Is It Really Interesting to Invest in Option rather than in Its Underlying Stock?

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Abstract

Despite the suggestion made by investment banks and brokerage firms for private investors to substitute one or all part of their stock investment by the associated option, the real benefit obtained from dealing with an option rather than with its underlying stock remains to be understood.

To clarify the situation, instead of founding our argumentation on specific past data, we build a general analysis of the option return and a comparison of this last with the underlying asset relative change. Using market and synthetic data on option prices, it appears that for many situations investing in an option may be less interesting than investing directly in the related underlying stock.

However there is no definitive answer about the superiority or not of investing in the option, since the situation depends on the level of asset change at the investment maturity. It is up to the investor to decide the best investment support to choose, depending on her views on the future asset relative change and on the option characteristics which are available on the markets. The formulas we derive here may be helping in performing this task.

Keywords: Investment decision, Stock options, Returns, Black-Scholes-Merton pricing

Classification: G11, G13.

1. Introduction

Many investment banks and brokerage firms, involved in selling speculative investment products, suggest to private investors to substitute one or all part of their stock investment into the investment in the associated option (see as an example [Os]). In the leaflets related to the derivative products they propose, very few (and most of time simplistic) examples are provided, and these last do not reflect seriously the risk and complexity behind the proposed instrument. For instance, the presentation is made in order to draw the client attention on the option leverage effect rather than on the real return itself. Therefore for the favorable case, the investment in option seems to be attractive due to the large amplification of the underlying asset return obtained in comparison with a direct investment in the stock itself. For the defavorable case, the sellers divert the client attention on the small size of the premium associated to the considered option and recall that only one part of the investor's wealth would be suitable for the option speculative investment. The question of real benefit obtained from investing in the option rather than in its associated underlying stock remains to be understood in full generality and deserves some analyzes.

Consequently, in this paper, we will focuse on analyzing the returns corresponding to the two alternative investments: in the option and its associated stock. Therefore any investor may get from our result, by incorporating her view on the future asset relative change, a first indicator on the possible return she could get related to her investement project. It is just a first indicator since, as done in financial literature and some commercial leaflets, we do not take into account the transaction costs. To get more realistic views on the consequence of any investment choice, market frictions such as transaction costs should be incorporated into the analyzes. Transaction costs are not taken into consideration here for clearness and also to avoid tedious technical development. It should not be forgotten that an investment with a theoretical positive return may be really less interesting when the transaction costs are taken into consideration.

The speculative and hedging features of option, for the seller-standpoint, are largely studied and developped over the financial literature. In this work, we will focus on the speculative aspect of option for the buyer-standpoint. Of course the option deals are not only intended for an immediate return but are also helpful for an objective of portfolio diversification. However for various investors (as those who are involved in a portfolio with small size and possibly made by one type of option), the question is essentially reduced to materialize a maximum profit and loss for a short or medium term horizon. The analyses we perform in this work may be used to help these investors to take their investment decisions. Next they can also viewed as a departure for further research on general return structure related to a portfolio containing derivatives. Indeed most of available results for such a portfolio are essentially simulation-based or just lying on very specific market situations.

In contrast with the various claims presented inside commercial leafts and intended for (private) investors, by using our general results (see Theorem 1 and Proposition 3), real market and synthetic data on option prices, it appears that for many situations investing in option may be less interesting than investing directly in the associated underlying asset. However, there is no definitive answer about the superiority or not of investing in option, since the situation depends on the level of asset change at the investment maturity. Our contribution here to the question is to provide an analysis of the return structures associated with the two alternative investments. Then it is up to the investor to decide the best investment support to choose depending on her views of the future asset relative change and on the option characteristics which are available on the markets. The return structures analyses we obtain here may be seen as a starting point for implementing a tool for investment decision.

Our main results are presented in Section 2. First we analyze in Theorem 1, the return structure of the call-option. The main point here is about finding suitable intervals containing the realized asset relative change at the maturity and for which the comparison between the two investments (in the option and in the underlying stock) becomes clear. The fact that the investment in option may be less interesting in comparison with the investment in the underlying stock leads us to ask about the returns associated with the well-known call-covered strategy. Especially the question of finding sufficient conditions ensuring a positive return is raised here. The answer is displayed in Proposition 2. Moreover in Proposition 3, we explicit the probability for which the investment in option is superior compared with the investment in the underlying stock when this last is assumed to follow the standard geometrical brownian motion process (GBMP). To clarify how interesting is our Theorem 1 when taking the investement decision, we give in Section 3 some numerical examples whose the corresponding Tables are display in Subsection 6.2 of the Appendix part, which also contains the proofs of our results.

For shortness, this paper is just devoted to the comparison of investments in a call and its underlying stock. Similar study for the put-option and the associated stock-short selling may be also done. However in this case the short-selling mechanism creates some difficulties both for the theoretical analysis and the real practice. For the reader convenience, the full details for the put framework are performed by the author in [Ra2].

2. Main Results

We consider european call-options with the exercise price K, maturity T and, whose the underlying asset is some stock XX. It is assumed that $0 < K, T$. Let us recall that a call-option is a contract which gives to its holder the right to buy, at maturity T from the call-seller, the underlying stocks XX at the unit price K. Let us denote by S_0 the stock value at the present time 0. The future time-T value of stock XX may be written as

$$S_T(\cdot) = \{1 + v_T(\cdot)\}S_0 \tag{1}$$

such that

$$v_T(\cdot) = \frac{S_T(\cdot) - S_0}{S_0}$$

represents the asset stock relative change during the time-period $[0, T]$. Of course, from time 0, either $v_T(\cdot)$ or $S_T(\cdot)$ may be considered as a random variable, and we always assume that

$$-1 < v_T(\cdot) \qquad \text{and} \qquad 0 < S_T(\cdot).$$

These last inequalities mean that bankruptcy of the firm issuing the stock is excluded to happen. For convenience we will make use of the quantity

$$w_0 \equiv (\frac{K}{S_0} - 1). \tag{2}$$

Using the stock relative change $v_T(\cdot)$ between $[0, T]$ and (2), it is clear that

the call-option exercise, i.e. $K < S_T(\cdot)$*, is equivalent to the realization of*

$$w_0 < v_T(\cdot). \tag{3}$$

This inequality means that the call-options may be exercised only when the stock relative change $v_T(\cdot)$ between $[0, T]$ is large enough. Our motivation in introducing (3) is that in practice, very often, people deal with the asset relative change rather than with the asset price itself. So we have immediatly here a suitable call-exercise criterion. Though $v_T(\cdot)$ remains completely unknown at the present time 0 (the truth will be revealed only at time-T), the investor should have some feeling (good or not) about a rough interval which contains it. It should not be forgotten that having some views on the future market change is a necessary prerequisite before taking any investment position. At the point where the investor is assumed to have a rough idea about the possible values of $v_T(\cdot)$, it may be useful to focuse on the value of w_0. To this end we observe that

- *for a in-the-money (ITM) call (i.e.* $K < S_0$ *), then* $w_0 < 0$*; for a at-the-money (ATM) call (i.e.* $K = S_0$ *) then* $w_0 = 0$*; for a out-of-the-money (OTM) call (i.e.* $S_0 < K$ *) then* $0 < w_0$.

Therefore intuitively, when thinking that $v_T(\cdot)$ will be certainly strictly positif, then we expect that an ITM or ATM-call will be easily exercised. For the OTM-call the status seems not clear since, by (3), it depends on the magnitude of w_0. Numerical values of w_0 may be seen over most of all Tables displayed in the Appendix Part.

Since we do not take into account the transaction costs, the investment return when taking a long position on the stock XX during the time-period $[0,T]$ is given by

$$\text{ret\_asset}(\cdot) = \frac{S_T(\cdot) - S_0}{S_0} = v_T(\cdot). \tag{4}$$

For shortness we use the notation $\text{ret\_asset}(\cdot)$ instead of the more complete one $\text{ret\_asset}_{0,T}(\cdot)$. Note that in real investment we have to deal with an integer number of stocks XX but not just with one-security as used over this paper. Similarly taking a long position on the associated call-option, during the time-period $[0,T]$, leads to the return

$$\text{ret\_call}(\cdot) = \frac{\{S_T(\cdot) - K\}1_{\{K<S_T(\cdot)\}} - C_0}{C_0} = (\frac{S_0}{C_0})(v_T(\cdot) - w_0)1_{\{w_0<v_T(\cdot)\}} - 1. \tag{5}$$

Here C_0 denotes the call premium which corresponds to the price of the right to exercise at the maturity T. To avoid arbitrage, we have always $0 \le C_0 < S_0$, and will focus on the significant case $0 < C_0$. The notation $1_{\{K<S_T(\cdot)\}}$ is used to indicate the characteristic function, that is $1_{\{K<S_T(\cdot)\}} = 1$ for $K < S_T(\cdot)$, else $1_{\{K<S_T(\cdot)\}} = 0$.

For an ATM-call, i.e. $w_0 = 0$, and when $(\frac{S_0}{C_0})v_T(\cdot)$ is large enough in comparison with 1 then one has

$$\text{ret\_call}(\cdot) \approx (\frac{S_0}{C_0})v_T(\cdot) = (\frac{S_0}{C_0})\text{ret\_asset}(\cdot).$$

Since $1 < \frac{S_0}{C_0}$ then the leverage effect linked to option, as emphasized and pointed by investment banks and brokerage firms appears here. The advantage with investing in the call-option for the other cases (as for no ATM-call, or $\frac{S_0}{C_0}v_T(\cdot)$ with small or moderate size, $\ldots$) deserves some analyses, we are performing from now.

In order to state a general result, not depending on specific situation or data, we introduce the following two quantities

$$w_0^* = w_0 + \frac{C_0}{S_0} = \frac{1}{S_0}(K + C_0) - 1 \tag{6}$$

and

$$w_0^{**} = \frac{w_0^*}{1 - \frac{C_0}{S_0}} \tag{7}$$

such that

$$w_0 < w_0^* < w_0^{**}. \tag{8}$$

Remind that the call can be exercised whenever the future asset change $v_T(\cdot)$ goes above w_0. Here w_0^* may be seen as the call-break-even point, that is the call return becomes nonnegative only when $v_T(\cdot)$ is above w_0^*.

Theorem 1 *Recall that*

$$\text{ret\_asset}(\cdot) = v_T(\cdot).$$

1. **Case**: $-1 < v_T(\cdot) \le w_0$. Here we have

$$\text{ret\_call}(\cdot) = -1 = -100\% \tag{9}$$

such that

$$\text{ret\_call}(\cdot) < \text{ret\_asset}(\cdot) \tag{10}$$

and

$$0 < \text{ret\_asset}(\cdot) - \text{ret\_call}(\cdot) \le 1 + w_0. \tag{11}$$

2. **Case**: $w_0 < v_T(\cdot) \le w_0^*$. Here we have

$$\text{ret\_call}(\cdot) = (\frac{S_0}{C_0})(v_T(\cdot) - w_0^*) \tag{12}$$

and

$$\text{ret\_asset}(\cdot) - \text{ret\_call}(\cdot) = (\frac{S_0}{C_0})\{1 - (\frac{C_0}{S_0})\}(-v_T(\cdot) + w_0^{**}) \tag{13}$$

such that

$$-1 < \text{ret\_call}(\cdot) \le 0 \tag{14}$$

$$(10) \text{ is satisfied} \tag{15}$$

and

$$0 < (\frac{S_0}{C_0})\{1 - (\frac{C_0}{S_0})\}(w_0^{**} - w_0^*) \le \text{ret\_asset}(\cdot) - \text{ret\_call}(\cdot) < \{1 - (\frac{C_0}{S_0})\}. \tag{16}$$

3. **Case**: $w_0^* < v_T(\cdot) \le w_0^{**}$. Identities (12) and (13) are also satisfied. Moreover we have

$$0 < \text{ret\_call}(\cdot) \le (\frac{S_0}{C_0})(w_0^{**} - w_0^*) \tag{17}$$

$$\text{ret\_call}(\cdot) \le \text{ret\_asset}(\cdot) \tag{18}$$

and

$$0 \le \text{ret\_asset}(\cdot) - \text{ret\_call}(\cdot) < (\frac{S_0}{C_0})\{1 - (\frac{C_0}{S_0})\}(w_0^{**} - w_0^*). \tag{19}$$

4. **Case**: $w_0^{**} < v_T(\cdot)$. Here we have

$$0 < (\frac{S_0}{C_0})(w_0^{**} - w_0^*) < \text{ret\_call}(\cdot) = (\frac{S_0}{C_0})(v_T(\cdot) - w_0^*) \tag{20}$$

$$\text{ret\_asset}(\cdot) < \text{ret\_call}(\cdot) \tag{21}$$

and

$$0 < \text{ret\_call}(\cdot) - \text{ret\_asset}(\cdot) = (\frac{S_0}{C_0})\{1 - (\frac{C_0}{S_0})\}(v_T(\cdot) - w_0^{**}). \tag{22}$$

This result is a useful tool in deciding the superiority or not of the call-option investment (in comparison with the investment in the underlying stock) when it is combined with the investor view on the possible level of asset relative change at the maturity T. The first step in comparing returns related to taking a long position on a call and a long position on the underlying stock is to compute the real numbers w_0, w_0^* and w_0^{**} as defined respectively from (2), (6) and (7). They depend just on the call characteristics: strike K, premium C_0 and the underlying spot S_0. These numbers may be sorted in increasing order as described in (8). As a consequence there are four cases to consider for the level of future stock relative change:

1) $-1 < v_T(\cdot) \le w_0$,
2) $w_0 < v_T(\cdot) \le w_0^*$,
3) $w_0^* < v_T(\cdot) \le w_0^{**}$ and
4) $w_0^{**} < v_T(\cdot)$.

Only one of these situations can occur at the future time T. The investor would have a clear idea about which interval among $]-1, w_0]$, $]w_0, w_0^*]$, $]w_0^*, w_0^{**}]$ and $]w_0^{**}, \infty[$ should contain the future asset value $v_T(\cdot)$. Introducing a model for the stock dynamic, as we will do in Proposition 3, may lead to determine the probability for which this future asset relative value $v_T(\cdot)$ belongs to a given interval. Having historical data on the stock past values is useful to this end. The idea of combining statistical approach and the investor views, as developed by A. Meucci [Me] as an extension of the Black-Litterman approach, may be also an interesting way to explore here. But we do not go further in such a direction.

Our Theorem1 describes the quantities involved in each situation. We try now to recast this result in a more qualitative understandable form.

The case 1): $-1 < v_T(\cdot) \le w_0$ corresponds to the fact that the asset relative change has not increased enough or probably decreased. Here we get the worst situation for the call. It cannot be exercised and the return is equal to -100%. Nevertheless the return for the asset position remains strictly large than that of this call, as we can read in (10). If $0 < w_0$ (as in the case of OTM-call) and $0 < v_T(\cdot) \le w_0$ then the position in the stock leads to a profit, which is not the case for the call position for which we loose all of the invested amount. Always for the first case 1), as we may see from (11), the distance between the returns of the call and underlying stock remains bounded by the fixed number $1 + w_0$.

The case 2): $w_0 < v_T(\cdot) \le w_0^*$, corresponds to the fact that the call may be exercised, but the stock growth is not sufficiently enough such that the call return remains negative (see (14)). Once again here, the return for the stock position remains more than that for this call, as we can read from (15). The position in the asset leads to a profit for $0 < w_0$ (as in the case of ATM or OTM-call). Moreover for this second case 2), as we may see from (16), the distance between the returns of the call and underlying asset remains bounded below and above by fixed numbers depending only on K, S_0 and C_0.

The case 3): $w_0^* < v_T(\cdot) \le w_0^{**}$, corresponds to the fact that the call may be exerciced and the asset increase is sufficiently enough in order that the call return becomes now positive (see (17)). Again here, the return for the asset position is above than that for this call, as we can read in (18). If $0 < w_0^*$ (which is already satisfied if $-\frac{C_0}{S_0} < w_0$) then the position in the asset leads to a profit. Though the call also wins a profit, this last remains less than that obtained directly with the asset position. As we may see from (19), the distance between the returns of the call and underlying asset remains bounded above by a fixed number depending only on K, S_0 and C_0.

The case 4): $w_0^{**} < v_T(\cdot)$ may be viewed as the case where the asset has done a strong increase. Not only the call may be exerciced but the stock increase is sufficiently enough in order that the call return becomes now positive (see (20)) and, as can be read in (21), this return is larger than the return linked to the stock position. It may be seen from (20) that the call position leads always to a profit and a nonnegative lower bound for the corresponding return may be given. The distance between the returns of the call and underlying stock moves as an affine function of the return asset as described in (22).

Numerical illustrative examples and more comments are displayed in the next section. Over these examples it can be seen that for many situations it is preferable to invest directly in the underlying stock XX rather than in the associated call-option. So it may be interesting to consider the strategy of covered call, which consists to sell a call and buy the stock. Selling a call means having a bearish or neutral view on the asset movement. The purchase of the stock is done here in order to cover an unexcepted upward asset change. The return associated to the covered-call strategy is given by the following.

Proposition 2 *Assume that we have sold a call with the premium* C_0, $0 < C_0 < S_0$, *and simultaneously bought the underlying stock XX at the unit price* S_0. *Then this covered-call strategy leads to the profit and loss*

$$\text{Profit}\,\&\,\text{Loss\_covered\_call}(\cdot) = S_0 \times \text{ret\_covered\_call}(\cdot)$$

where

$$\text{ret\_covered\_call}(\cdot) = \{v_T(\cdot) + (\frac{C_0}{S_0})\}\,1_{\{v_T(\cdot) \le w_0\}} + w_0^* 1_{\{w_0 < v_T(\cdot)\}} \tag{23}$$

where w_0^* is defined as in (6). As a consequence for $0 < w_0^*$, the covered call leads always to a profit whenever the stock relative change $v_T(\cdot)$ remains strictly above $-(\frac{C_0}{S_0})$.

The covered-call is a winning strategy as long as the future asset change $v_T(\cdot)$ does not move below $-\frac{C_0}{S_0}$ nor goes above w_0^*. For $v_T(\cdot) \in]-\frac{C_0}{S_0}, w_0]$ the call-covered strategy has an effect to enhance the return $v_T(\cdot)$ into $v_T(\cdot) + \frac{C_0}{S_0}$. For $v_T(\cdot) \in]w_0, w_0^*]$ the call-covered strategy has an effect to fix the return at the level w_0^*. This is also the return for the other cases $w_0^* < v_T(\cdot)$, corresponding to a bullish situation which is out of the call-covered strategy spirit. Moreover in these situations the return w_0^* is less than the one obtained from an outright stock investment.

In the framework of Black-Scholes-Merton, the underlying stock XX is assumed to follow the Geometrical Brownian Motion (GBM) whose the dynamic is given by the stochastic differential equation of the form

$$dS_t(\cdot) = \mu S_t(\cdot)dt + \sigma S_t(\cdot)dW_t(\cdot) \tag{24}$$

where μ is real number which represents the asset local tendency, and σ, with $0 < \sigma$, is the asset volatility.

For any w, with $-1 < w$, then let us introduce the quantity

$$x(w) = \frac{1}{\sigma\sqrt{T}}[\ln(1+w) - (\mu - \frac{1}{2}\sigma^2)T]. \tag{25}$$

Proposition 3 *Assume that the underlying asset stock XX follows the GBM process (24). Let us consider a strategy with taking a long position on the call. Then we have the following probabilities estimates:*

$$\mathsf{P}[-1 < v_T(\cdot) \le w_0] = \Phi(x(w_0)) \tag{26}$$

$$\mathsf{P}[w_0 < v_T(\cdot) \le w_0^*] = \Phi(x(w_0^*)) - \Phi(x(w_0)) \tag{27}$$

$$\mathsf{P}[w_0^* < v_T(\cdot) \le w_0^{**}] = \Phi(x(w_0^{**})) - \Phi(x(w_0^*)) \tag{28}$$

$$\mathsf{P}[w_0^{**} < v_T(\cdot)] = 1 - \Phi(x(w_0^{**})). \tag{29}$$

where Φ denotes the cumulative probability function of the standard gaussian normal law; w_0, w_0^* and w_0^{**} are defined respectively as in (2) (6) and (7).

More realistic model for the underlying asset, as with the jump diffusion model, may be considered. But we just limit here in the case of the Black-Scholes benchmark model. Indeed our purpose in this work is to focuse on analyzing the structure of the return rather than including a future view of the asset change.

3. Numerical Results

3.1 Examples for the call position

Table1 is related to the quote for options on Agricole Credit (AC3) as seen from the website of the Nyse-Euronext (www.euronext.com/trader/priceslistderivatives/derivativespriceslists-1930-FR.html) on December 24, 2009. Here we consider european options with the time-maturity September 2010. In columns 1 and 2 of Table 1 the strikes and premia of the considered call are reported respectively. Moreover, the computed values of w_0, w_0^* and w_0^{**} as defined in (2), (6) and (7) respectively are presented in columns 3 to 5. Following our result in Theorem1, these values and the investor view on the asset relative change level for the maturity $T = 276$ days can be mixed in order to shed light the best investment to perform (call-option or underlying stock).

For instance, we observe here that for a OTM-call with the strike-price 14 euros, the call-exercise may be done only if the asset increase is above $w_0 = 15.05\%$, and the investment in call-option is better than that in the underlying stock only for a level of asset change above $w_0^{**} = 24.45\%$. Clearly this last situation should not be realized unless the market has done an exceptional upward jump. Having a bullish market view for the underlying asset at the maturity, the best investment to perform here is to take a long position on the stock.

For an almost ATM-call, i.e. with the strike-price 12 euros, the call-exercise arises if at least the asset relative change level is above -1.45%. So it seems to be not too difficult to reach as long as the market remains slightly bullish at the maturity T. However, by part4 of our Theorem 1, the investment in the call is superior to that in the underlying stock only if the asset change increase level is above $w_0^{**} = 14.65\%$. Such a requirement is not so easy to realize in a neutral or slightly bullish market, so here the investment in stock may be preferable to choose. If the investor expects to get at least a stock relative increase of 3.5% then she may get a best return by investing in a ITM-call with the strike 8.8 euros (or 8 euros) rather than investing in the underlying stock.

The remaining examples presented from Table 2 are not directly drawn from real market data. But we have introduced them in order to better grasp how do the various related factors (as spot price, strike-level, $\ldots$) affect the decision investment. For this purpose, the call prices are generated by using the (practical) standard Black-Scholes-Merton approach for a given choice of implied volatility surface. As introduced in [Al], this volatility surface is defined from the following formula

$$\sigma \equiv \sigma(\tau, S, K, r) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 \tau + \beta_4 x \tau \tag{30}$$

with

$$x \equiv x(\tau, S, K, r) = \ln\left[\frac{S \exp(r\tau)}{K}\right]$$

where S is the underlying spot level, K is the strike-price, r is the risk-free interest-rate and τ is the remaining time-to-maturity. The constants $\beta_0, \ldots, \beta_4$ are inferred from the available option prices. In our example we take $\beta_0 = 0.0990$, $\beta_1 = 0.0408$, $\beta_2 = 0.4671$, $\beta_3 = 0.0272$ and $\beta_4 = 0.2762$.

Tables 2 and 3 are related to call-options associated with the spot $S_0 = 12$ euros, maturity $T = 30$ days, and interest rate $r = 2.85\%$. It is clear from Table 2 that for any ITM-call with the strike-price level less than 11.5 euros then the investment with the call is interesting if the investor expects that for the maturity T, the asset increase level is at least above 0.34%. For the OTM-call with the strike-price 12.5 euros, the investment with the underlying stock seems better than with the call whenever the investor forsees an asset level which cannot go beyond 4.20%. Similarly the call-investment with the strike price 13 euros is only interesting whenever one expect an asset growth of at least 8.35% for the remaining 30 days time-to-maturity.

Then these results seen in Table 2 would lead us to think that things are easy. However, in real exchange market situation any call-option with a given strike level may be not quoted. Moreover it is important to feel the asset change level at maturity.

To localize the interval which would contain the final asset change value $v_T(\cdot)$ is useful. This end is our target with Table 3. In columns 3 to 6, we can read respectively the probabilities

$$\mathsf{P}[-1 < v_T(\cdot) \le w_0], \mathsf{P}[w_0 < v_T(\cdot) \le w_0^*], \mathsf{P}[w_0^* < v_T(\cdot) \le w_0^{**}] \quad \text{and} \quad \mathsf{P}[w_0^{**} < v_T(\cdot)]$$

computed from (26) to (29) with the drift $\mu = r = 2.85\%$.

For the ITM-call with the strike-price 11.5 euros, we have seen in Table 2 that the call-investment is preferable if one expects at the maturity $T = 30$ days the asset increase level to be at least 0.34%. With Table 3, we see that such a situation may be realized with the probability 48.15%. Observe that $\mathsf{P}[w_0 < v_T(\cdot) \le w_0^*] = 44.5271\%$ such that the more likely situation is either $w_0 < v_T(\cdot) \le w_0^*$ or $w_0^{**} < v_T(\cdot)$. The event $w_0^{**} < v_T(\cdot)$ seems little bit difficult to obtain since $\mathsf{P}[-1 < v_T(\cdot) \le w_0^{**}] \approx 51.85\%$.

For the OTM call with the strike-price 13.5, by Table 2, the stock investment seems to preferable if for the remaining time-to-maturity $T = 30$ days the asset increase level does not exceed 12.50%. Probabilities values given in Table 3 go in this direction since for the same strike-price 13, one has $\mathsf{P}[-1 < v_T(\cdot) \le w_0] = 99.9969\%$.

Observe that in all of our Tables the values are rounded up to four digits. For instance with the strike 13.5, the call-price is 2.7086×10^{-6} but not zero as we present in this Table.

Tables 4 and 5 are given with the same attention and purpose as for Tables 4 and 5. The difference is that here we consider the time-maturity 180 days, but with the same spot price 12 and interest rate 2.85%.

As noted in the introduction, the numerical examples from Tables 1 to 5 are given in order to clarify how interesting is the Theorem 1 in taking the investement decision. But these examples do not necessarily reflect any given real market situation. Theorem 1 allows the investor to have a first sight of the consequence of her investment choice between the call or its underlying stock. However, as we have illustrated with the use of probabilities in Proposition 3, the other determing aspect in the investment decision is the future asset change level localization inside some interval. This is a matter of forecasting. A theoretical approach is to introduce a dynamic model for tthe stock price, as the Black-Scholes one presented in (24). This last is suitable for getting explicit expressions of the needed probabilities, though empirical studies show that the real market situation is not captured by such a benchmark model.

4. Conclusions and perspectives

(1). The main issue raised and analyzed in this paper is to decide about the superiority or not of the investment in option compared with its underlying stock, as claimed by various investment banks and brokerage firms when promoting their derivative products. To cope with this situation, in Theorem 1, we determine the return structures corresponding to a long position on the option and its underlying stock. The distance between the two returns is also estimated. The analysis perfomed here is general since it does not assume any specific situation or data. Applying our result to real market and synthetic option prices, it appears that for many situations the call-option investment may be worse than the investment in the underlying stock. This happens when the future asset change level at the investment maturity is not sufficiently high. The thresold level may be determined previously before investing from Theorem 1. However, there is no definitive answer about the superiority or not of investing in option. It is up to the investor to decide the best investment support to choose depending on her views on the future asset relative change and on the option characteristics which are available on the markets.

(2). It is seen, in Theorem 1, that only when the stock relative change has done a strong growth that the investment in call-option is really preferable. Moreover in this case, the option leverage amplifies the asset relative change. It would be also interesting to introduce the leverage effect with the stock position by borrowing one part of the amount required to buy the corresponding asset. Then the analysis is closed to the one used for the case of short-selling the underlying stock as presented in [Ra2].

(3). This paper is restricted to the case of european call-options and we have assumed that the investment takes end only at the option maturity. It may be also interesting to raise and investigate the question about the superiority or not of the investment in option when the investment maturity is shorter than the option maturity. This situation seems to be difficult to handle since specific models for the underlying asset process and the corresponding option pricing are needed.

(4). To obtain easy readable results and to simplify the analysis, we have not considered the related transaction costs. Of course to get more reliable investment decision, fees and liquidity factors should be included into the analyses. Technics and ideas introduced in [Ra1] may be used for this purpose, nevertheless further details are presented in [Ra2].

(5). As is seen over this work, considering a dynamic model for the asset evolution is not so useful when analyzing and comparing the returns related to the investment in option and the corresponding underlying stock. Indeed a clear separation between the return structure and forecasting of the future asset change level should be made. An asset price model and the investor view/feeling may be helping for this last point. Understanding the return structures, as we perform in this paper, is another thing which is at the core and sucess of the investment decision.

(6). The option premium is essential when comparing the returns between the investment in option and the correponding investment in the associated underlying stock. Conversely, when raising and solving incoherence and drawbacks related to the Black-Scholes-Merton pricing, I. Gikhman [Gi] has lead to compare the returns for the two investments in order to define the good notion of option fair price (in stochastice sense). One question, underlying the issue we consider in this paper, is to understand whether the considered option is correctly priced by the market or the model. If it is not the case, we may also ask about the kind of arbitrage to perform.

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6. Appendix

6.1 Proofs of Results

6.1.1 Proofs of Theorem 1

We need to make some preliminaries which are useful in the sequel of the proof. Assuming that $w_0 < v_T(\cdot)$ and making use of (5) then the call return is given by

$$\begin{aligned}\text{ret\_call}(\cdot) &= (\frac{S_0}{C_0})(v_T(\cdot) - w_0) - 1\\ &= (\frac{S_0}{C_0})\{v_T(\cdot) - w_0 - (\frac{C_0}{S_0})\} = (\frac{S_0}{C_0})(v_T(\cdot) - w_0^*).\end{aligned} \tag{31}$$

Using this last expression we get

$$\begin{aligned}&\text{ret\_asset\_SS}(\cdot) - \text{ret\_call}(\cdot) =\\ &= v_T(\cdot) - (\frac{S_0}{C_0})\{v_T(\cdot) - w_0^*\}\\ &= (\frac{S_0}{C_0})[\{(\frac{C_0}{S_0}) - 1\}v_T(\cdot) + w_0^*]\\ &= (\frac{S_0}{C_0})\{1 - (\frac{C_0}{S_0})\}(-v_T(\cdot) + w_0^{**}).\end{aligned} \tag{32}$$

Identities (31) and (32) are the main keys to derive Theorem 1. Particularly from (32) for $w_0 < v_T(\cdot)$ it appears that

$$\text{ret\_asset\_SS}(\cdot) < \text{ret\_call}(\cdot) \qquad \text{if and only if} \qquad w_0^{**} < v_T(\cdot).$$

1). **Case**: $-1 < v_T(\cdot) \le w_0$.

Using (5) with the fact that $v_T(\cdot) \le w_0$ then we get immediately (9). Moreover we have

$$v(\cdot) + 1 = \text{ret\_asset}(\cdot) - \text{ret\_call}(\cdot) \le 1 + w_0$$

which leads to (10) and (11).

2). **Case**: $w_0 < v_T(\cdot) \le w_0^*$.

For $w_0 < v_T(\cdot)$, the call return written in (5) is just reduced to (12) because of identity (31). Obviously the returns difference in (13) is obtained from (32). Using identity (31) associated with $w_0 < v_T(\cdot) \le w_0^*$ and the fact that $w_0 - w_0^* = -\dfrac{C_0}{S_0}$ then we get $-1 < \mathrm{ret\_call}(\cdot) \le 0$ which is (14). Since $-w_0^* \le -v_T(\cdot) < -w_0$, and using the identity (32), then the inequalities written in (16) appear. The fact that the put return is less than the asset return, as announced in (15), is an immediate consequence of the first two inequalities from (16).

3). **Case**: $w_0^* < v_T(\cdot) \le w_0^{**}$.

Using identity (31) associated with $w_0^* < v_T(\cdot) \le w_0^{**}$ then the call return estimates in (17) appear immediatly. Since $-w_0^{**} \le -v_T(\cdot) < -w_0^*$ then, using the identity (32), we obtain the inequalities written in (19). The fact that the put return is less than the asset return as announced in (18) is also an obvious consequence of the first inequality in (19).

4). **Case**: $w_0^{**} < v_T(\cdot)$.

The low bound of the call return written in (20) arises by using identity (31) and the fact that $w_0^{**} \le v_T(\cdot)$. Also from this last inequality and (32) then both (22) and (21) are satisfied.

6.1.2 Proof of Proposition 2

Using the notations as above then we have

$$\begin{aligned}
\mathrm{Profit\,\&\,Loss_covered_call}(\cdot) &= \{S_T(\cdot) - S_0\} - \{S_T(\cdot) - K\}_{\{K < S_T(\cdot)\}} + C_0 \\
&= v_T(\cdot)S_0 + S_0\{v_T(\cdot) - w_0\}_{\{w_0 < v_T(\cdot)\}} + C_0 \\
&= S_0[v_T(\cdot) - \{v_T(\cdot) - w_0\}_{\{w_0 < v_T(\cdot)\}} + (\frac{C_0}{S_0})] \\
&= S_0[\{v_T(\cdot) + (\frac{C_0}{S_0})\}_{\{v_T(\cdot) \le w_0\}} + w^*_{0\{w_0 < v_T(\cdot)\}}].
\end{aligned}$$

Since the amount invested for the call-covered strategy is reduced to S_0 then clearly the corresponding return $\mathrm{ret\_covered\_call}(\cdot)$ is reduced to (23).

6.1.3 Proof of Proposition 3

For the GBM process (24) and with the underlying probability P, the Itô lemma leads to the future asset value

$$S_T(\cdot) = S_0 \exp[(\mu - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}U_T(\cdot)]$$

where $U_T(\cdot)$ is a standard gaussian normal law under the probability P.

The main point to get results in Propositions 3 is to observe that for $-1 < a < b < \infty$

$$\{a < v_T(\cdot) \le b\} = \{x(a) < U_T(\cdot) \le x(b)\}.$$

Now identities (27) and (28) can be easily derived by taking respectively $a = w_0^*$, $b = w_0$ and $a = w_0^{**}$, $b = w_0^*$ since

$$\mathsf{P}[\{a < v_T(\cdot) \le b\}] = \Phi(x(b)) - \Phi(x(a)).$$

Identities (26) and (29) appear since

$$\lim_{a \to -1+} \Phi(x(a)) = \lim_{y \to 0} \Phi(y) = 0 \qquad \text{and} \qquad \lim_{b \to \infty} \Phi(x(b)) = \lim_{y \to \infty} \Phi(y) = 1.$$

Table 1.

strike	call-price	w_0 (%)	w_0^* (%)	w_0^{**} (%)
7.38	4.7200	-39.3591	-0.5752	-0.9396
8	4.2400	-34.2646	0.5752	0.8827
8.80	3.6300	-27.6910	2.1364	3.0445
9.60	3.1100	-21.1175	4.4371	5.9603
10	2.8500	-17.8307	5.5875	7.2961
11.07	2.2200	-9.0386	9.2030	11.2563
12	1.7000	-1.3969	12.5719	14.6132
13	1.3700	6.8200	18.0772	20.3704
14	0.9200	15.0370	22.5965	24.4444
14.76	0.7100	21.2818	27.1159	28.7958
16	0.4600	31.4708	35.2506	36.6354
17	0.3100	39.6878	42.2350	43.3390
18	0.2100	47.9047	49.6302	50.5017
20	0.0900	64.3385	65.0781	65.5629

Table 2.

strike	volatility (%)	call-price	w_0 (%)	w_0^* (%)	w_0^{**} (%)
10.5000	11.8567	1.5249	-12.5000	0.2076	0.2378
11.0000	11.0703	1.0264	-8.3333	0.2198	0.2404
11.5000	10.5077	0.5382	-4.1667	0.3180	0.3329
12.0000	10.1421	0.1547	0	1.2890	1.3059
12.5000	9.9504	0.0147	4.1667	4.2894	4.2947
13.0000	9.9128	0.0004	8.3333	8.3363	8.3366
13.5000	10.0123	0.0000	12.5000	12.5000	12.5000
14.0000	10.2341	0.0000	16.6667	16.6667	16.6667

Table 3.

strike	call-price	proba1 (%)	proba2 (%)	proba3 (%)	proba4 (%)
10.5000	1.5249	0.0038	50.3278	0.3514	49.3169
11.0000	1.0264	0.2709	50.1426	0.2560	49.3305
11.5000	0.5382	7.1297	44.5271	0.1955	48.1477
12.0000	0.1547	47.3497	17.1177	0.2111	35.3214
12.5000	0.0147	91.1948	0.6371	0.0265	8.1415
13.0000	0.0004	99.6819	0.0009	0.0001	0.3171
13.5000	0.0000	99.9969	0.0000	0.0000	0.0031
14.0000	0.0000	100.0000	0.0000	0.0000	0.0000

Table 4.

strike	volatility (%)	call-price	w_0 (%)	w_0^* (%)	w_0^{**} (%)
10.5000	14.9239	1.6916	-12.5000	1.5968	1.8588
11.0000	13.5505	1.2372	-8.3333	1.9767	2.2040
11.5000	12.4271	0.8228	-4.1667	2.6900	2.8880
12.0000	11.5244	0.4780	0	3.9837	4.1490
12.5000	10.8176	0.2316	4.1667	6.0968	6.2168
13.0000	10.2852	0.0898	8.3333	9.0814	9.1499
13.5000	9.9085	0.0273	12.5000	12.7274	12.7564
14.0000	9.6714	0.0066	16.6667	16.7217	16.7309

Table 5.

strike	call-price	proba1 (%)	proba2 (%)	proba3 (%)	proba4 (%)
10.5000	1.6916	8.8888	43.8157	0.9706	46.3249
11.0000	1.2372	15.6508	38.4701	0.9206	44.9585
11.5000	0.8228	27.3400	29.9535	0.8582	41.8483
12.0000	0.4780	44.6653	18.8395	0.7301	35.7652
12.5000	0.2316	65.0114	8.4113	0.4825	26.0948
13.0000	0.0898	82.6652	2.3164	0.2004	14.8181
13.5000	0.0273	93.4826	0.3582	0.0445	6.1146
14.0000	0.0066	98.1233	0.0314	0.0052	1.8401