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Demystifying the Concept of Probability

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Abstract

This paper attempts to demystify the concepts underlying the study of probability. It also attempts to simplify the processes that govern the pragmatic use of probability. The paper employs practical examples in order to enable the use of probability in numerous fields, including sports, medicine, engineering, education, business, gambling, weather patterns, etc. At some point or another, we all employ probability without being aware that we are doing so. The conscious recognition that we are using probability makes our choices clearer and our decisions more informed.

Keywords: experiment, sample space, outcome, event, impossibility, certainty, axiomatic

Probability Theory

According to Lightner (1991), Probability theory had its root in the 16th century when Gerolamo Cardano, an Italian Mathematician and Physician, addressed the first work on the topic, ***The Book on Games of Chance (Liber de Ludo Aleae)***. Cardano's book contains the foundations of Mathematical Probability Theory about one hundred years before Pascal and Fermat. After its inception, the knowledge of probability was brought to the attention of great Mathematicians. Probability is the branch of Mathematics concerning numerical descriptions of how likely an event is to occur. The probability of an event is a number between **0** and **1**, where **0** indicates impossibility of the event and **1** indicates certainty of the event. All possible outcomes, $\boldsymbol{a}$, of an experiment is referred to as Sample Space, $\boldsymbol{U}$. That is, a is a subset of **U**, $\boldsymbol{A \subset U}$.

Equally Likely Events

When the outcomes of a random experiment have an equal likelihood of occurrence, they are called *equally likely events*. Like during a coin toss, the probability of getting a head or a tail is equally likely. Therefore, equally likely events have the same theoretical probability of occurring.

If the outcomes of an event are equally likely then we can calculate the probability using the formula:

$$\boldsymbol{Probability\ of\ an\ event = \frac{Number\ of\ successful\ outcomes}{Total\ number\ of\ possible\ outcomes}}$$

$$\boldsymbol{P(A) = \frac{n(A)}{n(U)}}$$

Example:

A bag contains 1 red, 3 green, 4 yellow, and 2 black marbles. What is the probability of pulling a green marble from the bag without looking?

Solution:

P(green) = $\frac{Number\ of\ successful\ outcomes}{Total\ number\ of\ possible\ outcomes}$

$=\frac{3}{10}$

= 0.3 (30%)

The Impossible Event

If P(A) = 0, then the event is an absolute impossibility, that is, the event will never occur, for example, the probability of a person walking on the sun.

That is, if A = Ø (empty set).

Then, $P\ (A) = \frac{n(A)}{n(U)} = \frac{n(\emptyset)}{n\ (U)} = \frac{0}{n(U)}$ = 0 = P (Ø)

Hence, P (Ø) = 0

The Certain Event

If P (A) = 1, then the event is an absolute certainty, that is, the event will occur, for example, the probability of a person walking in the park.

That is, if A = U (universal set).

Then, $P\ (A) = \frac{n(A)}{n(U)} = \frac{n(U)}{n\ (U)} = \frac{1}{n(U)}$ = 0 = P (U)

Hence, P (U) = 1

Types of Probability

There are three major types of probabilities:

1. Experimental Probability
2. Theoretical Probability
3. Axiomatic Probability

Experimental Probability

Experimental Probability (relative frequency) is found by repeating an experiment and observing the outcomes. Therefore, experimental probability is the result of an experiment. The experimental probability can be calculated based on the number of possible outcomes by the total number of trials.

$$\boldsymbol{P(event) = \frac{number\ of\ times\ event\ occurs}{total\ number\ of\ trials}}$$

Example:

A coin is tossed 10 times. A head is recorded 7 times and tail 3 times.

Solution:

P(head)=$\frac{number\ of\ times\ event\ occurs}{total\ number\ of\ trials}$

$=\frac{7}{10}$

P(tail)= $\frac{number\ of\ times\ event\ occurs}{total\ number\ of\ trials}$

$= \frac{3}{10}$

Theoretical Probability

Theoretical probability is what is expected to happen after an event. It is mainly based on the reasoning behind probability.If the number of favorable outcomes and the number of possible outcomes can be determined, the probability can be calculated using the following formula:

$$\boldsymbol{Theoretical\ Probability\ = \frac{number\ of\ favourable\ outcomes}{total\ size\ of\ sample\ space}}$$

Example:

What is the probability of rolling a 3 on a number cube?

Solution:

$$P(3) = \frac{number\ of\ favourable\ outcomes}{total\ size\ of\ sample\ space}$$

$$= \frac{1}{6}$$

$$= 0.16$$

What is the probability of rolling a number less than 3 on a number cube?

Solution:

$$P(\text{less than } 3) = \frac{number\ of\ favourable\ outcomes}{total\ size\ of\ sample\ space}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

$$= 0.33$$

Axiomatic Probability

In *axiomatic probability*, a set of rules or axioms are set which applies to all types of probabilities. Shafer and Vovk (2012) posit that these axioms are set by Andrey Nikolaevich Kolmogorov and are known as Kolmogorov's three axioms. With the axiomatic approach to probability, the chances of occurrence or non-occurrence of the events can be quantified.

Let ***S*** be the sample space of a random experiment. If a number ***P (A)*** assigned to each event $A \in S$ satisfies the following axioms, then ***P (A)*** is called the probability of ***A.***

Axiom 1: $P(A) \geq 0$

Axiom 2: $P(S) = 1$

Axiom 3: If $\{A_1, A_2, \ldots\}$ is a sequence of mutually exclusive events i.e., $A_i \cap A_j = \phi$

When, $i \neq j$

Then,

$$P\left(\cup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Finding Probability by using the Complement

The complement **A'** of the event A consists of all the elements in the sample space that are not in **A**. The complement rule states that the sum of the probabilities of an event and its complement must be equal to **1**.

Given an event A,

$$P(U) = P(A) + P(A')$$

So, $1 = P(A) + P(A')$

That is $P(A) = 1 - P(A')$

Since $P(U) = 1$

$$P(A) = \frac{n(A)}{n(U)}$$

And $(PA') = \frac{n(A')}{n(U)}$

Example:

1. A coin is tossed once and the results are observed and noted.

Calculate the probability that:

I. a head appears

II. a tail appears

Solution

Sample space $U = \{H, T\}$

$n(U) = 2$

Let A = $\{head\} = \{H\}$

$n(A) = 1$

Let A'= $\{tail\} = \{T\}$

$n(A') = 1$

I. $P(A) = \frac{n(A)}{n(U)} = \frac{1}{2} = 0.5$

The probability of a head appearing is 0.5.

II. $P(A') = \frac{n(A')}{n(U)} = \frac{1}{2} = 0.5$

The probability of a tail appearing is 0.5.

2. A bag contains only red, yellow, and green marbles. The probability of choosing a red marble is $\frac{1}{4}$, the probability of choosing a yellow marble is $\frac{1}{2}$. What is the probability of choosing a green marble?

P (red) + P (yellow) + P (green) = 100%

25% + 50% + P (green) = 100%

75% + P (green) = 100%

P (green) = 100% - 75%

P (green) = 25%

Mutually Exclusive Events

Two events are ***mutually exclusive*** or ***disjoint*** events, if they both cannot occur in the same trial of an experiment. For example, rolling a 4 and an odd number on a number cube are mutually exclusive events because they both cannot happen at the same time. Suppose both ***A*** and ***B*** is two mutually exclusive events:

$A \cap B = \emptyset$ (disjoint set)

So $P(A \cup B) = P(A) + P(B)$

Where A and B are mutually exclusive events.

That is, P (both ***A*** and ***B*** will occur) = 0

P (either ***A*** or ***B*** will occur) = P (***A***) + P (***B***)

Example:

A fair number cube is rolled once and the result observed. What is the probability that a 2 or a 3 appears?

Solution:

The probability of a 2 appearing is $P(E_1) = \frac{1}{6}$

The probability of a 3 appearing is $P(E_2) = \frac{1}{6}$

The probability that a 2 or a 3 appearing is $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

The Addition Law of Probability

The general rule for mutually exclusive events is called the addition law of probability. The addition law of probability states that if E_1, E_2, E_3,…E_n are mutually exclusive events, then the probability of any one of the events occurring is given by:

$$P(E_1, \text{ or } E_2, \text{ or } E_3, \ldots \text{ or } E_n) = P(E_1 \cup E_2 \cup E_3 \ldots \cup E_n)$$
$$= P(E_1) + P(E_2) + P(E_3) + \ldots P(E_n)$$

That is, the probability of any one of the several mutually exclusive events occurring is equal to the sum of their individual probabilities.

Independent Events

An event ***B*** is said to be independent of another event ***A***, if the probability of ***B*** occurring is not influenced by whether ***A*** has or has not occurred.

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

Where ***A*** and ***B*** are independent events.

Example:

A coin and a number cube are tossed at the same time. Determine the probability that a tail and a 2 will result.

Solution:

The probability of a tail appearing is: $P(T) = \frac{1}{2}$

The probability of a 2 appearing is: $P(2) = \frac{1}{6}$

Therefore, the probability of a tail and a 2 appearing is: $P(T \cap 2) = P(T) \times P(2)$

$$= \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1}{12}$$

Hence, the probability of a tail and a 2 appearing is $\frac{1}{12}$

The Multiplication Law of Probability

The general rule for independent events is called the multiplication law of probability. The multiplication law of probability states that if E_1, E_2, E_3, … E_n are independent events, then the probability of all the events occurring simultaneously is given by:

$$P(E_1 \text{ and } E_2 \text{ and } E_3 \text{ and } \ldots E_n)$$
$$= P(E_1 \cap E_2 \cap E_3 \ldots \cap E_n)$$
$$= P(E_1) \times P(E_2) \times P(E_3) \times \ldots P(E_n)$$

That is, the probability of all the events occurring simultaneously is equal to the product of their individual probabilities.

Dependent Events

An event ***A*** and ***B*** is said to be dependent on another event ***A***, if the probability of ***B*** occurring is influenced by whether A has or has not occurred. The conditional probability of ***B*** given ***A*** is:

$$P\left(\frac{B}{A}\right) = \frac{P(A\ and\ B)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A)\ X\ P(\frac{B}{A})$$

Example:

A box contains 10 similar balls. Four balls are green. Calculate the probability that a ball drawn at random is green. If the ball is green and not replaced, calculate the probability that a second ball drawn at random is also green. Hence, determine the probability of drawing two green balls.

Solution:

The number of green balls, n(G) = 4

The total number of balls, n(U) = 10

Therefore, $P(G) = \frac{n(G)}{n(U)} = \frac{4}{10} = \frac{2}{5}$

Hence, the probability of drawing a green ball is $\frac{2}{5}$.

The total number of green balls remaining, n(G) = 4 – 1 = 3

The total number of balls remaining, n(U) = 10 – 1 = 9

Therefore, P(G/G) = $\frac{n(G)}{n(U)} = \frac{3}{9} = \frac{1}{3}$

Hence, the probability of drawing a second green ball is $\frac{1}{3}$.

Thus, P (G and G) = P(G) X P(G/G)

$\frac{2}{5}$ X $\frac{1}{3}$

$\frac{2}{15}$

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