Intrinsic Bubbles in the American Stock Exchange: The case of the S&P 500 Index

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Abstract

The aim of this paper is to test the presence of rational intrinsic bubbles in the S&P 500 index. To this effect, we used two econometric techniques. The first technique applies stationarity and cointegration tests to real prices and dividends series. The second technique consists in directly estimating intrinsic bubbles coefficients. Studying a sample of annual real price and dividends indices, observed during the 1871 to 2009 period, we note the presence of a bubble with features consistent with intrinsic bubbles theory.

Keywords: Rational bubbles, Intrinsic bubbles, Fundamentals-dependent bubbles, Stationarity and cointegration test.

JEL classification: C22; E31; G12

1. Introduction

The Efficient Market Hypothesis (EMH) stipulates that the observed price entirely and accurately reflects all the information disclosed on the market. From this perspective, the observed price should be compatible with its fundamental value. However, when a stock price deviates from this value, arbitrage mechanisms prevail in terms of selling overvalued stocks and buying undervalued stocks. The aim of such behavior is to permanently adjust prices to their fundamental values. Shiller (1981) and LeRoy and Porter (1981), studying the US market and using variance bound tests, note that market prices exhibit an excessive volatility compared to their fundamental values. The idea behind this test is that in an efficient market stock prices variance should be bound by a theoretical value which depends solely on the variability of the price's fundamental determinants. Mankiw, Romer and Shapiro (1985) developed an alternative measure of stock prices volatility, called the second generation test. This latter is a new reformulation of the variance bound test based on a naive prediction of fundamental values that are issued from naive information. Market prices' excessive volatility is the main reason for the emergence of the speculative bubbles theory. Accordingly, we distinguish between two categories of rational bubbles. Those exogenous to the economic fundamentals and those directly issued from these fundamentals.

As for the exogenous rational bubbles, they exhibit an evolution pattern bound by time. This type of bubbles rests on the idea that prices are guided by self-fulfilling predictions causing the bubble to increase exponentially to interest rate (Blanchard and Watson (1982), Fung (1999a, 1999b), Schaller and Norden (2002) and Evans (1991)). Several studies have been conducted on exogenous rational bubbles. The use of stationarity and cointegration tests is pervasive in these studies (Diba and Grossman (1987, 1988), Craine (1993), Campbell, Lo and Mackinlay (1997), Sarno and Taylor (1999), Psaradakis , Sola and Spagnolo (2001) and Gürkaynak (2005)). The obtained results often reject the absence of bubbles hypothesis without nevertheless confirming their presence. Gurkaynak (2008) proposes an excellent review of the different tests used to detect rational bubbles. Geiecke and Trede (2010), studying the Dow Jones Euro Stoxx 50 Price Index, note that the presence of rational bubbles is consistent with investors' rationality hypothesis. Watanapalachaikul and Islam (2003) checked for the presence of rational bubbles in the Thai market using the duration technique. The authors find out that this market is influenced by a rational bubble, specifically after the 1997 Asian crisis.

Despite their contribution in explaining deviation of prices from their fundamental value, exogenous deterministic and multi-regime rational bubbles are unable to explain several speculation-related behaviours, notably in a case where market prices fluctuations are with minimum effects. Furthermore, the absence of a measure for the different classes of exogenous rational bubbles is a major difficulty facing researchers. Consequently, research has refocused attention on a new venue with a double concern of developing, on the one hand, a new class of rational bubbles able to accurately reproduce fluctuation of prices and, on the other hand, including fundamentals in this development process. It took then some years to see the emergence of fundamentals-dependent bubbles (intrinsic bubbles) thanks mainly to the works of Ikeda and Shibata (1992) and Froot and Obstfeld (1991).

The dynamics, properties and shape of these bubbles, labelled endogenous, greatly depend on fundamentals. In order to detect the presence of fundamentals-dependent bubbles, it is enough to assume that fundamentals' random fluctuations (essentially dividends) carry information reflected both in the fundamental value and the bubble. Moreover, to meet the growth restriction, it is convenient to assume that investors use information disclosed by dividends so as to feed their predictions of the direction of the volatility of prices.

This paper is structured as follows. Section two presents the mathematical formulation used to compute the stock's fundamental value and the intrinsic bubble. Section three describes the sample and the study period. Section four reports the results and their discussion. Section five concludes the paper.

II. Rational Bubbles Specifications

By definition, the return rate R_{t+1}, of a stock is given by the sum of the most valued $(P_{t+1} - P_t)$, and of the dividend, D_{t+1}, adjusted to the stock price in t. Then,

$$R_t = \frac{P_{t+1} - P_t + D_{t+1}}{P_t} \tag{1}$$

where, R_{t+1} denotes the return on the stock held from time t to t + 1 and Dt+1 is the dividend in period t+1. The subscript t+1 denotes that only the return becomes known in period t + 1. Taking the mathematical expectation of (1), based on information available at time t, $E_t(.)$, we obtain:

$$E_t(R_{t+1}) = \frac{E(P_{t+1} + D_{t+1}) - P_t}{P_t} = R \tag{2a}$$

Where again $$E_t\left[P_{t+1}\right] - P_t + D_{t+1} = RP_t \tag{2b}$$

Rearranging (2), we obtain:

$$P_t = E_t\left[\left(\frac{1}{1+R}\right)D_{t+1}\right] + E_t\left[\left(\frac{1}{1+R}\right)P_{t+1}\right] \tag{3}$$

with $\left(\frac{1}{1+R}\right)$, denoting a discounting factor

Solving (2) forward k periods yield the semi-reduced form:

$$P_t = E_t\left[\sum_{i=1}^{k}\left(\frac{1}{1+R_{t+i}}\right)^i D_{t+i}\right] + E_t\left[\left(\frac{1}{1+R_{t+k}}\right)^k P_{t+k}\right] \tag{4}$$

In order to obtain a unique solution to (4) we need to assume that the expected discounted value of the stock in the indefinite future converges to zero:

$$\lim_{k\to\infty} E_t\left[\left(\frac{1}{1+R_{t+k}}\right)^k P_{t+k}\right] = 0 \tag{5}$$

The convergence assumption allows us to obtain the so-called fundamental value of the stock as the sum of the expected discounted dividend sequence:

$$F_t = E_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+R_{t+i}}\right)^i D_{t+i}\right] \tag{6}$$

Abandoning the convergence assumption - equation (5) - leads to an infinite number of solutions any one of which can be written in the form of:

$$P_t = F_t + B_t \tag{7}$$

with $$B_t = E_t\left[\frac{B_{t+1}}{1+R_{t+1}}\right] \tag{8}$$

where F_t, denotes the dividend's future real value or the stock's fundamental value. The term B_t called a "rational bubble", as it is entirely consistent with rational expectations and the time path of expected returns. Blanchard and Watson (1981) define rational bubble as the difference between the observed price on the market and its fundamental value. In this regard, Gilles and Leroy (1992) insist that the term bubble translates the high increase in stock prices resulting from promises made by companies about future dividends. The higher the level of dividends is the higher will be the demand for the stock in such a way which intensifies pressures on prices. A dramatic decrease results in the non-fulfilment of these promises.

The literature distinguishes between several rational bubbles measures. Blanchard and Watson (1982) are the first to specify measures of exogenous rational bubbles. They proposed deterministic bubbles having an exponential increase and stochastic bubbles having an exponential inflation followed by a brutal collapse. Evans (1991) proposed the periodically collapsing bubbles which integrate the possibility of repetitive crashes. Fukuta (1998,

2002) proposed the incompletely bursting bubbles which are a generalisation of Blanchard and Watson's (1982) deterministic and stochastic bubbles and Evans' (1991) periodically collapsing bubbles.

Froot and Obstfeld (1991) specified a new set of bubbles, called intrinsic bubbles, which are exclusively bound in a nonlinear fashion to fundamentals, specifically dividends. Their deviation is explained by the fact that the component of stock prices which is unexplained by fundamental values is highly correlated with the dividends process. The authors insist that intrinsic bubbles provide an empirical measure of deviation of prices from their fundamental values. Froot and Obstfeld's intrinsic bubbles assume that the dividends' logarithmic function follows a geometric shape. Then,

$$d_{t+1} = \mu + d_t + \xi_{t+1} \tag{9}$$

where ;

μ , denotes the dividend's growth rate;

d_t , denotes the dividend's logarithm;

ξ_{t+1} , denotes a random null conditional prediction variable with a variance equal to σ^2.

Then, when a dividend D_t , of a coming period is known at a moment t and if P_t is fixed by the market, the fundamental value of a stock will be directly proportional to dividends

$$P_t = kD_t \tag{10}$$

with $$k = \left(e^{r} - e^{\mu + \frac{\sigma^2}{2}} \right)^{-1} \quad \text{et} \quad r \succ \mu + \frac{\sigma^2}{2}$$

The condition $r \succ \mu + \frac{\sigma^2}{2}$, indicates that interest rate, which is constant, should be superior to the dividends' growth rate.

The function of the intrinsic bubble specified by Froot and Obstfeld (1991) is written as

$$B(D_t) = cD_t^{\lambda} \tag{11}$$

where ;

c , is an arbitrary constant;

λ ,is the positive root of the following equation

$$\lambda^2 \frac{\sigma^2}{2} + \lambda\mu - r = 0 \tag{12}$$

At this level, it seems that the growth anticipation restriction imposed by equation (8) allows dividends to contribute in self-fulfilling predictions. Then, it is convenient to admit that dividends transmit information that investors use to ground their predictions.

By summing up the dividends' observed value, function (10), with the intrinsic bubble, function (11), we obtain the equation of the stock's fundamental price.

$$P_t = F_t + B_t = kD_t + cD_t^{\lambda} \tag{13}$$

Equation (13) indicates that the stock value is derived exclusively from fundamentals even in the presence of a speculative bubble. The presence of the intrinsic bubble allows, as suggested by equation (13), limiting the nonlinear dependencies that stock prices may exhibit. Likewise, it is clear that when the fundamental value varies, the stock price overreacts because of the bubble term which tends to amplify movement. Then, this bubble may cause an important and persistent deviation, yet it may remain stable during some periods.

3. Data and empirical results

3.1. Data

In this paper, we test the null hypothesis of no rational speculative bubbles in the US stock exchanges against the alternative hypothesis that bubbles do exist. This paper includes data for the years 1871 through 2009 of the US Stock Exchange. Data consist of real prices and real dividends of the S&P 500 index. Data is obtained from Robert Shiller's web page.

3.2. Empirical results

We test the presence of intrinsic bubbles for the S&P 500 index. First, we conduct a stationarity test. Then, we estimate the intrinsic bubble specification.

3.2.1. Descriptive Statistics

[Insert Table 1 here]

The real stock price series (S&P 500 composite stock price index) show a skewness coefficient different from zero and a kurtosis superior to 3. Consequently, the distribution of the real price is not normally distributed. It has rather a leptokurtic shape. Moreover, the Jaque Bera test rejects the normality hypothesis. It is possible to see that the real dividends series show a symmetry coefficient close to zero (skewness=0,75) and a flatness coefficient close to 3 (kurtosis=2,94). However, the jaque bera test rejects the normality hypothesis for the real dividends series.

3.2.2. Stationarity and cointegration

The main relationship between the cointegration test and the bubble is the following: presence of bubbles, which induces prices to deviate from their fundamental value, is assumed by an absence of cointegration between these two variables. Thus, testing the presence of cointegration (null hypothesis) is testing the absence of bubbles hypothesis. Cointegration and thus long-term equilibrium between prices and dividends, consequently exclude the presence of a speculative bubbles hypothesis.

Applying the cointegration technique on rational bubbles dates back to the works of Diba and Grossman (1988a). These authors noted that absence of cointegration may be due to the presence of a rational bubble which provoked a persistent deviation between the stock price and its fundamental value. Craine (1993), Campbell et al, (1997), Sarno and Taylor (1999) and Raymond (2001) further developed cointegration test techniques to adjust them to the rational bubbles theory. Table (2) reports the Phillips and Perron stationarity test applied on the two prices and real dividends series.

[Insert Table 2 here]

The PP test indicates that the two real prices and dividends series are non-stationary in level, yet they are stationary in first difference. Consequently, the two series are integrated at a 1, I(1) order. Prices and dividends stationarity in first difference excludes an explosive price hypothesis. According to Hamilton and Whiteman (1985), this assumption allows removing exogenous bubbles having an explosive growth. Indeed, Hamilton and Whiteman (1985) suggest that the presence of this type of explosive behaviour within stock prices, like Blanchard and Watson's deterministic bubble (1982, 1984), tends to make their process explosive.

Table (3) reports the results of the cointegration test in line with Johensen (1991, 1995).

[Insert Table 3 here]

The trace test indicates the absence of a cointegration relationship between the real price and the real dividend. This observation points to the presumption of the presence of a rational bubble.

At this level and in line with Diba and Grossman (1988) and Campell and Shiller (1987) and Sarno and Taylor (1999) and Raymond (2001), it is convenient to assume that these cointegration tests can only give a presumption of the presence of bubbles. It is necessary then to further refine the empirical specification through estimating the bubble's parameters. To this effect, we retain the intrinsic bubble's formal specification initially proposed by Froot and Obstfeld (1991). In order to assess the presence of this type of bubble, it is enough to assume that random fluctuations (essentially dividends) transmit information reflected in both the fundamental value and the bubble. Moreover, to be in line with the growth anticipation constraint, it is convenient to assume as well that investors use information transmitted by dividends to base their anticipation of stock prices' future evolution.

3.2.3. Intrinsic bubbles

From an econometric perspective, testing the presence of intrinsic bubbles is testing the following regression;

$$P_t = c_0 D_t + cD_t^{\lambda} + \varepsilon$$

where,

$$c_0 = K = \left(e^r - e^{\mu + \frac{\sigma^2}{2}} \right)^{-1} \quad \text{and} \quad \lambda = \frac{-\mu \pm \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2}$$

In order to avoid the multi-collinearity problem facing the regression, it is necessary to estimate the following modified regression.

$$\frac{P_t}{D_t} = c_0 + cD_t^{\lambda - 1} + \eta_t$$

Where η_t are independent from dividends.

The null hypothesis of the absence of a bubble is H_0: $c_O = K$ and c=0 against the alternative hypothesis of the presence of a bubble H_1: $c_O = K$ et $c > 0$.

The retained methodology is that of Froot and Obstfeld's (1991). We estimate the intrinsic bubbles model by imposing the root λ in the regression. It is however necessary to estimate the priori market process by a geometric random imposed on the dividends to determine μ et σ^2.

The Dividends Process: the hypothesis of a geometric martingale plays a major role in the study of intrinsic bubbles. For this reason, we should be sure of its validity before moving ahead with our test.

$$d_{t+1} = \mu + d_t + \varepsilon_{t+1} \quad , \varepsilon_{t+1} \to N(0,\sigma^2)$$

The estimation of the process of dividends indicates that $\mu = 0{,}0137$ and that $\sigma = 0{,}1166$. These values, to which we add up the average return rate of the stocks which approximates 8,20 % during the whole study period, allow us to determine the roots of λ :

$$\lambda_1 = 2{,}608 \text{ et } \lambda_2 = -\ 4{,}622$$

Taking into account these parameters, the theoretical K given by

$K = \left(e^r - e^{\mu + \frac{\sigma^2}{2}} \right)^{-1}$, is evaluated at 15,433. This value indicates, following equation (3.20), that the price should be 15,433 times higher than the dividend.

[Insert Table 4 and table 5 here]

It is possible then to conclude that the obtained results differ from the value of the λ parameter. Differently put, when λ_1 = 2,608 (table 4), the constant of the model c_0 is significantly different from zero and approximates the theoretical value (c_0 = 12,47 and K= 15,433). The intrinsic bubble coefficient is significant at the 1% level. The model shows an explanatory power of 53,4%. However, when λ_2 = - 4,622 (table 5), the constant takes a value very far from the theoretical value. The explanatory power of the model is very low (Adjusted R^2 = 3,73%). Then, we retain only the root λ_1 = 2,608.

4. Conclusion

The theoretical predictions of the EMH seem to be hardly reconcilable with the reality of financial markets' mechanisms. Speculative incidents throughout the economic and financial history and more specifically the periodic stock market crashes hitting international financial markets, starting from the "Tulip Bulb Mania" in Holland, the "South Sea Bubble", the 1929 or 1987 crisis, till the repetitive collapses of the stock markets during mars 2000, October 2002 and Mars 2003, are examples of anomalies inherent mainly to speculation mania. Moreover, the recent subprime crisis which first hit the real estate market in 2007, before spreading over the stock market is indeed another example of a speculative bubble explosion. With regard to this paper, we tested the presence of a rational intrinsic bubble in the S&P 500 index. Using a sample of real prices and dividends series observed over the 1871 to 2009 period, we noted the presence of an intrinsic bubble in line with the specifications suggested initially by Froot and Obstfeld (1992).

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Table (1). Descriptive Statistics

	P	DIV
Mean	339.8814	11.16683
Median	199.8170	9.945235
Maximum	1709.492	27.21658
Minimum	65.67299	4.052238
Std. Dev.	346.6378	5.142829
Skewness	2.192548	0.750292
Kurtosis	7.350007	2.942634
Jarque-Bera	219.3718	12.96651
Probability	0.000000	0.001529
Sum	46903.63	1541.022
Sum Sq. Dev.	16461611	3623.470
Observations	138	138

Table 2. Testing for stationarity

Real Price : level		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-2.396896	0.3795
Test critical values:	1% level	-4.025924	
	5% level	-3.442712	
	10% level	-3.146022	
Real Price : first difference		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-5.472102	0.0001
Test critical values:	1% level	-4.026429	
	5% level	-3.442955	
	10% level	-3.146165	
Real dividend: Level		**Adj. t-Stat**	**Prob.***
Phillips-Perron test statistic		-1.890196	0.6542
Test critical values:	1% level	-4.026429	
	5% level	-3.442955	
	10% level	-3.146165	
*MacKinnon (1996) one-sided p-values.			
Residual variance (no correction)			1.015502
HAC corrected variance (Bartlett kernel)			1.267939
Real dividend: First difference		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-8.779100	0.0000
Test critical values:	1% level	-4.026942	
	5% level	-3.443201	
	10% level	-3.146309	

Table 3. Johansen Cointegration Test

Date: 06/07/10 Time: 16:42
Sample (adjusted): 1876 2008
Included observations: 133 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: P DIV
Lags interval (in first differences): 1 to 4
Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.084048	17.21833	20.26184	0.1246
At most 1	0.040814	5.542105	9.164546	0.2292

Trace test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None	0.084048	11.67622	15.89210	0.2056
At most 1	0.040814	5.542105	9.164546	0.2292

Max-eigenvalue test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values
Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):

P	DIV	C
0.005881	-0.307445	1.959895
0.002458	-0.286570	1.610360

Unrestricted Adjustment Coefficients (alpha):

D(P)	-2.720114	-13.33925
D(DIV)	0.260235	-0.081382

1 Cointegrating Equation(s): Log likelihood -922.7402

Normalized cointegrating coefficients (standard error in parentheses)

P	DIV	C
1.000000	-52.27532	333.2434
	(11.5860)	(133.071)

Adjustment coefficients (standard error in parentheses)

D(P)	-0.015998
	(0.03519)
D(DIV)	0.001531
	(0.00050)

Table 4. intrinsic bubbles (λ_1 = 2,608)

$$\frac{P_t}{D_t} = c_0 + cD_t^{\lambda-1} + \eta_t$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
c_0	12.47335	1.795953	6.945255	0.0000
c	0.254180	0.051443	4.941031	0.0000
R-squared	0.537485	Mean dependent var		26.02589
Adjusted R-squared	0.534085	S.D. dependent var		13.62504
S.E. of regression	9.300181	Akaike info criterion		7.312332
Sum squared resid	11763.10	Schwarz criterion		7.354756
Log likelihood	-502.5509	Hannan-Quinn criter.		7.329572
F-statistic	158.0448	Durbin-Watson stat		0.306705
Prob(F-statistic)	0.000000			

Table 5 : intrinsic bubbles (λ_2 = - 4,622)

$$\frac{P_t}{D_t} = c_0 + cD_t^{\lambda-1} + \eta_t$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
c_0	27.20041	2.713767	10.02312	0.0000
c	-49758.75	20435.88	-2.434872	0.0162
R-squared	0.044345	Mean dependent var		26.02589
Adjusted R-squared	0.037318	S.D. dependent var		13.62504
S.E. of regression	13.36840	Akaike info criterion		8.038051
Sum squared resid	24305.11	Schwarz criterion		8.080475
Log likelihood	-552.6255	Hannan-Quinn criter.		8.055291
F-statistic	6.310743	Durbin-Watson stat		0.164389
Prob(F-statistic)	0.013170			