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Comparative Analysis of the Traditional Models for Capital Budgeting

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Abstract

Financial decision making for investing firms requires metric tools for comparison and analysis. The managerial choice among many investment alternatives with complex possibilities has made it easier to rely on the now more advanced computer programs of simulation that considers the uncertainty and stochastic changes in the cash flow and risk levels. The major drawback here, especially in the academic world, is the increasing dependency on software and departing from the underlying mathematical reasoning that is most practically fathomed by the manual problem solving. This paper goes back to the tradition on analyzing and comparing the major models of capital budgeting.

Keywords: capital budgeting, project evaluation, net present value, internal rate of return, profitability index, payback method, crossover rate, ranking reversal

1. Introduction

Capital budgeting focuses on long-term and strategic plans to sustain a stable and more productive future for the firm. In the core of this long-term thinking is the ability and capacity to make decisions to replace, improve, and expand the firm's major existing resource base, as compared to only seeking the efficiency of the use of that resource base. The emphasis would primarily be placed on assessing the firm's investment opportunities in order to choose the most viable alternative among them. Given that alternative investment opportunities differ in many aspects such as the level of risk associated with each one and their capacities to yield future returns, the criteria for choice would be for the upper management to employ objective, quantitative, and credible methods to evaluate the proposed alternatives and select the best, especially in terms of higher profitability and less risk.

Over the years, many methods and techniques have been developed, tested, and modified for the best ways to evaluate the worth of investment projects. Most of the literature today, especially in the academic journals, seems to skip over the foundation of these methods and go directly to the computerized applications of them. It is analogous to using the financial calculators before knowing the mathematical reasoning of the operations performed by the embedded software. It is essential to connect between the theory and applications, not only to make practical sense of the theoretical ideas by rendering them useful in real life, but also to explore the ways to modify, adjust, and adapt the theory to different applications and a diversity of conditions and circumstances. The major objective of this paper is to discuss the approach and technicality of the decision-making process in evaluating investment projects. The method would be to differentiate among the actual and most traditional ways of capital budgeting followed in the business world by exposing their mathematical analyses as they are tied to real life scenarios.

2. Discussion

2.1 The Fundamental View to Capital Budgeting

Capital expenditures is an outlay of funds which the firm would rely on to generate a stream of returns enough to cover and exceed the initial investment spending. Capital budgeting, therefore, would be the process to review, assess, and select the business projects that promise to be the most rewarding in the medium and long runs. In other words, it is a long-term planning and evaluating capital allocations which are expected to generate cash inflows over a future period of time. Figure 1 illustrates the major elements of the process of capital budgeting on a timeline. It shows the typical dual cash flow scenario, where the first of these flows is the cash outflow, which consists mainly of the initial capital fund allocated for an investment project in the current time, as well as the capital expenditures made throughout the life of assets in their productive life. The second flow is the cash inflow which includes the future annual returns of the project during its five years. While the cash outflow is normally measured in its current value, the cash inflows have to be brought from their maturity time back to the current time. They are discounted at the firm's cost of capital rate and added up to form the present value of all returns which has to be either equal or larger than the cash outflow.

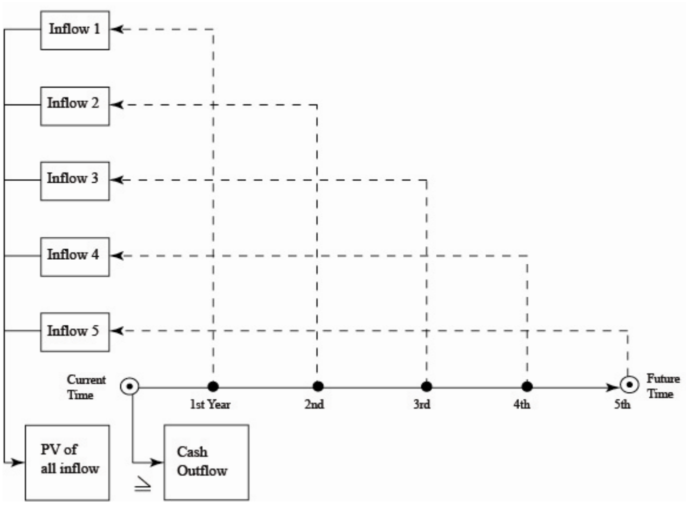

Figure 1. Timeline of the cash flow for capital budgeting

Typical examples of capital outlay for the firm's investment projects are funds to finance the purchase of land, buildings, equipment and business expansion with what is required to enhance the working capital such as inventories and accounts receivable as well as funds to finance research and development and pay for promotion and advertisement. Generally, capital budgeting is often associated with the following most common investment projects:

- Expansion: Includes investment projects to move into the phase of developing and producing either a new product or adding a new line of product, or venturing into a new market. It also includes the extension of service outlets and the distribution and storage facilities.

- Replacement: Includes projects to replace worn-out and damaged machines and equipment as well as renewing structures and technology for more efficient production and services.

- Cost Reduction: Includes all the projects aiming at reducing the cost of production through lower labor cost, raw material, energy, as well as employing higher technology that increases production and ultimately lowers costs. Also, moving production to less expensive locations and spending on training programs could be done to eventually lower the total cost of production.

- Conforming: Includes projects which comply with federal or state regulations and standards on issues such as health, safety, pollution, reservation, and alike.

2.2 The Basic Model of Capital Budgeting

The underlying theoretical framework for capital budgeting is based on the economic equi-marginal principle. It states that a firm would pursue and continue to engage in an activity until the marginal cost equals its marginal benefit. As for the firm's decision on investment projects, this principle is applied in a way that capital would continue to be allocated for investment projects to the point where the marginal return on investment, as represented by the incremental cash flows, would equal the marginal cost, as represented by the added expenditures of new investment capital. Applying this principle would assure the firm's maximum value. Figure 2 shows an example of Firm X that is deciding on the worthiness of eight proposed investment projects, I through VIII, which are requiring a total of $25 million, which is distributed among different capital funds yielding different rates of returns:

Project I:	3 million dollars, with 15% return.
Project II:	2 million dollars, with 13% return.
Project III:	3 million dollars, with 11% return.
Project IV:	4 million dollars, with 9% return.
Project V:	2 million dollars, with 8% return.
Project VI:	5 million dollars, with 6% return.
Project VII:	3 million dollars, with 5% return.
Project VIII:	3 million dollars, with 4% return.
Total Capital Demanded: 25 million dollars.	

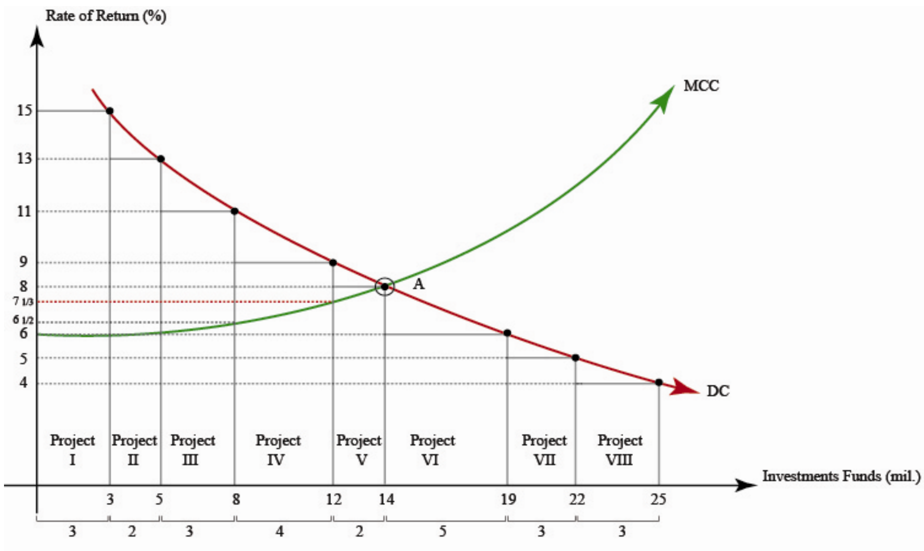

Figure 2. Competition of several investment alternatives by their rates of return

Curve DC is the firm's demand for capital, and curve MCC is the firm's marginal cost of capital. The intersection between the two curves at point A would determine that at maximum, the firm can only allocate capitals for the first five projects, I through V, at a total of 14 million dollars. The firm's initial cost of capital rate is 6% which would continue to rise after allocating the first 3 million. So, the first project yields a rate of 15% but it costs 6%; the second project yields 13% and it costs about 6¼%; the third project costs 6½% but it returns 11%; the fourth project costs 71/3 % and it returns 9%; and the fifth costs 8% and barely returns the same rate back. All the remaining Projects, VI, VII, and VIII, would be rejected since they cost more than the firm's cost of capital and all of their returns are below MCC. Project VI returns 6% but it costs 10½%, Project VII returns 5% but it costs 12½%; and Project VIII returns 4% but it costs 15%. The optimal capital allocation for the firm is 14 million.

2.3 Selection Process and Project Evaluation

Capital budgeting involves a standard, logical and consistent decision-making process that may consist of several steps:

1. Exploring a pool of proposed investment projects and generating a list of most qualified proposals to form the alternative projects under consideration. By focusing on the initially strong and promising projects, this step would also be a screening procedure to keep all of the clearly unfeasible or unworthy proposals out so that they won't go further in the process.

2. Estimating the project cash flow which is a stream of returns that would occur in a future time. This estimation should, therefore, be considered with the appropriate level of uncertainty, risk, and biasedness. A major biasedness is the natural subjective tendency to be over-optimistic which may end up underestimating the costs and overestimating the benefits, especially if there is a certain desire to adopt a specific project. It is essential to rely on unbiased professionals who use objective measures to minimize any over or under estimation. This step usually emphasizes three important criteria:

a. Cash flow measurement should be done on an incremental basis. This is to say that a project cash flow is taking a marginal sense as the difference between the firm's cash flow before and after the construction of that project.

b. Cash flow estimation should be considered on after-tax basis using the firm's marginal tax rate and including the effect of depreciation and all other non-cash expenses which would be considered for income tax purposes.

c. Cash flow estimation should include all of the indirect effects of the project throughout its lifetime such as the possible interference and overlap with other products, services, or functions of the firm.

3. Determining the firm's cost of capital which would serve as the discount rate for converting the value of cash flow from the future to the present time. It is equivalent to the required rate of return by the firm's investors.

4. Evaluating the alternative investment projects in order to choose and accept the best alternative project that would yield the most value for the firm. The central criterion that has been established to determine a measure of desirability and preference for a specific alternative project is the comparison of the present value of the expected cash inflows with the initial cash outflow. The project that wins the allocation of capital has to have the present value of its expected returns larger than the initial capital outlay. There are two groups of methods to evaluate the worthiness and desirability of any investment project under consideration.

2.4 Methods of Evaluation for Proposed Investment Projects

As decision-making tools for the firm's capital budgeting, there are two groups of methods of evaluation. The first group, which is the most common, is called the *value-adjusted method* for its utilization of the time value of money in establishing reliable criteria of project worthiness. This group includes three models: the net present value (NPV), the internal rate of return (IRR), and the profitability index (PI). The second group can be called the *value-unadjusted methods* for not using any time value of money adjustment. This group is represented by the payback model.

2.4.1 Net Present Value (NPV) Model

The net present value of an investment project is the present value of all the future returns or cash inflows minus the initial capital invested in the project. The basic premise is that the stream of future cash inflows must be discounted back to the current time using the firm's cost of capital as the discount rate. This rate is basically determined by the firm based on its assessment of the risk involved in each undertaken investment project. High risk projects are assigned higher discount rate while the low risk projects are assigned low discount rate. Net present value (NPV) is probably the most common technique used to assess how worthy a project is and whether it would be accepted for funding or not. If the present value of all future returns (the cash inflows) is larger than the initial cost of the project (the cash outflows), the net present value would be positive and the project would be deemed acceptable. Otherwise, if the present value of the cash inflows is smaller than the initial capital outlay or the cash outflows, the net present value would be negative and the project cannot be accepted.

Let's recall that the current or discounted value (CV) of any future return (FV) can be obtained by:

$$CV = \frac{FV}{(1+r)^n}$$

where r is the interest rate used for discounting or bringing the value of return from future back to the present and n is the number of terms such as years. If we refer to an annual return or cash inflow of a project by CF, and to the project cash outflow by I, then the net present value (NPV) would be the difference between the discounted stream of both flows throughout the life of the project (t):

$$\boxed{NPV = \sum_{i=1}^{t}\left[\frac{CF_i}{(1+r)^n}\right] - \sum_{i=1}^{t}\left[\frac{I_i}{(1+r)^n}\right]}$$

If the project takes only the initial capital outlay or the starting investment fund only, then that initial amount would be in its current value already (I_0), and does not need to be discounted.

$$\boxed{NPV = \sum_{i=1}^{t}\left[\frac{CF_i}{(1+r)^n}\right] - I_0}$$

The criteria for project acceptability is for the net present value to be non-negative:

$$\boxed{\mathrm{NPV} \geq 0}$$

Let's suppose that a proposal to expand a fast food restaurant calls for the investment of an initial capital of \$42,000 and promises to deliver a return of at least \$14,000 per year during the next five years. Would the franchise company approve such an expansion project, given that its cost of capital is 11½%?

$$\mathrm{NPV} = \sum_{i=1}^{t}\left[\frac{CF_i}{(1+r)^n}\right] - I_0$$

$$= \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^5} - I_0$$

$$= \frac{14,000}{(1+.115)^1} + \frac{14,000}{(1+.115)^2} + \frac{14,000}{(1+.115)^3} + \frac{14,000}{(1+.115)^4} \frac{14,000}{(1+.115)^5} - 42,000$$

= 12,556.05 + 11,261.03 + 10,099.58 + 9,057.92 + 8,123.69 - 42,000

= 51,098.27 - 42,000

= 9,098.27

This expansion project would be accepted since the net present value turned out to be positive.

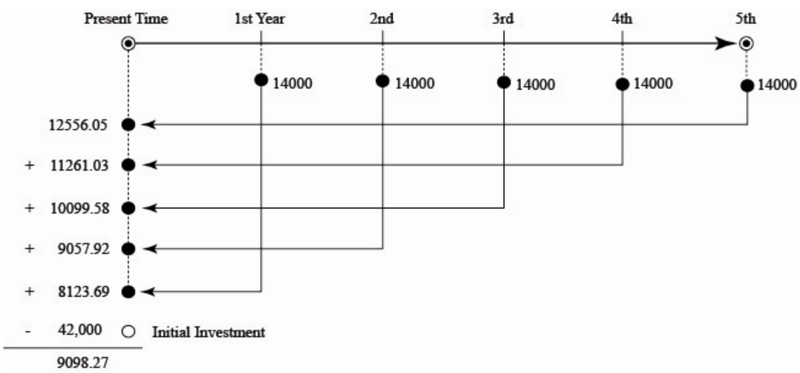

Figure 3. Five-year-return investment proposal for fast food franchise

Let's consider another example where the development committee in a construction company is studying two investment proposals whose cash inflows for the next four years are projected in the table below. Both projects require a capital allocation of \$200,000 given that the cost of capital for the first project is 8% and for the second is 7½%. Which of the two proposals would get an approval?

Year	Project I		Project II	
	Cash Inflows r = 8%	Cash Outflows	Cash Inflows r = 7.5%	Cash Outflow
1	35,000	200,000	40,000	200,000
2	40,000		40,000	
3	50,000		95,000	
4	120,000		100,000	

Project I:

$$NPV = \sum_{i=1}^{4}\left[\frac{CF_i}{(1+r)^n}\right] - I_0$$

$$= \frac{35,000}{(1+.08)^1} + \frac{40,000}{(1+.08)^2} + \frac{50,000}{(1+.08)^3} + \frac{120,000}{(1+.08)^4} - 200,000$$

= 32,407.40 + 34,293.55 + 39,691.61 + 88,203.58 - 200,000

= 194,596 - 200,000

= -5,404.26

Project II:

$$NPV = \frac{40,000}{(1+.075)^1} + \frac{40,000}{(1+.075)^2} + \frac{95,000}{(1+.075)^3} + \frac{100,000}{(1+.075)^4} - 200,000$$

= 37,209.30 + 34,613.30 + 76,471.25 + 74,880.05 - 200,000

= 223,173.90 - 200,000

= 23,173.90

Project I has a negative NPV and Project II has a positive NPV. The committee would accept the second and reject the first project.

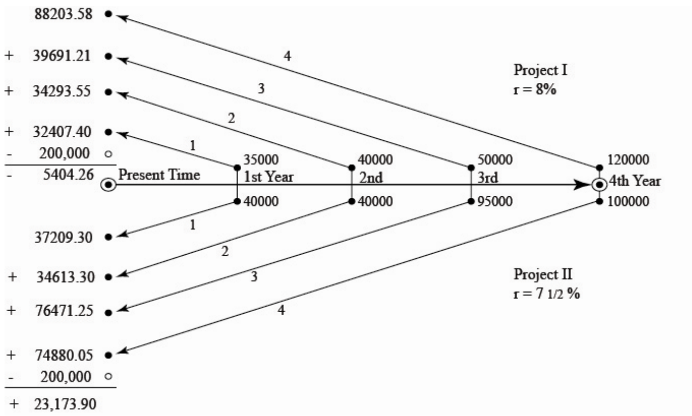

Figure 4. Two alternative investment for construction projects

The cash inflow in the net present value formula could be replaced by the firm's profit for any period (π_i), and can be adjusted for depreciation and taxes:

Since π_i = TR_i - TC_i

$$\boxed{NPV = \sum_{i=1}^{t}\left\{\frac{[TR_i - TC_i](1-T) + D_i}{(1+r)^n}\right\} - I_0}$$

where TR_i and TC_i are the firm's total revenue and total cost for the i^{th} period; T is the firm's marginal tax rate; and D_i is the firm's capital depreciation; and I_0 is the initial investment capital allocated for the project. The following table shows the estimated projections for the gross profit based on the total cost, total revenue, and depreciation allowances of a proposed project during the first five years. How would a funding decision be made on accepting or rejecting the request for an initial capital of $650,000, given that the cost of capital is 8¼% and the marginal income tax is 32%?

1	2	3	4	5	6	7
Year	TR_i	TC_i	π_i (3-2)	(1-T) = 1-.32	D_i	$\pi_i(1\text{-}t)+D_i$ (4x5+6)
1	600,000	460,000	140,000	.68	29,400	124,600
2	660,000	475,000	185,000	.68	31,500	157,300
3	625,000	420,000	205,000	.68	33,000	172,400
4	620,000	400,000	220,000	.68	34,100	183,700
5	668,000	418,000	250,000	.68	35,000	205,000

$$\text{NPV} = \frac{124,600}{(1+.0825)^1}+\frac{157,300}{(1+.0825)^2}+\frac{172,400}{(1+.0825)^3}+\frac{183,700}{(1+.0825)^4}+\frac{205,000}{(1+.0825)^5}$$

$$= 115,103.93 + 134,237.21 + 135,910.67 + 133,781.95 + 137,915.90 - 650,000$$

$$= 656,949.66 - 650,000$$

$$= 6,949.66$$

The proposal would be approved for yielding a non-negative NPV.

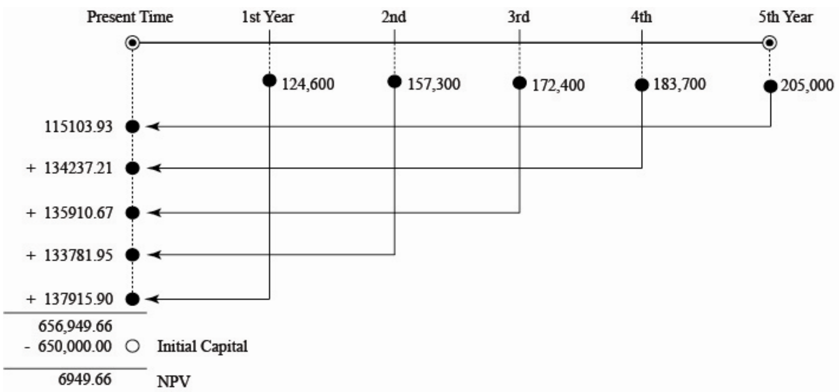

Figure 5. Estimated projections for the gross profit with depreciation allowances of a proposed project during the first five years

2.4.2 Internal Rate of Return (IRR) Model

Another method used to determine the acceptability of a proposed investment is designed to compare the internal rate of return with the firm's cost of capital. The central criterion is that in order for the project to be accepted, it must yield an internal rate of return at least equal or larger than the firm's cost of capital.

$$\boxed{\text{IRR} \geq r}$$

The internal rate of return is sometimes called "*the profit rate*" or "*the marginal efficiency of investment*". It is defined as the rate that equates between the present value of cash inflows and the cash outflows. In the net present value formula, such a rate that makes the two cash flows equal must make the net present value (NPV) equal to zero. This would mean that the project is not capable of delivering an earning rate higher than the cost of capital.

Using the net present value (NPV) formula, we can now replace the firm's cost of capital (r) with the internal rate of return (IRR) and set the net present value to zero.

$$\text{NPV} = \sum_{i=1}^{n}\left[\frac{CF_i}{(1+\text{IRR})^n}\right] - I_0 = 0$$

To find the right internal rate of return that makes the net present value to be zero, we have to solve the equation above for IRR. Since a mathematical solution is not easy, IRR can be found by many ways such as using the table values, trial and error, interpolation, and other ways. Computers and sophisticated business calculators can find IRR easily. However, a preliminary ballpark estimation could be made, and an equation was developed to get at least a starting point in such an estimation of the IRR. Once we get that estimate, we can keep iterating back and forth until we get the exact rate that makes the value of NPV zero.

$$IRR = \left[\frac{\sum_{i=1}^{n} CF_i - I_0}{\sum iCF_i}\right]$$

where CF is cash inflow, and i is the number of any year of the period n: i = 1, 2, 3, ……, n, and I_0 is the initial investment.

Suppose that a firm is studying an investment proposal that is asking for $15,000 as initial capital. The projections for cash inflows during the five years are shown in the table below:

1	2	3
i	Cash Inflows CF_i	i CF_i (1 x 2)
1	3,600	3,600
2	4,200	8,400
3	5,500	16,500
4	6,300	25,200
5	7,500	37,500
	27,100	91,200
	$\sum_{i=1}^{5} CF_i$	$\sum_{i=1}^{5} i\ CF_i$

$$IRR \approx \left[\frac{\sum_{i=1}^{5} CF_i - I_0}{\sum iCF_i}\right]$$

$$IRR \approx \left[\frac{27{,}100 - 15{,}000}{91{,}200}\right]$$

$$IRR \approx 13.3\%$$

But, this is only a rough estimate. However, we can reach to the exact rate with some trial and error attempts, guided by the calculations of the net present value (NPV) until it reaches zero. That would be when the present value of all cash inflows are exactly equal the initial investment, in this case $15,000.

The first step we take here is to calculate the present value of cash inflows at the discount rate of 13.3% to see how close it would make the present value to the initial investment of $15,000.

$$\text{PV} = \frac{3{,}600}{(1+.133)^1} + \frac{4{,}200}{(1+.133)^2} + \frac{5{,}500}{(1+.133)^3} + \frac{6{,}300}{(1+.133)^4} + \frac{7{,}500}{(1+.133)^5}$$

$$= 3{,}177 + 3{,}272 + 3{,}782 + 3{,}823 + 4{,}017$$

$$= 18{,}071$$

So, a rate of 13.3 produces a present value of the cash inflows larger than the initial investment of $15,000. Since the rate of discount has a reverse relationship with the present value, we have to increase the rate in the next try to reduce the present value hoping to let it reach $15,000.

At a rate of 18%, the present value of cash inflows would be:

$$PV = \frac{3,600}{(1+.18)^1} + \frac{4,200}{(1+.18)^2} + \frac{5,500}{(1+.18)^3} + \frac{6,300}{(1+.18)^4} + \frac{7,500}{(1+.18)^5}$$

$$= 3,051 + 3,016 + 3,347 + 3,249 + 3,278$$

$$= 15,942$$

Now, we are getting much closer to the $15,000. We need to try to raise the rate a few more times to get the present value to go down to exactly $15,000. Recall that we can also use the $PVIF_{r,n}$ table value (also comes in another notation: v^n) to ease up the tedious calculations with multiple tries. Any table book would show the discount factor of a dollar for many combinations of r and n.

$$PVIF = v^n = \frac{1}{(1+r)^n}$$

To get the discounted cash inflow in the third year above, we can either divide the 5,500 by $(1+.18)^3$ or get the discount factor from the table by looking up across r = 18% and n = 3, and get the value 0.6086 for the discount factor and multiply it by 5,500.

$$PV = FV[PVIF_{r,n}]$$

$$= 5,500\ [PVIF_{.18,3}]$$

$$= 5,500\ [.6086]$$

$$= 3,347$$

Other few tries to get the exact rate revealed that:

- at 20%, PV = 15,151

- and at exactly 20.4%, PV = 15,000, and that is the internal rate of return (IRR) that brings about the equality between the present value of cash inflows and the initial investment and, therefore, makes the net present value equal to zero.

$$NPV = \left[\frac{3,600}{(1+.204)^1} + \frac{4,200}{(1+.204)^2} + \frac{5,500}{(1+.204)^3} + \frac{6,300}{(1+.204)^4} + \frac{7,500}{(1+.204)^5}\right] - 15,000$$

$$NPV = [2,990 + 2,897 + 3,151 + 2,998 + 2,964] - 15,000$$

$$NPV = 15,000 - 15,000 = 0$$

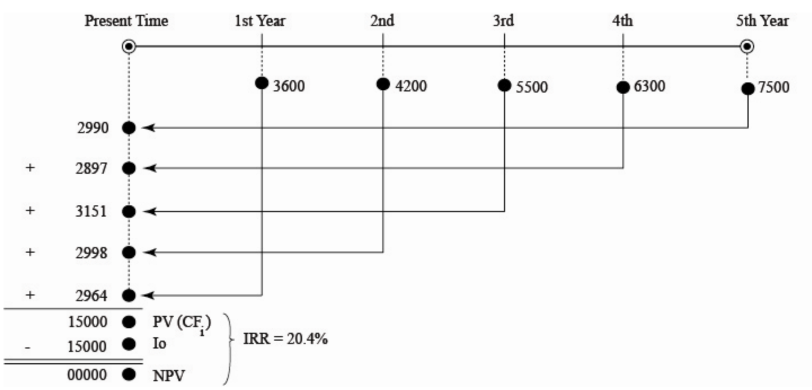

Figure 6. A 5-year-return project with a $15,000 capital

2.4.3 Comparing NPV to IRR for the Mutually Exclusive Projects

The criterion for accepting or rejecting an investment project can either be based on the highest net present value (NPV) or the highest internal rate of return (IRR). It would make no difference to the firm as which of these two measures is followed, simply because they reflect each other consistently. Having a positive value for the net present value means having an internal rate of return that exceeds the firm's cost of capital, and having a negative value of the net present value refers to having an internal rate of return lower than the firm's cost of capital:

$$\text{If IRR} > \text{MCC} \rightarrow \text{NPV} > 0$$

$$\text{IRR} < \text{MCC} \rightarrow \text{NPV} < 0$$

Therefore, either measure would be fine if followed, but that is especially true if the firm is assessing only a single independent project. But if the firm wants to assess two or more projects which are mutually exclusive, then the measures of NPV and IRR may not mean the same thing! Mutually exclusive projects are those projects which compete to earn the only one decision of approval. In other words, the firm can only choose one project among the alternatives.

Year	Project I	Project II	
0	250,000	250,000	Initial Capital
1	-25,000	87,500	Cash Inflows
2	0	87,500	
3	125,000	87,500	
4	125,000	87,500	
5	350,000	87,500	
NPV	131,653	85,973	Discount Rate
IRR	20.2%	22.1%	9½%

Let's calculate both the net present value (NPV) and the internal rate of return (IRR) for the two projects whose 5-year cash inflows are shown in the table, given that both are competing to get the $250,000 initial capital that is to be invested at least at the firm's 9½% cost of capital.

$$\text{NPCV} = \sum_{i=I}^{n}\left[\frac{\text{CF}_i}{(1+r)^n}\right] - I_0$$

$$\text{NPV}_I = \left[\frac{-25,000}{(1+.095)^1} + \frac{0}{(1+.095)^2} + \frac{125,000}{(1+.095)^3} + \frac{125,000}{(1+.095)^4} + \frac{350,000}{(1+.095)^5}\right] - 250,000$$

$$\text{NPV}_I = [-22,831 + 0 + 95,207 + 86,947 + 222,330] - 250,000$$

$$\boxed{\text{NPV}_I = 131,653}$$

$\boxed{\text{IRR}_I = 20.2\%}$ (obtained using a calculator).

$$\text{NPV}_{II} = \left[\frac{87,500}{(1+.095)^1} + \frac{87,500}{(1+.095)^2} + \frac{87,500}{(1+.095)^3} + \frac{87,500}{(1+.095)^4} + \frac{87,500}{(1+.095)^5}\right] - 250,000$$

$$\text{NPV}_{II} = [79,908 + 72,976 + 66,645 + 60,862 + 55,582] = -250,000$$

$$\boxed{\text{NPV}_{II} = 85,973}$$

$\boxed{\text{IRR}_{II} = 22.1\%}$ (obtained using a calculator).

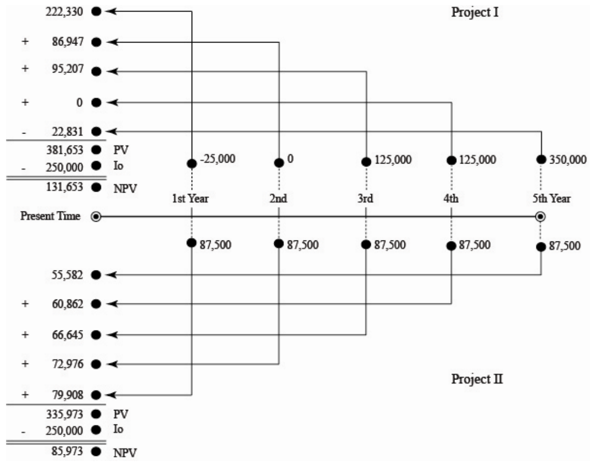

Figure 7. Mutually exclusive investment projects

The calculation shows that the net present value and the internal rate of return are not consistent across the two projects. While Project I have a higher net present value (NPV_I = 131,653 compared to NPV_{II} = 85,973), Project II have a higher internal rate of return (IRR_{II} - 22.1 compared to IRR_I = 20.2). Therefore, the firm has to choose which measure would be better to follow and which one to ignore. Theoretically, it would be better for the firm to decide acceptability based on the higher net present value than the higher internal rate of return. One of the theoretical justifications for that is the assumption that the earned cash inflows are to be reinvested at the usually reasonable firm's cost of capital rate, and there would be no guarantees on the reinvestment of the cash inflows earned by the other project at its higher rate of return. However, practically most financial managers in the business market tend to favor decisions based on the higher internal rate of return. One interpretation for such a tendency is the common reliance on relative change than on absolute change which makes "rates" more preferable than actual dollar amounts as it is in the case of the net present value amount. The relative measures can still remain reliable for comparison, especially when the firm faces many investment proposals which cannot afford to fund except the most profitable, due to some budgetary constraints.

Let's consider a firm receiving six investment project proposals requiring different capitals and promising different net present values as shown in the following table. Suppose that the firm can only allocate a maximum of $800,000 and one of the proposals is asking for an initial capital of $800,000 which is the total investing capacity for the firm. The rest of the proposals are asking for different funding, ranging from $150,000 to $400,000. In this case, it would be clear that it is better not to look at the absolute amounts of the net present value but to find the **yield** or the net present value per dollar, obtained by dividing the net present value by the amount invested or the initial capital allocated (NPV/I_0).

Projects	I_0	NPV	NPV/I_0		
I.	800,000	2,560,000	3.2%		
II.	290,000	725,000	2.5%		
III.	250,000	850,000	3.4%	←	
IV.	350,000	770,000	2.2%		3.6125
V.	400,000	1,440,000	3.6%	←	
VI.	150,000	600,000	4.0%	←	
Firm's Total Funding Capacity: $800,000					

The firm can invest in Project I only giving all it has and getting $3.2 per dollar invested, but a combination of more than one project, not only reduces the risk by diversification but also increases the return. So the available capital of $800,000 can be shared by Projects III, V, and VI generating a total of $2,890,000 of a net present value (850,000 + 1,440,000 + 600,000) which would be translated into a 3.6125% earning per dollar invested

$\left(\frac{2,890,000}{800,000}\right)$. It is higher than the 3.2% from the first project that required the entire investment budget. Project I would be dismissed as will Projects II and IV.

2.4.4 NPV Profile, Crossover Rate and the Ranking Reversal

A net present value profile is the relationship between a project's net present value and several alternative cost of capital rates. It is expressed in a table and graph such as the ones we see below for Projects I and II. The two expressions, table and graph, show how the net present value changes in response to changes in the firm's cost of capital rate.

Cost of Capital	Net Present Value I	Net Present Value II
0%	20,000	25,000
2.5%	15,972	20,656
4%	12,270	15,662
6%	10,527	12,498
7.5%	9,057	10,775
8%	8,765	8,765
10%	5,695	4,307
11.5%	2,772	240
13%	60	-3,480
15%	-2,462	-6,887
IRR	13.3%	11.7%

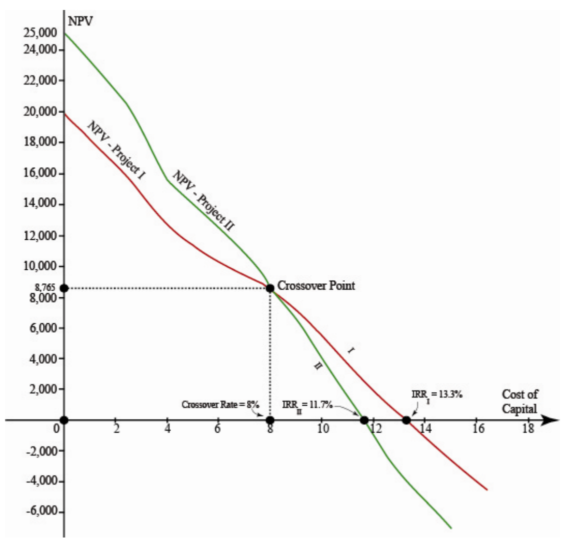

Figure 8. The crossover point

Looking at both the table and graph above, we can make the following observations:

- Since the relationship between the net present value and the cost of capital is negative, both curves, NPV_I and NPV_{II} are sloping downwards: starting from their initial values of \$20,000 and \$25,000 at zero rate, and ending at negative values at the highest rate, above 11.7%. That is to confirm that as the firm's cost of capital increases, its net present value decreases.
- The curve for Project II is steeper than the curve for Project I. This reflects that Project II is more sensitive to the change in cost of capital rate than Project I.

$$\frac{\partial NPV_{II}}{\partial r} > \frac{\partial NPV_{I}}{\partial r}$$

We can observe, for example, that as cost of capital rate goes up by 92% (from 6% to 11.5%), the net present value for Project II goes down by 98%, while the net present value for Project I declines by 74%.

$$\frac{11.5-6}{6}=92\%\,;\ \frac{10{,}527-2{,}772}{10{,}527}=74\%\,;\ \frac{12{,}498-240}{12{,}498}=98\%$$

The difference in the project sensitivity to changes in the cost of capital is due to the differences in the magnitude as well as the timing of the cash inflows. Because the discounting process is just a reverse of the compounding process, the present value of cash inflows coming in later years would decrease more than those coming in earlier years. This is the reason for the curves to intersect at the crossover point.

- When the two net present values of the two projects equal each other, their curves intersect at the crossover point. The cost of capital rate corresponding to that point is called the crossover rate, 8% in this case. So the crossover rate is defined as the rate at which the net present value of two projects get equal to each other and where their curves intersect.

- The crossover rate serves as the turning point where the reversal of project ranking occurs. At any level of cost of capital below the crossover rate, Project II would be preferred for its higher net present value. For any cost of capital level higher than the crossover rate, Project I would be preferred for having its net present value higher this time than the net present value of Project II.

- The net present value of both projects would decline to zero when they cross the x-axis at their firm's internal rate of return. The graph shows that the internal rate of return for Project I is 13.3%, and for Project II is 11.7%, and those where the two curves cross the x-axis respectively.

2.4.5 Profitability Index and Capital Rationing

We hinted earlier to the relative measure of net present value and the net present value per dollar invested. Another relative measure can serve as one of the criteria for project acceptability. It is the measure of Profitability Index (PI) which is a ratio of the present value of cash inflows and the present value of cash outflows:

$$PI=\frac{\sum_{i=1}^{t}\left[\frac{CF_i}{(1+r)^n}\right]}{\sum_{i=1}^{t}\left[\frac{I_i}{(1+r)^n}\right]}$$

It can also be expressed as a ratio between the present value of cash inflows and the initial investment.

$$PI=\frac{PV(CF_i)}{I_0}\qquad\qquad PI\geq 1$$

The criterion for project acceptability is for the profitability index to be equal or larger than 1 since being equal to 1 would indicate the equality between the cost and benefit.

In two of the projects mentioned previously, the net present value for one was \$770,000, and the other was \$600,000 but because the investment capitals required for both respectively were \$350,000 and \$150,000, their values to the firm would differ dramatically. Calculating the profitability index for both would reveal that difference. But, we need to restore the present value for cash flows by combining the net present value and the initial investment for both:

$$PV = NPV + I_0$$

$$PV_1 = 770{,}000 + 350{,}000 = 1{,}120{,}000$$

$$PV_2 = 600{,}000 + 150{,}000 = 750{,}000$$

$$PI_1 = \frac{PV_1(CF_i)}{I_0}$$

$$= \frac{1{,}120{,}000}{350{,}000}$$

$$= 3.2$$

$$PI_2 = \frac{750,000}{150,000}$$

$$= 5$$

2.4.6 Payback Method

Among the most common methods to evaluate investment projects, especially in the past (in the pre-computer age), was the payback method. It may still be in use in some business corners for its simplicity and straightforwardness. Payback period refers to the expected number of operational years during which the initial investment can be recovered. In this sense, as a criterion for project selection, the shorter the payback period the better. The short recovery time would be a crude measure of liquidity of the project. It can also be an indication for less potential risk. The fewer the number of years in which the initial capital can be fully recovered, the more the project can cut off the potential risk that may lie ahead.

The payback period (PB) can be obtained by the following two techniques:

1. For projects yielding an equal amount of cash inflow during the project life time, payback period is obtained by dividing the initial investment or the proposed capital (I) by the annual cash flow (CIF):

$$\boxed{PB = \frac{I}{CIF}}$$

2. For projects yielding variable cash inflows throughout the years of the project operation. The payback period (PB) can be obtained by:

$$\boxed{PB = (f-1) + \left[\frac{I - \sum_{t=1}^{f-1} CIF_t}{CIF_f}\right]}$$

where:

f: is the year in which the initial investment can be fully recovered.

f-1: the year before the year of full recovery of the initial investment.

I: is the initial investment or the capital proposed for allocation.

CIF_t: the cash inflow during the period t up to the year before the year of full recovery.

CIF_f: the cash inflow in the year of full recovery of the initial investment.

Let's calculate the payback period for the following two proposed projects:

The sunshine company is considering the following two projects for capital allocation. Project X is asking for \$64,000 and Project Y is asking for \$68,000. Calculate the payback period for both projects:

Year	Project X (\$64,000)		Project Y (\$68,000)	
	Expected Profits (after taxes)	Cash Inflows	Expected Profits (after taxes)	Cash Inflows
1	9,500	16,000	21,500	40,800
2	10,200	16,000	9,000	12,200
3	10,200	16,000	5,500	10,000
4	8,000	16,000	4,500	10,000
5	7,100	16,000	4,000	8,000
	9,000	16,000	8,900	16,000

For Project X:

$$PB = \frac{I}{CIF}$$

$$= \frac{64,000}{16,000}$$

$$= 4 \text{ years}$$

For Project Y:

$$PB = (f-1) + \left[\frac{I - \sum_{t=1}^{f-1} CIF_f}{CIF_f} \right]$$

$$= 3 + \left[\frac{68,000 - (40,800 + 12,200 + 10,000)}{10,000} \right]$$

$$= 3.5 \text{ years}$$

f: is the fourth year since the 68,000 would be within the accumulated cash inflows by the end of the fourth year, that is (40,800 + 12,200 + 10,000 + 10,000 = 73,0000).

f-1: is the third year (the year before the fourth).

CIF_f : is the annual cash inflow ($10,000) in the fourth year as the year in which the initial investment would be fully recovered.

In addition to the major shortcoming of the payback method in ignoring the time value of money changes, it has also been criticized for ignoring the dynamics of the cash inflows obtained during the years after the year of full recovery.

3. Conclusion

While we compared the Net Present Value to the Internal Rate of Return, we can conclude that adding the profitability index would make the picture clearer. While the net present value of the first project ($770,000) is 28% larger than the net present value of the second project ($600,000), the profitability index of the second project (5) is 56% larger than the profitability index of the first project (3.2). This would illustrate the benefits of the profitability index as a tool, especially when judgment by the net present value alone would not be conclusive. The relative measure of the net present value may rise again when the firm has some constraints on its capacity of investment. Often there is a certain limit as to how much any firm can finance its all available feasible projects. High management of the firms may place such a limit and determine the maximum capacity of capital investment whether project financing is done by borrowing from banks and financial institutions or from the public in terms of corporate bonds and equity shares. Declaring and recognizing the limit on investment may mean recognizing some sort of capital scarcity, which should most likely lead to seeking allocation efficiency, and that is what is called capital rationing. **Capital rationing** is defined as the process of allocating scarce financial capital as efficiently as the firm's conditions and circumstances permit. It is obvious that capital rationing would be exercised when the total funds requested to finance all eligible projects exceeds the firm's affordability as set by the maximum level of funding. Capital rationing involves contemplating every possible combination of projects that can be funded, and choose the best combination of projects that satisfy:

1) Their total required capital would not exceed the firm's maximum level of funding.
2) Their total net present value per dollar is largest among the alternative combinations.

The following two tables show how capital is being rationalized. The first table shows five competing projects all of which were considered worthy, but the firm cannot fund all because the required total capital is $300,000 and the firm's maximum level of funding is $200,000. The second table lists the possible combinations of projects with their combined capital and combined net present values. It also lists the remaining funds out of the available $200,000 (column 4) and their compounded future values (column 5). Column 6 adds up the combined net present value to the accumulated future value, and finally column 7 calculates the adjusted net present value per dollar of funded capital. This is the column that shows which combination of project is the best based on the highest adjusted net present value per dollar, and that would be the combination containing projects (1, 4, and 5) which shows 79% net present value for each dollar of initial investment.

Project	Initial Capital	NPV
1	100,000	60,000
2	75,000	40,000
3	50,000	35,000
4	45,000	30,000
5	30,000	15,000
	300,000	180,000

1	2	3	4	5	6	7
Project Combination	Combined Capital	Combined NPV	Remaining Fund (RM) 200 - (2)	Value of Invested Remaining Funds $RM(1+r)^n$	Final NPV (3 + 5)	NPV Per $ Invested (6 ÷ 2)
1, 2	175	100	25	33.45	133	.75
1, 3, 5	195	125	5	6.70	132	.68
1, 3, 5	180	110	20	26.76	137	.76
1, 4, 5	175	105	25	33.45	138	.79
2, 3, 4, 5	200	120	0	0	120	.60

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