Liu Estimator in Semiparametric Partially Linear Varying Coefficient Models

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Abstract

This paper considers biased estimation for partially linear varying coefficient model to overcome the problem of multi-collinearity. By the Liu estimation approach, we construct a profile Liu estimator for the constant coefficients. Furthermore, a restricted profile-Liu estimator is proposed for the situation that some additional linear restrictions are available. The properties of the proposed estimators are investigated.

Keywords: Liu estimation, partially linear varying coefficient model, profile least-squares, restricted estimation

1. Introduction

Over the last three decades, nonparametric and semiparametric models have attracted much attention. We consider the following semiparametric partially linear varying coefficient model

$$y_i = \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} + \mathbf{z}_i^{\mathrm{T}}\boldsymbol{\alpha}(u_i) + \varepsilon_i, i = 1, 2, \cdots, n \tag{1.1}$$

with $y_i s$ are responses; $\mathbf{x}_i = (x_{i1}, x_{i2}, \cdots, x_{ip})^{\mathrm{T}}$, $\mathbf{z}_i = (z_{i1}, z_{i2}, \cdots, z_{iq})^{\mathrm{T}}$ and u_i are associated covariates; $\boldsymbol{\beta} = (\beta_1, \beta_2, \cdots, \beta_p)^{\mathrm{T}}$ is a vector of unknown regression coefficients and $\boldsymbol{\alpha}(\cdot) = (\alpha_1(\cdot), \alpha_1(\cdot), \cdots, \alpha_q(\cdot))^{\mathrm{T}}$ are unknown nonparametric functions; $\varepsilon_i s$ are independent random errors with $\mathrm{E}(\varepsilon_i|\mathbf{x}_i, \mathbf{z}_i, u_i) = 0$ and $\sigma^2 = \mathrm{E}[\varepsilon_i^2|\mathbf{x}_i, \mathbf{z}_i, u_i]$.

For model (1.1), how to estimate the unknown regression coefficients is an interesting and important problem. Several methods have been proposed, such as local polynomial estimator of Zhang *et al.* (2002), series estimator of Ahmad *et al.* (2005), and local linear based profile least-squares approach of Fan and Huang (2005). However, the problem of multi-collinearity was not considered by the above literatures. The problem of multi-collinearity of linear regression model may cause serious problem for least squares estimators of the regression coefficients. To solve this problem, some biased estimators including ridge estimator and Liu estimator as well as principal components regression estimator have been proposed.

In the last two decades, biased estimation in semiparametric models has received much attention. For partially linear models, Hu (2005), Akdeniz and Tabakan (2009) and Roozbeh and Arashi (2013) proposed the ridge estimators of the parametric components. Akdeniz and Akdeniz Duran (2010) developed a Liu-type estimator. For partially linear additive models, Wei and Wang (2016) proposed Liu estimator and restricted Liu estimator for the parametric components. For partially linear varying coefficient model (1.1), Liu *et al.* (2017) constructed a stochastic restricted ridge estimator for the parametric components. Following the idea of Akdeniz and Akdeniz Duran (2010) and Wei and Wang (2016), we will construct Liu estimator for the parametric component $\boldsymbol{\beta}$.

The Liu estimator is constructed in Section 2. In Section 3, the restricted Liu estimator is proposed. Conclusion is given in Section 4. Section 5 offers the proofs of the main results.

2. Liu Estimator of the Parametric Components

To apply the Liu estimation approch for model (1.1), we first introduce profile least-squares approach.

If $\boldsymbol{\beta}$ is known, then model (1.1) can be rewritten as the following varying coefficient model

$$y_i^* = \alpha_1(u_i)z_{i1} + \cdots + \alpha_q(u_i)z_{iq} + \varepsilon_i, \quad i = 1, 2, \cdots, n \tag{2.1}$$

with $y_i^* = y_i - \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta}$. The local linear method was applied to estimate the varying coefficient functions of model (2.1). For

u in a small neighborhood of u_0, then we have

$$\alpha_j(u) \approx \alpha_j(u_0) + \alpha'_j(u_0)(u - u_0) \quad j = 1, 2, \cdots, q \tag{2.2}$$

Then, the estimators of $\{(\alpha_j(u_0), \alpha'_j(u_0)), \quad j = 1, 2, \cdots, q\}$ can be obtained by minimizing

$$\sum_{i=1}^{n} \left[y_i^* - \sum_{j=1}^{q} \{\alpha_j(u_0) + \alpha'_j(u_0)(u - u_0)\} z_{ij} \right]^2 K_h(u_i - u_0) \tag{2.3}$$

where K is a kernel function, h is a bandwidth and $K_h(\cdot) = K(\cdot/h)/h$.

Denote

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_n^T \end{bmatrix} = \begin{bmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & \cdots & x_{2p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} \mathbf{z}_1^T \\ \mathbf{z}_2^T \\ \vdots \\ \mathbf{z}_n^T \end{bmatrix} = \begin{bmatrix} z_{11} & \cdots & z_{1q} \\ z_{21} & \cdots & z_{2q} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nq} \end{bmatrix},$$

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \ \mathbf{M} = \begin{bmatrix} \mathbf{z}_1^T \alpha(u_1) \\ \mathbf{z}_2^T \alpha(u_2) \\ \vdots \\ \mathbf{z}_n^T \alpha(u_n) \end{bmatrix}, \ \mathbf{D}_{u_0} = \begin{bmatrix} \mathbf{z}_1^T & \mathbf{z}_1^T \frac{u_1-u_0}{h} \\ \mathbf{z}_2^T & \mathbf{z}_2^T \frac{u_2-u_0}{h} \\ \vdots & \vdots \\ \mathbf{z}_n^T & \mathbf{z}_n^T \frac{u_n-u_0}{h} \end{bmatrix}$$

and

$$\boldsymbol{\varepsilon} = (\varepsilon_1, \cdots, \varepsilon_n)^T, \mathbf{W}_{u_0} = \text{diag}\{K_h(u_1 - u_0), K_h(u_2 - u_0), \cdots, K_h(u_n - u_0)\}.$$

Then varying coefficient model (2.1) can be written as

$$\mathbf{Y} - \mathbf{X}\boldsymbol{\beta} = \mathbf{M} + \boldsymbol{\varepsilon}. \tag{2.4}$$

The solution the problem (2.3) is given by

$$[\hat{\alpha}_1(u_0), \cdots, \hat{\alpha}_p(u_0), h\hat{\alpha}'_1(u_0), \cdots, h\hat{\alpha}'_p(u_0)]^T = \{\mathbf{D}_{u_0}^T \mathbf{W}_{u_0} \mathbf{D}_{u_0}\}^{-1} \mathbf{D}_{u_0}^T \mathbf{W}_{u_0}(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}), \tag{2.5}$$

We can define the estimator of $\mathbf{M}$ as

$$\bar{\mathbf{M}} = \mathbf{S}(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}),$$

where

$$\mathbf{S} = \begin{bmatrix} (\mathbf{z}_1^T \ \mathbf{0})\{\mathbf{D}_{u_1}^T \mathbf{W}_{u_1} \mathbf{D}_{u_1}\}^{-1} \mathbf{D}_{u_1}^T \mathbf{W}_{u_1} \\ (\mathbf{z}_2^T \ \mathbf{0})\{\mathbf{D}_{u_2}^T \mathbf{W}_{u_2} \mathbf{D}_{u_2}\}^{-1} \mathbf{D}_{u_2}^T \mathbf{W}_{u_2} \\ \vdots \\ (\mathbf{z}_n^T \ \mathbf{0})\{\mathbf{D}_{u_n}^T \mathbf{W}_{u_n} \mathbf{D}_{u_n}\}^{-1} \mathbf{D}_{u_n}^T \mathbf{W}_{u_n} \end{bmatrix}.$$

Replacing $\mathbf{M}$ of (2.4) by $\bar{\mathbf{M}}$, we can get the following linear regression model

$$\bar{\mathbf{Y}} = \bar{\mathbf{X}}\boldsymbol{\beta} + \boldsymbol{\varepsilon}. \tag{2.6}$$

where $\bar{\mathbf{Y}} = (\mathbf{I}_n - \mathbf{S})\mathbf{Y}, \bar{\mathbf{X}} = (\mathbf{I}_n - \mathbf{S})\mathbf{X}$. Then, the profile least-squares (PLS) estimator for parametric component $\boldsymbol{\beta}$ can be obtained as

$$\hat{\boldsymbol{\beta}}_P = \arg \min_{\boldsymbol{\beta} \in R^p} \left[(\bar{\mathbf{Y}} - \bar{\mathbf{X}}\boldsymbol{\beta})^T (\bar{\mathbf{Y}} - \bar{\mathbf{X}}\boldsymbol{\beta}) \right] = (\bar{\mathbf{X}}^T \bar{\mathbf{X}})^{-1} \bar{\mathbf{X}}^T \bar{\mathbf{Y}}. \tag{2.7}$$

Based on the linear model (2.6) and the Liu estimation approach, we define the following objective function:

$$F_1(\boldsymbol{\beta}) = (\bar{\mathbf{Y}} - \bar{\mathbf{X}}\boldsymbol{\beta})^T (\bar{\mathbf{Y}} - \bar{\mathbf{X}}\boldsymbol{\beta}) + (d\hat{\boldsymbol{\beta}}_P - \boldsymbol{\beta})^T (d\hat{\boldsymbol{\beta}}_P - \boldsymbol{\beta}). \tag{2.8}$$

Then, we have

$$\frac{\partial F_1(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = -2\bar{\mathbf{X}}^T \bar{\mathbf{Y}} + 2\bar{\mathbf{X}}^T \bar{\mathbf{X}}\boldsymbol{\beta} - 2d\hat{\boldsymbol{\beta}}_P + 2\boldsymbol{\beta} = \mathbf{0}. \tag{2.9}$$

We can obtain the following profile least-squares Liu estimator by solving equation (2.9) as

$$\hat{\boldsymbol{\beta}}_{PL}(d) = (\bar{\mathbf{X}}^T \bar{\mathbf{X}} + \mathbf{I}_p)^{-1}(\bar{\mathbf{X}}^T \bar{\mathbf{Y}} + d\hat{\boldsymbol{\beta}}_P). \tag{2.10}$$

For $\hat{\beta}_{\text{PL}}(d)$, we have the following properties.

Theorem 2.1 *For the profile least-squares Liu estimator $\hat{\beta}_{\text{PL}}(d)$, denote $\mathbf{M} = \mathbf{m}_1 + \mathbf{m}_2$, we have*

$$\text{Bias}(\hat{\beta}_{\text{PL}}(d)) = -(1-d)(\bar{\mathbf{X}}^{\text{T}}\bar{\mathbf{X}} + \mathbf{I}_{\text{p}})^{-1}\beta + \Delta\bar{\mathbf{X}}^{\text{T}}(\mathbf{I}_n - \mathbf{S})\mathbf{M}, \quad \text{Cov}(\hat{\beta}_{\text{PL}}(d)) = \Delta\Phi\Delta^{\text{T}},$$

where $\Delta = (\bar{\mathbf{X}}^{\text{T}}\bar{\mathbf{X}} + \mathbf{I}_{\text{p}})^{-1}(\bar{\mathbf{X}}^{\text{T}}\bar{\mathbf{X}} + d\mathbf{I}_{\text{p}})(\bar{\mathbf{X}}^{\text{T}}\bar{\mathbf{X}})^{-1}$, $\Phi = \sigma^2\bar{\mathbf{X}}^{\text{T}}(\mathbf{I}_n - \mathbf{S})(\mathbf{I}_n - \mathbf{S})^{\text{T}}\bar{\mathbf{X}}$.

Theorem 2.2 *Under the assumptions 1-5 in Section 5, $\beta_{\text{PL}}(d)$ is asymptotically normal, namely,*

$$\sqrt{n}(\hat{\beta}_{\text{PL}}(d) - \beta) \xrightarrow{D} N(\mathbf{0}, \sigma^2\Sigma^{-1}),$$

where $\Sigma = \text{E}(\mathbf{XX}^{\text{T}}) - \text{E}[\Phi(U)\Gamma^{-1}(U)\Phi(U)^{\text{T}}]$, $\Gamma(U) = \text{E}\left[\mathbf{ZZ}^{\text{T}}|U\right]$, $\Phi(U) = \text{E}\left[\mathbf{X}^{\text{T}}\mathbf{Z}|U\right]$.

3. Restricted Profile Least-Squares Liu Estimator

In this section, we consider the following additional restrictions on the parametric component

$$\mathbf{A}\beta = \mathbf{b}, \tag{3.1}$$

with $\mathbf{A}$ is a $k \times p$ matrix of known constants, $\mathbf{b}$ is a $k-$vector of known constants, we assume that rank$(\mathbf{A}) = k$.

Based on the linear model (2.6) with restriction conditions (3.1), we define the following Lagrange function

$$F_2(\beta, \lambda) = (\bar{\mathbf{Y}} - \bar{\mathbf{X}}\beta)^{\text{T}}(\bar{\mathbf{Y}} - \bar{\mathbf{X}}\beta) + (d\hat{\beta}_{\text{P}} - \beta)^{\text{T}}(d\hat{\beta}_{\text{P}} - \beta) + 2\lambda^{\text{T}}(\mathbf{A}\beta - \mathbf{b}).$$

By differentiating F_2 with respect to β and λ, then we obtain the following system of vector equations:

$$\begin{cases} \frac{\partial F_3(\beta,\lambda)}{\partial \beta} = -2\bar{\mathbf{X}}^{\text{T}}\bar{\mathbf{Y}} + 2\bar{\mathbf{X}}^{\text{T}}\bar{\mathbf{X}}\beta - 2d\hat{\beta}_{\text{P}} + 2\beta + 2\mathbf{A}^{\text{T}}\lambda = \mathbf{0}, \\ \frac{\partial F_3(\beta,\lambda)}{\partial \lambda} = 2(\mathbf{A}\beta - \mathbf{b}) = \mathbf{0}. \end{cases}$$

Solving the above equations, the restricted profile least-squares Liu estimator of β is obtained as

$$\hat{\beta}_{\text{PL}}^{\text{R}} = \hat{\beta}_{\text{PL}}(d) - (\bar{\mathbf{X}}^{\text{T}}\bar{\mathbf{X}} + \mathbf{I}_{\text{p}})^{-1}\mathbf{A}^{\text{T}}\left[\mathbf{A}(\bar{\mathbf{X}}^{\text{T}}\bar{\mathbf{X}} + \mathbf{I}_{\text{p}})^{-1}\mathbf{A}^{\text{T}}\right]^{-1}\left[\mathbf{A}\hat{\beta}_{\text{PL}}(d) - \mathbf{b}\right].$$

The following theorem gives the asymptotic normality of $\hat{\beta}_{\text{PL}}^{\text{R}}$.

Theorem 3.1 *Under the assumptions 1-5 in Section 5, the restricted profile least-squares Liu estimator $\hat{\beta}_{\text{PL}}^{\text{R}}$ is asymptotically normal, namely,*

$$\sqrt{n}(\hat{\beta}_{\text{PL}}^{\text{R}} - \beta) \xrightarrow{D} N(\mathbf{0}, \sigma^2\Omega),$$

with $\Omega = \Sigma^{-1} - \Sigma^{-1}\mathbf{A}^{\text{T}}\left(\mathbf{A}\Sigma^{-1}\mathbf{A}^{\text{T}}\right)^{-1}\mathbf{A}\Sigma^{-1}$.

4. Conclusions

For partially linear varying coefficient models, this paper proposed a Liu-type biased estimator for the parametric component as well as a restricted Liu estimator for multi-collinearity and linear restrictions. The properties of the proposed estimators are given.

5. Proof of the Main Results

We consider the following assumptions of Fan and Huang (2005).

Assumption 1. The random variable U has a bounded support Ψ. Its density function $f(\cdot)$ is Lipschitz continuous and bounded away from 0 on its support.

Assumption 2. The $q \times q$ matrix $E(\mathbf{ZZ}^{\text{T}}|U)$ is non-singular for each $U \in \Psi$. $E(\mathbf{ZZ}^{\text{T}}|U)$, $E(\mathbf{ZZ}^{\text{T}}|U)^{-1}$ and $E(\mathbf{ZX}^{\text{T}}|U)$ are all Lipschitz continuous.

Assumption 3. There is an $s > 2$ such that $E \parallel \mathbf{X} \parallel^{2s} < \infty$ and $E \parallel \mathbf{Z} \parallel^{2s} < \infty$ and $n^{2k-1}h \to \infty$ for some $k < 2 - s^{-1}$.

Assumption 4. $\{\alpha_j(\cdot), j = 1, \cdots, q\}$ have continuous second derivatives in $U \in \Psi$.

Assumption 5. The function $K(\cdot)$ is a symmetric density function with compact support and the bandwidth h satisfies $nh^8 \to 0$ and $nh^2/(\log n)^2 \to \infty$.

Lemma 5.1 *Under the assumptions 1-4, the profile least-squares estimator of β_{P} is asymptotically normal, namely,*

$$\sqrt{n}(\hat{\beta}_{\text{P}} - \beta) \xrightarrow{D} N(\mathbf{0}, \sigma^2\Sigma^{-1})),$$

This lemma is Theorem 4.1 of Fan and Huang (2012).

Proof of Theorem 2.1. By the definition of $\hat{\boldsymbol{\beta}}_{\mathrm{PL}}(d)$, we can get

$$
\begin{aligned}
\hat{\boldsymbol{\beta}}_{\mathrm{PL}}(d) &= (\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + \mathbf{I}_{\mathrm{p}})^{-1}(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{Y}} + d\hat{\boldsymbol{\beta}}_{\mathrm{P}}) = (\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + \mathbf{I}_{\mathrm{p}})^{-1}(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + d\mathbf{I}_{\mathrm{p}})\hat{\boldsymbol{\beta}}_{\mathrm{P}} \\
&= \boldsymbol{\beta} - (1-d)(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + \mathbf{I}_{\mathrm{p}})^{-1}\boldsymbol{\beta} + \boldsymbol{\Delta}\bar{\mathbf{X}}^{\mathrm{T}}(\mathbf{I}_n - \mathbf{S})\mathbf{M} + \boldsymbol{\Delta}\bar{\mathbf{X}}^{\mathrm{T}}(\mathbf{I}_n - \mathbf{S})\boldsymbol{\varepsilon}.
\end{aligned}
$$

Then, it is easy to prove that

$$
\mathrm{Bias}(\hat{\boldsymbol{\beta}}_{\mathrm{PL}}(d)) = -(1-d)(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + \mathbf{I}_{\mathrm{p}})^{-1}\boldsymbol{\beta} + \boldsymbol{\Delta}\bar{\mathbf{X}}^{\mathrm{T}}(\mathbf{I}_n - \mathbf{S})\mathbf{M}, \quad \mathrm{Cov}(\hat{\boldsymbol{\beta}}_{\mathrm{PM}}(d)) = \boldsymbol{\Delta}\boldsymbol{\Phi}\boldsymbol{\Delta}^{\mathrm{T}}.
$$

Proof of Theorem 2.2. Like the proof of Theorem 2.2 of Wei and Wang (2016), by the definition of $\hat{\boldsymbol{\beta}}_{\mathrm{PL}}(d)$, we can get

$$
\sqrt{n}(\hat{\boldsymbol{\beta}}_{\mathrm{PL}}(d) - \boldsymbol{\beta}) = (\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + \mathbf{I}_{\mathrm{p}})^{-1}(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + d\mathbf{I}_{\mathrm{p}})\sqrt{n}(\hat{\boldsymbol{\beta}}_{\mathrm{P}} - \boldsymbol{\beta}) - (1-d)\sqrt{n}(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + \mathbf{I})^{-1}\boldsymbol{\beta}
$$

and

$$
\frac{1}{n}\left(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + \mathbf{I}_{\mathrm{p}}\right) \xrightarrow{P} \boldsymbol{\Sigma}, \quad \frac{1}{n}\left(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + d\mathbf{I}_{\mathrm{p}}\right) \xrightarrow{P} \boldsymbol{\Sigma}, \quad (1-d)\sqrt{n}(\bar{\mathbf{X}}^{\mathrm{T}}\bar{\mathbf{X}} + \mathbf{I}_{\mathrm{p}})^{-1}\boldsymbol{\beta} = O_p(n^{-1/2}), \tag{12}
$$

By the Lemma 5.1 and the Slutsky theorem, we have

$$
\sqrt{n}(\hat{\boldsymbol{\beta}}_{\mathrm{PL}}(d) - \boldsymbol{\beta}) \xrightarrow{D} N(\mathbf{0}, \sigma^2 \boldsymbol{\Sigma}^{-1}).
$$

Proof of Theorem 3.1. By the similar way in the proof of Theorem 3.1 of Wei and Wang (2016), we can proof Theorem 3.1, we omit the details.

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