

# Time Series Analysis of the US Term Structure of Interest Rates Using a Bayesian Markov Switching Cointegration Model

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## Abstract

This paper examines the US term structure of interest rates using a Bayesian Markov switching cointegration model that allows the cointegrating vectors, the number of cointegrating rank, the risk premium, and other parameters to change when regime shifts. We find that for any pair of the interest rates there is a strong support for the cointegrating implication of the expectation hypothesis at least in a stable regime, while for some pairs of the interest rates the cointegration does not occur in a high volatility regime. We find that a Markov switching cointegration model captures regime shifts that are corresponding to high inflation regime. In high inflation regime, variance is much higher for both the long and short rates and adjustment toward equilibrium is much faster than those in the other regime.

**Keywords:** Bayesian, cointegration, markov-switching, term structure, expectations hypothesis

## 1. Introduction

This paper examines the term structure of interest rates for the United States over the period 1960-2016. For this purpose, we employ a Bayesian multivariate nonlinear cointegration model that allows any set of parameters in the model, including the cointegrating vectors, the number of cointegrating rank, and the risk premium, to shift with the regime according to the first order unobservable Markov process.

Clarida et al. (2006) analyze the term structure of interest rates for the US, Germany, and Japan using a multivariate vector error correction model (VECM) with a Markov switching. Tillmann (2007) extends Clarida et al. (2006) and considers a Markov switching cointegration model, that allows the risk premium and some other parameters in the model to change when the regime shifts, and finds interest rate volatility distinguishes two regimes. These authors find that the high variance regime is resulted by the Federal Reserve's non-borrowed reserve targeting policy in 1979-1982 and other phases of rising inflation expectations.

These classical approaches assume that both the cointegrating vectors and the number of cointegrating rank are constant over the regime. Classical method for a Markov switching cointegration model used by Psaradakis et al. (2004), Sarno et al. (2005), Sarno and Valente (2005), Clarida et al. (2006), and Tillmann (2007) requires a two-step procedure – as the first step testing and estimating for cointegration using Johansen's maximum likelihood method assuming a model is linear, and then as the second step imposing the cointegrating properties obtained in the first step to analyze the Markov switching model. Thus, it is not possible for this two-stage method to consider a Markov switching cointegration model, where the number of the cointegrating rank and/or the cointegrating vectors are subject to regime shifts.

Sugita (2008) analyzes a Bayesian temporal cointegration model where the adjustment toward the equilibrium might not occur in one regime. Since this temporal cointegration might be occurred for the term structure of interest rates, we use a Bayesian Markov switching cointegration model developed by Jochmann and Koop (2015) and Sugita (2016). By using a Bayesian Markov switching cointegration model, it is possible to consider a model that allows both the cointegrating vectors and the number of cointegrating rank to change when regime shifts. Sugita (2016) uses different prior densities from those used by Jochmann and Koop (2015) and employs the multi-move Gibbs sampling method to sample the state variable, which is widely used for a Markov switching model, e.g. Carter and Kohn (1994), Shephard (1994), Kim and Nelson (1998), and so on.

A Bayesian method by Jochmann and Koop (2015) and Sugita (2016) has another advantage over the classical

method. The method does not require a two-step procedure, that is, it can estimate the models efficiently within the non-linear framework by using the Markov chain Monte Carlo method (see Sugita, 2008).

In this paper, we investigate the US term structure of interest rates using Bayesian approach to a Markov switching cointegration model which is more general model for the term structure of interest rates than model by classical approach (Clarida et al., 2006; Tillmann, 2007) in terms of allowing for regime shifts in the number of cointegrating rank as well as any parameter including the cointegrating vectors, the adjustment terms, the risk premium, the variance-covariance matrix, the risk premium, and so on.

The plan of the paper is as follows. Next section presents a brief review of the term structure of interest rates. Section 3 presents model selection using Chib (1995)'s marginal likelihood calculation method. Empirical results follow in section 4. We conclude in section 5. All results reported in this paper are generated using OX version 7.00 for Linux (Doornik, 2013).

### 2. U.S. Term Structure of Interest Rates

In this section, we give a brief overview of the term structure of interest rates. The expectations hypothesis of the term structure of interest rates implies a long-run relationship between long and short maturity interest rates, see Shiller and McCulloch (1990) in detail. Let  $R_t(f)$  be yield to maturity for an  $f$ -period at time  $t$  and  $\lambda_t(f)$  is the risk premium, then the hypothesis implies:

$$R_t(f) = f^{-1} \sum_{i=1}^f E_t R_{t+i-1}(1) + \lambda_t(f) \tag{1}$$

Arranging equation (1) leads to the interest rate spread  $S_t(f)$  as follows:

$$S_t(f) \equiv R_t(f) - R_t(1) = f^{-1} \sum_{i=1}^{f-1} \sum_{j=1}^i E_t \Delta R_{t+j}(1) + \lambda_t(f) \tag{2}$$

where  $\Delta$  denotes the difference operator. Campbell and Shiller (1987) show that if both  $R_t(1)$  and  $R_t(f)$  are integrated of order one, then  $R_t(f)$  and  $R_t(1)$  are cointegrated with cointegrating vector (1, -1). Assuming the risk premium is  $I(0)$ , it follows that the hypothesis implies that  $R_t(f) - R_t(1) - \lambda_t(f)$  is stationary. Thus, the expectations hypothesis in equation (2) implies the following vector error correction model with  $p$  lag length:

$$\Delta x_t = \sum_{l=1}^p \Delta x_{t-l} \Gamma_l + \mu + x_{t-1} \beta \alpha + \varepsilon_t \tag{3}$$

where  $x_t = (R_t(f) \ R_t(1))$  and  $\varepsilon_t \sim iidN(0, \Omega)$ .

There are several studies that find nonlinearity of U.S. term structure of interest rates caused by changing in monetary policy. Tsay (1998), Hansen and Seo (2002), Clements and Galvao (2003), Clarida et al. (2006) and Tillmann (2007) employ nonlinear cointegration model to analyze regime switching. These researches confirm nonlinearity caused by high volatility of interest rates between 1979 and 1982 as a potential factor of shifts. In this period between 1979 and 1982, which is known as the *non-borrowed reserves operating procedure*, the Federal Reserve shifts monetary policy from interest rate targeting to money growth targeting, and it follows that the interest rate is allowed to fluctuate freely. Other studies of regime shifts in interest rates caused by shifts in monetary policy include Kugler (1996), Engsted and Nyholm (2000), Ang and Bekaert (2002), and so on. Nonlinearity of the term structure of interest rates is a stylized fact.

In this paper, we apply a Markov switching cointegration model to U.S. term structure of interest rate to explain the regime shifts as:

$$\Delta x_t = \sum_{l=1}^p \Delta x_{t-l} \Gamma_l(s_t) + \mu(s_t) + x_{t-1} \beta(s_t) \alpha(s_t) + \varepsilon_t \tag{4}$$

where  $\varepsilon_t \sim iidN(0, \Omega(s_t))$ .

We also impose the restriction  $\beta(s_t) = \beta^* = (1, -1)'$  on the system for the implications of the expectations hypothesis.

$$\Delta x_t = \sum_{l=1}^p \Delta x_{t-l} \Gamma_l(s_t) + \mu(s_t) + x_{t-1} \beta^* \alpha(s_t) + \varepsilon_t \tag{5}$$

With this restriction, we also consider the cointegration relation such as  $x_{t-1} \beta^* - \lambda(s_t)$ , where  $\lambda(s_t)$  can be interpreted as the estimated risk premium, thus the model can be written as:

$$\Delta x_t = \sum_{l=1}^p \Delta x_{t-l} \Gamma_l(s_t) + \eta(s_t) + [x_{t-1} \beta^* - \lambda(s_t)] \alpha(s_t) + \varepsilon_t \tag{6}$$

where  $\eta(s_t) = \mu(s_t) + \lambda(s_t) \alpha(s_t)$ .

We estimate a Markov switching VECM in equation (4) and a Markov switching VECM in equation (5), with various combination of cointegrating rank in each regime for all given pairs of interest rates. We also compute the marginal likelihood for all models to select the most appropriate model for each pairs of interest rates. If the restricted model in equation (5) is selected as the most appropriate model by the marginal likelihood, a Markov switching VECM in equation (6) is considered to analyze how the risk premium changes with regime shifts.

### 3. Data, Unit-Root, and Model Selection

The data set consists of monthly observations on three-month ( $f = 3$ ) treasury bill ( $R_t(3)$ ), six-month ( $f = 6$ ) treasury bill ( $R_t(6)$ ), one-year ( $f = 12$ ) treasury constant maturity rate ( $R_t(12)$ ), three-year ( $f = 36$ ) treasury constant maturity rate ( $R_t(36)$ ), and five-year ( $f = 60$ ) treasury constant maturity rate ( $R_t(60)$ ) US bonds covering the period 1960:1 to 2016:03 with 675 observations. We obtain these data from the Federal Reserve Bank of St. Louis. A pairwise plot of the series and the spread, using three-month ( $f = 3$ ) rate as a short rate, are presented in Figure 1. Table 1 shows the descriptive statistics for these variables. All series show positive skew, that is, they are right-skewed. We find that kurtosis is lower with longer maturities. The Jarque-Bera test shows that the hypothesis that these series are normally distributed is rejected for all these variables. The ARCH (1) statistic tests for heteroscedasticity indicate that all series are rejected the null hypothesis of no ARCH effect. The last column in Table 1 shows the Pearson correlation between 3-month treasury bill and other longer rates. While the correlations for each pair of interest rates are very high, the correlation is lower as maturities are longer. Table 2 reports standard unit root tests using the ADF, the GLS-ADF, and the KPSS tests. Both the ADF and the GLS-ADF tests cannot reject the null hypothesis of unit root for each interest rate, while the KPSS test rejects the null of stationarity.

We consider bivariate models for the term structure because of not only a parsimoniousness of model as Tillmann (2007) notes, but also the fact that it is possible to consider different model specification for each pairs of interest rates. Thus, we model Markov switching cointegration models for four pairs of interest rates -  $y_t = [R_t(6), R_t(3)]$ ,  $[R_t(12), R_t(3)]$ ,  $[R_t(36), R_t(3)]$ , and  $[R_t(60), R_t(3)]$  pair of interest rates. For each pair of interest rates, we consider models with the cointegrating rank  $r = 0$  or 1 for each regime, that is,  $(\tau_0, \tau_1) = (0, 0)$ ,  $(0, 1)$ ,  $(1, 0)$  and  $(1, 1)$ . We also consider a model with  $(\tau_0, \tau_1) = (1, 1)$  where  $\beta$  is unaffected by regime shifts

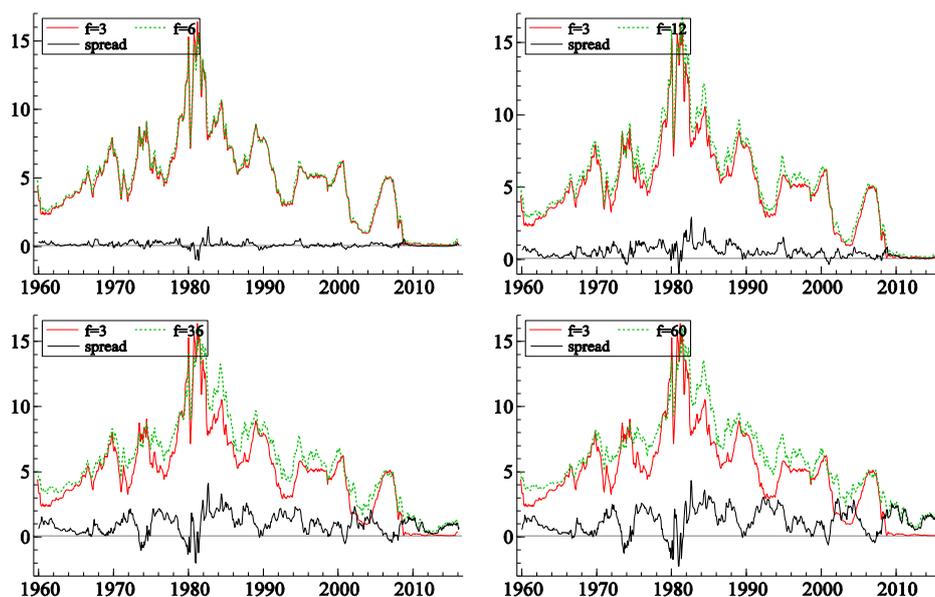


Figure 1. Interest rates (in % p.a.) on US bonds of maturity  $f$  (in months) and the spread

Table 1. Descriptive statistics

Series	Mean	Standard Deviation	Skewness	Kurtosis	J-B $p$ -value	ARCH(1) $p$ -value	Pearson correlation
$R_t(3)$	4.746	3.143	0.6744	0.9140	0.0000	0.0000	-----
$R_t(6)$	4.884	3.127	0.5743	0.6208	0.0000	0.0000	0.9982
$R_t(12)$	5.269	3.339	0.6095	0.6141	0.0000	0.0000	0.9934
$R_t(36)$	5.701	3.183	0.5494	0.4029	0.0000	0.0000	0.9706
$R_t(60)$	5.968	3.026	0.6289	0.4246	0.0000	0.0000	0.9524

Note. J-B stands for the Jarque-Bera statistic. The Pearson correlation is the correlation between  $R_t(3)$  and  $R_t(f)$ , where  $f = 6, 12, 36$ , and 60.

Table 2. Unit-root tests

Series	ADF const.	ADF no const.	ADF-GLS	KPSS
$R_t(3)$	-1.4353 (0.5665)	-1.2277 (0.202)	-1.4570 (0.1359)	1.7021 (<.01)
$R_t(6)$	-1.3712 (0.5980)	-1.0835 (0.2528)	-1.7674 (0.0733)	2.6938 (<.01)
$R_t(12)$	-1.3380 (0.6139)	-0.9408 (0.3094)	-1.3591 (0.1619)	3.4301 (<.01)
$R_t(36)$	-1.3382 (0.6139)	-0.9408 (0.3094)	-1.4043 (0.1494)	3.4490 (<.0)
$R_t(60)$	-1.2649 (0.6481)	-0.8318 (0.3558)	-1.2934 (0.1812)	7.5878 (<.01)

Note. ADF (augmented Dickey-Fuller) tests with and without constant term. ADF-GLS test is proposed by Elliott, Rothenberg and Stock (1996). KPSS is proposed by Kwiatkowski, Phillips, Schmidt, and Shin (1992). Both the ADF and the ADF-GLS tests the null of unit-root, while KPSS tests the null of stationary. The lag length for the ADF and the ADF-GLS is chosen by the Bayesian Schwartz criterion. We use the same lag length for the KPSS as the ADF and the ADF-GLS.

as  $\beta(s_t) = \beta$ , denoting  $1^C$ . For the implications of the expectations hypothesis, we also consider a model with restriction on the cointegrating vector  $\beta = (1, -1)'$  for one of the regime or both, denoting  $1^R$ . For the lag length of each model, we consider the case  $p = 1$  to 3. Thus, we consider a total of 30 bivariate models for each pair of the interest rates to select the most appropriate model among them. To estimate these models and obtain the posterior model probabilities we implement the collapsed Gibbs sampling algorithm (see Jochmann & Koop, 2015; Sugita, 2016). For prior hyperparameters, we use the same values that used in Sugita (2016), favoring the absence of cointegration.

We employ Chib (1995)'s method for the marginal likelihood calculation to choose model. Chib (1996), Chib (1998), Kim and Nelson (1999), and Sugita (2016) choose Chib's method for a Markov switching model. The full Gibbs sampler is run with  $G = 10,000$  and additional  $3 \times G = 30,000$  draws for the reduced Gibbs sampler after 1,000 discarded to calculate the marginal likelihood by the Chib's method. Table 1 gives logarithms of marginal likelihood using Chib's method for all 30 models for each pair of the interest rates. The highest marginal likelihood is given to the model with  $(r_0, r_1) = (0, 1^R)$  and  $p = 2$  for the  $[R_t(6), R_t(3)]$  pair,  $(r_0, r_1) = (1^R, 1^R)$  and  $p = 1$  for the  $[R_t(12), R_t(3)]$  pair,  $(r_0, r_1) = (0, 1^R)$  and  $p = 2$  for the  $[R_t(36), R_t(3)]$  pair, and  $(r_0, r_1) = (1^R, 1^R)$  and  $p = 3$  for the  $[R_t(60), R_t(3)]$  pair. These selected models suggest that there is a strong support for the cointegrating implication of the expectations hypothesis with  $\beta = (1, -1)'$  in one or both regimes. For both  $[R_t(6), R_t(3)]$  and  $[R_t(36), R_t(3)]$  pairs, cointegration relationship occurs temporarily in regime 2, that is, the adjustment toward the equilibrium does not occur in regime 0.

Table 3. Markov switching cointegration model: Model selection

Cointegration		$[R_t(6), R_t(3)]$ Lag length			$[R_t(12), R_t(3)]$ Lag length		
$r_0$	$r_1$	$p = 1$	$p = 2$	$p = 3$	$p = 1$	$p = 2$	$p = 3$
0	0	173.90	195.58	167.46	-110.53	-90.78	-93.02
0	1	234.23	246.84	243.74	-50.245	-47.879	-63.047
0	$1^R$	243.61	<b>251.39</b>	245.88	-42.347	-33.374	-45.406
1	0	187.92	205.80	197.37	-68.072	-69.231	-81.523
1	1	224.63	219.37	161.64	-23.830	-35.765	-49.111
$1^C$	$1^C$	195.31	223.15	168.62	14.441	-6.154	-13.212
1	$1^R$	212.54	230.85	221.71	11.158	2.630	-16.811
$1^R$	0	184.50	191.67	193.73	-42.781	-33.389	-49.271
$1^R$	1	211.52	222.21	223.31	10.377	5.238	-13.319
$1^R$	$1^R$	225.31	233.15	228.62	<b>25.383</b>	12.680	9.272

Cointegration		$[R_t(36), R_t(3)]$ Lag length			$[R_t(60), R_t(3)]$ Lag length		
$r_0$	$r_1$	$p = 1$	$p = 2$	$p = 3$	$p = 1$	$p = 2$	$p = 3$
0	0	-176.00	-195.63	-217.04	-279.18	-278.99	-280.32
0	1	-140.47	-144.29	-169.38	-262.11	-264.98	-261.16
0	$1^R$	-148.09	<b>-125.10</b>	-140.18	-253.32	-247.97	-252.60
1	0	-158.42	-152.03	-165.40	-272.19	-277.81	-274.46
1	1	-156.91	-143.24	-156.24	-266.26	-262.19	-269.76
$1^C$	$1^C$	-149.05	-141.72	-150.98	-254.35	-255.48	-260.15
1	$1^R$	-141.26	-139.16	-144.06	-260.59	-251.50	-253.37
$1^R$	0	-168.29	-155.88	-187.25	-274.27	-271.83	-280.92
$1^R$	1	-139.21	-137.20	-156.38	-259.01	-252.28	-257.65
$1^R$	$1^R$	-136.55	-132.35	-142.89	-243.20	-244.61	<b>-239.51</b>

Note. Superscript C in  $1^C$  denotes that the cointegrating vector is constant over the regime. Superscript R in  $1^R$  denotes that the restriction  $\beta = (1, -1)'$  is imposed.

#### 4. Empirical Results

As shown in Table 3, the marginal likelihoods select models where the restriction on the cointegrating vector is imposed in one or both regimes for all pairs of the interest rates. Thus, we consider the model in equation (6) where the risk premium is included in the cointegrating relations. Figure 2a – 2d shows the posterior expectation of the state variables for each interest rate pair, and find that they are almost identical. The non-borrowed reserves operating procedure between 1979 and 1982 is detected as the regime shift. Regime shift also occurs between 1973 and 1976, between 1984 and 1985, and between 2007 and 2008. These regime shifts are related to higher inflation regime (Goodfriend, 1998) with a much higher variance of interest rates in regime 0 than those of regime 1. Regime 0 is also influenced by other aspects of rising inflation expectations as shown in Goodfriend (1998). The posterior means and standard deviations of parameters for each interest rate pair are reported in Table 4. We find that in regime 0 the variance of both the long and short rate are a much higher than those of regime 1. In regime 0 the variance of the short rate is higher than that of the long rate, while in regime 1 the variance of the short rate is lower than that of the long rate.

In regime 0 the adjustment (in absolute terms) toward the equilibrium is much faster for pair of  $[R_t(12), R_t(3)]$  and  $[R_t(60), R_t(3)]$  than in regime 1. Thus, interest rates adjust much faster in regime of high volatility that correspond to periods of rising inflation expectations and aggressive disinflation. We also find that when the volatility is low in regime 1 the adjustment terms of short rate is faster and significantly different from zero with positive sign. Thus, the adjustment toward the equilibrium occurs mainly through the short rate  $R_t(3)$  for all pairs of interest rate.

As for the risk premium, longer the maturity higher the risk premium for regime 1. Regime 0 shows a higher risk premium for pair of  $[R_t(12), R_t(3)]$  and a lower risk premium for pair of  $[R_t(60), R_t(3)]$  than in regime 1. This is consistent with the results by Hansen (2003) and Tillmann (2007). They find that the regime 0 leads to a higher risk premium for  $R_t(f)$  with  $f \leq 12$  and a lower risk premium for  $R_t(f)$  with  $f > 12$ .

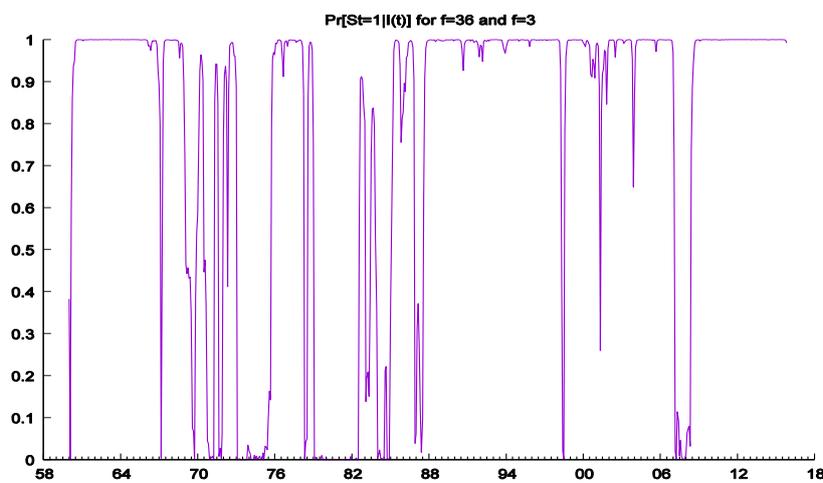
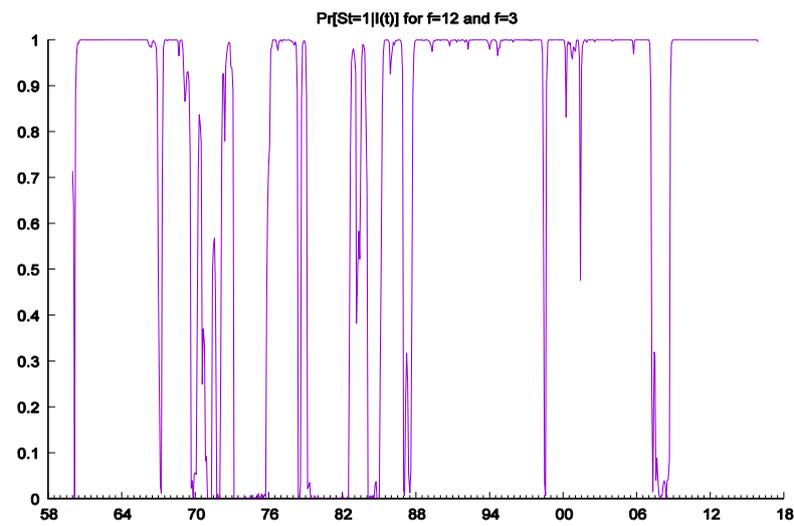
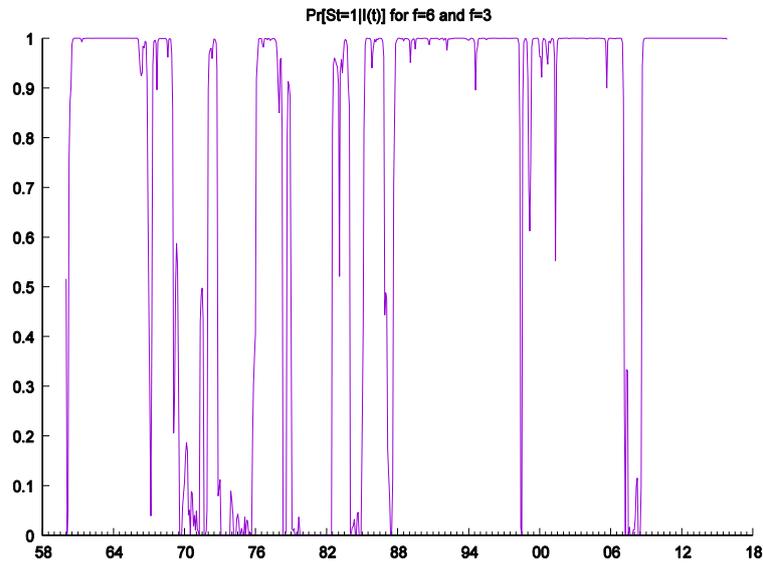
Table 4. Posterior parameter estimates

Model	$R_t(6)$	$R_t(12)$	$R_t(36)$	$R_t(60)$
	$R_t(3)$	$R_t(3)$	$R_t(3)$	$R_t(3)$
	$(\tau_0, \tau_1) = (0, 1^R)$	$(\tau_0, \tau_1) = (1^R, 1^R)$	$(\tau_0, \tau_1) = (0, 1^R)$	$(\tau_0, \tau_1) = (1^R, 1^R)$
$\alpha(0)$	-----	-0.1684 (0.1102)	-----	-0.0646 (0.0342)
	-----	0.0287 (0.1153)	-----	0.0351 (0.0515)
$\alpha(1)$	0.1796 (0.0661)	0.0328 (0.0311)	-0.0215 (0.0103)	-0.0204 (0.0127)
	0.3497 (0.0582)	0.1039 (0.0245)	0.0289 (0.0119)	0.0272 (0.0084)
$\lambda(0)$	-----	0.7976 (0.1592)	-----	0.2669 (0.0209)
$\lambda(1)$	0.0991 (0.0249)	0.3578 (0.0724)	0.7903 (0.1518)	0.9575 (0.0299)
$p_{00}$	0.9706 (0.0092)	0.9707 (0.0084)	0.9672 (0.0092)	0.9647 (0.0098)
$p_{11}$	0.9410 (0.0177)	0.9381 (0.0173)	0.9298 (0.0186)	0.9266 (0.0195)
$\Omega_{11}(0)$	0.5637 (0.0646)	0.6283 (0.0682)	0.5844 (0.0615)	0.5632 (0.0549)
$\Omega_{22}(1)$	0.0210 (0.0015)	0.0225 (0.0016)	0.0221 (0.0018)	0.0189 (0.0014)

Note. Standard errors are in parentheses. The diagonal elements (the variances) of the regime-dependent variance-covariance matrices are given by  $\Omega_{ii}(s_i)$ , where  $i = 0$  or  $1$ .

#### 5. Conclusion

In this paper, we examined the dynamics relationship between short and long interest rate using data since 1960 at monthly frequency for the 3-month US bond rate as a short rate and the 6-month, 1-year, 3-year and 5-year bond rate as a long rate. We employed a Bayesian Markov switching cointegration model to analyze the nonlinearity of the US term structure of interest rates. By using Bayesian cointegration analysis it is possible to allow any set of parameters in the model, including the cointegrating vectors and the number of cointegrating rank, to shift with the regime according to the first order unobservable Markov process. We also considered a model where the risk premium is subject to change when the regime shifts. We found that for any pair of the interest rates there is a strong support for the cointegrating implication of the expectation hypothesis at least in the stable regime, while for some pair of the interest rates the cointegration does not occur in the high volatility regime. We found that these regime shifts are related to higher inflation regime, e.g., caused by the non-borrowed reserves operating procedure between 1979 and 1982. We found that in the inflation regime variance is much higher and the adjustment to the long-run relationship is much faster than in the other regime.



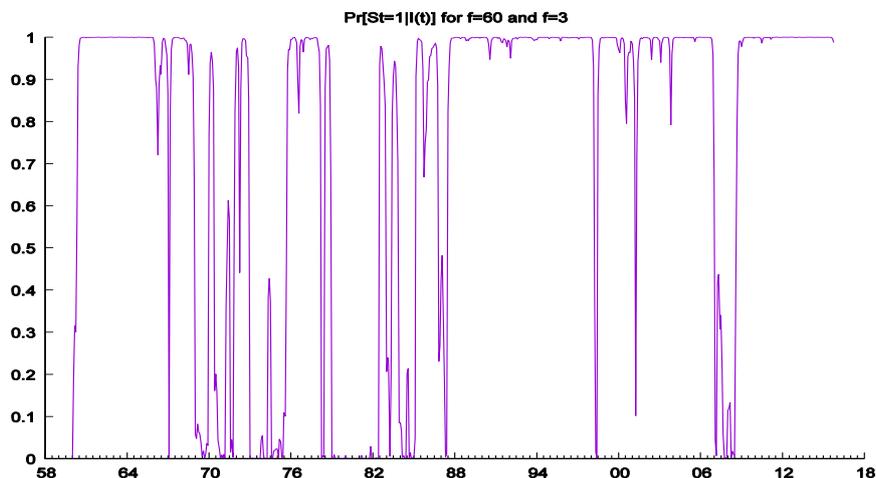


Figure 2d. Posterior expectation of the regime variable for  $[R_t(60), R_t(3)]$

## References

- Ang, A., & Bekaert, G. (2002). Regime switches in interest rates. *Journal of Business & Economic Statistics*, 20(2), 163-182. <http://dx.doi.org/10.1198/073500102317351930>
- Campbell, J. Y., & Shiller, R. J. (1987). Cointegration and tests of present value models. *Journal of Political Economy*, 108, 205-251.
- Carter, C. K., & Kohn, R. (1994). On Gibbs sampling for state space models. *Biometrika*, 81(3), 541-553. <http://dx.doi.org/10.1093/biomet/81.3.541>
- Chib, S. (1995). Marginal likelihood from the Gibbs output. *Journal of the American Statistical Association*, 90(432), 1313-1321. <http://dx.doi.org/10.1080/01621459.1995.10476635>
- Chib, S. (1996). Calculating posterior distributions and modal estimates in Markov mixture models. *Journal of Econometrics*, 75(1), 79-97. [http://dx.doi.org/10.1016/0304-4076\(95\)01770-4](http://dx.doi.org/10.1016/0304-4076(95)01770-4)
- Chib, S. (1998). Estimation and comparison of multiple change-point models. *Journal of Econometrics*, 86(2), 221-241. [http://dx.doi.org/10.1016/S0304-4076\(97\)00115-2](http://dx.doi.org/10.1016/S0304-4076(97)00115-2)
- Clarida, R. H., Sarno, L., Taylor, M. P., & Valente, G. (2006). The Role of Asymmetries and Regime Shifts in the Term Structure of Interest Rates. *The Journal of Business*, 79(3), 1193-1224. <http://dx.doi.org/10.1086/500674>
- Clements, M. P., & Galvao, A. B. C. (2003). Testing the expectations theory of the term structure of interest rates in threshold models. *Macroeconomic Dynamics*, 7(04), 567-585. <http://dx.doi.org/10.1017/S1365100502020163>
- Doornik, J. (2013). *Object-Oriented Matrix Programming Using Ox*. London, Timberlake Consultants Press.
- Elliot, G., Rothenberg, T. J., & Stock, J. H. (1996). Efficient tests for an autoregressive unit root. *Econometrica*, 64, 813-836.
- Engsted, T., & Nyholm, K. (2000). Regime shifts in the Danish term structure of interest rates. *Empirical Economics*, 25(1), 1-13. <http://dx.doi.org/10.1007/s001810050001>
- Goodfriend, M. (1998). Using the term structure of interest rates for monetary policy. *FRB Richmond Economic Quarterly*, 84(3), 13-30. [http://dx.doi.org/10.1007/978-1-349-26891-7\\_11](http://dx.doi.org/10.1007/978-1-349-26891-7_11)
- Hansen, B. E., & Seo, B. (2002). Testing for two-regime threshold cointegration in vector error-correction models. *Journal of Econometrics*, 110(2), 293-318. [http://dx.doi.org/10.1016/S0304-4076\(02\)00097-0](http://dx.doi.org/10.1016/S0304-4076(02)00097-0)
- Hansen, P. R. (2003). Structural changes in the cointegrated vector autoregressive model. *Journal of Econometrics*, 114(2), 261-295. [http://dx.doi.org/10.1016/S0304-4076\(03\)00085-X](http://dx.doi.org/10.1016/S0304-4076(03)00085-X)
- Jochmann, M., & Koop, G. (2015). Regime-switching cointegration. *Studies in Nonlinear Dynamics & Econometrics*, 19(1), 35-48. <http://dx.doi.org/10.1515/sn-de-2012-0064>
- Kim, C. J., & Nelson, C. R. (1998). Business cycle turning points, a new coincident index, and tests of duration

- dependence based on a dynamic factor model with regime switching. *Review of Economics and Statistics*, 80(2), 188-201. <http://dx.doi.org/10.1162/003465398557447>
- Kim, C. J., & Nelson, C. R. (1999). Has the US economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle. *Review of Economics and Statistics*, 81(4), 608-616.
- Koop, G., León-González, R., & Strachan, R. W. (2010). Efficient posterior simulation for cointegrated models with priors on the cointegration space. *Econometric Reviews*, 29(2), 224-242. <http://dx.doi.org/10.1080/07474930903382208>
- Kugler, P. (1996). The term structure of interest rates and regime shifts: Some empirical results. *Economics Letters*, 50(1), 121-126. [http://dx.doi.org/10.1016/0165-1765\(95\)00731-8](http://dx.doi.org/10.1016/0165-1765(95)00731-8)
- Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., & Shin, Y. (1992). Testing the null of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54, 159-178.
- Psaradakis, Z., Sola, M., & Spagnolo, F. (2004). On Markov error-correction models, with an application to stock prices and dividends. *Journal of Applied Econometrics*, 19(1), 69-88. <http://dx.doi.org/10.1002/jae.729>
- Sarno, L., & Valente, G. (2005). Modelling and forecasting stock returns: Exploiting the futures market, regime shifts and international spillovers. *Journal of Applied Econometrics*, 20(3), 345-376. <http://dx.doi.org/10.1002/jae.787>
- Sarno, L., Thornton, D. L., & Valente, G. (2005). Federal funds rate prediction. *Journal of Money, Credit, and Banking*, 37(3), 449-471. <http://dx.doi.org/10.1353/mcb.2005.0035>
- Shephard, N. (1994). Partial non-Gaussian state space. *Biometrika*, 81(1), 115-131. <http://dx.doi.org/10.1093/biomet/81.1.115>
- Shiller, R. J., & McCulloch, J. H. (1990). The term structure of interest rates. *Handbook of Monetary Economics*, 1, 627-722.
- Sugita, K. (2008). Bayesian analysis of a Markov switching temporal cointegration model. *Japan and the World Economy*, 20(2), 257-274. <http://dx.doi.org/10.1016/j.japwor.2006.12.002>
- Sugita, K. (2016). Bayesian inference in Markov switching vector error correction model. *Economics Bulletin*, 36(3), 1534-1546.
- Tillmann, P. (2007). Inflation regimes in the US term structure of interest rates. *Economic Modelling*, 24(2), 203-223. <http://dx.doi.org/10.1016/j.econmod.2006.07.001>
- Tsay, R. S. (1998). Testing and modeling multivariate threshold models. *Journal of the American Statistical Association*, 93(443), 1188-1202. <http://dx.doi.org/10.1016/j.econmod.2006.07.001>

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