International Journal of Statistics and Probability; Vol. 10, No. 1; January 2021
ISSN 1927-7032 E-ISSN 1927-7040
Published by Canadian Center of Science and Education

An Empirical Evaluation of a Test Procedure for the Median of Symmetrical and Asymmetrical Populations Using an Interpolated Nonparametric Confidence Interval

Mohammad Ibrahim Ahmmad Soliman Gaafar

Correspondence: Department of Statistics, Faculty of Commerce, Alexandria University, Alexandria, Egypt. E-mail: aswanybakkar@yahoo.com

Received: October 10, 2020 Accepted: December 25, 2020 Online Published: December 28, 2020

doi:10.5539/ijsp.v10n1p85 URL: https://doi.org/10.5539/ijsp.v10n1p85

Abstract

This paper investigates, evaluates, and highlights the performance of a test procedure for the median of a single population using an old nonparametric interpolated confidence interval. Simulation results show that the test procedure under investigation strictly maintains the size at its nominal level and has generally higher empirical power under both symmetrical heavy-tailed and asymmetrical populations.

Keywords: interpolated confidence intervals, the Wilcoxon signed ranks test, the sign test, the Shapiro-Wilk test, the triples test, the normal scores test

1. Introduction

Hettmansperger and Sheather (1986) proposed a nonlinear interpolation of adjacent order statistics to form a confidence interval for the population median (M) with intermediate value of the confidence coefficient. They investigated the performance of their confidence interval under four symmetrical models (Normal, Cauchy, Uniform, Double Exponential) and just one asymmetrical model formed by piecing together double exponential and logistic distributions. Hence, they did not detect the effect of skewness on their proposed confidence interval. In this paper, the performance of Hettmansperger and Sheather's confidence interval in the context of testing hypotheses for the population median is evaluated under different asymmetrical models and compared with six competitor test procedures.

When testing the null hypothesis ($H_\circ: M = M_\circ$) against a composite alternative H_1, it is known that the t-test is the uniformly most powerful unbiased test under the normal model while the Fraser (1957b) Normal Scores test is the asymptotically most powerful rank test. If there is a severe departure from normality, there are two possible approaches to follow. The first is to correct the data for departure from normality, by applying some power transformation, and then apply one of the classical test procedures to the transformed data. The second is to consider some alternative procedures that do not require normality. The sign, the Wilcoxon signed ranks, the Baklizi (2005) tests, or some adaptive tests are possible such alternatives. Various adaptive test procedures have been suggested in the literature. These tests are mainly based on a preliminary test of normality or symmetry or some measure of asymmetry, and then choosing the sign test, the Wilcoxon signed ranks test, or generally a data-driven score function to be used in constructing the linear signed ranks test accordingly.

Lemmer (1993) proposed two adaptive test procedures to choose between the sign test and the Wilcoxon signed ranks test. The first test procedure depends on a measure of symmetry introduced by Randles and Wolfe (1979), while the second test procedure depends on the runs test statistic of symmetry introduced by McWilliams (1990). A shortcoming of Lemmer's two test procedures is the discontinuous nature of the test selection method. Another shortcoming is that the runs test may reject the null hypothesis of symmetry not because the distribution is asymmetric but because it is symmetric about another value different from the hypothesized value of the median. Freidlin et al. (2003) proposed an adaptive test procedure that uses the p-value of the Shapiro-Wilk normality test to choose an appropriate linear signed ranks test (the Wilcoxon scores, the T(2) scores, or the Cauchy scores) to analyze the pairs. A shortcoming with this test procedure is the inflation of the type(I) error due to the dependence between the Shapiro-Wilk test and the signed ranks test. Another shortcoming is that as the sample size increases the Shapiro-Wilk and other tests of normality will detect minor departures from normality and thus selecting a test that is good for a very heavy-tailed distribution even if the data are not far from normal. Baklizi (2005) proposed an adaptive test procedure that is based on modifying the Wilcoxon scores according to the evidence of asymmetry of the distribution present in the data as indicated by the magnitude of the p-value from the triples test of symmetry introduced by Randles et al. (1980). As the p-value of the triples test approaches zero the scores approach that of the sign test. As the p-value of the triples test approaches one the scores approach that of the Wilcoxon signed ranks test. The main advantage of this procedure is that it adapts its scores smoothly and continuously while the main disadvantage is its sensitivity to minor inappropriately detected departures from symmetry. Bandyopadhyay and Dutta (2007) proposed two adaptive test procedures, one is a probabilistic approach while the other is a deterministic approach. The deterministic approach is based on calculating a measure of symmetry and using it as a basis for choosing between the sign test and the Wilcoxon signed ranks test. The probabilistic approach uses the p-value of the triples test of symmetry to choose between the sign test and the Wilcoxon signed ranks test. The major shortcoming of these two procedures is the discontinuous nature of the test selection method. Gastwirth and Miao (2009) proposed an adaptive test procedure that uses a measure of tail-heaviness of the underlying distribution to choose the appropriate score function (the Wilcoxon scores, the normal scores, the T(2) scores, or the Cauchy scores) to be used in the construction of the linear signed ranks test. The major shortcoming of this procedure is that it is constructed for only symmetric populations. Gaafar (2016) proposed using the Yeo-Johnson family of power transformations and the Shapiro-Wilk test of normality to modify the classical normal scores test. The simulation results showed that this test is superior to all competitor tests under consideration in terms of preserving the empirical size of the test at its nominal level and having higher empirical powers.

Based on the Hettmansperger and Sheather's (1986), a test procedure for right-tailed, left-tailed, and two-tailed tests for the median of symmetrical and asymmetrical populations is introduced. A simulation study is conducted to evaluate the empirical performance of the recommended test procedure (HS) and compare it with six competitor test procedures: the t-test (T), the normal scores test (NS), the Wilcoxon signed ranks test (W), the Gastwirth and Miao (2009) test (GM), the sign test (Sign), the Baklizi (2005) test (Baklizi).

Section 2 introduces the triples test for symmetry used in the Baklizi (2005) test. Section 3 introduces six competitor test procedures. Section 4 introduces the recommended test procedure. Section 5 describes the design of the simulation experiment. Section 6 reports the main results and section 7 gives concise conclusions of the study.

2. The Triples Test

The Randles et al. (1980) triples test for the symmetry of the distribution of the random variable X about some unknown value is to reject the null hypothesis of symmetry if $|TR| > z_{\alpha/2}$, where $Z_{\alpha/2}$ is the $(1-(\alpha/2))$ quantile of the standard normal distribution and

$$TR = \psi/\Delta \ ,$$

$$\psi = \sum_{1\le i}\sum_{<j}\sum_{<k\le n} f^*(x_i, x_j, x_k)\ ,$$

$$f^*(x_i, x_j, x_k) = sgn(x_i + x_j - 2x_k) + sgn(x_i + x_k - 2x_j) \ + sgn(x_j + x_k - 2x_i),$$

$$\Delta^2 = \Delta_1 + \Delta_2 - \Delta_3 + \frac{n(n-1)(n-2)}{6}\ ,$$

$$\Delta_1 = \frac{(n-3)(n-4)}{(n-1)(n-2)}\sum_{t=1}^{n} B_t^2\ ,$$

$$\Delta_2 = \frac{(n-3)}{(n-4)}\sum_{s=1}^{n-1}\sum_{t=s+1}^{n} B_{s,t}^2\ ,$$

$$\Delta_3 = \left\{1 - \frac{(n-3)(n-4)(n-5)}{n(n-1)(n-2)}\right\}\psi^2\ ,$$

$$B_t = \sum_{j=t+1}^{n-1}\sum_{k=j+1}^{n} f^*(x_t, x_j, x_k) + \sum_{j=1}^{t-1}\sum_{k=t+1}^{n} f^*(x_j, x_t, x_k) + \sum_{j=1}^{t-2}\sum_{k=j+1}^{t-1} f^*(x_j, x_k, x_t),$$

$$B_{s,t} = \sum_{j=1}^{s-1} f^*(x_j, x_s, x_t) + \sum_{j=s+1}^{t-1} f^*(x_s, x_j, x_t) + \sum_{j=t+1}^{n} f^*(x_s, x_t, x_j)$$

3. Competitor Test Procedures

This section presents six test procedures and their exact or approximate distributions under the null hypothesis to be tested ($H_\circ: M = M_\circ$).

Let $x_1, x_2, \dots\dots, x_n$ be a random sample selected from a population with median M.

- When the sampled population is assumed to be normal:

1) the uniformly most powerful unbiased test statistic of the above hypothesis is given as:

$$T = \frac{\sqrt{n}(\bar{x} - M_\circ)}{S_x} \quad (1)$$

where $\bar{x}$ and S_x denote the mean and the standard deviation of the sample observations, respectively. Under $H_\circ$, T has a t-distribution with (n-1) degrees of freedom.

2) the van der Waerden type of the asymptotically most powerful rank test of the above hypothesis has the following test statistic:

$$NS_V = \sum_{i=1}^{n} I(x_i - M_\circ) \times \Phi^{-1}\left(\frac{1}{2} + \frac{R(|x_i - M_\circ|)}{2(n+1)}\right) \quad (2)$$

where $R_i^+ (= R(|x_i - M_\circ|))$ denotes the rank of the absolute deviation of the ith sample observation from $M_\circ$ and

$$I(x_i - M_\circ) = \begin{cases} 1 & \text{for } x_i > M_\circ \\ 0 & for\ x_i < M_\circ \end{cases}$$

Under H_0, the statistic Z_V has approximately a standard normal distribution.

Where,

$$Z_V = \frac{NS_V - E(NS_V)}{\sqrt{Var(NS_V)}},$$

$$E(NS_V) = 0.5 \sum_{i=1}^{n} \Phi^{-1}\left(\frac{1}{2} + \frac{R(|x_i - M_\circ|)}{2(n+1)}\right),$$

$$Var(NS_V) = 0.25 \sum_{i=1}^{n} \left[\Phi^{-1}\left(\frac{1}{2} + \frac{R(|x_i - M_\circ|)}{2(n+1)}\right)\right]^2$$

- When the sampled population is not normal but assumed to be symmetrical:

1) the Wilcoxon signed ranks test is one choice. The test statistic can be defined as:

$$W = \sum_{i=1}^{n} I(x_i - M_\circ) \times R(|x_i - M_\circ|) \quad (3)$$

Under $H_\circ$, the statistic Z_W has approximately a standard normal distribution.

Where,

$$Z_W = \frac{(W - E(W)) - 0.5 \times sgn(W - E(W))}{\sqrt{Var(W)}},$$

$$E(W) = 0.5 \sum_{i=1}^{n} R(|x_i - M_\circ|),$$

$$\mathrm{Var(W)} = 0.25\sum_{i=1}^{n}[\mathrm{R}(|x_i - \mathrm{M}_\circ|)]^2$$

2) another choice is the adaptive two-stage test procedure proposed by Gastwirth and Miao (2009) defined as:

$$\mathrm{GM} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{I}(x_i - \mathrm{M}_\circ) \times \mathrm{J}^*\left(\frac{\mathrm{R}(|x_i - \mathrm{M}_\circ|)}{(\mathrm{n+1})}\right) \quad (4)$$

Where,

$$\mathrm{J}^*(\mathrm{u}) = \begin{cases} \Phi^{-1}((\mathrm{u}+1)/2) & \text{if} \quad \mathrm{sM} < 1.02 \\ \mathrm{u} & \text{if} \quad 1.02 \le \mathrm{sM} < 1.2 \\ (3\sqrt{2}/2)\mathrm{u}\sqrt{1-\mathrm{u}^2} & \text{if} \quad 1.2 \le \mathrm{sM} < 2.7 \\ 2\tan\left[\frac{1}{2}\pi\mathrm{u}\right] \Big/ \left(1 + \tan^2\left[\frac{1}{2}\pi\mathrm{u}\right]\right) & \text{if} \quad \mathrm{sM} \ge 2.7 \end{cases}$$

And

$$\mathrm{sM} = (\tilde{\mathrm{s}}/\tilde{\mathrm{M}}), \quad \tilde{\mathrm{s}}^2 = \frac{1}{\mathrm{n}}\sum_{\mathrm{i=1}}^{\mathrm{n}}(x_i - \mathrm{M}_\circ)^2 \ , \quad \tilde{\mathrm{M}} = \mathrm{median}(|x_i - \mathrm{M}_\circ|)/\Phi^{-1}(0.75)$$

Under $\mathrm{H}_\circ$, the statistic Z_{GM} has approximately a standard normal distribution.
Where,

$$\mathrm{Z}_{GM} = \frac{(\mathrm{GM} - \mathrm{E(GM)})}{\sqrt{Var(\mathrm{GM})}} \ ,$$

$$\mathrm{E(GM)} = 0.5\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{J}^*\left(\frac{\mathrm{R}(|x_i - \mathrm{M}_\circ|)}{(\mathrm{n+1})}\right) \ ,$$

$$\mathrm{Var(GM)} = 0.25\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{J}^{*2}\left(\frac{\mathrm{R}(|x_i - \mathrm{M}_\circ|)}{(\mathrm{n}+1)}\right)$$

- When the sampled population is assumed to be asymmetrical:

1) the sign test is one choice whose test statistic is given as:

$$\mathrm{K} = \sum_{i=1}^{n} \mathrm{I}(x_i - \mathrm{M}_\circ) \quad (5)$$

Under $\mathrm{H}_\circ$, k has a binomial distribution Bin(n, 0.5).

2) another choice is the test introduced by Baklizi (2005). This test is based on modifying the Wilcoxon scores according to the evidence of asymmetry of the population present in the data as indicated by the magnitude of the P-value from the triples test of symmetry. Given that the P-value of the triples test of symmetry is "p", the Baklizi test statistic is given as:

$$Bak = \sum_{i=1}^{n} \mathrm{I}(x_i - \mathrm{M}_\circ) \times [\mathrm{R}(|x_i - \mathrm{M}_\circ|)]^p \quad (6)$$

Under $\mathrm{H}_\circ$, the statistic Z_{Bak} has approximately a standard normal distribution.
Where,

$$Z_{Bak} = \frac{Bak - \mathrm{E}(Bak|p)}{\sqrt{Var(Bak|p)}} \ ,$$

$$\mathrm{E}(Bak|p) = 0.5\sum_{i=1}^{n}[\mathrm{R}(|x_i - \mathrm{M}_\circ|)]^p \ ,$$

$$Var(Bak|p) = 0.25\sum_{i=1}^{n}[\mathrm{R}(|x_i - \mathrm{M}_\circ|)]^{2p}$$

4. The Recommended Test Procedure

Hettmansperger and Sheather (1986) proposed the following $100(1-\alpha)\%$ equal-tailed two-sided nonparametric interpolated confidence interval for the population median (M):

$$\left((1-\lambda)x_{(k)} + \lambda x_{(k+1)} \quad , \qquad (1-\lambda)x_{(n-k+1)} + \lambda x_{(n-k)}\right)$$

Where k is chosen such that $p\left(X_{(k)} > M\right) = \pi_k = \sum_{i=0}^{k-1} 2^{-n} \times \binom{n}{i} \approx \frac{1}{2}\alpha$ and λ is defined as:

$$\lambda = \left[1 + \frac{k\left(\pi_{k+1} - \frac{1}{2}\alpha\right)}{(n-k)\left(\frac{1}{2}\alpha - \pi_k\right)}\right]^{-1}$$

They showed that their confidence interval gives exactly the desired coverage probability for the double exponential distribution and approximately the desired coverage probability for several other common symmetric distributions.

To conduct a right-tailed, left-tailed, or two-tailed test for the median of symmetrical and asymmetrical populations at a significance level (α), the following steps are proposed:

[1] Given a nominal significance level (α), calculate (r_1) for one-sided tests or (r_2) for two-sided tests, where

$$r_1 = inf\left\{r: \sum_{j=0}^{r} \binom{n}{j} > 2^n \times \alpha\right\} \quad and$$

$$r_2 = inf\left\{r: \sum_{j=0}^{r} \binom{n}{j} > 2^{n-1} \times \alpha\right\}$$

[2] Calculate (ρ_{r_1} and $\rho_{r_{1-1}}$) or (ρ_{r_2} and $\rho_{r_{2-1}}$), where

$$\rho_i = 2^{-n} \times \sum_{j=0}^{i} \binom{n}{j}$$

[3] Calculate (λ_1) for one-sided tests or (λ_2) for two-sided tests, where

$$\lambda_1 = \left[1 + \frac{r_1(\rho_{r_1} - \alpha)}{(n-r_1)(\alpha - \rho_{r_{1-1}})}\right]^{-1} \text{ and}$$

$$\lambda_2 = \left[1 + \frac{r_2\left(\rho_{r_2} - \frac{\alpha}{2}\right)}{(n-r_2)\left(\frac{\alpha}{2} - \rho_{r_{2-1}}\right)}\right]^{-1}$$

[4] The rejection region depends on the type of the test as follows:

- <u>For the right-tailed tests</u>, reject $H_\circ$ at a significance level (α) if:

$$\frac{M_\circ - x_{(r_1)}}{x_{(1+r_1)} - x_{(r_1)}} \le \lambda_1$$

- <u>For the left-tailed tests</u>, reject $H_\circ$ at a significance level (α) if:

$$\frac{x_{(1+n-r_1)} - M_\circ}{x_{(1+n-r_1)} - x_{(n-r_1)}} \ge \lambda_1$$

- For the two-tailed tests, reject $H_\circ$ at a significance level (α) if:

$$\frac{M_\circ - x_{(r_2)}}{x_{(1+r_2)} - x_{(r_2)}} \le \lambda_2 \qquad or \qquad \frac{x_{(1+n-r_2)} - M_\circ}{x_{(1+n-r_2)} - x_{(n-r_2)}} \ge \lambda_2$$

5. The Simulation Study

The simulation study was executed using **R version 4.0.3.** For each experimental situation described below, 5000 pseudo random samples, of sizes n=20, n=30, and n=50, were generated with initial seed 9831815. These samples are then used to evaluate the empirical levels of the 7 tests described in Sections 3 and 4. This number of replicates makes the upper limit of estimating the 5% nominal level within 0.007 at probability 99%. That is, tests with empirical levels exceeding 0.057 are considered liberal and invalid for the underlying testing problem.

The null hypothesis ($H_\circ: M = M_\circ$) is tested against both right-tailed and left-tailed alternatives. To evaluate the empirical powers, the medians of the simulated distributions are shifted gradually from the true value on both directions. The hypothesized values to be tested ($M_\circ$) are determined as $M_\circ = M \pm \delta d$, where M and d are respectively, the population median and standard deviation (σ). When the standard deviation (σ) does not exist, as for the T(2) and Cauchy distributions, the median absolute deviations from the median ,"MAD", is used instead of (σ). Values of δ are selected to cover different spots over the range of the power function, (0.05, 1). The empirical power is computed as the percentage of times each test statistic exceeded its corresponding 5% cutoff point.

The simulation study covered the uniform, the standard normal, the standard logistic, the double exponential, the T(2), and the Cauchy distributions to represent symmetrical models and some other distributions to represent positively skewed models as described in table 1.

Table 1. The probability distributions covered in the simulation study

The distribution	Median(M)	σ or MAD
Standard Uniform	0.5	$\sigma = 0.288675$
Standard Normal	0	$\sigma = 1$
Standard Logistic	0	$\sigma = 1.8138$
Standard Double Exponential	0	$\sigma = 1.414$
T(2)	0	$MAD = 0.693$
Standard Cauchy	0	$MAD = 1$
Weibull(2)	0.833	$\sigma = 0.463$
Gamma(4)	3.672	$\sigma = 2$
Standard Extreme Value	0.367	$\sigma = 1.283$
Gamma(2)	1.678	$\sigma = 1.414$
Standard Lognormal	1	$\sigma = 2.1612$

6. Results

Table 2 gives the empirical levels, where the data is generated according to the null model, associated with the recommended test procedure, HS, and the other six competitors described in section 3. Tables 3 & 4 summarize the empirical powers, of valid tests, associated with some symmetrical and asymmetrical models respectively for just two sample sizes and two values of the alternative hypotheses under each model. The empirical powers of invalid tests are replaced by asterisks (*). The Appendix contains the full empirical levels and powers associated with all symmetrical and asymmetrical models under all configurations of the sample sizes and the values of the alternative hypotheses as described in section 5.

Using table 2, the following results can be deduced concerning the significance level associated with each of the seven test procedures under investigation:

(1) The sign test is the most conservative test procedure, in the sense of rejecting the true model with a lower percentage of times than the specified nominal level, among the seven test procedures under both symmetrical and asymmetrical models.

(2) Under symmetrical models, the recommended test procedure, HS, and the other six competitors are all valid.

(3) Under asymmetrical models,

(a) When the alternative hypothesis takes the direction of the longer tail, the HS, Sign, and Baklizi test procedures are the only valid test procedures under investigation. The other four test procedures are liberal and invalid, in the sense of rejecting the true model with a higher percentage of times than the specified nominal level.

(b) When the alternative hypothesis takes the direction of the shorter tail, the T, W, and NS test procedures are too conservative, while the recommended HS test procedure strictly keeps the size at its nominal level with the Baklizi then the sign test procedures trailing it.

Table 2. Empirical levels under some symmetrical and asymmetrical models (n = 20, n = 50)

	Dist.	n	H_1	T	W	Sign	GM	NS	Baklizi	HS
Symmetrical Models	Standard Uniform	20	Right	0.05	0.047	0.0204	0.0474	0.047	0.0348	0.0456
			Left	0.0542	0.0524	0.023	0.0516	0.0516	0.0378	0.054
		50	Right	0.0472	0.047	0.033	0.047	0.047	0.0412	0.0526
			Left	0.046	0.0442	0.03	0.0454	0.0456	0.0408	0.047
	Standard Normal	20	Right	0.0504	0.0492	0.021	0.0502	0.0508	0.0456	0.0488
			Left	0.0484	0.0462	0.0214	0.0462	0.048	0.0394	0.0482
		50	Right	0.0536	0.0528	0.0326	0.0554	0.0542	0.051	0.0534
			Left	0.053	0.0508	0.0326	0.0528	0.0516	0.0488	0.0494
	Standard Logistic	20	Right	0.0474	0.0468	0.0224	0.0516	0.0474	0.0414	0.0454
			Left	0.0486	0.0462	0.0192	0.0456	0.0478	0.0402	0.0462
		50	Right	0.052	0.0566	0.036	0.054	0.0548	0.0546	0.0542
			Left	0.0508	0.0528	0.034	0.052	0.0516	0.0498	0.0538
	Laplace	20	Right	0.0522	0.0508	0.0196	0.05	0.0532	0.0384	0.0532
			Left	0.0476	0.0436	0.0178	0.0482	0.047	0.0356	0.046
		50	Right	0.0474	0.0494	0.0336	0.0476	0.0492	0.0416	0.0516
			Left	0.0566	0.0506	0.0332	0.0506	0.0534	0.0458	0.052
	T (2)	20	Right	0.0404	0.0476	0.0226	0.0492	0.0498	0.0374	0.0494
			Left	0.038	0.0472	0.0172	0.0458	0.0478	0.0334	0.0472
		50	Right	0.0412	0.0486	0.031	0.0472	0.0488	0.0408	0.049
			Left	0.0432	0.0522	0.0356	0.052	0.052	0.0462	0.0528
	Standard Cauchy	20	Right	0.03	0.0464	0.0224	0.0514	0.0458	0.0344	0.0504
			Left	0.0286	0.0504	0.022	0.0518	0.0526	0.0338	0.049
		50	Right	0.0318	0.0494	0.0328	0.0504	0.0496	0.038	0.052
			Left	0.032	0.0456	0.0344	0.045	0.0492	0.038	0.051
Asymmetrical Models	Weibull (2)	20	Right	0.1046	0.077	0.0198	0.0806	0.0956	0.052	0.05
			Left	0.0226	0.0314	0.02	0.028	0.0234	0.0326	0.0494
		50	Right	0.1794	0.1044	0.0278	0.1292	0.1612	0.0576	0.0488
			Left	0.0108	0.021	0.028	0.0178	0.012	0.0302	0.0476
	Gamma (4)	20	Right	0.1304	0.0856	0.022	0.084	0.1124	0.0532	0.048
			Left	0.0142	0.0252	0.0224	0.0294	0.018	0.0292	0.0532
		50	Right	0.2808	0.1348	0.0306	0.135	0.2202	0.0486	0.0484
			Left	0.0046	0.017	0.03	0.0202	0.0072	0.0306	0.0488
	Standard Extreme Value	20	Right	0.1298	0.0888	0.0214	0.0786	0.1098	0.053	0.0516
			Left	0.0122	0.0212	0.02	0.0248	0.0158	0.0278	0.0472
		50	Right	0.2862	0.1378	0.0366	0.1188	0.2178	0.0508	0.0538
			Left	0.0048	0.018	0.0314	0.0264	0.008	0.0348	0.053
	Gamma (2)	20	Right	0.2042	0.1244	0.023	0.1042	0.1656	0.055	0.0526
			Left	0.0094	0.0172	0.0164	0.026	0.0124	0.0206	0.046
		50	Right	0.4598	0.1972	0.0314	0.148	0.3354	0.0398	0.0488
			Left	0.0016	0.0092	0.0332	0.0258	0.0022	0.0332	0.0492
	Standard Lognormal	20	Right	0.373	0.1854	0.017	0.0742	0.2588	0.034	0.0528
			Left	0.0022	0.0096	0.021	0.0588	0.0052	0.0224	0.052
		50	Right	0.8268	0.3544	0.0332	0.0644	0.558	0.0334	0.0544
			Left	0	0.0026	0.0324	0.1048	2.00E-04	0.0324	0.0512

Using table 3, the following results can be deduced concerning the power associated with each of the seven test procedures under symmetrical models:

(1) Under the normal model, the HS test is trailing the five tests (T, W, GM, NS, and Baklizi), but as the sample size increases the performance of the HS test approaches that of the uniformly most powerful test (T) and that of the asymptotically most powerful rank test (NS).

(2) Under the double exponential model, where the Wilcoxon signed ranks test is known to be highly correlated with the maximum efficiency robust test (Gastwirth, 1966), the HS test outperforms all the six competitor test procedures, especially under contiguous alternatives (those which are close to $H_\circ$).

(3) As the sample size increases, the performance of the HS test procedure approaches that of the best test procedure.

(4) As the degree of tail-heaviness increases, the outperformance of the HS test procedure increases and trails that of the GM test procedure.

Using table 4, the following results can be deduced concerning the power associated with each of the seven test procedures under asymmetrical models:

(1) The HS test procedure performs well under contiguous alternatives.

(2) When the alternative hypothesis takes the direction of the longer tail, the HS test procedure outperforms the other

two valid tests, the Sign and Baklizi test procedures, as the sample sizes and degrees of skewness increase.

(3) When the alternative hypothesis takes the direction of the shorter tail, the HS test procedure outperforms all other test procedures under consideration, especially as the degrees of skewness increase.

7. Conclusions

In practice, the following notes should be considered:

(1) When the evidence is in favor of the normality assumption, the T test or the NS test should be used.

(2) When the normality assumption is violated but the evidence is in favor of the symmetry or near symmetry assumption, the GM test or HS test is recommended.

(3) When both the normality assumption and the symmetry or near symmetry assumption are strongly violated, the HS or the Baklizi tests are recommended.

Table 3. Empirical powers under some symmetrical models (n = 20, n = 50)

Dist.	n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
Standard Uniform	20	0.05	Right	0.076	0.0726	0.0302	0.08	0.0806	0.0494	0.0692
			Left	0.0678	0.0688	0.0256	0.0784	0.0788	0.0438	0.0576
		0.5	Right	0.6804	0.6208	0.2286	0.6742	0.695	0.5396	0.372
			Left	0.6912	0.6242	0.2226	0.6838	0.7048	0.5338	0.366
	50	0.05	Right	0.0972	0.0946	0.0504	0.1136	0.1138	0.0736	0.0732
			Left	0.0934	0.0928	0.0462	0.112	0.1122	0.074	0.0666
		0.5	Right	0.9744	0.9382	0.5874	0.9736	0.9798	0.8322	0.6758
			Left	0.9712	0.9436	0.5904	0.9746	0.9786	0.8334	0.6734
Standard Normal	20	0.05	Right	0.0776	0.0766	0.0332	0.0792	0.08	0.0694	0.073
			Left	0.0754	0.0708	0.0296	0.0752	0.0784	0.0628	0.0716
		0.5	Right	0.6904	0.6704	0.3844	0.6634	0.6832	0.6096	0.5582
			Left	0.6918	0.6658	0.3766	0.6596	0.6794	0.6038	0.5566
	50	0.05	Right	0.1002	0.0942	0.0598	0.0954	0.0966	0.0892	0.0864
			Left	0.1016	0.1014	0.0614	0.1024	0.1022	0.096	0.0886
		0.5	Right	0.9684	0.9596	0.8284	0.9562	0.9668	0.9284	0.8766
			Left	0.9688	0.9606	0.8294	0.9596	0.9678	0.9302	0.883
Standard Logistic	20	0.05	Right	0.0778	0.074	0.0352	0.0752	0.0782	0.0642	0.0772
			Left	0.0756	0.074	0.0306	0.0738	0.0776	0.0666	0.0724
		0.5	Right	0.7062	0.7186	0.4734	0.7074	0.7152	0.6726	0.6448
			Left	0.7074	0.717	0.4672	0.702	0.711	0.671	0.6362
	50	0.05	Right	0.0968	0.0974	0.0626	0.0974	0.096	0.09	0.091
			Left	0.1106	0.1102	0.0676	0.112	0.1142	0.0994	0.0958
		0.5	Right	0.9668	0.9762	0.8996	0.967	0.972	0.9628	0.9362
			Left	0.9662	0.9766	0.8982	0.9702	0.972	0.9624	0.935
Laplace	20	0.05	Right	0.0758	0.0816	0.0416	0.0814	0.0798	0.0688	0.0888
			Left	0.072	0.0762	0.0408	0.081	0.0774	0.067	0.0856
		0.5	Right	0.7096	0.7766	0.6202	0.7732	0.746	0.773	0.7814
			Left	0.7158	0.7786	0.6282	0.7734	0.7516	0.769	0.779
	50	0.05	Right	0.1018	0.1154	0.0864	0.1212	0.112	0.1106	0.129
			Left	0.1058	0.114	0.0856	0.1172	0.113	0.1084	0.1264
		0.5	Right	0.9606	0.986	0.9718	0.985	0.9782	0.9882	0.9846
			Left	0.9632	0.9886	0.9742	0.9874	0.979	0.9896	0.9868
T (2)	20	0.05	Right	0.0552	0.0654	0.0284	0.0638	0.0654	0.0482	0.0622
			Left	0.0536	0.0612	0.0276	0.0648	0.061	0.0518	0.0698
		0.5	Right	0.2624	0.3528	0.2128	0.3718	0.335	0.3248	0.356
			Left	0.2566	0.3574	0.212	0.3734	0.3282	0.33	0.3648
	50	0.05	Right	0.0588	0.0726	0.046	0.0788	0.0728	0.0642	0.0736
			Left	0.0602	0.0734	0.0502	0.0782	0.0718	0.0624	0.074
		0.5	Right	0.3942	0.6346	0.5524	0.682	0.5748	0.6396	0.6326
			Left	0.4058	0.6426	0.5662	0.6818	0.5892	0.6472	0.6448
Standard Cauchy	20	0.05	Right	0.0346	0.061	0.031	0.0696	0.062	0.046	0.0688
			Left	0.0366	0.064	0.0322	0.0718	0.0634	0.0468	0.0718
		0.5	Right	0.1374	0.3184	0.2444	0.4086	0.2824	0.3182	0.3932
			Left	0.123	0.3062	0.2388	0.4038	0.2726	0.307	0.3814
	50	0.05	Right	0.0352	0.0742	0.0504	0.083	0.0706	0.0598	0.0756
			Left	0.04	0.074	0.0484	0.078	0.069	0.058	0.0788
		0.5	Right	0.1334	0.5774	0.6238	0.762	0.4876	0.6512	0.7042
			Left	0.1396	0.5912	0.6124	0.7558	0.4946	0.6462	0.6988

Table 4. Empirical powers under some asymmetrical models (n = 20, n = 50)

Dist.	n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
Weibull (2)	20	0.05	Right	*	*	0.029	*	*	0.08	0.0692
			Left	0.0354	0.047	0.0314	0.0434	0.0372	0.0474	0.0706
		0.5	Right	*	*	0.417	*	*	0.6476	0.584
			Left	0.52	0.5406	0.3346	0.5152	0.5082	0.4912	0.5038
	50	0.05	Right	*	*	0.0558	*	*	0.105	0.087
			Left	0.0218	0.0446	0.0562	0.0348	0.024	0.0614	0.0846
		0.5	Right	*	*	0.8514	*	*	0.8878	0.9026
			Left	0.8292	0.8748	0.7722	0.8308	0.8232	0.8484	0.8332
Gamma (4)	20	0.05	Right	*	*	0.034	*	*	0.0936	0.0806
			Left	0.0298	0.0438	0.03	0.0462	0.0334	0.0446	0.0778
		0.5	Right	*	*	0.4874	*	*	0.6846	0.6624
			Left	0.4522	0.5122	0.3568	0.4706	0.4522	0.472	0.5312
	50	0.05	Right	*	*	0.062	*	*	0.0908	0.0878
			Left	0.0134	0.035	0.0584	0.0376	0.0166	0.0606	0.0886
		0.5	Right	*	*	0.9294	*	*	0.937	0.9554
			Left	0.7484	0.8594	0.807	0.792	0.7722	0.847	0.8602
Standard Extreme Value	20	0.05	Right	*	*	0.0346	*	*	0.078	0.0778
			Left	0.0304	0.0476	0.0332	0.0544	0.0358	0.051	0.0798
		0.5	Right	*	*	0.5496	*	*	0.7286	0.7168
			Left	0.4632	0.5406	0.3964	0.5104	0.4842	0.5128	0.564
	50	0.05	Right	*	*	0.061	*	*	0.0866	0.0902
			Left	0.0116	0.0402	0.063	0.0524	0.0178	0.0666	0.09
		0.5	Right	*	*	0.9478	*	*	0.9568	0.9696
			Left	0.7472	0.8846	0.8392	0.8232	0.7924	0.8764	0.8892
Gamma (2)	20	0.05	Right	*	*	0.0354	*	*	0.0836	0.0778
			Left	0.0208	0.0382	0.0348	0.0416	0.0252	0.0432	0.0762
		0.5	Right	*	*	0.5858	*	*	0.7128	0.75
			Left	0.3842	0.4768	0.3688	0.4188	0.3886	0.4384	0.5426
	50	0.05	Right	*	*	0.0638	*	*	0.0722	0.0898
			Left	0.005	0.0224	0.0668	0.0432	0.008	0.067	0.093
		0.5	Right	*	*	0.9692	*	*	0.97	0.9834
			Left	0.6102	0.8156	0.8192	0.695	0.6568	0.83	0.871
Standard Lognormal	20	0.05	Right	*	*	0.0656	*	*	0.09	0.1424
			Left	0.0124	0.0352	0.0584	*	0.0178	0.0622	0.1192
		0.5	Right	*	*	1	*	*	1	1
			Left	0.6246	0.8332	0.8396	*	0.7232	0.849	0.9302
	50	0.05	Right	*	*	0.1524	*	*	*	0.2066
			Left	0	0.025	0.1366	*	0.0038	0.1368	0.1922
		0.5	Right	*	*	1	*	*	*	1
			Left	0.7776	0.9912	0.998	*	0.9524	0.998	0.9996

References

Baklizi, A. (2005). A Continuously Adaptive Rank Test for Shift in Location. *Australian and New Zealand Journal of Statistics, 47,* 203-209. https://doi.org/10.1111/j.1467-842X.2005.00384.x

Bandyopadhyay, U., & Dutta, D. (2007). Adaptive Nonparametric Tests for a Single Sample Location Problem. *Journal of Statistical Methodology*, *4*, 423-433. https://doi.org/10.1016/j.stamet.2007.01.004

Fraser, D. A. S. (1957b). Most Powerful Rank-Type Tests. *Annals of Mathematical Statistics, 28,* 1040-1043. https://doi.org/10.1214/aoms/1177706809

Freidlin, B., Miao, W., & Gastwirth, J. L. (2003). On the Use of the Shapiro-Wilk Test in Two-Stage Adaptive Inference for Paired Data from Moderate to Very Heavy Tailed Distributions. *Biometrical Journal, 45,* 887-900. https://doi.org/10.1002/bimj.200390056

Gaafar, M. I. A. S. (2016). A New Non-Parametric Test Procedure for the Median of an Asymmetrical Population. *American Journal of Theoretical and Applied Statistics, 5*(6), 376-386. https://doi.org/10.11648/j.ajtas.20160506.17

Gastwirth, J. L. (1966). On Robust Procedures. *Journal of the American Statistical Association, 61,* 929-948. https://doi.org/10.1080/01621459.1966.10482185

Gastwirth, J. L., & Miao, W. (2009). A New Two-Stage Adaptive Nonparametric Test for Paired Differences. *Statistics and Its Interface, 2*, 213-221. https://doi.org/10.4310/SII.2009.v2.n2.a11

Hettmansperger, T. P., & McKean, J. W. (2011). *Robust Nonparametric Statistical Methods* (2nd ed.). Chapman & Hall. https://doi.org/10.1201/b10451

Hettmansperger, T. P., & Sheather, S. J. (1986). Confidence Intervals Based on Interpolated Order Statistics. *Statistics and Probability Letters, 4*, 75-79. https://doi.org/10.1016/0167-7152(86)90021-0

Hollander, M., Wolfe, D. A., & Chicken, E. (2014). *Nonparametric Statistical Methods* (3rd ed.). New York, NY: John Wiley & Sons. https://doi.org/10.1002/9781119196037

Lemmer, H. H. (1993). Adaptive Tests for the Median. *Journal of IEEE Transactions on Reliability, 42,* 442-448. https://doi.org/10.1109/24.257828

McWilliams, T. P. (1990). A Distribution-Free Test for Symmetry Based on a Runs Statistic. *Journal of the American Statistical Association, 85,* 1130-1133. https://doi.org/10.1080/01621459.1990.10474985

Randles, R. H., & Wolfe, D. A. (1979). *Introduction to the Theory of Nonparametric Statistics*. New York, NY: John Wiley & Sons.

Randles, R. H., Fligner, M., Policello, G., & Wolfe, D. (1980). An Asymptotically Distribution-Free Test for Symmetry versus Asymmetry. *Journal of the American Statistics Association, 75,* 168–172. https://doi.org/10.1080/01621459.1980.10477448

Shapiro, S. S., & Wilk, M. B. (1965). An Analysis of Variance Test for Normality (Complete Samples). *Biometrika, 52,* 591 – 611. https://doi.org/10.1093/biomet/52.3-4.591

Yeo, I. K., & Johnson, R. A. (2000). A New Family of Power Transformations to Improve Normality and Symmetry. *Biometrika, 87,* 954 – 959. https://doi.org/10.1093/biomet/87.4.954

Appendix

Full Empirical Levels and Powers Under Some Symmetrical and Asymmetrical Models

Table A1. Empirical levels and Powers of the Standard Uniform distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.05	0.047	0.0204	0.0474	0.047	0.0348	0.0456
		Left	0.0542	0.0524	0.023	0.0516	0.0516	0.0378	0.054
	0.05	Right	0.076	0.0726	0.0302	0.08	0.0806	0.0494	0.0692
		Left	0.0678	0.0688	0.0256	0.0784	0.0788	0.0438	0.0576
	0.10	Right	0.1072	0.1062	0.0358	0.119	0.1198	0.0696	0.0812
		Left	0.1062	0.1016	0.0366	0.1148	0.116	0.0716	0.0832
	0.20	Right	0.2104	0.2008	0.0636	0.2344	0.2382	0.1402	0.1368
		Left	0.212	0.2044	0.0656	0.2378	0.2402	0.1422	0.1364
	0.30	Right	0.3542	0.3336	0.1008	0.3968	0.4024	0.2466	0.2056
		Left	0.348	0.3256	0.1036	0.3788	0.3856	0.239	0.1968
	0.40	Right	0.5232	0.4772	0.1586	0.5402	0.5518	0.3958	0.2852
		Left	0.5166	0.476	0.1654	0.5346	0.5484	0.3898	0.2902
	0.50	Right	0.6804	0.6208	0.2286	0.6742	0.695	0.5396	0.372
		Left	0.6912	0.6242	0.2226	0.6838	0.7048	0.5338	0.366
30	0	Right	0.0482	0.0482	0.05	0.051	0.0512	0.0438	0.0506
		Left	0.0544	0.0524	0.052	0.0544	0.0542	0.047	0.0522
	0.05	Right	0.085	0.0868	0.0668	0.095	0.0954	0.066	0.0676
		Left	0.0808	0.0836	0.0704	0.0952	0.0956	0.0702	0.0712
	0.10	Right	0.1334	0.131	0.0904	0.153	0.1546	0.101	0.0918
		Left	0.1302	0.131	0.0862	0.1482	0.149	0.0952	0.0878
	0.20	Right	0.2824	0.2742	0.159	0.3352	0.3378	0.2128	0.161
		Left	0.2652	0.2544	0.1508	0.3164	0.3208	0.198	0.1516
	0.30	Right	0.4764	0.4458	0.2468	0.5248	0.5296	0.3704	0.2484
		Left	0.4772	0.4494	0.2454	0.5352	0.5408	0.372	0.248
	0.40	Right	0.6766	0.6244	0.3616	0.707	0.7192	0.5484	0.364
		Left	0.678	0.6308	0.3628	0.7122	0.721	0.5532	0.3648
	0.50	Right	0.846	0.7902	0.4846	0.8466	0.86	0.702	0.4874
		Left	0.8366	0.7804	0.4844	0.843	0.8558	0.6924	0.486
50	0	Right	0.0472	0.047	0.033	0.047	0.047	0.0412	0.0526
		Left	0.046	0.0442	0.03	0.0454	0.0456	0.0408	0.047
	0.05	Right	0.0972	0.0946	0.0504	0.1136	0.1138	0.0736	0.0732
		Left	0.0934	0.0928	0.0462	0.112	0.1122	0.074	0.0666
	0.10	Right	0.1664	0.1636	0.0706	0.2066	0.207	0.1236	0.104
		Left	0.1724	0.1676	0.0778	0.2166	0.2176	0.1314	0.1156
	0.20	Right	0.3986	0.3776	0.143	0.4838	0.4864	0.316	0.201
		Left	0.3932	0.3756	0.1496	0.4778	0.4788	0.3156	0.205
	0.30	Right	0.6672	0.6222	0.264	0.739	0.7424	0.5444	0.3376
		Left	0.6662	0.613	0.2694	0.7408	0.7436	0.5452	0.347
	0.40	Right	0.8704	0.8192	0.408	0.9014	0.907	0.7122	0.4918
		Left	0.883	0.829	0.4434	0.9106	0.9154	0.7298	0.5268
	0.50	Right	0.9744	0.9382	0.5874	0.9736	0.9798	0.8322	0.6758
		Left	0.9712	0.9436	0.5904	0.9746	0.9786	0.8334	0.6734

Table A2. Empirical levels and Powers of the Standard Normal distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.0504	0.0492	0.021	0.0502	0.0508	0.0456	0.0488
		Left	0.0484	0.0462	0.0214	0.0462	0.048	0.0394	0.0482
	0.05	Right	0.0776	0.0766	0.0332	0.0792	0.08	0.0694	0.073
		Left	0.0754	0.0708	0.0296	0.0752	0.0784	0.0628	0.0716
	0.10	Right	0.1034	0.1008	0.0454	0.1026	0.1042	0.0898	0.0916
		Left	0.1154	0.1102	0.043	0.112	0.1144	0.0972	0.1
	0.20	Right	0.2196	0.2112	0.0884	0.2112	0.22	0.1874	0.1834
		Left	0.2162	0.2092	0.0872	0.2094	0.2162	0.1802	0.1808
	0.30	Right	0.3604	0.3412	0.1636	0.3446	0.3588	0.3062	0.2878
		Left	0.3628	0.3496	0.161	0.3506	0.3632	0.3016	0.283
	0.40	Right	0.5348	0.5128	0.2608	0.5052	0.5268	0.455	0.418
		Left	0.5374	0.5194	0.255	0.5142	0.531	0.4556	0.4242
	0.50	Right	0.6904	0.6704	0.3844	0.6634	0.6832	0.6096	0.5582
		Left	0.6918	0.6658	0.3766	0.6596	0.6794	0.6038	0.5566
30	0	Right	0.0464	0.0488	0.0462	0.0482	0.0468	0.0458	0.0468
		Left	0.054	0.053	0.0474	0.0538	0.0534	0.0498	0.048
	0.05	Right	0.0876	0.083	0.0768	0.0838	0.0856	0.0786	0.0782
		Left	0.0868	0.0874	0.0768	0.086	0.0876	0.082	0.0778
	0.10	Right	0.1344	0.132	0.115	0.1328	0.1346	0.1246	0.116
		Left	0.1374	0.1334	0.1126	0.1322	0.138	0.1192	0.1138
	0.20	Right	0.2874	0.2818	0.2188	0.2782	0.289	0.2544	0.2208
		Left	0.2774	0.2738	0.2146	0.2728	0.2816	0.2442	0.2162
	0.30	Right	0.4912	0.4764	0.3594	0.4696	0.488	0.426	0.3632
		Left	0.4818	0.4698	0.3574	0.461	0.4808	0.4258	0.3596
	0.40	Right	0.682	0.6646	0.5336	0.6572	0.675	0.6134	0.5366
		Left	0.6978	0.678	0.5318	0.6696	0.6894	0.6216	0.5342
	0.50	Right	0.8454	0.8304	0.692	0.8208	0.8414	0.7764	0.6956
		Left	0.8504	0.8332	0.6876	0.8254	0.8454	0.78	0.6914
50	0	Right	0.0536	0.0528	0.0326	0.0554	0.0542	0.051	0.0534
		Left	0.053	0.0508	0.0326	0.0528	0.0516	0.0488	0.0494
	0.05	Right	0.1002	0.0942	0.0598	0.0954	0.0966	0.0892	0.0864
		Left	0.1016	0.1014	0.0614	0.1024	0.1022	0.096	0.0886
	0.10	Right	0.1686	0.162	0.0934	0.168	0.1714	0.1512	0.1328
		Left	0.1698	0.1664	0.1004	0.1684	0.171	0.1558	0.1396
	0.20	Right	0.41	0.3918	0.241	0.3906	0.4058	0.3616	0.3164
		Left	0.4042	0.3896	0.2414	0.3916	0.404	0.358	0.3082
	0.30	Right	0.6788	0.6636	0.4496	0.6622	0.6742	0.6108	0.5342
		Left	0.6632	0.6436	0.4288	0.645	0.6564	0.59	0.5102
	0.40	Right	0.874	0.8614	0.653	0.8526	0.8698	0.8098	0.7328
		Left	0.8836	0.8718	0.6616	0.87	0.8832	0.8242	0.7408
	0.50	Right	0.9684	0.9596	0.8284	0.9562	0.9668	0.9284	0.8766
		Left	0.9688	0.9606	0.8294	0.9596	0.9678	0.9302	0.883

Table A3. Empirical levels and Powers of the Standard Logistic distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.0474	0.0468	0.0224	0.0516	0.0474	0.0414	0.0454
		Left	0.0486	0.0462	0.0192	0.0456	0.0478	0.0402	0.0462
	0.05	Right	0.0778	0.074	0.0352	0.0752	0.0782	0.0642	0.0772
		Left	0.0756	0.074	0.0306	0.0738	0.0776	0.0666	0.0724
	0.10	Right	0.1096	0.1052	0.0444	0.106	0.1088	0.0922	0.104
		Left	0.114	0.1162	0.05	0.117	0.1184	0.1012	0.1114
	0.20	Right	0.2282	0.2332	0.1094	0.2266	0.229	0.2058	0.2072
		Left	0.2244	0.2276	0.1064	0.2288	0.2326	0.1982	0.2056
	0.30	Right	0.3752	0.3804	0.1968	0.3748	0.379	0.3344	0.3332
		Left	0.3868	0.3888	0.2078	0.38	0.3904	0.35	0.339
	0.40	Right	0.5474	0.5554	0.326	0.5418	0.5554	0.5084	0.4928
		Left	0.5532	0.5572	0.3238	0.543	0.5536	0.5064	0.4866
	0.50	Right	0.7062	0.7186	0.4734	0.7074	0.7152	0.6726	0.6448
		Left	0.7074	0.717	0.4672	0.702	0.711	0.671	0.6362
30	0	Right	0.048	0.0474	0.0468	0.0498	0.0484	0.042	0.0478
		Left	0.0478	0.0496	0.0474	0.0486	0.049	0.045	0.0478
	0.05	Right	0.0854	0.0872	0.084	0.0864	0.0856	0.0782	0.0856
		Left	0.0852	0.0862	0.0774	0.0858	0.088	0.0782	0.0788
	0.10	Right	0.1356	0.1402	0.1276	0.1378	0.1402	0.128	0.1298
		Left	0.1352	0.1362	0.1258	0.1372	0.137	0.1236	0.126
	0.20	Right	0.2908	0.2944	0.2566	0.2834	0.2914	0.2746	0.2588
		Left	0.3	0.3076	0.2566	0.2926	0.3032	0.2784	0.2596
	0.30	Right	0.4978	0.5178	0.4214	0.503	0.5094	0.4784	0.4248
		Left	0.5062	0.521	0.4332	0.5064	0.512	0.4792	0.436
	0.40	Right	0.6894	0.7108	0.609	0.6978	0.7036	0.6704	0.6122
		Left	0.6928	0.7188	0.6218	0.6972	0.701	0.6786	0.6244
	0.50	Right	0.8486	0.87	0.7832	0.8564	0.8584	0.8424	0.787
		Left	0.8546	0.869	0.7732	0.8558	0.8642	0.8294	0.7748
50	0	Right	0.052	0.0566	0.036	0.054	0.0548	0.0546	0.0542
		Left	0.0508	0.0528	0.034	0.052	0.0516	0.0498	0.0538
	0.05	Right	0.0968	0.0974	0.0626	0.0974	0.096	0.09	0.091
		Left	0.1106	0.1102	0.0676	0.112	0.1142	0.0994	0.0958
	0.10	Right	0.1826	0.188	0.1126	0.1824	0.1866	0.1724	0.1632
		Left	0.1862	0.1874	0.1224	0.1852	0.1872	0.1802	0.1684
	0.20	Right	0.4038	0.4252	0.2942	0.4092	0.4132	0.3964	0.3688
		Left	0.4258	0.4386	0.2882	0.429	0.4414	0.4072	0.3664
	0.30	Right	0.677	0.7018	0.5334	0.6842	0.6916	0.6638	0.6184
		Left	0.6746	0.7008	0.5164	0.6832	0.6876	0.657	0.605
	0.40	Right	0.8728	0.8914	0.7522	0.8806	0.883	0.8664	0.8162
		Left	0.8756	0.8968	0.7592	0.891	0.887	0.8722	0.8218
	0.50	Right	0.9668	0.9762	0.8996	0.967	0.972	0.9628	0.9362
		Left	0.9662	0.9766	0.8982	0.9702	0.972	0.9624	0.935

Table A4. Empirical levels and Powers of the Double Exponential distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.0522	0.0508	0.0196	0.05	0.0532	0.0384	0.0532
		Left	0.0476	0.0436	0.0178	0.0482	0.047	0.0356	0.046
	0.05	Right	0.0758	0.0816	0.0416	0.0814	0.0798	0.0688	0.0888
		Left	0.072	0.0762	0.0408	0.081	0.0774	0.067	0.0856
	0.10	Right	0.1158	0.1302	0.0774	0.1346	0.127	0.1176	0.145
		Left	0.119	0.1312	0.0772	0.1344	0.132	0.1188	0.1464
	0.20	Right	0.2302	0.265	0.1734	0.2718	0.2536	0.2526	0.2944
		Left	0.2512	0.2896	0.1802	0.2998	0.2772	0.2714	0.3124
	0.30	Right	0.3988	0.461	0.3168	0.467	0.4396	0.4476	0.486
		Left	0.399	0.4554	0.3202	0.4606	0.4346	0.4446	0.4852
	0.40	Right	0.5594	0.6344	0.4762	0.636	0.6058	0.628	0.6552
		Left	0.5592	0.6308	0.4762	0.6342	0.596	0.6226	0.649
	0.50	Right	0.7096	0.7766	0.6202	0.7732	0.746	0.773	0.7814
		Left	0.7158	0.7786	0.6282	0.7734	0.7516	0.769	0.779
30	0	Right	0.0468	0.0466	0.044	0.0454	0.0488	0.0398	0.0442
		Left	0.0496	0.0498	0.0498	0.0486	0.048	0.0432	0.0504
	0.05	Right	0.0826	0.0938	0.0946	0.094	0.0906	0.0834	0.0952
		Left	0.0902	0.097	0.0948	0.0956	0.0928	0.0856	0.0964
	0.10	Right	0.1414	0.1626	0.1828	0.1694	0.1506	0.1572	0.1844
		Left	0.1488	0.1784	0.1864	0.1808	0.1686	0.1654	0.1886
	0.20	Right	0.3028	0.3674	0.3866	0.3742	0.3422	0.3656	0.3896
		Left	0.3048	0.3738	0.3934	0.3772	0.342	0.3726	0.396
	0.30	Right	0.5144	0.597	0.6146	0.6074	0.56	0.6062	0.6188
		Left	0.4956	0.5928	0.6078	0.601	0.5482	0.601	0.6102
	0.40	Right	0.7134	0.8012	0.8044	0.8058	0.7644	0.8126	0.807
		Left	0.7028	0.7834	0.7842	0.7866	0.7492	0.7942	0.7868
	0.50	Right	0.8532	0.9116	0.8984	0.907	0.8868	0.9158	0.9006
		Left	0.8516	0.9138	0.9032	0.911	0.8884	0.9206	0.905
50	0	Right	0.0474	0.0494	0.0336	0.0476	0.0492	0.0416	0.0516
		Left	0.0566	0.0506	0.0332	0.0506	0.0534	0.0458	0.052
	0.05	Right	0.1018	0.1154	0.0864	0.1212	0.112	0.1106	0.129
		Left	0.1058	0.114	0.0856	0.1172	0.113	0.1084	0.1264
	0.10	Right	0.1708	0.2106	0.1718	0.2232	0.1964	0.2074	0.2374
		Left	0.1754	0.212	0.181	0.2188	0.2	0.2178	0.2466
	0.20	Right	0.4032	0.5054	0.4588	0.5248	0.462	0.5304	0.5462
		Left	0.4138	0.5178	0.4756	0.5424	0.4786	0.5372	0.5602
	0.30	Right	0.6778	0.7918	0.7354	0.797	0.7478	0.8066	0.801
		Left	0.6846	0.797	0.748	0.806	0.7508	0.8138	0.8116
	0.40	Right	0.865	0.9348	0.9088	0.9372	0.91	0.9448	0.9414
		Left	0.8656	0.9346	0.9072	0.9358	0.9084	0.9424	0.9396
	0.50	Right	0.9606	0.986	0.9718	0.985	0.9782	0.9882	0.9846
		Left	0.9632	0.9886	0.9742	0.9874	0.979	0.9896	0.9868

Table A5. Empirical levels and Powers of the T (2) distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.0404	0.0476	0.0226	0.0492	0.0498	0.0374	0.0494
		Left	0.038	0.0472	0.0172	0.0458	0.0478	0.0334	0.0472
	0.05	Right	0.0552	0.0654	0.0284	0.0638	0.0654	0.0482	0.0622
		Left	0.0536	0.0612	0.0276	0.0648	0.061	0.0518	0.0698
	0.10	Right	0.0692	0.0862	0.0412	0.089	0.0848	0.0708	0.0894
		Left	0.0728	0.0864	0.0374	0.085	0.0872	0.068	0.084
	0.20	Right	0.099	0.116	0.0562	0.132	0.1176	0.0976	0.1246
		Left	0.1122	0.1364	0.0686	0.1364	0.1302	0.1158	0.139
	0.30	Right	0.141	0.1818	0.1052	0.1936	0.176	0.1616	0.1982
		Left	0.1428	0.187	0.1004	0.203	0.1774	0.1656	0.1964
	0.40	Right	0.1932	0.2588	0.1476	0.2778	0.2472	0.2348	0.2676
		Left	0.195	0.2582	0.1528	0.2756	0.2438	0.2342	0.268
	0.50	Right	0.2624	0.3528	0.2128	0.3718	0.335	0.3248	0.356
		Left	0.2566	0.3574	0.212	0.3734	0.3282	0.33	0.3648
30	0	Right	0.0522	0.0554	0.0518	0.0516	0.0576	0.0436	0.0524
		Left	0.0444	0.0514	0.0508	0.054	0.0522	0.0434	0.0516
	0.05	Right	0.0558	0.069	0.0664	0.0698	0.0658	0.057	0.0678
		Left	0.0604	0.0742	0.0682	0.069	0.0716	0.0584	0.069
	0.10	Right	0.0776	0.1002	0.1008	0.106	0.0972	0.0882	0.1018
		Left	0.073	0.0908	0.0924	0.0946	0.0894	0.0782	0.094
	0.20	Right	0.1204	0.1594	0.1522	0.1632	0.1488	0.1414	0.1542
		Left	0.1124	0.1546	0.152	0.1622	0.1486	0.1402	0.153
	0.30	Right	0.1716	0.2436	0.2374	0.2536	0.2262	0.2258	0.2392
		Left	0.1698	0.244	0.2374	0.2512	0.2266	0.2228	0.2382
	0.40	Right	0.242	0.3494	0.3428	0.3804	0.3192	0.3378	0.3468
		Left	0.2406	0.3484	0.3426	0.3704	0.3202	0.339	0.345
	0.50	Right	0.3116	0.4642	0.4636	0.4902	0.42	0.4608	0.4654
		Left	0.3074	0.4536	0.4578	0.4822	0.4104	0.4514	0.4614
50	0	Right	0.0412	0.0486	0.031	0.0472	0.0488	0.0408	0.049
		Left	0.0432	0.0522	0.0356	0.052	0.052	0.0462	0.0528
	0.05	Right	0.0588	0.0726	0.046	0.0788	0.0728	0.0642	0.0736
		Left	0.0602	0.0734	0.0502	0.0782	0.0718	0.0624	0.074
	0.10	Right	0.0786	0.1036	0.0722	0.1104	0.0996	0.0906	0.1062
		Left	0.0822	0.099	0.0706	0.105	0.0966	0.0902	0.1034
	0.20	Right	0.1402	0.202	0.156	0.2206	0.1926	0.1962	0.2118
		Left	0.1424	0.2048	0.147	0.216	0.1866	0.191	0.2014
	0.30	Right	0.2122	0.3312	0.2594	0.3544	0.3068	0.3234	0.3274
		Left	0.2084	0.3394	0.2652	0.3662	0.3042	0.329	0.3384
	0.40	Right	0.313	0.5008	0.4022	0.5358	0.4538	0.4938	0.488
		Left	0.3062	0.4832	0.405	0.529	0.4352	0.4884	0.4916
	0.50	Right	0.3942	0.6346	0.5524	0.682	0.5748	0.6396	0.6326
		Left	0.4058	0.6426	0.5662	0.6818	0.5892	0.6472	0.6448

Table A6. Empirical levels and Powers of the Cauchy distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.03	0.0464	0.0224	0.0514	0.0458	0.0344	0.0504
		Left	0.0286	0.0504	0.022	0.0518	0.0526	0.0338	0.049
	0.05	Right	0.0346	0.061	0.031	0.0696	0.062	0.046	0.0688
		Left	0.0366	0.064	0.0322	0.0718	0.0634	0.0468	0.0718
	0.10	Right	0.0414	0.0728	0.0332	0.0826	0.0756	0.0536	0.0822
		Left	0.0362	0.0712	0.0348	0.0872	0.0686	0.0546	0.0822
	0.20	Right	0.0596	0.1236	0.0734	0.1512	0.1168	0.1014	0.1432
		Left	0.0628	0.1182	0.0678	0.141	0.1126	0.0962	0.14
	0.30	Right	0.0762	0.1772	0.1132	0.2162	0.1606	0.1566	0.2076
		Left	0.077	0.1754	0.1098	0.2284	0.1664	0.155	0.2172
	0.40	Right	0.101	0.2474	0.1698	0.3118	0.221	0.2324	0.2994
		Left	0.1034	0.2376	0.1668	0.3032	0.218	0.2184	0.2882
	0.50	Right	0.1374	0.3184	0.2444	0.4086	0.2824	0.3182	0.3932
		Left	0.123	0.3062	0.2388	0.4038	0.2726	0.307	0.3814
30	0	Right	0.0292	0.0478	0.0488	0.0486	0.0476	0.0372	0.0494
		Left	0.025	0.0426	0.0412	0.0462	0.0408	0.0338	0.0422
	0.05	Right	0.0378	0.0708	0.0728	0.074	0.0696	0.0586	0.0736
		Left	0.036	0.0658	0.0644	0.0646	0.066	0.0504	0.0652
	0.10	Right	0.041	0.0856	0.0908	0.0984	0.08	0.0688	0.092
		Left	0.0418	0.0898	0.0958	0.0998	0.0852	0.0782	0.0972
	0.20	Right	0.0608	0.149	0.1718	0.1906	0.134	0.1382	0.1724
		Left	0.0612	0.1516	0.1694	0.1768	0.1344	0.1358	0.1708
	0.30	Right	0.0778	0.2246	0.2624	0.2892	0.1894	0.221	0.2638
		Left	0.082	0.2278	0.266	0.2888	0.201	0.2246	0.2682
	0.40	Right	0.0972	0.3036	0.3606	0.4144	0.2618	0.3098	0.3638
		Left	0.1078	0.3268	0.385	0.4276	0.2762	0.3378	0.3882
	0.50	Right	0.1388	0.426	0.5082	0.5522	0.3646	0.4454	0.5108
		Left	0.1296	0.4092	0.4956	0.5438	0.3474	0.4424	0.498
50	0	Right	0.0318	0.0494	0.0328	0.0504	0.0496	0.038	0.052
		Left	0.032	0.0456	0.0344	0.045	0.0492	0.038	0.051
	0.05	Right	0.0352	0.0742	0.0504	0.083	0.0706	0.0598	0.0756
		Left	0.04	0.074	0.0484	0.078	0.069	0.058	0.0788
	0.10	Right	0.0468	0.1084	0.0834	0.123	0.099	0.0962	0.1184
		Left	0.0456	0.1098	0.082	0.121	0.0988	0.0966	0.115
	0.20	Right	0.066	0.1966	0.1674	0.2544	0.174	0.188	0.2296
		Left	0.0596	0.189	0.1662	0.2426	0.164	0.1864	0.2248
	0.30	Right	0.0792	0.3074	0.3084	0.4202	0.259	0.33	0.3904
		Left	0.0832	0.304	0.2962	0.414	0.2518	0.321	0.3714
	0.40	Right	0.1036	0.4438	0.4468	0.5964	0.3754	0.4824	0.5414
		Left	0.1072	0.453	0.4564	0.5886	0.3748	0.4948	0.5434
	0.50	Right	0.1334	0.5774	0.6238	0.762	0.4876	0.6512	0.7042
		Left	0.1396	0.5912	0.6124	0.7558	0.4946	0.6462	0.6988

Table A7. Empirical levels and Powers of the Weibull (2) distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.1046	0.077	0.0198	0.0806	0.0956	0.052	0.05
		Left	0.0226	0.0314	0.02	0.028	0.0234	0.0326	0.0494
	0.05	Right	0.1448	0.109	0.029	0.113	0.1358	0.08	0.0692
		Left	0.0354	0.047	0.0314	0.0434	0.0372	0.0474	0.0706
	0.10	Right	0.194	0.1492	0.0452	0.1548	0.1832	0.1092	0.097
		Left	0.058	0.0746	0.0464	0.0696	0.0608	0.0698	0.1014
	0.20	Right	0.3702	0.2886	0.0948	0.2986	0.3464	0.2106	0.1794
		Left	0.128	0.1466	0.0822	0.1384	0.129	0.1326	0.17
	0.30	Right	0.5456	0.4474	0.1604	0.436	0.5104	0.3326	0.2784
		Left	0.216	0.2436	0.14	0.2312	0.2182	0.219	0.26
	0.40	Right	0.7332	0.6366	0.2634	0.6098	0.7038	0.4886	0.4226
		Left	0.361	0.382	0.2306	0.3626	0.353	0.345	0.373
	0.50	Right	0.88	0.81	0.417	0.7654	0.8526	0.6476	0.584
		Left	0.52	0.5406	0.3346	0.5152	0.5082	0.4912	0.5038
30	0	Right	0.1284	0.0832	0.0492	0.0936	0.1146	0.0586	0.0498
		Left	0.0164	0.0282	0.05	0.028	0.0192	0.0362	0.0506
	0.05	Right	0.2042	0.1446	0.072	0.1554	0.1872	0.0984	0.0726
		Left	0.0346	0.0514	0.075	0.0456	0.0364	0.0588	0.0754
	0.10	Right	0.2964	0.209	0.1128	0.2224	0.2712	0.141	0.1144
		Left	0.0562	0.0832	0.113	0.0724	0.0582	0.0962	0.114
	0.20	Right	0.5132	0.3946	0.2174	0.3988	0.478	0.2714	0.22
		Left	0.1482	0.1908	0.2136	0.1646	0.1558	0.2012	0.2152
	0.30	Right	0.7436	0.6192	0.3734	0.5944	0.7036	0.4426	0.3754
		Left	0.2832	0.3388	0.3378	0.2972	0.2822	0.333	0.3404
	0.40	Right	0.8908	0.8056	0.544	0.769	0.8638	0.6094	0.5476
		Left	0.4652	0.5216	0.4802	0.4808	0.4664	0.5072	0.4832
	0.50	Right	0.9734	0.9278	0.7236	0.8896	0.958	0.7716	0.7268
		Left	0.6322	0.6832	0.6226	0.6398	0.6304	0.658	0.626
50	0	Right	0.1794	0.1044	0.0278	0.1292	0.1612	0.0576	0.0488
		Left	0.0108	0.021	0.028	0.0178	0.012	0.0302	0.0476
	0.05	Right	0.3036	0.1868	0.0558	0.2164	0.2746	0.105	0.087
		Left	0.0218	0.0446	0.0562	0.0348	0.024	0.0614	0.0846
	0.10	Right	0.4348	0.2948	0.0992	0.3228	0.4	0.1664	0.1414
		Left	0.0452	0.0796	0.0922	0.0584	0.0484	0.1022	0.1334
	0.20	Right	0.7338	0.5682	0.236	0.5764	0.6916	0.3338	0.305
		Left	0.1672	0.2402	0.2134	0.1878	0.17	0.2462	0.284
	0.30	Right	0.9158	0.8062	0.4316	0.7782	0.8906	0.5352	0.519
		Left	0.374	0.4634	0.3918	0.3872	0.3692	0.4544	0.4804
	0.40	Right	0.9828	0.946	0.6678	0.9076	0.9762	0.7386	0.7456
		Left	0.6316	0.7092	0.5906	0.646	0.6302	0.6792	0.6746
	0.50	Right	0.9986	0.9944	0.8514	0.9754	0.9976	0.8878	0.9026
		Left	0.8292	0.8748	0.7722	0.8308	0.8232	0.8484	0.8332

Table A8. Empirical levels and Powers of the Gamma (4) distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.1304	0.0856	0.022	0.084	0.1124	0.0532	0.048
		Left	0.0142	0.0252	0.0224	0.0294	0.018	0.0292	0.0532
	0.05	Right	0.2026	0.1438	0.034	0.1326	0.177	0.0936	0.0806
		Left	0.0298	0.0438	0.03	0.0462	0.0334	0.0446	0.0778
	0.10	Right	0.2614	0.1888	0.0536	0.1808	0.236	0.1222	0.108
		Left	0.0416	0.0584	0.0438	0.0576	0.0432	0.058	0.0944
	0.20	Right	0.4392	0.3348	0.0988	0.3082	0.3984	0.2132	0.1888
		Left	0.107	0.1402	0.0906	0.1338	0.1152	0.1342	0.1824
	0.30	Right	0.6546	0.5316	0.1868	0.4788	0.602	0.3586	0.3262
		Left	0.1902	0.2354	0.154	0.2212	0.197	0.2228	0.283
	0.40	Right	0.8286	0.7254	0.3326	0.6612	0.783	0.5242	0.4904
		Left	0.3098	0.371	0.2492	0.347	0.322	0.3436	0.3944
	0.50	Right	0.933	0.8734	0.4874	0.8144	0.9118	0.6846	0.6624
		Left	0.4522	0.5122	0.3568	0.4706	0.4522	0.472	0.5312
30	0	Right	0.1878	0.1114	0.049	0.1082	0.1604	0.0616	0.05
		Left	0.0078	0.0172	0.0448	0.0196	0.0106	0.0304	0.0456
	0.05	Right	0.2838	0.1798	0.0814	0.1682	0.2428	0.0984	0.0826
		Left	0.0188	0.038	0.0742	0.0424	0.0216	0.0536	0.0754
	0.10	Right	0.3846	0.2548	0.1256	0.2308	0.3314	0.1406	0.1262
		Left	0.034	0.0646	0.1116	0.0636	0.0434	0.0822	0.1122
	0.20	Right	0.628	0.48	0.2394	0.4194	0.568	0.2792	0.2416
		Left	0.0976	0.1636	0.221	0.1426	0.1124	0.1892	0.2222
	0.30	Right	0.8358	0.708	0.4144	0.6206	0.7852	0.458	0.4164
		Left	0.2164	0.3238	0.367	0.275	0.2348	0.3396	0.3692
	0.40	Right	0.9536	0.891	0.635	0.805	0.927	0.665	0.6394
		Left	0.3996	0.5056	0.5306	0.448	0.4186	0.5156	0.5338
	0.50	Right	0.9918	0.9694	0.8012	0.9146	0.9848	0.8162	0.8038
		Left	0.5694	0.6734	0.665	0.5982	0.5732	0.6666	0.6682
50	0	Right	0.2808	0.1348	0.0306	0.135	0.2202	0.0486	0.0484
		Left	0.0046	0.017	0.03	0.0202	0.0072	0.0306	0.0488
	0.05	Right	0.4186	0.2288	0.062	0.2154	0.3432	0.0908	0.0878
		Left	0.0134	0.035	0.0584	0.0376	0.0166	0.0606	0.0886
	0.10	Right	0.5798	0.3722	0.1016	0.3264	0.501	0.1466	0.1452
		Left	0.0268	0.071	0.0988	0.0644	0.0326	0.105	0.1422
	0.20	Right	0.8444	0.6602	0.263	0.5468	0.7752	0.3196	0.337
		Left	0.1022	0.2014	0.2378	0.1604	0.122	0.2514	0.3052
	0.30	Right	0.9684	0.8926	0.511	0.7824	0.943	0.5676	0.6048
		Left	0.2738	0.438	0.4274	0.357	0.3062	0.4546	0.5186
	0.40	Right	0.9976	0.9822	0.7712	0.9286	0.9946	0.8018	0.8316
		Left	0.504	0.6804	0.638	0.5796	0.5364	0.6812	0.7168
	0.50	Right	1	0.9992	0.9294	0.986	0.9996	0.937	0.9554
		Left	0.7484	0.8594	0.807	0.792	0.7722	0.847	0.8602

Table A9. Empirical levels and Powers of the Standard Extreme Value distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.1298	0.0888	0.0214	0.0786	0.1098	0.053	0.0516
		Left	0.0122	0.0212	0.02	0.0248	0.0158	0.0278	0.0472
	0.05	Right	0.1888	0.13	0.0346	0.1126	0.168	0.078	0.0778
		Left	0.0304	0.0476	0.0332	0.0544	0.0358	0.051	0.0798
	0.10	Right	0.274	0.1962	0.0496	0.1756	0.2408	0.1218	0.1088
		Left	0.044	0.0632	0.0498	0.0706	0.0502	0.0718	0.1016
	0.20	Right	0.4658	0.3528	0.1126	0.313	0.4178	0.2212	0.2078
		Left	0.099	0.1328	0.0974	0.1328	0.1068	0.1362	0.1814
	0.30	Right	0.6576	0.551	0.2154	0.4958	0.6122	0.3854	0.3618
		Left	0.1848	0.2468	0.1728	0.2366	0.2006	0.245	0.2996
	0.40	Right	0.8422	0.7576	0.3722	0.6944	0.8088	0.5722	0.548
		Left	0.3208	0.392	0.2812	0.3692	0.3356	0.375	0.4332
	0.50	Right	0.9416	0.8978	0.5496	0.8374	0.9238	0.7286	0.7168
		Left	0.4632	0.5406	0.3964	0.5104	0.4842	0.5128	0.564
30	0	Right	0.1796	0.1038	0.0488	0.0924	0.146	0.0566	0.049
		Left	0.0088	0.0174	0.049	0.0282	0.0094	0.03	0.0502
	0.05	Right	0.3006	0.189	0.083	0.1532	0.249	0.0978	0.0846
		Left	0.0212	0.0428	0.0784	0.0476	0.0276	0.0556	0.0796
	0.10	Right	0.3756	0.2426	0.1212	0.2102	0.3142	0.1408	0.1224
		Left	0.0372	0.0726	0.1166	0.076	0.0458	0.0932	0.1176
	0.20	Right	0.6386	0.486	0.251	0.407	0.573	0.2862	0.2528
		Left	0.1022	0.1756	0.2376	0.168	0.1242	0.2018	0.2398
	0.30	Right	0.847	0.7284	0.461	0.6296	0.7968	0.4938	0.4642
		Left	0.2186	0.3292	0.3866	0.299	0.2468	0.3636	0.3906
	0.40	Right	0.956	0.9018	0.6706	0.8276	0.9358	0.7072	0.6748
		Left	0.3882	0.5216	0.56	0.468	0.4204	0.5434	0.5626
	0.50	Right	0.9902	0.9744	0.8438	0.934	0.9856	0.8586	0.8464
		Left	0.5762	0.7022	0.7076	0.6458	0.608	0.7082	0.7108
50	0	Right	0.2862	0.1378	0.0366	0.1188	0.2178	0.0508	0.0538
		Left	0.0048	0.018	0.0314	0.0264	0.008	0.0348	0.053
	0.05	Right	0.4132	0.2278	0.061	0.1872	0.3328	0.0866	0.0902
		Left	0.0116	0.0402	0.063	0.0524	0.0178	0.0666	0.09
	0.10	Right	0.5686	0.3534	0.121	0.291	0.478	0.1616	0.1654
		Left	0.027	0.0762	0.111	0.085	0.0372	0.116	0.1564
	0.20	Right	0.8452	0.6726	0.288	0.5472	0.7834	0.3428	0.3694
		Left	0.099	0.2168	0.251	0.193	0.1278	0.2698	0.3268
	0.30	Right	0.9712	0.8982	0.546	0.7876	0.9488	0.6038	0.6382
		Left	0.2868	0.4706	0.478	0.4112	0.3348	0.5104	0.5602
	0.40	Right	0.9956	0.9798	0.7914	0.9318	0.9904	0.8184	0.8456
		Left	0.515	0.7034	0.6682	0.6314	0.574	0.7122	0.748
	0.50	Right	1	0.9988	0.9478	0.9906	0.9998	0.9568	0.9696
		Left	0.7472	0.8846	0.8392	0.8232	0.7924	0.8764	0.8892

Table A10. Empirical levels and Powers of the Gamma (2) distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.2042	0.1244	0.023	0.1042	0.1656	0.055	0.0526
		Left	0.0094	0.0172	0.0164	0.026	0.0124	0.0206	0.046
	0.05	Right	0.2746	0.173	0.0354	0.148	0.2296	0.0836	0.0778
		Left	0.0208	0.0382	0.0348	0.0416	0.0252	0.0432	0.0762
	0.10	Right	0.3824	0.2476	0.0544	0.2052	0.3152	0.1212	0.1116
		Left	0.0312	0.0552	0.0468	0.0534	0.0352	0.0558	0.0998
	0.20	Right	0.601	0.4446	0.118	0.3558	0.529	0.2308	0.2218
		Left	0.0792	0.117	0.102	0.1146	0.0834	0.1208	0.1932
	0.30	Right	0.8038	0.666	0.235	0.5464	0.7388	0.373	0.3816
		Left	0.1384	0.2018	0.1662	0.1846	0.1516	0.2008	0.2892
	0.40	Right	0.9322	0.8468	0.3966	0.73	0.8902	0.5424	0.578
		Left	0.2586	0.3328	0.2626	0.3012	0.265	0.32	0.4162
	0.50	Right	0.9878	0.9522	0.5858	0.8724	0.972	0.7128	0.75
		Left	0.3842	0.4768	0.3688	0.4188	0.3886	0.4384	0.5426
30	0	Right	0.2884	0.1474	0.0494	0.1166	0.219	0.0558	0.05
		Left	0.0052	0.0166	0.0508	0.0262	0.0058	0.0302	0.0514
	0.05	Right	0.419	0.233	0.0762	0.1726	0.3256	0.089	0.0776
		Left	0.0114	0.0312	0.0854	0.0398	0.0132	0.0488	0.0866
	0.10	Right	0.543	0.3188	0.1188	0.233	0.433	0.1266	0.1202
		Left	0.0196	0.0504	0.1172	0.0568	0.0242	0.0776	0.1186
	0.20	Right	0.789	0.59	0.2676	0.4346	0.6918	0.2748	0.2702
		Left	0.0646	0.139	0.2368	0.1266	0.0804	0.1718	0.2392
	0.30	Right	0.9404	0.8196	0.4806	0.6636	0.8872	0.48	0.4852
		Left	0.148	0.2716	0.3862	0.2238	0.1728	0.31	0.3906
	0.40	Right	0.9916	0.9498	0.7062	0.8512	0.9744	0.6882	0.7102
		Left	0.2954	0.4504	0.5376	0.3714	0.3216	0.4764	0.54
	0.50	Right	0.9986	0.9942	0.8934	0.9634	0.9968	0.8686	0.896
		Left	0.4576	0.6194	0.6832	0.5242	0.4884	0.6386	0.6852
50	0	Right	0.4598	0.1972	0.0314	0.148	0.3354	0.0398	0.0488
		Left	0.0016	0.0092	0.0332	0.0258	0.0022	0.0332	0.0492
	0.05	Right	0.62	0.328	0.0638	0.2192	0.4794	0.0722	0.0898
		Left	0.005	0.0224	0.0668	0.0432	0.008	0.067	0.093
	0.10	Right	0.769	0.4732	0.1078	0.312	0.644	0.1236	0.1512
		Left	0.011	0.0514	0.112	0.0684	0.015	0.1138	0.1602
	0.20	Right	0.9502	0.7862	0.2962	0.544	0.888	0.3182	0.3806
		Left	0.0544	0.16	0.2562	0.1392	0.0704	0.2586	0.3272
	0.30	Right	0.9952	0.9568	0.5844	0.8006	0.9868	0.5966	0.6684
		Left	0.1672	0.37	0.4594	0.2846	0.204	0.4646	0.5522
	0.40	Right	0.9998	0.9964	0.8352	0.9494	0.9988	0.8396	0.8856
		Left	0.3698	0.6146	0.6582	0.4864	0.422	0.6696	0.7376
	0.50	Right	1	1	0.9692	0.9974	1	0.97	0.9834
		Left	0.6102	0.8156	0.8192	0.695	0.6568	0.83	0.871

Table A11. Empirical levels and Powers of the Standard Lognormal distribution

n	δ	H_1	T	W	Sign	GM	NS	Baklizi	HS
20	0	Right	0.373	0.1854	0.017	0.0742	0.2588	0.034	0.0528
		Left	0.0022	0.0096	0.021	0.0588	0.0052	0.0224	0.052
	0.05	Right	0.6034	0.39	0.0656	0.1896	0.4922	0.09	0.1424
		Left	0.0124	0.0352	0.0584	0.1148	0.0178	0.0622	0.1192
	0.10	Right	0.81	0.6582	0.1758	0.3582	0.7412	0.2104	0.3066
		Left	0.0344	0.095	0.1226	0.1896	0.0582	0.1286	0.2246
	0.20	Right	0.9842	0.9938	0.8022	0.911	0.9968	0.824	0.905
		Left	0.1208	0.2736	0.31	0.3506	0.1746	0.3194	0.4748
	0.30	Right	0.9966	1	1	1	1	1	1
		Left	0.2972	0.5064	0.544	0.5212	0.3758	0.5574	0.7064
	0.40	Right	0.999	1	1	1	1	1	1
		Left	0.456	0.6984	0.7044	0.672	0.5658	0.7158	0.8396
	0.50	Right	0.9986	1	1	1	1	1	1
		Left	0.6246	0.8332	0.8396	0.7974	0.7232	0.849	0.9302
30	0	Right	0.5794	0.2556	0.0506	0.0718	0.3706	0.0366	0.0508
		Left	2.00E-04	0.0056	0.0458	0.0782	0.0016	0.0218	0.0462
	0.05	Right	0.8198	0.5336	0.1714	0.1992	0.6654	0.1164	0.1726
		Left	0.0052	0.0324	0.1422	0.1626	0.0112	0.0752	0.1434
	0.10	Right	0.9568	0.8358	0.4278	0.448	0.9094	0.3272	0.4314
		Left	0.0202	0.101	0.2846	0.266	0.0466	0.1736	0.2866
	0.20	Right	0.9958	0.9998	0.9752	0.9624	1	0.9428	0.976
		Left	0.111	0.365	0.6122	0.4936	0.2068	0.4808	0.6158
	0.30	Right	0.9994	1	1	1	1	1	1
		Left	0.3042	0.6642	0.85	0.6792	0.4644	0.7516	0.8516
	0.40	Right	1	1	1	1	1	1	1
		Left	0.509	0.8574	0.9474	0.8048	0.6928	0.9006	0.949
	0.50	Right	0.9998	1	1	1	1	1	1
		Left	0.6898	0.9414	0.9832	0.901	0.8476	0.9626	0.9838
50	0	Right	0.8268	0.3544	0.0332	0.0644	0.558	0.0334	0.0544
		Left	0	0.0026	0.0324	0.1048	2.00E-04	0.0324	0.0512
	0.05	Right	0.9734	0.7176	0.1524	0.221	0.8674	0.1526	0.2066
		Left	0	0.025	0.1366	0.2552	0.0038	0.1368	0.1922
	0.10	Right	0.9974	0.9588	0.4906	0.5214	0.9894	0.491	0.5772
		Left	0.0062	0.108	0.325	0.4254	0.0308	0.325	0.4002
	0.20	Right	0.9994	1	0.9978	0.9954	1	0.9978	0.9992
		Left	0.0904	0.4886	0.7518	0.7048	0.2458	0.752	0.814
	0.30	Right	1	1	1	1	1	1	1
		Left	0.3088	0.8376	0.941	0.8496	0.602	0.941	0.9652
	0.40	Right	1	1	1	1	1	1	1
		Left	0.585	0.9656	0.9902	0.9392	0.854	0.9902	0.9954
	0.50	Right	1	1	1	1	1	1	1
		Left	0.7776	0.9912	0.998	0.9732	0.9524	0.998	0.9996