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On Forecasting Taiwanese Stock Index Option Prices: The Role of Implied Volatility Index

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Abstract

This paper aims to propose four volatility measures: The first is the GARCH model advocated by Bollerslev (1986); the second is the $\text{GARCH}_{\text{VIX}}$ model which extends the GARCH model by including the volatility index (VIX) as explanatory variable for volatility; the last two are $\text{HS}_{20\text{D}}$ and $\text{HS}_{252\text{D}}$, which represent the historical volatilities generated by traditional rolling window technique with 20- and 252-day historical index returns data, respectively. We examine the price information on VIX to improve the predictive performance of GARCH model for valuing TAIEX stock index call options (TXO) over the period from January 2014 to May 2015. Empirical results firstly indicate that both the GARCH and $\text{GARCH}_{\text{VIX}}$ models consistently perform better than the historical volatility models for forecasting call value of TXO under different moneynesses. Secondly, the $\text{GARCH}_{\text{VIX}}$ model significantly outperforms the GARCH model for most cases, indicating that the GARCH-based option price forecasts can be effectively improved with the additional information contained in VIX. Finally, the use of $\text{GARCH}_{\text{VIX}}$ model can greatly reduce model mispricing especially for out-the-money TXO option case. Thus, volatility index is crucial for option traders to efficiently predict TXO option value with GARCH model.

Keywords: VIX, TXO, option, BS model, GARCH

1. Introduction

Chicago Board Options Exchange (CBOE) develops the volatility index (or VIX) that is derived from the S&P500 stock index option prices via an option pricing formula in 1993. The VIX has been considered a proxy measure of market’s expectation of future stock market volatility over the next 30 day period. Meanwhile, the VIX is also called the fear index or implied volatility index in order to reflect sentiments of investors. The introduction of VIX in 1993 has inspired researchers to explore its practical applications for financial markets.

Chu and Freund (1996) examine the mispricing of option valuation models for a sample of calls on the S&P 500 and S&P 100 stock indices when volatility estimates are generated by rolling window historical index returns, GARCH model (Note 1), IGARCH model (Note 2) and index option prices implied volatility. Blair et al. (2001) explore the incremental information content of implied volatilities and intraday returns in the context of forecasting S&P 100 index volatility over 1 to 20 days forecast horizons. González-Rivera et al. (2004) investigate the forecasting performance of various volatility models for stock returns in terms of several loss functions (including an option pricing function) for which volatility estimation is of paramount importance. We deal with two economic loss functions. Koopman et al. (2005) compare the predictive ability of historical volatility (extracted from daily returns), implied volatility (extracted from option data) and realized volatility (the cumulative sum of squared high frequency returns within a day) for forecasting daily variability of the S&P 100 stock index returns. Corrado and Truong (2007) augment the GJR-GARCH model of Glosten et al. (1993) by intraday high-low price range and VIX in order to investigate their additional information for improving GJR-GARCH volatility forecasting accuracy. Recently, Wang et al. (2016) propose the augmented GJR model by including various volatility estimators (overnight volatility, daily prices range, and VIX) as explanatory variables for the variance equations in GJR model. These models are used to estimate their daily VaR values for the

Standard & Poor's Depositary Receipts (SPDRs). Kim and Ryu (2015) propose a modified value-at-risk (VaR) model that utilizes the implied volatilities extracted from the KOSPI 200 options. They find that the model-free implied volatility index of the KOSPI 200 (VKOSPI) does not greatly enhance the performance of suggested VaR models.

However, despite an extensive literature on volatility forecasting, relatively little research investigates the prices information on VIX to improve the predictive performance of GARCH model for valuing stock index options. This paper aims to use four volatility estimates which are generated by the following models to forecast daily call values of TAIEX options: The first is the GARCH model advocated by Bollerslev (1986); the second is the $GARCH_{VIX}$ model which extends the GARCH model by including the VIX as explanatory variable for volatility; the last two are HS_{20D} and HS_{252D}, which represent the historical volatilities extracted by traditional rolling window estimation with 20- and 252-day historical index returns data, respectively. We empirically calculate the value of each call on the TAIEX options (TXO) via Black and Scholes (1973)' formula with each of the four volatility estimates, and evaluate their mispricing over the period from January 2014 to May 2015.

The rest of this paper is arranged as follows. Section 2 describes the sample data and Section 3 introduces econometric methodology employed. Section 4 presents the forecasting performance for TXO options under different moneyness cases, while conclusions drawn from this study are also summarized in the same section.

2. Data

The data examined in this study comprises of the daily closing prices of Taiwan Weighted Stock Index (TAIEX) and the VIXs of the TAIEX options obtained from the CMONEY database. The sample period for these daily data spans from 2 January 2012 to 29 May 2015 for a total of 839 trading days. The first two years are used as the in-sample period for estimation purpose, while the remaining 1.5 years (343 observations) are taken as the out-of-sample for forecast evaluation. In order to calculate call values of TAIEX options and compare the model mispricing, we also retrieve option data including stock prices, strike prices, maturity day, risk-free rate and settlement prices from the CMONEY database.

Table 1 shows the descriptive statistics of the daily returns for the Taiwan Weighted Stock Index. As shown in Table 1, the average daily return is positive, and approaches close to zero. The returns series exhibits significant evidence of skewness and kurtosis, which means that the series is skewed to the left, and the distribution of the daily returns is more fat-tailed and high-peaked than normal distribution. The J-B test statistic further confirms that the daily returns series is not normal distribution. Finally, the Ljung-Box test statistic exhibits linear dependence for the squared returns and strong ARCH effects.

Table 1. Descriptive statistics of daily returns for the Taiwanese stock index

Mean (%)	Std.	Min.	Max.	Skew.	Kurt.	JB	$Q_s(12)$
0.039	0.797	-3.020	3.052	-0.192*	1.197**	55.232**	92.081**

Note: JB is the statistic of Jarque and Bera (1987)'s normal distribution test. $Q_s(12)$ refers to the Ljung-Box Q test statistic of the squared return series for up to the 12th order serial correlation. * and ** indicate significance at the 5%, and 1% levels.

3. Econometric Methodology

3.1 Forecasting Volatility of Underlying Asset Returns

We augment the GARCH model Bollerslev (1986) with implied volatility as follows:

$$R_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \Big|_{\Omega_{t-1}} \sim NID(0,1) \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \rho VIX_{t-1} \tag{2}$$

where R_t is daily return; μ denotes the conditional mean of returns; ε_t is the innovation process; z_t is the standardized residual with zero mean and unit variance; σ_t^2 is the conditional variance; VIX_{t-1} is the implied volatility at the end of TXO options trading on day $t-1$. Thus, marginal contribution of the implied volatility to predicting conditional volatility σ_t^2 is measured by the coefficient ρ. Obviously, the traditional GARCH model applies the restriction $\rho = 0$ to equation (2) with no exogenous regressors.

The annualized standard deviation of the asset returns can be also estimated using historical volatility as follows:

$$\sigma_{t,D} = \sqrt{252} \cdot \left[\frac{1}{D}\sum_{i=1}^{D}(R_{t-i} - \bar{R})^2\right]^{0.5} \tag{3}$$

where $\bar{R}\ (= \frac{1}{D}\sum_{i=1}^{D} R_{t-i})$ denotes the average returns for the past D days, and D = 20 or 252.

3.2 Black-Scholes Option Price Forecast

The theoreatical price of an European call option can be directly calculated by Black and Scholes (1976)'s formula as follows:

$$C_{t,k} = S_t \cdot \Phi(d_{1,k}) - \mathrm{X} \cdot e^{-r_t(\tau-t)} \cdot \Phi(d_{2,k}) \tag{4}$$

$$d_{1,k} = \frac{\ln\left(\frac{S_t}{X}\right) + (r_t + 0.5 \cdot \sigma_{t,k}^2)(\tau - t)}{\sigma_{t,k} \cdot (\tau - t)^{0.5}} \tag{5}$$

$$d_{2,k} = d_{1,k} - \sigma_{t,k} \cdot (\tau - t)^{0.5} \tag{6}$$

where $C_{t,k}$ denotes the daily forecasting price of TAIEX call options using BS call option formula based on k volatility model at time t that expires in time $(\tau - t)$; S_t is the price of Taiwan Stock Exchange Capitalization Weighted Stock Index at time t; X is the option striking price; $(\tau - \mathrm{t})$ is the call option time to maturity in years; r_t is the risk-free interest rate at time t; $\Phi(\cdot)$ is the cumulative probability density function of the normal distribution; $\sigma_{t,k}$ represents the annualized standard deviation of the index returns forecasted by volatility model, k.

3.3 Evaluation of Forecasting Accuracy

To examine the option mispricing of four competing models, we calculate mean absolute error (MAE) and mean squared error (MSE) as follows:

$$MAE_k = \frac{1}{T}\sum_{t=1}^{T}\left|C_t^{MP} - C_{t,k}\right| \tag{7}$$

$$MSE_k = \frac{1}{T}\sum_{t=1}^{T}(C_t^{MP} - C_{t,k})^2 \tag{8}$$

where T denotes the number of forecast data points; C_t^{MP} denotes the market price of TAIEX call options on day t. As shown in Table 2, we examine out-of-sample predictive performance of TAIEX call options across the various models under different moneynesses for empirical illustration.

Table 2. The moneyness of TXO call options

Case	Moneyness interval
In-the-money, ITM	$1.03 \le \mathrm{S/X} < 1.06$
At-the-money, ATM	$0.97 \le \mathrm{S/X} < 1.03$
Out-the-money, OTM	$0.94 \le \mathrm{S/X} < 0.97$

Note. S denotes the price of the underlying stock, and X is the strike price of the stock.

4. Empirical Results and Conclusions

Tables 3 presents out-of-sample daily option forecasting performance across the various models by reporting MAE, MSE and Benefit statistics, under at-the-money, out-the-money and in-the-money moneyness cases.

Empirical results indicate that the conditional GARCH-type models consistently perform better than the historical volatility models for forecasting call value of TXO under different moneynesses. Meanwhile, the $\mathrm{GARCH_{VIX}}$ model significantly outperforms the GARCH model for most cases, suggesting that the GARCH-based option price forecasts can be effectively improved with the additional information contained in VIX. In addition, the use of $\mathrm{GARCH_{VIX}}$ model can vastly reduce model mispricing especially for out-the-money TXO option case. Thus, volatility index is crucial for option traders to efficiently predict TXO option value with GARCH model.

Table 3. Out-of-sample forecasting performance for call values of TAIEX options

Model	MAE	Rank	Benefit	MSE	Rank	Benefit
Panel A. At-the-money case						
$\mathrm{HS_{252D}}$	19.2227	4	-	643.5562	4	-
$\mathrm{HS_{20D}}$	16.5746	3	13.78%	456.4446	3	29.07%
GARCH	15.4219	2	19.77%	421.2543	2	34.54%
$\mathrm{GARCH_{VIX}}$	14.4951	1	24.59%	372.0200	1	42.19%
Panel B. Out-the-money case						
$\mathrm{HS_{252D}}$	6.6708	4	-	90.8518	3	-
$\mathrm{HS_{20D}}$	6.2277	3	6.64%	95.9793	4	-5.64%
GARCH	5.2299	2	21.60%	50.8562	2	44.02%
$\mathrm{GARCH_{VIX}}$	3.8193	1	42.75%	32.4124	1	64.32%

Panel C. In-the-money case						
HS_{252D}	20.6462	4	-	757.8975	4	-
HS_{20D}	18.3933	3	10.91%	573.8943	3	24.28%
GARCH	17.3761	1	15.84%	552.8949	2	27.05%
$GARCH_{VIX}$	17.5604	2	14.95%	540.7973	1	28.65%

Note. 1. MAE and MSE denote the mean absolute error and the mean squared error, respectively. 2. Benefit refers to the percentage forecast error reduction that a forecasting model brings relative to the worst-performing model.

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Notes

Note 1. GARCH refers to the generalized autoregressive conditional heteroskedastic model proposed by Bollerslev (1986).

Note 2. IGARCH refers to the integrated GARCH model of Engle and Bollerslev (1986).

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