Extended Poisson-Log-Logistic Distribution

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Abstract

In this work, we introduce a new Poisson-log-logistic distribution with a physical interpretation and some applications. Some essential properties are derived. Modeling of four real data sets are provided to illustrate the wide applicability of the new model in differnt fields like finance, reliability, economy and medicine. The new compound model is better than other well-known competitive models which have at least the same number of parameters.

Keywords: log-logistic distribution, generating function, moments, truncated poisson, maximum likelihood

1. Introduction and Physical Motivation

The cumulative distribution function (CDF) of Burr type XII (BXII) is given as

$$G_{(\alpha,\beta)}(x) = 1 - \frac{1}{(x^{\alpha}+1)^{\beta}}, \tag{1}$$

both α and β are shape parameters (for more details about the model of BXII and other related models see Tadikamalla (1980), Rodriguez (1977), Burr and Cislak (1968), Burr (1942 and 1973)).

Setting $\beta = 1$ we obtain the well known one parameter log-logistic (LL) model

$$G_{(\alpha)}(x) = 1 - \frac{1}{x^{\alpha}+1}.$$

Scale parameter can easily be added for getting other versions of the LL models as follows

$$G_{(\alpha,\phi)}(x) = 1 - \frac{1}{\left(\frac{x}{\phi}\right)^{\alpha}+1} \text{ and } G_{(\alpha,\phi)}(x) = 1 - \frac{1}{\frac{x^{\alpha}}{\phi}+1},$$

the corresponding probability density function (PDF) of $G_{(\alpha)}(x)$ is given by

$$g_{(\alpha)}(x) = \frac{\alpha x^{\alpha-1}}{(x^{\alpha}+1)^{2}}. \tag{2}$$

Upon following Yousof et al. (2016), we propose a new model called the Burr type X LL (BXLL) model defined by the CDF given by

$$H_{(\vartheta,\alpha)}(x) = \left(1 - \mathrm{e}^{-x^{2\alpha}}\right)^{\vartheta}, \tag{3}$$

where $\vartheta > 0$ is a shape parameter. Suppose that we have a system has N subsystems functioning independently at a given time where N has zero truncated Poisson (ZTP) distribution with parameter λ. The probability mass function (PMF) of N is given by

$$p_{\mathrm{ZTP}}^{(\lambda)}(N=n)\,|_{(n=1,2,\ldots)} = \frac{\mathrm{e}^{-\lambda}\lambda^{n}}{n!\,(1-\mathrm{e}^{-\lambda})}. \tag{4}$$

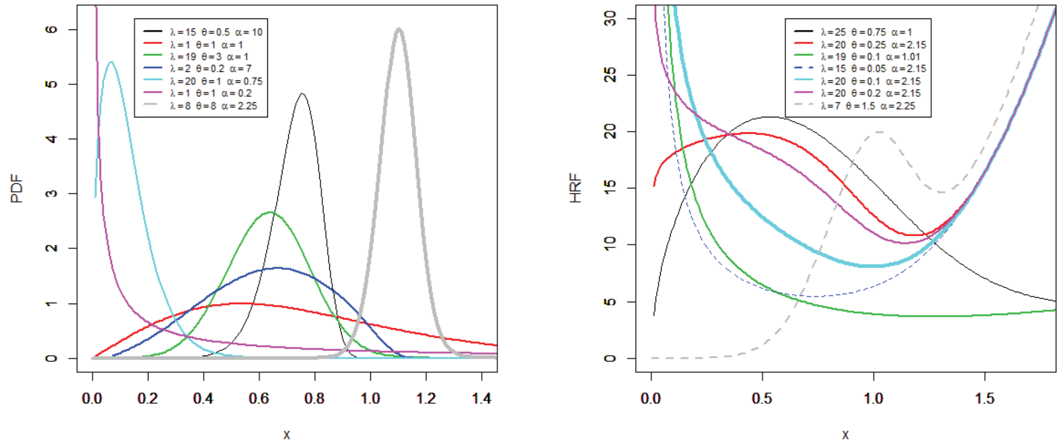
Note that for ZTP random variable (r.v.), the expected value $\mathbf{E}(N|\lambda)$ and the variance $Var(N|\lambda)$ are, respectively, given by $\mathbf{E}(N|\lambda) = \lambda/\left(1-\mathrm{e}^{-\lambda}\right)$ and $Var(N|\lambda) = \frac{(1+\lambda)\lambda}{1-\mathrm{e}^{-\lambda}} - \left[\frac{\lambda}{1-\mathrm{e}^{-\lambda}}\right]^{2}$.

Assume that the failure time of each subsystem has the BXLL(ϑ,α). Let Y_i denote the failure time of the i^{th} subsystem, let

$$X = \min\{Y_1, Y_2, \cdots Y_{N-1}, Y_N\}.$$

Then, the conditional CDF of $X \mid N$ is

$$F(x \mid N) = 1 - \Pr(X > x \mid N) = 1 - \left[1 - H_{(\vartheta,\alpha)}(x)\right]^{N}.$$



(a) (b)

Figure 1. PDFs and HRFs plots for the PBXLL model

So, the unconditional CDF of the Poisson Burr type X log-logistic (PBXLL) can be expressed as

$$F_{(\lambda,\vartheta,\alpha)}(x) = \frac{1 - \mathrm{e}^{-\lambda\left(1-\mathrm{e}^{-x^{2\alpha}}\right)^{\vartheta}}}{1 - \mathrm{e}^{-\lambda}}, \tag{5}$$

with the corresponding PDF as

$$f_{(\lambda,\vartheta,\alpha)}(x) = 2\alpha\lambda\vartheta x^{2\alpha-1}\left(1 - \mathrm{e}^{-\lambda}\right)^{-1}\left(1 - \mathrm{e}^{-x^{2\alpha}}\right)^{\vartheta-1}\mathrm{e}^{-x^{2\alpha}-\lambda\left(1-\mathrm{e}^{-x^{2\alpha}}\right)^{\vartheta}}. \tag{6}$$

The PBXLL reduces to Poisson Rayleigh log-logistic when $\vartheta = 1$. After some algebra the PDF of the PBXLL can be expressed as

$$f_{(\lambda,\vartheta,\alpha)}(x) = \sum_{r=0}^{\infty}\upsilon_r\,\pi_{[\alpha,(1+r)]}(x), \tag{7}$$

where

$$\upsilon_r = \sum_{h,w,m,i=0}^{\infty}\frac{2\vartheta\lambda^{1+m}(-1)^{h+r+m+i}(i+1)^h\,\Gamma(3+2h+w)\,\Gamma((1+m)\vartheta)\,\Gamma(2(1+h)+w)}{h!w!i!(1+r)!(1-\mathrm{e}^{-\lambda})\,\Gamma(3+2h)\,\Gamma(2h+w+2-r)\,\Gamma((1+m)\vartheta-i)},$$

and $\pi_{[\alpha,(1+r)]}(x)$ is the LL density with parameters α and $(1 + r)$. Similarly, the CDF of the PBXLL can also be expressed as

$$F_{(\lambda,\vartheta,\alpha)}(x) = \sum_{r=0}^{\infty}\upsilon_r\Pi_{[\alpha,(1+r)]}(x), \tag{8}$$

where $\Pi_{[\alpha,(1+r)]}(x)$ is the LL CDF with parameters α and $(1 + r)$. The hazard rate function (HRF) of the new model can be calculated from $f_{(\lambda,\vartheta,\alpha)}(x)/\left[1 - F_{(\lambda,\vartheta,\alpha)}(x)\right]$. The PBXLL density can be left-skewed, right-skewed, unimodal and symmetric (see Figure 1(a)) while the PBXLL HRF can be bathtub or unimodal or unimodal then bathtub or decreasing or unimodal then increasing (see Figure 1(b)).

The quantile function (QF) of X, where X ~PBXLL$(\lambda, \vartheta, \alpha)$, is obtained by inverting (5) as

$$Q(u) = \left\{\left[1 - \left(1 + \left\{-\ln\left[1 - \left(\frac{-\ln\left\{1 - u\left(1 - \mathrm{e}^{-\lambda}\right)\right\}}{\lambda}\right)^{\frac{1}{\vartheta}}\right]\right\}^{\frac{1}{2}}\right)\right] - 1\right\}^{\frac{1}{\alpha}},$$

Simulating the PBXLL r.v. is straightforward. If U is a uniform variate on the unit interval $(0, 1)$, then the r.v. $X = Q(U)$ follows (5).

The nth ordinary moment of X, say μ'_r, follows from (7) as

$$\mu'_n|_{[n<(1+r)\alpha\beta]} = \mathbf{E}\left(X^n\right) = \sum_{r=0}^{\infty}\upsilon_r\ (1+r)\ B\left(1+\frac{n}{\alpha},(1+r)-\frac{n}{\alpha}\right), \tag{9}$$

Setting $n = 1$ in (9) gives the mean of X. The nth incomplete moment of X is defined by $m_n(t) = \int_{-\infty}^{t} x^r f(x)dx$. We can write from (7)

$$m_n(t)|_{[n<(1+r)\alpha\beta]} = \sum_{r=0}^{\infty}\upsilon_r\ (1+r)\ B\left(t^{\alpha};1+\frac{n}{\alpha},(1+r)-\frac{n}{\alpha}\right),$$

where

$$\int_0^{\infty}(1+x)^{-(\alpha+\beta)}x^{\alpha-1}dx = B(\alpha,\beta)$$

and

$$\int_0^{t}(1+x)^{-(\alpha+\beta)}x^{\alpha-1}dx = B(t;\alpha,\beta)$$

are beta and incomplete beta functions from the second type, respectively.

2. Parameter Estimation

The log-likelihood function ($\ell_n(\phi)$) for ϕ is given by

$$\begin{aligned}\ell_n(\phi) \quad = \quad & n\log(2)+n\log(\vartheta)+n\log(\lambda)+n\log(\alpha)-n\log\left(-\mathrm{e}^{-\lambda}+1\right)\\ & +(-1+\alpha)\sum_{i=1}^{n}\log x_i+\sum_{i=1}^{n}\log s_i+\sum_{i=1}^{n}\log\left(1-\frac{1}{s_i}\right)\\ & -\lambda\sum_{i=1}^{n}(1-z_i)^{\vartheta}+\sum_{i=1}^{n}\log z_i+(\vartheta-1)\sum_{i=1}^{n}\log(1-z_i)\end{aligned} \tag{1}$$

where

$$s_i = x^{\alpha}+1 \text{ and } z_i = \exp\left[-\left(s_i-1\right)^2\right].$$

The $\ell_n(\mathbf{\Phi})$ in (10) can be numerically maximized via `SAS`or `R` or `Ox` programs. The components of the score vector, $\mathbf{U}(\phi) = \frac{\partial\ell}{\partial\phi} = \left(\frac{\partial\ell_n(\mathbf{\Phi})}{\partial\lambda}, \frac{\partial\ell_n(\mathbf{\Phi})}{\partial\vartheta}, \frac{\partial\ell_n(\mathbf{\Phi})}{\partial\alpha}\right)^{\intercal}$are easily to be derived.

3. Applications

For the four data sets, we will compare the PBXLL distribution with other well-known generalizations of the LL model such as the BXII, Zografos-Balakrishnan BXII (ZBBXII), Marshall-Olkin BXII (MOBXII), the Five Parameters beta BXII (FBBXII), BBXII, Beta exponentiated BXII (BEBXII), Five Parameters Kumaraswamy BXII (FKumBXII), Topp Leone BXII (TLBXII) and KumBXII distributions (for more details about the competitive models see Altun et al. 2018 a, b and Yousof et al. 2018 a, b). Data Set **I** called breaking stress data. This data set consists of 100 observations of breaking stress of carbon fibres (in Gba) given by Nichols and Padgett (2006). Data Set **II** called survival times. In this application, we work with the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, originally observed and reported by Bjerkedal (1960). Data Set **III** called taxes revenue data. The actual taxes revenue data (in 1000 million Egyptian pounds). Data set **IV** called leukaemia data. This real data set gives the survival times, in weeks, of 33 patients suffering from acute Myelogeneous Leukaemia (see the appendix).

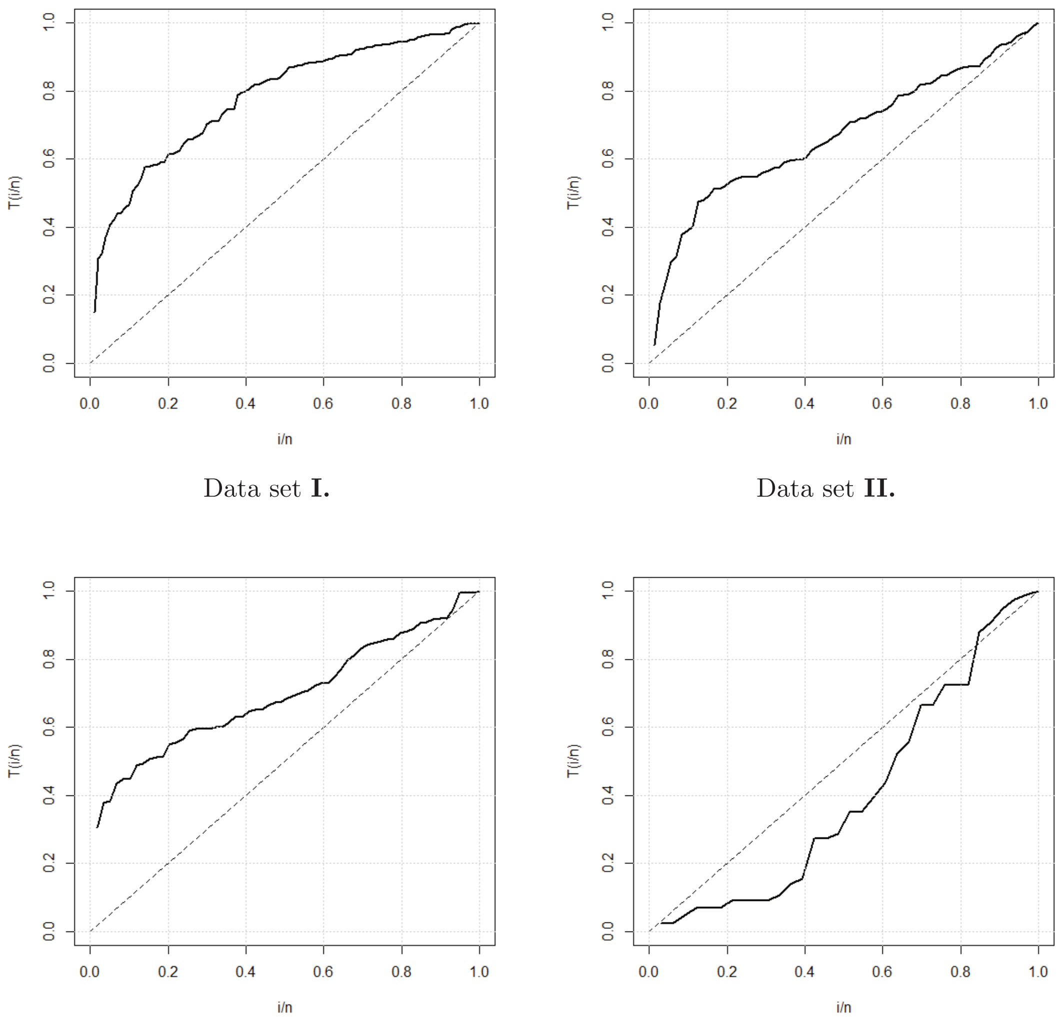
The total time test ($T.T.T.$) plots for the four real data sets is presented in Figure 2. This plot indicates that the empirical HRFs of the four data sets are increasing, increasing, increasing and **U**-shaped (for more details about the $T.T.T.$ see Aarset (1987)).

We will consider the following goodness-of-fit statistics:

1-Akaike Information Criterion ($\mathrm{AI_c}$);

2-Bayesian Information Criterion ($\mathrm{BI_c}$);

3-Hannan-Quinn Information Criterion ($\mathrm{HQI_c}$);



Data set **I.** Data set **II.**

Data set **III.** Data set **IV.**

Figure 2. T.T.T. plots

4-Consistent Akaike Information Criterion ($\mathrm{CAI_c}$), where

$$\begin{aligned} \mathrm{AI_c} &= 2\left[-\ell\left(\widehat{\phi}\right)+K\right],\ \mathrm{BI_c}=2\left[-\ell\left(\widehat{\phi}\right)+\frac{1}{2}K\log(n)\right], \\ \mathrm{HQI_c} &= 2\left\{-\ell\left(\widehat{\phi}\right)+K\log[\log(n)]\right\} \text{ and } \mathrm{CAI_c}=2\left[-\ell\left(\widehat{\phi}\right)+\frac{Kn}{n-K-1}\right], \end{aligned}$$

where K is the number of parameters, n is the sample size, $-2\ell\left(\widehat{\phi}\right)$ is the maximized log-likelihood. Generally, the smaller these statistics are, the better the fit. Table 1, 2, 3 and 4 gives the MLEs and standard errors, confidence interval (in parentheses) with $\mathrm{AI_c}$, $\mathrm{BI_c}$, $\mathrm{CAI_c}$ and $\mathrm{HQI_c}$ values for the data set **I, II, III** and **IV** respectively. .Figures 3, 4 and 5 gives the estimated density, the estimated CDF and the estimated HRF for the four data sets. Figure 6 gives the p-p plot for the four data sets. Figure 7 gives the Kaplan-Meier survival plots for the four data sets. From these two figure we note the new model could be chosen to fit these four data sets. Based on the values in Tables 1, 2, 3 and 4 and Figure 3-7 the PBXLL model provides the best fits as compared to other BXII models in the four applications with small values for $\mathrm{BI_c}$, $\mathrm{AI_c}$, $\mathrm{CAI_c}$ and $\mathrm{HQI_c}$.

Table 1. MLEs and standard errors, confidence interval (in parentheses) with AI_c, BI_c, CAI_c and HQI_c values for the data set **I**

Model	$\widehat{\lambda}, \widehat{\vartheta}, \widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}$	AI_c, BI_c, CAI_c, HQI_c
BXII	—,—, 5.941, 0.187,— —,—, (1.279) ,(0.044),— —,—, (3.43,8.45),(0.10,0.27),—	382.94, 388.15, 383.06, 385.05
MOBXII	—,—, 1.192,4.834,838.73 —,—, (0.952),(4.896),(229.34) —,—, 0, 3.06),(0, 14.43),(389.22,1288.24)	305.78, 313.61, 306.03, 308.96
TLBXII	—,—, 1.350,1.061,13.728 —,—, 0.378) ,(0.384) ,(8.400) —,—, (0.61, 2.09), (0.31,1.81) ,(0, 30.19)	323.52, 331.35, 323.77, 326.70
KumBXII	48.103 ,79.516 ,0.351 ,2.730,— (19.348) ,(58.186) ,(0.098) ,(1.077) ,— (10.18,86.03) ,(0,193.56) ,(0.16,0.54), (0.62,4.84),—	303.76, 314.20, 304.18, 308.00
BBXII	359.683 ,260.097 ,0.175 ,1.123 ,— (57.941) ,(132.213),(0.013),(0.243),— (246.1,473.2), (0.96,519.2), (0.14,0.20), (0.65,1.6),—	305.64, 316.06, 306.06, 309.85
BEBXII	0.381, 11.949, 0.937, 33.402, 1.705 (0.078), (4.635), (0.267), (6.287),(0.478) (0.23,0.53) ,(2.86,21), (0.41,1.5), (21,45), (0.8,2.6)	305.82, 318.84, 306.46, 311.09
FBBXII	0.421, 0.834, 6.111, 1.674, 3.450 (0.011), (0.943), (2.314), (0.226), (1.957) (0.4,0.44), (0. 2.7), (1.57, 10.7), (1.23, 2.1), (0, 7)	304.26, 317.31, 304.89, 309.56
FKumBXII	0.542,4.223, 5.313, 0.411, 4.152 (0.137), (1.882), (2.318), (0.497), (1.995) (0.3, 0.8), (0.53,7.9), (0.9,9), (0, 1.7), (0.2,8)	305.50, 318.55, 306.14, 310.80
ZBBXII	123.101,—,0.368, 139.247,— (243.011), —, (0.343), (318.546),— (0, 599.40), —,(0, 1.04), (0, 763.59),—	302.96, 310.78, 303.21, 306.13
PBXLL	−3.21, 3.67, 0.56,—,— 1.72, 1.837, 0.023,—,— (−6.4,0.4),(0.13,7.33),(0.52,0.6),—,—	**290.6, 298.4, 290.8, 293.7**

Table 2. MLEs and standard errors, confidence interval (in parentheses) with AI_c, BI_c, CAI_c and HQI_c values for the data set **II**

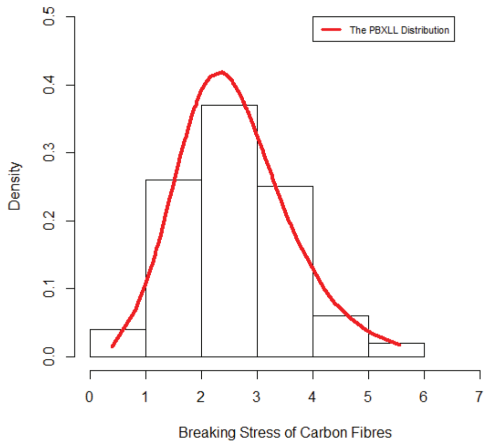
Model	$\widehat{\lambda}, \widehat{\vartheta}, \widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}$	AI_c, BI_c, CAI_c, HQI_c
BXII	—,—, 3.102, 0.465, — —,—, (0.538), (0.077),— —,—, (2.05,4.16), (0.31,0.62),—	209.60, 214.15, 209.77, 211.40
MOBXII	—,—, 2.259,1.533, 6.760 —,—, (0.864), (0.907), (4.587) —,—, (0.57,3.95), (0,3.31), (0, 15.75)	209.74, 216.56, 210.09, 212.44
TLBXII	—,—, 2.393,0.458,1.796 —,—, (0.907), (0.244),(0.915) —,—, (0.62,4.17),(0, 0.94),(0.002,3.59)	211.80, 218.63, 212.15, 214.52
KumBXII	14.105,7.424, 0.525, 2.274,— (10.805), (11.850), (0.279),(0.990),— (0, 35.28), (0.30.65), (0, 1.07),(0.33, 4.21),—	208.76, 217.86, 209.36, 212.38
BBXII	2.555, 6.058,1.800,0.294, — (1.859), (10.391), (0.955),(0.466),— (0, 6.28), (0, 26.42), (0, 3.67),(0, 1.21), —	210.44, 219.54, 211.03, 214.06
BEBXII	1.876,2.991, 1.780, 1.341, 0.572 (0.094), (1.731), (0.702), (0.816), (0.325) (1.7,2.06), (0, 6.4), (0.40, 3.2), (0, 2.9), (0, 1.21)	212.10, 223.50, 213.00, 216.60
FBBXII	0.621, 0.549,3.838, 1.381, 1.665 (0.541), (1.011), (2.785), (2.312), (0.436) (0, 1.7), (0, 2.5), (0, 9.3), (0, 5.9), (0.8, 4.5)	206.80, 218.20, 207.71, 211.30
FKumBXII	0.558,0.308, 3.999, 2.131, 1.475 (0.442), (0.314), (2.082), (1.833), (0.361) (0, 1.4), (0, 0.9), (0, 3.1), (0, 5.7), (0.76, 2.2)	206.50, 217.90, 207.41, 211.00
PBXLL	1.5, 4.34, 0.45,—,— (2.2), (1.7), (0.1),—,— (0,5.9),(0.94,7.7),(0.25,0.65),—,—	**204.8, 211.6, 205.1, 207.5**

Table 3. MLEs and standard errors, confidence interval (in parentheses) with AI_c, BI_c, CAI_c and HQI_c values for the data set **III**

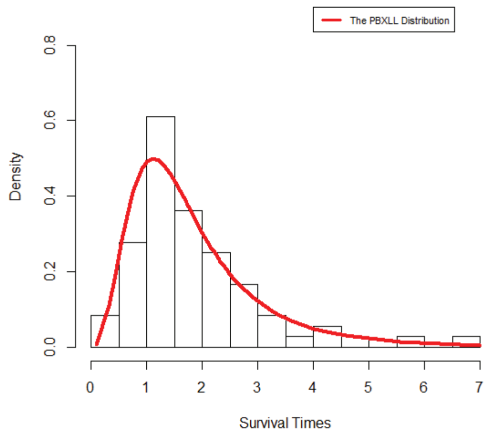
Model	$\widehat{\lambda}, \widehat{\vartheta}, \widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}$	AI_c, BI_c, CAI_c, HQI_c
BXII	—,—, 5.615, 0.072,— —,—, (15.048), (0.194),— —,—, (0, 35.11), (0, 0.45),—	518.46, 522.62, 518.67, 520.08
MOBXII	—,—, 8.017, 0.419, 70.359 —,—, (22.083), (0.312), (63.831) —,—, (0, 51.29), (0, 1.03), (0, 195.47)	387.22, 389.38, 387.66, 389.68
TLBXII	—,—, 91.320, 0.012, 141.073 —,—, (15.071), (0.002), (70.028) —,—, (61.78,120.86) (0.008, 0.02) (3.82,278.33)	385.94, 392.18, 386.38, 388.40
KumBXII	18.130, 6.857, 10.694, 0.081,— (3.689), (1.035), (1.166), (0.012),— (10.89,25.36), (4.83,8.89), (8.41,12.98), (0.06,0.10),—	385.58, 393.90, 386.32, 388.86
BBXII	26.725, 9.756, 27.364, 0.020,— (9.465), (2.781), (12.351), (0.007),— (8.17,45.27), (4.31,15.21), (3.16,51.57), (0.006,0.03),—	385.56, 394.10, 386.30, 389.10
BEBXII	2.924, 2.911, 3.270, 12.486, 0.371 (0.564), (0.549), (1.251), (6.938), (0.788) (1.82,4.03), (1.83,3.99), (0.82,5.72), (0, 26.08), (0, 1.92)	387.04, 397.42, 388.17, 391.09
FBBXII	30.441, 0.584, 1.089, 5.166, 7.862 (91.745), (1.064), (1.021), (8.268), (15.036) (0, 210.26), (0, 2.67), (0, 3.09), (0, 21.37), (0, 37.33)	386.74, 397.14, 387.87, 390.84
FKumBXII	12.878, 1.225, 1.665, 1.411, 3.732 (3.442), (0.131), (0.034), (0.088), (1.172) (6.13,19.62), (0.97,1.48), (1.56,1.73), (1.24,1.58), (1.43,6.03),—	386.96, 397.36, 388.09, 391.06
PBXLL	−91.5, 0.43, 0.29,—,— (0.0), (0.0), (0.0), —,— —	**383.9, 390.1, 384.3, 386.3**

Table 4. MLEs and standard errors, confidence interval (in parentheses) with AI_c, BI_c, CAI_c and HQI_c values for the data set **IV**

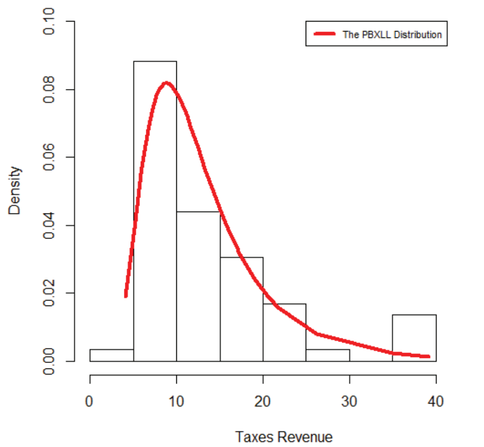
Model	$\widehat{\lambda}, \widehat{\vartheta}, \widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}$	AI_c, BI_c, CAI_c, HQI_c
BXII	—,—, 58.711,0.006,— —,—, (42.382), (0.004),— —,—, (0, 141.78), (0, 0.01),—	328.20, 331.19, 328.60, 329.19
MOBXII	—,—, 11.838, 0.078, 12.251 —,—, (4.368), (0.013), (7.770) —,—, (0, 141.78), (0, 0.01), (0, 27.48)	315.54, 320.01, 316.37, 317.04
TLBXII	—,—,0.281, 1.882 ,50.215 —,—, (0.288), (2.402), (176.50) —,—, (0, 0.85), (0, 6.59), (0, 396.16)	316.26, 320.73, 317.09, 317.76
KumBXII	9.201, 36.428, 0.242,0.941,— (10.060), (35.650), (0.167), (1.045),— (0, 28.912), (0, 106.30), (0, 0.57), (0, 2.99),—	317.36, 323.30, 318.79, 319.34
BBXII	96.104, 52.121, 0.104, 1.227,— (41.201), (33.490), (0.023), (0.326),— (15.4,176.8),(0, 117.8), (0.6, 0.15), (0.59,1.9),—	316.46, 322.45, 317.89, 318.47
BEBXII	0.087, 5.007, 1.561, 31.270, 0.318 (0.077), (3.851), (0.012), (12.940), (0.034) (0, 0.3), (0, 12.6), (1.5, 1.6), (5.9, 56.6), (0.3,0.4)	317.58, 325.06, 319.80, 320.09
FBBXII	15.194, 32.048, 0.233, 0.581, 21.855 (11.58), (9.867), (0.091), (0.067), (35.548) (0, 37.8), (12.7,51.4), (0.05,0.4), (0.45,0.7), (0, 91.5)	317.86, 325.34, 320.08, 320.36
FKumBXII	14.732, 15.285, 0.293, 0.839, 0.034 (12.390), (18.868), (0.215), (0.854), (0.075) (0, 39.02), (0, 52.27), (0, 0.71), (0, 2.51), (0, 0.18)	317.76, 325.21, 319.98, 320.26
ZBBXII	41.973,—,0.157, 44.263,— (38.787),—,(0.082), (47.648),— (0, 117.99),—,(0, 0.32,) (0, 137.65),—	313.86, 318.35, 314.39, 315.36
PBXLL	−0.48, 6.594, 0.16,—,— (1.4), (2.37), (0.015),—,— (−3.3,2.3),(1.8,11.4),(0.13,0.19),—,—	**313.4, 317.9, 314.29, 314.9**



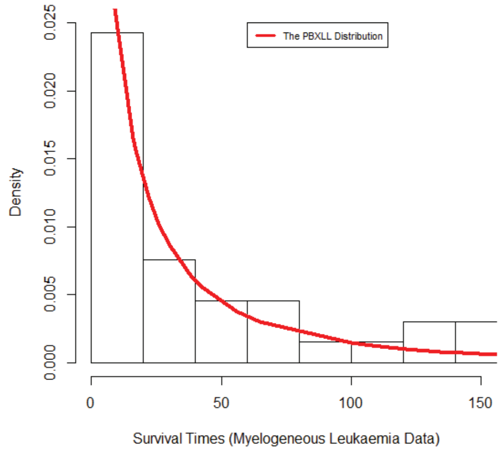
Data set **I**



Data set **II**

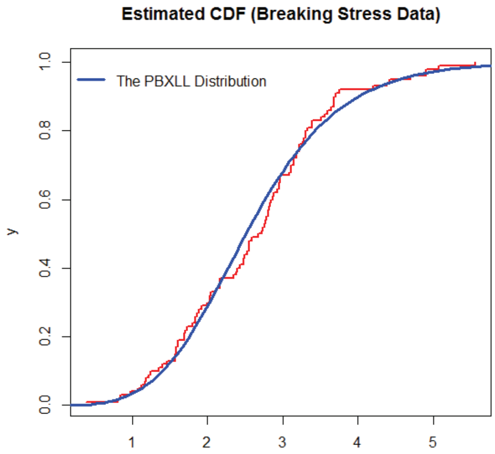


Data set **III**

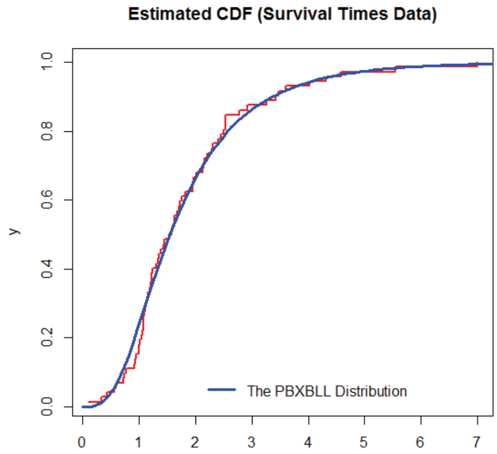


Data set **IV**

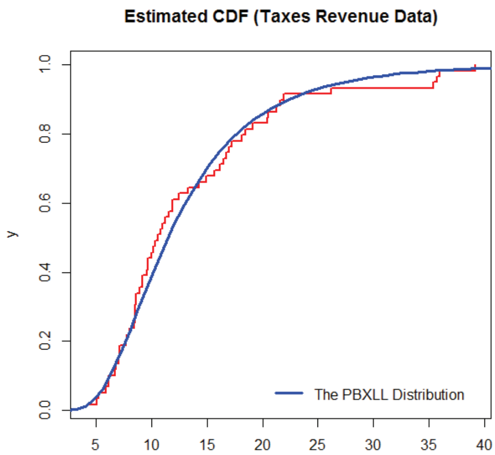
Figure 3. Histograms



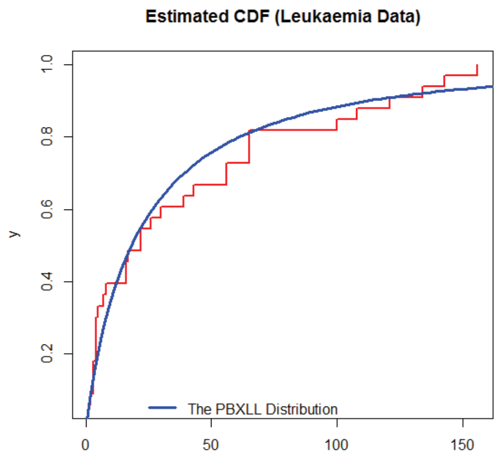
Data set **I.**



Data set **II.**

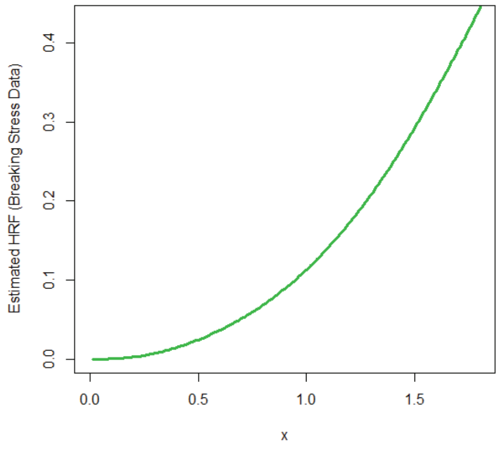


Data set **III.**

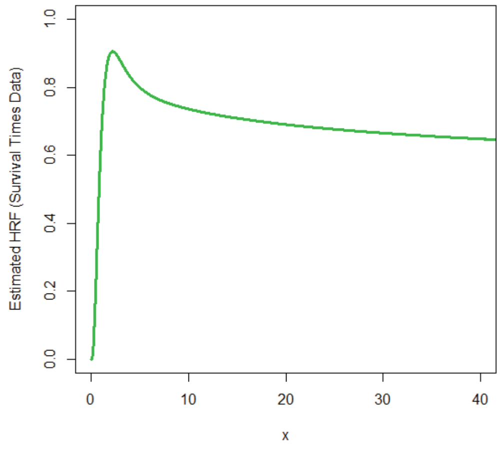


Data set **IV.**

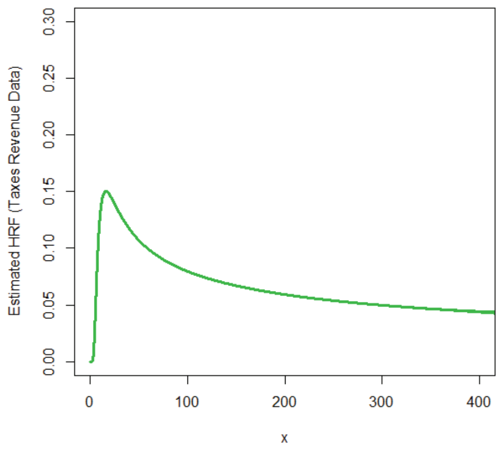
Figure 4. Estimated CDFs



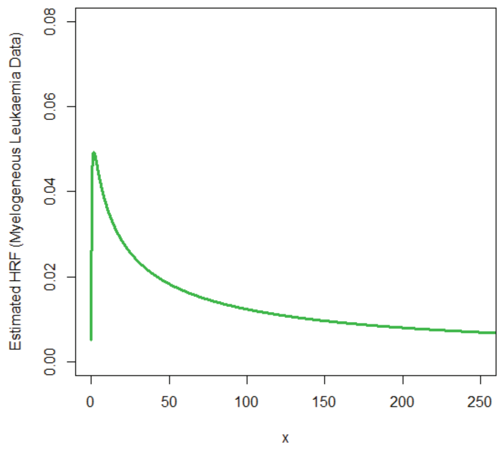
Data set **I.**



Data set **II.**

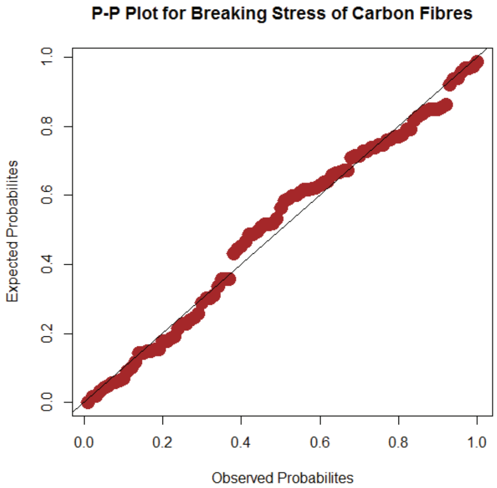


Data set **III.**

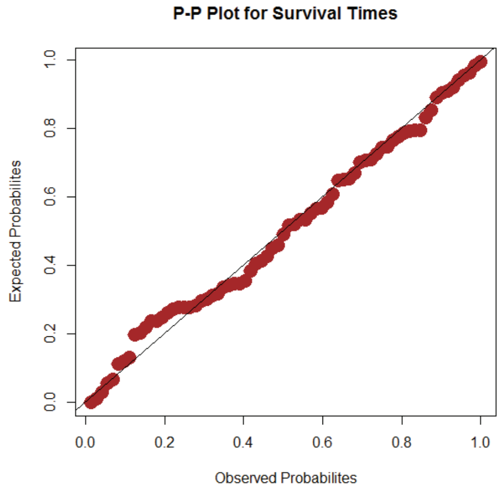


Data set **IV.**

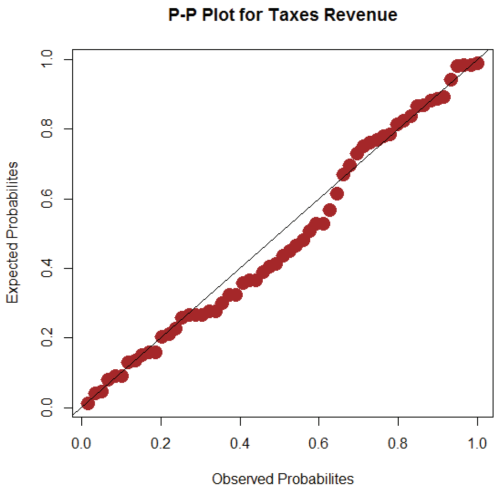
Figure 5. Estimated HRFs



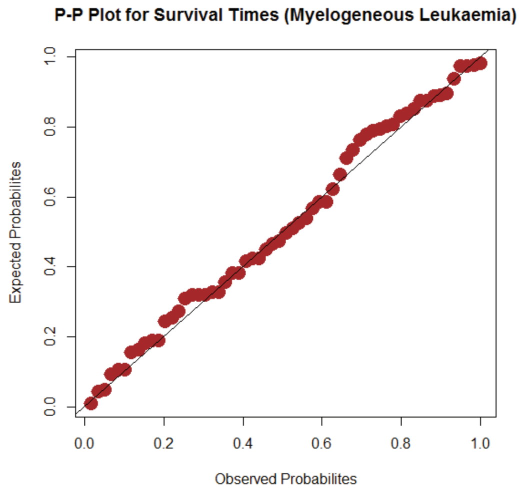
Data set **I**



Data set **II**

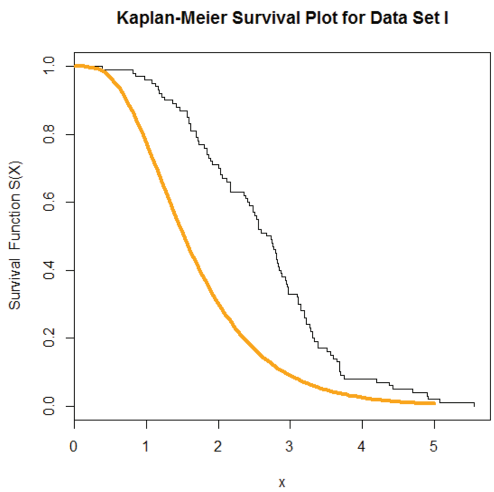


Data set **III**

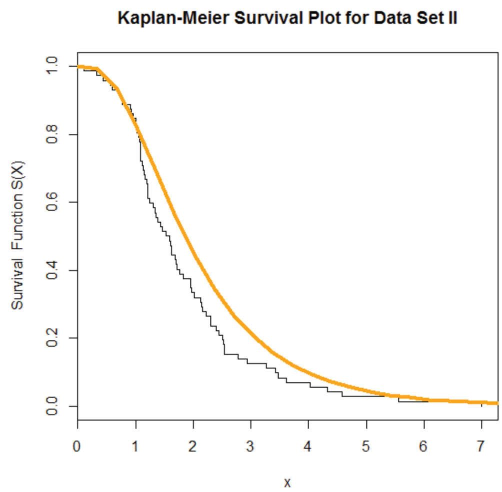


Data set **IV**

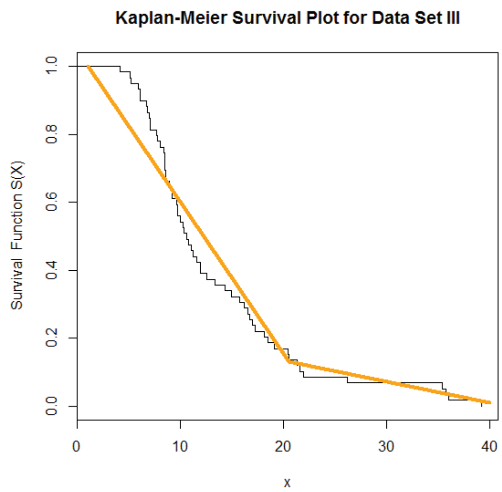
Figure 6. P-P plots



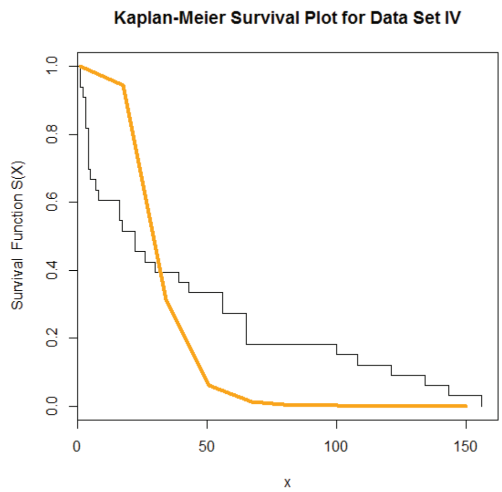
Data set **I.**



Data set **II.**



Data set **III.**



Data set **IV.**

Figure 7. Kaplan-Meier Survival plots

4. Conclusions

In this paper, we introduce a new Poisson-log-logistic distribution with a physical interpretation and capacious applications. Some essential properties are derived. Modeling of four real data sets are provided to illustrate the wide applicability of the new model in differnt fields like finance, reliability, economy and medicine. The new compound model is better than other well-known competitive models which have at least the same number of parameters.

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Appendix

Data Set **I** {0.98, 5.56, 5.080, 0.39, 1.570, 3.19, 4.90, 2.930, 2.85, 2.77, 2.760, 1.73, 2.48, 3.680, 1.08, 3.220, 3.75, 3.22, 3.70, 2.74, 2.730, 2.50, 3.60, 3.110, 3.27, 2.87, 1.47, 3.11, 4.91, 1.590, 1.18, 2.480, 2.03, 1.690, 2.43, 3.390, 3.56, 2.830, 3.68, 2.0, 3.510, 0.85, 1.61, 3.280, 2.95, 2.810, 3.150, 1.920, 1.840, 1.22, 2.170, 1.61, 4.420, 2.40, 4.20, 2.350, 1.410, 1.59, 1.12, 1.69, 2.79, 1.89, 1.87, 3.39, 3.33, 2.55, 3.68, 3.19, 1.71, 1.25, 4.70, 2.88, 2.96, 2.55, 2.59, 2.97, 3.15, 2.67,3.31, 2.81, 2.56, 2.17, 2.12, 3.09, 2.97, 1.57, 2.17, 4.38, 2.03, 2.82, 2.53, 3.310, 2.38, 1.360, 0.81, 1.170, 1.84, 1.80, 2.050, 3.650}.

Data Set **II** {0.10, 0.33, 0.440, 0.56, 0.590, 0.72, 0.740, 0.77, 0.920, 0.93, 0.960, 1.0, 1.0, 1.020, 1.05, 1.070, 0 2.02, 1.360, 1.39, 1.440, 1.46, 1.530, 1.59, 1.60, 1.63, 1.630, 1.680, 1.71, 1.720, 1.760, 1.83, 1.95, 2.130, 2.15, 2.160, 2.220, 2.3, 2.310, 2.4, 2.450, 2.51, 2.530, 7.0, 1.080, 1.08, 1.080, 1.09, 1.120, 1.13, 1.150, 1.16, 1.20, 1.21, 1.220, 1.220, 1.24, 1.30, 1.34, 1.960, 1.970, 2.540, 2.54, 2.780, 2.930, 3.270, 3.420, 3.47, 3.610, 4.020, 4.32, 4.580, 5.55}.

Data Set **III** {5.90, 20.40, 14.9, 16.20, 17.2, 12.50, 10.30, 11.2, 6.10, 8.4, 11.0, 11.6, 7.80, 6.10, 9.2, 10.20, 9.6, 8.50, 8, 9.2, 26.20, 21.9, 16.70, 21.30, 13.30, 8.5, 21.6, 18.5, 5.1, 6.7, 17, 8.60, 9.7, 39.2, 35.7, 15.70, 9.7, 10.0, 4.1, 36, 35.40, 14.3, 8.50, 10.6, 19.10, 20.5, 7.1, 7.70, 18.10, 16.50, 11.9, 7.0, 8.6, 11.90, 5.2, 6.80, 8.90, 7.1, 10.80}.

Data set **IV** {65, 121, 4, 39, 143, 56, 26, 22, 1, 1, 156, 100, 65, 17, 7, 16, 22, 3, 4, 2, 3, 8, 4, 3, 30, 134, 16, 108, 5, 65, 56, 4, 43}.