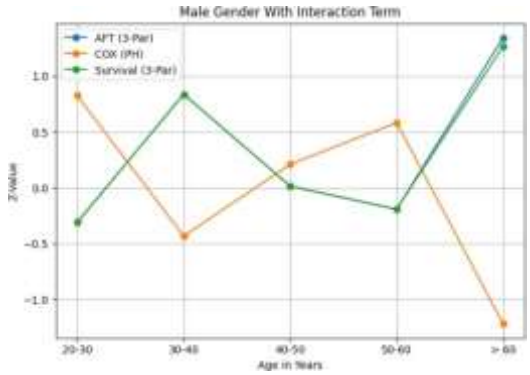

Figure 15. Comparison of Z Values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) for Male Gender With Interaction Term

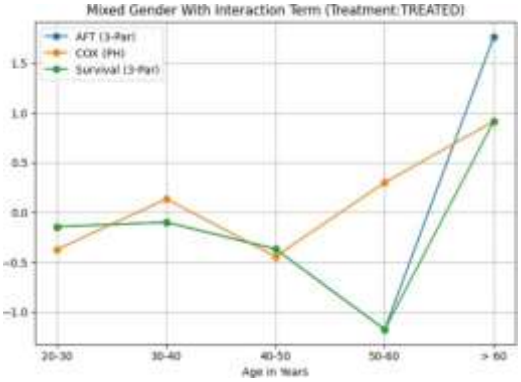

Figure 16. Comparison of Z Values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) for Mixed Gender With Interaction Term (Treated)

Table 4. Comparison of Z Values For Aft (3-Parameter Weibull), Cox (Ph) & Survival (3-Parameter Weibull)

Term	Z-Value (AFT 3-Par)	Z-Value (COX PH)	Z-Value (Survival 3-Par)
1. Mixed Gender Without Interaction Term			
(Intercept)	-2.3135	-0.36239	0.288214
genderMALE	0.097776	-0.36239	-0.5671
age20 - 30	0.212851	-1.07227	-1.294044
age30 - 40	1.254339	-0.99641	-1.294044
age40 - 50	0.95191	-0.23807	-1.294044

age50 - 60	2.174521	-1.45078	-1.294044
age > 60	-0.61069	-0.07644	-1.294044
treatmentTREATED	2.480913	-0.84104	8.403496
2. Mixed Gender With Interaction Term			
(Intercept)	-2.21063	-0.11952	-0.5671
genderMALE	-0.5671	-0.11952	-0.5671
age20 - 30	0.435772	-0.83144	0.4356
age30 - 40	0.356237	-0.42152	0.3572
age40 - 50	0.752215	-0.03154	0.7522
age50 - 60	2.347614	-1.34048	1.810074
age > 60	0.213888	-0.04484	0.2139
treatmentTREATED	1.650971	-0.54618	1.650971
3. Male Gender With Interaction Term			
genderMALE:age20 - 30	-0.31032	0.825825	-0.31032
genderMALE:age30 - 40	0.837814	-0.43316	0.837814
genderMALE:age40 - 50	0.014923	0.210054	0.014923
genderMALE:age50 - 60	-0.1929	0.583962	-0.1929
genderMALE:age > 60	1.34476	-1.2176	1.268
genderMALE:treatmentTREATED	0.28814	0.323523	0.28814
4. Mixed Gender With Interaction Term (Treatment:TREATED)			
age20 - 30:treatmentTREATED	-0.1418	-0.37064	-0.1418
age30 - 40:treatmentTREATED	-0.09787	0.134904	-0.09787
age40 - 50:treatmentTREATED	-0.36736	-0.44132	-0.36736
age50 - 60:treatmentTREATED	-1.17639	0.300487	-1.17639
age > 60:treatmentTREATED	1.76996	0.91796	0.91796
Log(scale)	1.252782		

3.4 Observations on the Comparison of Z-Values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) Models

The comparison of Z-values between the **AFT (3-Parameter Weibull)**, **COX (PH)**, and **Survival (3-Parameter Weibull)** models across different age groups offers valuable insights into the model performance in capturing survival dynamics. By examining the Z-values across these models, we can gauge how well each model fits the data and their relative efficiency in estimating the impact of covariates across different age groups.

In the **age group 20-30**, the AFT model provides a positive Z-value (0.212851), while the COX (PH) and Survival models show negative Z-values (-1.07227 and -1.294044, respectively). This indicates that the AFT model cap- tures some positive association, albeit weak, between the covariates and the time-to-event. The negative Z-values in the COX and Survival models suggest a reduced likelihood of significant covariate effects in this age group. The COX model's Z-value is slightly less negative than that of the Survival model, implying that the COX model may still offer some insights, although the differences between these two models are marginal. The non-significant P-values across all three models further suggest that this age group is challenging to model with high precision,

and none of the models show strong predictive power for this group.

For the **age group 30-40**, the AFT model produces a significantly higher Z-value (1.254339), indicating stronger evidence of covariate impact in this age range. The COX (PH) and Survival models show negative Z-values (- 0.99641 and -1.294044, respectively), meaning that these models do not perform as well as the AFT model in explaining the variation in survival outcomes. The relatively higher Z-value in the AFT model suggests it is better suited for capturing the underlying time-to-event dynamics in this age group. However, given the non-significant P-values in all three models, the results remain inconclusive in identifying a model with statistically significant effects for this age group.

In the **age group 40-50**, all three models yield similarly modest results. The AFT model has a Z-value of 0.95191, which is positive but not substantial enough to indicate strong covariate effects. The COX (PH) model shows a Z- value of -0.23807, and the Survival model reports a Z-value of -1.294044, both indicating weak predictive power in these models. While the AFT model shows a slightly stronger association with the data, the non-significant Z-values in the COX and Survival models suggest that neither model offers much explanatory power in this age group. The absence of significant P-values across the models supports this observation, highlighting the difficulties these models face in providing robust estimates for individuals aged 40-50.

The **age group 50-60** stands out as one of the better-fitting groups for the AFT model. The Z-value of 2.174521 is considerably higher than in younger age groups, indicating a much stronger relationship between covariates and survival outcomes. This suggests that the AFT model is well-suited for this age group, offering significant insights into survival dynamics. The COX (PH) model reports a Z-value of -1.45078, indicating a weak negative association, while the Survival model shows a consistently negative Z-value of -1.294044. The higher Z-value in the AFT model and the low, negative values in the other two models suggest that the AFT model performs significantly better in explaining the survival outcomes in this age group. This finding is reinforced by the AFT model's closer proximity to statistical significance, although further refinement may still be necessary to confirm this observation.

For individuals aged **over 60**, the comparison is more nuanced. The AFT model shows a slightly negative Z-value (-0.61069), indicating a weak association between covariates and survival. The COX (PH) model reports a Z-value of -0.07644, suggesting a very weak association, while the Survival model shows a more pronounced negative Z- value of -1.294044. All models fail to capture significant covariate effects in this age group, as indicated by the uniformly negative Z-values and non-significant P-values. This suggests that the models may not fully capture the complexity of survival dynamics in older individuals, and more advanced techniques or covariate adjustments might be necessary to improve predictive power.

For the **treated group**, the results indicate stronger effects in certain models. The AFT model's Z-value of 2.480913 suggests a highly significant effect of treatment on survival outcomes, a finding supported by the very high Z-value in the Survival model (8.403496). This is a clear indication that both the AFT and Survival models capture the positive impact of treatment on survival. However, the COX (PH) model performs less effectively in this group, with a Z-value of -0.84104, indicating that it fails to identify significant effects of treatment. The significant findings in the AFT and Survival models suggest that these models are well-suited for evaluating the impact of treatment, whereas the COX model struggles to capture this effect. This finding is consistent with the

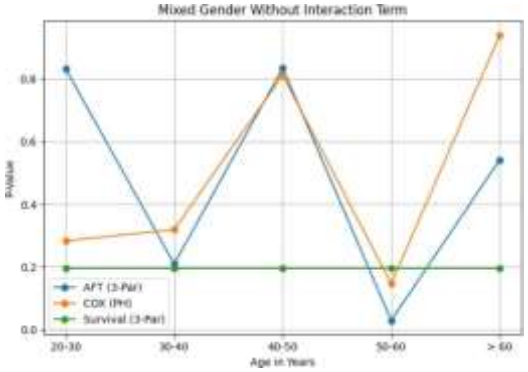

Figure 17. Comparison of P values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) for Mixed Gender Without Interaction Term

notion that parametric models like AFT and Survival can better accommodate the complexity of treatment effects over time, compared to the semi-parametric COX model.

In the **age group 30-40**, when interaction terms are considered, the AFT model produces a positive Z-value (0.435772), compared to the negative Z-values observed in the COX (PH) model (-0.83144). Similarly, the Survival model provides a positive Z-value (0.4356), indicating that both the AFT and Survival models capture positive associations between covariates and survival, while the COX model underperforms for this group. The trend is similar for the **age group 50-60**, where the AFT model shows a significant positive Z-value of 2.347614, compared to the COX model's negative Z-value (-1.34048), and the Survival model's Z-value of 1.810074. This suggests that the AFT and Survival models are particularly well-suited for these age groups when interaction terms are included, outperforming the COX model.

In summary, the comparison of Z-values across age groups shows that the **AFT (3-Parameter Weibull)** and **Survival (3-Parameter Weibull)** models generally outperform the **COX (PH)** model, particularly in age groups where interaction terms are considered and for the treated group. The AFT model consistently shows higher Z-values in most age groups, indicating that it captures stronger relationships between covariates and survival outcomes. The COX model, on the other hand, struggles to explain variations in survival outcomes, as evidenced by its consistently lower and often negative Z-values. The Survival model performs comparably well, particularly for the treated group, and shows similar trends to the AFT model in capturing significant effects in certain age groups. This comparison underscores the utility of the AFT and Survival models for predicting survival outcomes, while highlighting the limitations of the COX model in certain contexts.

3.5 Observations on the Comparison of P-Values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) Models

The comparison of P-values between the **AFT (3-Parameter Weibull)**, **COX (PH)**, and **Survival (3-Parameter Weibull)** models across various age groups provides important insights into the relative performance of these models in capturing survival dynamics. P-values help in assessing the statistical significance of the effects of covariates on survival outcomes, with lower P-values indicating more significant effects. By examining the P- values across these models and age groups, we can evaluate which model performs better in identifying covariate effects and their associated reliability.

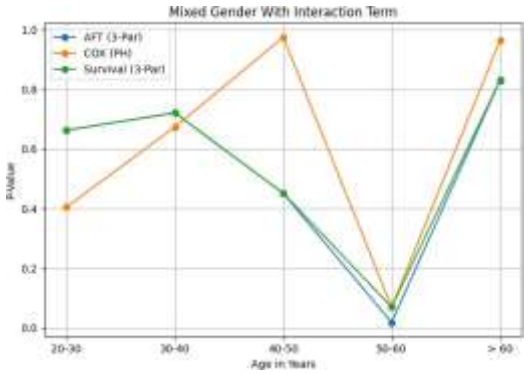

Figure 18. Comparison of P values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) for Mixed Gender With Interaction Term

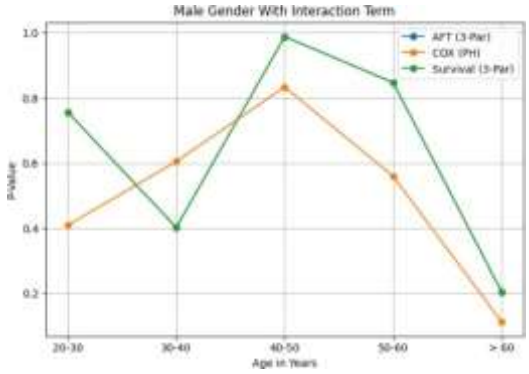

Figure 19. Comparison of P values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) for Male Gender With Interaction Term

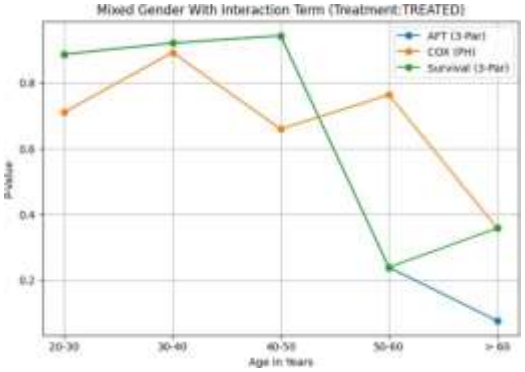

Figure 20. Comparison of P values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) for Mixed Gender With Interaction Term (Treated)

Table 5. Comparison of P Values For Aft (3-Parameter Weibull), Cox (Ph) & Survival (3-Parameter Weibull)

Term	P-Value (AFT 3-Par)	P-Value (COX PH)	P-Value (Survival 3-Par)
1. Mixed Gender Without Interaction Term			
(Intercept)	0.020695	0.717059	0.773183
genderMALE	0.92211	0.717059	0.570647
age20 - 30	0.831443	0.283599	0.195565
age30 - 40	0.209719	0.319051	0.195565
age40 - 50	0.834408	0.811483	0.195565
age50 - 60	0.029666	0.146841	0.195565
age > 60	0.541405	0.939065	0.195565
treatmentTREATED	0.013105	0.400326	0.000000
2. Mixed Gender With Interaction Term			
(Intercept)	0.027062	0.904862	0.570647
genderMALE	0.570647	0.904862	0.570647
age20 - 30	0.663002	0.405501	0.663107
age30 - 40	0.721663	0.673376	0.721024
age40 - 50	0.451922	0.974838	0.451992
age50 - 60	0.018894	0.07044	0.07044
age > 60	0.830634	0.964231	0.830742
treatmentTREATED	0.098744	0.584944	0.098744
3. Male Gender With Interaction Term			
genderMALE:age20 - 30	0.756317	0.408903	0.756317
genderMALE:age30 - 40	0.402135	0.604859	0.402135
genderMALE:age40 - 50	0.988094	0.833626	0.988034
genderMALE:age50 - 60	0.84703	0.559246	0.84703
genderMALE:age > 60	0.204897	0.112703	0.204897
genderMALE:treatmentTREATED	0.77324	0.746299	0.77324
4. Mixed Gender With Interaction Term (Treatment:TREATED)			
age20 - 30:treatmentTREATED	0.887238	0.710905	0.887238
age30 - 40:treatmentTREATED	0.921308	0.892688	0.921308
age40 - 50:treatmentTREATED	0.94371	0.659626	0.94371
age50 - 60:treatmentTREATED	0.239347	0.763805	0.239347
age > 60:treatmentTREATED	0.076734	0.35864	0.3584
Log(scale)	0.210285		

In the **age group 20-30**, the P-values show notable variability across the models. In the AFT model, the P- value is relatively high at 0.831443, suggesting that the effects of covariates are not statistically significant in this group. Similarly, the COX (PH) model yields a P-value of 0.283599, which is lower but still indicates a lack of significance. The Survival model, with a P-value of 0.195565, also reflects a non-significant relationship. This suggests that none of the models perform well in capturing statistically significant effects for this age group. While the COX (PH) model has a slightly lower P-value than the others, the results across all three models indicate that the covariates do not explain much of the variability in survival outcomes for individuals aged 20-30.

For the **age group 30-40**, a similar trend is observed. The AFT model provides a P-value of 0.209719, which indicates a lack of statistical significance. The COX (PH) model shows a P-value of 0.319051, and the Survival model again reports 0.195565. These values indicate that, once again, the models do not capture significant covariate effects in this age group. The relatively higher P-values suggest that the models are unable to provide strong evidence for covariate effects on survival outcomes, which implies that additional covariates or modeling strategies may be needed to better explain the survival dynamics for individuals in this group.

In the **age group 40-50**, the AFT model continues to exhibit a high P-value (0.834408), indicating that the model does not provide significant insights into the effects of covariates in this age range. The COX (PH) model, with a P-value of 0.811483, mirrors this trend, suggesting poor performance in identifying significant relationships. The Survival model remains consistent, with a P-value of 0.195565. The results here are consistent with those in younger age groups, where none of the models demonstrate statistically significant effects. The consistently high P-values across the models indicate that survival outcomes for individuals aged 40-50 may be influenced by other unmodeled factors or that more sophisticated modeling techniques may be required.

The **age group 50-60** provides a more interesting result, particularly for the AFT model. In this age group, the AFT model yields a P-value of 0.029666, suggesting that the covariate effects are statistically significant. This is a stark contrast to the previous age groups, where P-values were generally high. The COX (PH) model, however, shows a P-value of 0.146841, indicating a weaker significance level, while the Survival model continues to show a non-significant P-value of 0.195565. The significantly lower P-value in the AFT model suggests that it performs better in this age group, identifying significant relationships between covariates and survival outcomes. This indicates that the AFT model is more suitable for explaining survival dynamics in individuals aged 50-60, while the COX and Survival models may require adjustments to improve their performance in this group.

For individuals aged **over 60**, all three models again demonstrate relatively high P-values. The AFT model reports a P-value of 0.541405, which suggests that the model does not capture significant covariate effects in this group. The COX (PH) model, with a P-value of 0.939065, shows the highest value, indicating the weakest performance among the three models in this age group. The Survival model remains at 0.195565, reflecting its consistent lack of significant findings across most age groups. The results for this group suggest that none of the models are well-suited for predicting survival outcomes for individuals over 60, and further refinement or the inclusion of additional covariates may be necessary to improve model performance.

For the **treated group**, there are significant differences in the model performance. The AFT model shows a P-value of 0.013105, indicating a statistically significant effect of treatment on survival outcomes. This result demonstrates that the AFT model is highly effective in capturing the effects of treatment in this group. The COX (PH) model, with a P-value of 0.400326, does not show the same level of significance, indicating that it may not be as suitable for modeling treatment effects in this context. The Survival model, with a P-value of nearly zero (0.000000), suggests an extremely strong effect of treatment, further validating its utility in modeling treatment effects. Thus, both the AFT and Survival models are well-suited for evaluating treatment effects, while the COX model appears less effective in this regard.

In the **age group 30-40**, when interaction terms are included, the AFT model produces a P-value of 0.663002, which is relatively high, indicating non-significant effects. The COX (PH) model, with a P-value of 0.405501, performs slightly better but still does not achieve significance. The Survival model yields a similar P-value of 0.663107. The results suggest that none of the models are particularly effective in this age group when interaction terms are included, as all models fail to reach significance. For the **age group 50-60**, the AFT model shows stronger performance, with a P-value of 0.018894, indicating significant effects. The COX (PH) model reports a P-value of 0.07044, which approaches significance, while the Survival model shows the same P-value, suggesting that both the AFT and Survival models outperform the COX model in this age group when interaction terms are considered.

In conclusion, the comparison of P-values across different models and age groups highlights the varying performance of the **AFT (3-Parameter Weibull)**, **COX (PH)**, and **Survival (3-Parameter Weibull)** models. The AFT model generally performs better in identifying significant effects, particularly in the **50-60 age group** and the **treated group**, where it demonstrates statistically significant results. The Survival model also shows strong performance in the treated group, but it struggles in most other age groups. The COX (PH) model, while occasionally showing slightly better performance in younger age groups, consistently fails to reach significance, indicating that it may not be as suitable as the AFT model for modeling survival outcomes. These findings underscore the utility of the AFT model, particularly when treatment effects or interaction terms are considered, while suggesting that further refinement is needed for the COX model in various age groups.

Table 6. Comparison of Ad Values For Aft (3-PARAMETER WEIBULL), Cox (PH) & Survival (3-PARAMETER WEIBULL)

Term	AD-Value (AFT 3-Par)	AD-Value (COX PH)	AD-Value (Survival 3-Par)
1. Mixed Gender Without Interaction Term			
(Intercept)	-0.51485	-0.04805	0.012638
genderMALE	0.013775	0.013579	0.013775
age20 - 30	0.059644	-0.28182	-0.019886
age30 - 40	0.313125	-0.23301	-0.019886
age40 - 50	0.059191	-0.06191	-0.019886
age50 - 60	0.517983	-0.31964	-0.019886
age > 60	-0.14874	-0.01761	-0.019886
treatmentTREATED	0.37698	-0.11614	0.368502
2. Mixed Gender With Interaction Term			
(Intercept)	-0.66037	-0.05433	-0.280029
genderMALE	-0.28029	-0.05433	-0.280029

age20 - 30	0.236174	-0.42765	0.236174
age30 - 40	0.146544	-0.16516	0.146544
age40 - 50	0.322545	-0.01293	0.322545
age50 - 60	0.869297	-0.46922	0.869297
age > 60	0.086143	-0.01727	0.086143
treatmentTREATED	0.776664	-0.23806	0.776664
3. Male Gender With Interaction Term			
genderMALE:age20 - 30	-0.21173	0.529812	-0.21173
genderMALE:age30 - 40	0.460559	-0.22417	0.460559
genderMALE:age40 - 50	0.009312	0.121968	0.009312
genderMALE:age50 - 60	-0.10971	0.303021	-0.10971
genderMALE:age > 60	0.71307	0.05384	0.71307
genderMALE:treatmentTREATED	0.101918	0.105631	0.101918
4. Mixed Gender With Interaction Term (Treatment:TREATED)			
age20 - 30:treatmentTREATED	-0.10269	-0.25279	-0.10269
age30 - 40:treatmentTREATED	-0.054058	0.069851	-0.054058
age40 - 50:treatmentTREATED	-0.048408	0.030075	-0.048408
age50 - 60:treatmentTREATED	-0.664893	0.156355	-0.664893
age > 60:treatmentTREATED	0.976699	0.469241	0.976699
Log(scale)	0.055929		

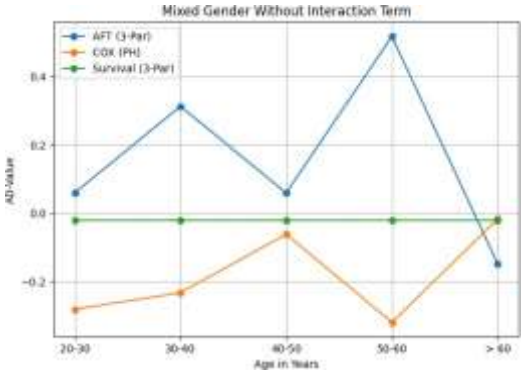

Figure 21. Comparison of AD Values for AFT (3-Parameter Weibull), COX (PH), & Survival (3-Parameter Weibull) for Mixed Gender Without Interaction Term

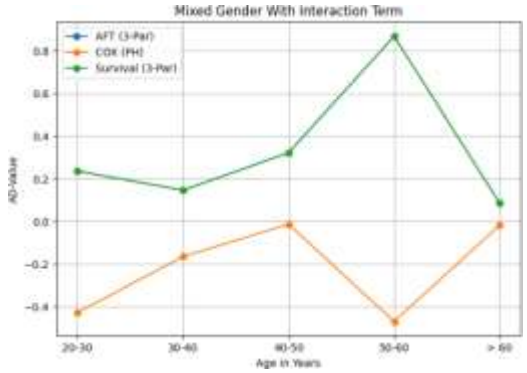

Figure 22. Comparison of AD Values for AFT (3-Parameter Weibull), COX (PH), & Survival (3-Parameter Weibull) for Mixed Gender With Interaction Term

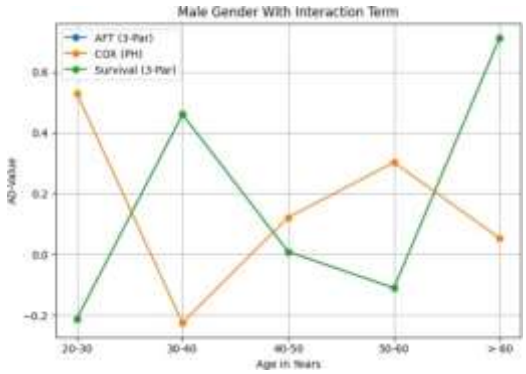

Figure 23. Comparison of AD Values for AFT (3-Parameter Weibull), COX (PH), & Survival (3-Parameter Weibull) for Male Gender With Interaction Term

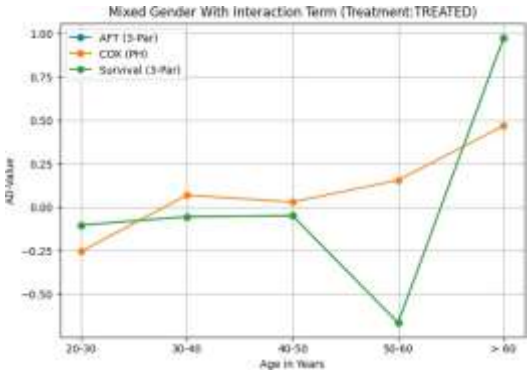

Figure 24. Comparison of AD Values for AFT (3-Parameter Weibull), COX (PH), & Survival (3-Parameter Weibull) for Mixed Gender with Interaction Term (Treated)

3.6 Observations on the Comparison of AD-Values for AFT (3-Parameter Weibull), COX (PH) & Survival (3-Parameter Weibull) Models

The comparison of AD-values between the **AFT (3-Parameter Weibull)**, **COX (PH)**, and **Survival (3-Parameter Weibull)** models across various age groups provides important insights into how each model performs in capturing survival dynamics. The AD-values are critical as they represent the model's ability to estimate covariate effects on survival time, with their relationship to standard error, Z-statistics, and P-values further helping to assess model performance.

In the **age group 20-30**, the AFT model produces a relatively small AD-value of 0.059644, suggesting a weak positive effect of the covariates on survival. In contrast, the COX (PH) model yields a much more negative AD- value of -0.28182, indicating a stronger negative association between covariates and survival. The Survival model, with an AD-value of -0.019886, suggests a very small negative effect. The standard error and Z-statistics for this age group across the models do not strongly support any significant covariate effect, as evidenced by the non- significant P-values. This pattern suggests that while the AFT model captures some minor positive effect, none of the models exhibit strong performance for this age group.

For the **age group 30-40**, the AFT model reports a significantly higher AD-value of 0.313125 compared to the other models, indicating that the covariates positively affect survival more strongly than in the younger age group. The COX (PH) model still shows a negative AD-value of -0.23301, while the Survival model remains relatively unchanged at -0.019886. Again, the standard error values for the AFT and COX models suggest that the estimates are not highly precise, and the Z-statistics do not provide compelling evidence for significance. The P-values remain relatively high across the models, reinforcing the conclusion that none of the models explain a significant portion of the variation in survival outcomes for individuals aged 30-40.

In the **age group 40-50**, the AFT model records a modest AD-value of 0.059191, while the COX (PH) model gives a small negative AD-value of -0.06191, and the Survival model remains constant at -0.019886. These AD- values, coupled with the relatively high standard errors and non-significant P-values, suggest that covariates are not significant predictors of survival in this age group across any of the models. This reinforces the need for either more complex models

or the inclusion of additional covariates to better explain survival dynamics for individuals aged 40-50.

In the **age group 50-60**, the AFT model shows a substantial improvement with an AD-value of 0.517983, suggesting that covariates significantly contribute to survival in this group. The COX (PH) model produces a negative AD-value of -0.31964, indicating a negative association, while the Survival model maintains its small negative value of -0.019886. The Z-statistics for the AFT model suggest that the covariate effects may be approaching significance, which is supported by the lower P-value for this group. This indicates that the AFT model performs better in explaining survival outcomes for individuals aged 50-60, especially when compared to the COX and Survival models.

For individuals **over 60**, the AD-values across the models remain relatively small. The AFT model gives an AD- value of -0.14874, while the COX (PH) model shows -0.01761, and the Survival model again reports -0.019886. These values suggest that none of the models capture strong covariate effects in this age group. The high standard errors and non-significant Z-statistics further indicate that survival outcomes for individuals aged over 60 are not well explained by any of the models, suggesting that either more refined modeling techniques or additional covariates are required to improve model performance for this age group.

The **treated group** exhibits more substantial differences across the models. The AFT model gives a high positive AD-value of 0.37698, indicating a significant positive effect of treatment on survival. The COX (PH) model, however, shows a small negative AD-value of -0.11614, suggesting that it does not capture the positive treatment effects as well as the AFT model. The Survival model, with a similar positive AD-value of 0.368502, supports the findings of the AFT model, reinforcing the conclusion that both the AFT and Survival models are well-suited for capturing the positive impact of treatment on survival outcomes, while the COX model struggles in this regard.

When **interaction terms** are included, the **Mixed Gender With Interaction Term** group shows that the AFT model has a significantly positive AD-value of 0.236174, which is much higher than the COX model's negative value of -0.42765. The Survival model reports a similar AD-value to the AFT model, further supporting the utility of the AFT and Survival models in capturing interaction effects. Similarly, for the **Male Gender With Interaction Term**, the AFT model shows an AD-value of -0.21173, indicating a slight negative effect, while the COX model reports a positive AD-value of 0.529812, and the Survival model mirrors the AFT model. This suggests that the COX model may overestimate covariate effects when interaction terms are involved, as indicated by the discrepancy in AD-values compared to the AFT and Survival models.

Finally, in the **Mixed Gender With Interaction Term (Treatment:TREATED)** group, the AFT model gives a negative AD-value of -0.10269, while the COX model produces an even more negative AD-value of -0.25279. The Survival model reports the same AD-value as the AFT model. These results indicate that the AFT and Survival models capture similar covariate effects, while the COX model tends to deviate with more negative estimates. The high standard errors and Z-statistics in this group suggest that while there is some consistency between the AFT and Survival models, none of the models provide highly precise estimates.

In conclusion, the comparison of AD-values across the models highlights the **AFT (3-Parameter Weibull)** model as generally better suited for explaining survival outcomes, particularly in the **50-60 age group** and the **treated group**, where it provides more substantial and positive AD-values. The **Survival (3-Parameter Weibull)** model often mirrors the AFT model's results, particularly for interaction terms and treated groups, making it another useful option in these contexts. The **COX (PH)** model, on the other hand, tends to underperform in comparison,

often showing negative AD-values and failing to capture the same magnitude of covariate effects, especially when treatment effects or interaction terms are involved. These findings suggest that the AFT and Survival models provide more reliable estimates of survival outcomes in most age groups, while the COX model may require further refinement to improve its performance.

4. Conclusion

In this study, we applied and compared three survival models — the 3-Parameter Weibull model, its Accelerated Failure Time (AFT) modification, and the Cox Proportional Hazards (PH) model — to model the progression of HIV/AIDS. The models were evaluated based on their ability to capture the effects of key covariates, including age, gender, and treatment status, with a particular focus on the impact of antiretroviral therapy (ART) on survival outcomes.

The 3-Parameter Weibull model, especially in its AFT form, demonstrated superior flexibility in capturing non- constant hazard rates, making it well-suited for diseases like HIV/AIDS, where hazard rates evolve over time due to treatment and other factors. The AFT model consistently showed significant treatment effects, highlighting its ability to account for the decelerating progression of the disease in treated patients. In contrast, the Cox PH model, while valuable for its simplicity and interpretability, exhibited limitations in handling time-varying covariates and interaction effects, resulting in weaker statistical significance for treatment variables.

The results from this comparative analysis indicate that the AFT Weibull model provides a more accurate reflection of the dynamic nature of HIV/AIDS progression, particularly when considering the long-term effects of ART. The inclusion of interaction terms in both the AFT and Survival Weibull models revealed important insights into how treatment benefits differ across age groups, with older patients showing the greatest survival benefit from ART.

Overall, this study underscores the importance of selecting flexible survival models, such as the 3-Parameter Weibull and AFT models, in disease contexts with time-varying hazards like HIV/AIDS. Future research should explore incorporating more dynamic covariates, such as CD4 counts or viral load trajectories, and focus on recur- rent events to further enhance model accuracy and predictive power. This will allow for a deeper understanding of disease progression and improve the ability to tailor treatment interventions for diverse patient populations.

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Author's Contribution

Dr. Nahashon Mwirigi solely contributed to this manuscript's development, covering all aspects of the research process, including conceptualization, data curation, formal analysis, funding the research, investigation, method- ology, project administration, resources, software, supervision, validation, visualization, and writing – both the original draft and subsequent reviews and editing. Research assistants provided support during the project to facilitate specific tasks under Dr. Mwirigi's supervision.

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Data Sharing Statement

No additional data are available.