International Journal of Economics and Finance; Vol. 9, No. 2; 2017
ISSN 1916-971X E-ISSN 1916-9728
Published by Canadian Center of Science and Education

Forecasting Volatility Stock Return: Evidence from the Nordic Stock Exchanges

Nikolaos Dritsakis[1] & Georgios Savvas[2]

[1] Professor, Department of Applied Informatics, School of Information Sciences, University of Macedonia, Thessaloniki, Greece

[2] Post-Graduate Student, Master of Science in Applied Informatics, Department of Applied Informatics, School of Information Sciences, University of Macedonia, Greece

Correspondence: Georgios Savvas, Department of Applied Informatics, School of Information Sciences, University of Macedonia, Thessaloniki, Greece. Tel: 306-987-281-898. E-mail: mai16058@uom.edu.gr

Received: November 19, 2016 Accepted: December 15, 2016 Online Published: January 10, 2017

doi:10.5539/ijef.v9n2p15 URL: http://dx.doi.org/10.5539/ijef.v9n2p15

Abstract

The purpose of this study is to explore the volatility and secondary effects in the four Nordic stock exchanges of Norway: Oslo Børs Linked all-share index AXLT Denmark: OMX Copenhagen 20, Sweden: OMX Stockholm 30 and Finland: OMX Helsinki 25. Keeping in mind that there is an ARCH effect in the returns of the four stock exchanges, we move on to the evaluation to the evaluation of models ARCH (q), GARCH (p, q) και GARCH-M (p, q). Evaluating the parameters became possible through the use of the maximum likelihood method using the BHHH algorithm of (Berndt et al., 1974) and the three distributions (normal, t-Student, and the Generalized normal distribution GED). The results of this study indicate model ARMA(0,1)-GARCH-M(1,1) with t-student distribution as the appropriate one to describe the returns of the all Nordic stock exchanges except that of Sweden, where model ARMA(0,3)-GARCH-M(1,1) describes it best. Lastly, for forecasting the models ARMA(0,1)-GARCH-M(1,1) and ARMA(0,3)-GARCH-M(1,1) of the current stock exchanges we use both the dynamic and static process. The results of this study indicate that the static process forecasts better than the corresponding dynamic.

Keywords: stock returns, GARCH models, forecasting volatility, Nordic stock exchanges, BHHH algorithm

1. Introduction

In recent years the financial world is in a serious instability. Modeling financial series is a complex matter for most economists. This complexity derives not only from the various financial market products (interest rate, exchange rates, reserves etc.) but also from the effect of political incidents. These incidents create volatility in time-series, resulting to the difficulty of creating various stochastic models. Assets evaluation becomes possible using their return, which is conventionally defined as the logarithmic price changes, which is close to the relative price change. This return is depicted as follows:

$$R_t = ln(X_t / X_{t-1}) \quad (1)$$

where R_t is a financial asset with value X_t at time t and X_{t-1} at time t-1. The return is scale-free, which facilitates comparisons between assets.

The theory of option pricing is an important topic in the financial literature. The Black, and Scholes (1973) study was the beginning of the European-style purchase options. Consequently it was discovered that the prices of Black and Scholes models differ from the market prices, therefore the literature for evaluating the purchase options formulated a series of theoretical models designed to capture these empirical biases. Various empirical studies related to the price dynamics of current assets, indicated that the assets features in time is the volatility, the non-normality and leverage effect, thus, they should be taken into consideration in financial data. Therefore the various models and developed techniques should incorporate some or all of the above properties.

As previously stated volatility clustering has been used instead of a constant variable in volatility functions depended on the asset value and time. Furthermore time-series are regarded as a stochastic process which can be analysed in two elements:

$$R_t = m_t + u_t \quad (2)$$

$$u_t = \sigma\varepsilon_t \quad (3)$$

Where m_t is a predictable process, and u_t is a nondeterministic process driven by a random noise ε_t is iid with zero mean value and variance one.

GARCH symmetrical models cannot define asymmetry and leptokurtosis in financial data. For this particular reason the GARCH-in-mean model is used in this study, which adds a heteroscedastic term in the equation mean which is determined as a risk premium and incorporates a type of assymetry.

The research on the return and volatility of the Nordic stock markets was carried out for the following reasons:

- There is a strong financial cooperation between those countries.
- The monetary and tax systems of the researched markets are similar and a close association exists between these countries.
- These four Nordic countries have similar industries, therefore their stock markets display a strong correlation.
- They have common trade partners.
- Their stock markets opening and working times coincide.

This research differs from past ones which researched the Nordic market for the following reasons:

1) The past studies of researching volatility and the relationship between the Nordic markets used data from 1988 to 1994 Booth et al. (1997), while Hyytinen (1999) used weekly data and only for three markets (Finnish, Norwegian and Swedish market) from 1983 to 1997. The expansion of data in this study and the conclusions analysis are important for investors who want to diversify their portfolios.

2) The results of this study indicate that every one of these markets are described better from model ARMA(0,1)-GARCH-M(1,1) with t-Student distribution, unlike Booth's et al. (1997) study which uses the EGARCH asymmetric model, and Hyytinen's (1999) which uses the TGARCH asymmetric model for Sweden and GARCH symmetric models for Finland and Norway.

3) Furthermore, the results of this study agree with those of Booth's et al. (1997) that the volatility in the four markets indicate that ill news are stronger than good news. The research on the returns and the volatility of secondary effects in the Nordic markets dictate the enactment of a common Nordic stock market, to create the fourth bigger stock market in Europe after the ones of London, Paris and Frankfurt.

In this paper a short introduction of financial return is presented in section 1. The rest of this paper is as follows: section 2 contains the literature review, section 3 presents the methodological analysis, section 4 describes the data and the descriptive statistics, in section 5 the empirical results of this study are given, while in section 6 the forecasting results are presented. Last but not least, in section 7 the conclusions of this study are given.

2. Literature Review

Time-series performance analysis has troubled financial researchers in recent years. One of the first evaluation tries was conducted from Eun and Shim (1989) when they researched the daily returns of the Australia, Hong Kong, Japan, France, Canada, Switzerland, Germany, the United States and Britain stock exchanges. The results of their study indicated that there are substantial interdependencies among these markets, most of which are with the U.S. stock market. European and Asian stock exchange dependencies are strong but with one day delay.

Hamao, et al. (1990) used daily data from April 1985 to March 1988 from three stock indexes, Nikkei 225 of the Japan stock exchange, FTSE of the Great Britain stock exchange and S & P 500 of the U.S.A. stock exchange. Using model GARCH-M(1,1) they resulted that Nikkei 225 is influenced both from S & P 500 and FTSE, while FTSE is influenced only from S & P 500.

Booth et al. (1997) in their study used a multivariate exponential generalized autoregressive conditionally heteroscedastic (EGARCH) model to research the interaction of the four Scandinavic stock markets in period May 2nd 1988 to June 30th 1994. The results of their study indicate that volatility in the asymmetric model used is being more pronounced for bad than good news.

Hyytinen (1999) studies the development of conditional volatility of returns in three Scandinavian countries (Finland, Norway and Sweden) using weekly data from 1983-1997. The results of the paper shown that the asymmetric EGARCH model is most suitable for Sweden's data whereas the symmetric model GARCH (1,1) is more preferable for the other two countries.

Ng (2000) examined the magnitude and changing nature of volatility spillovers from Japan and the US to six

Pacific–Basin equity. He used indexes Hang Seng of Hong Kong, the Korean Composite Stock Price Index, the Kuala Lumpur Stock Exchange Composite Index (Malaysia), the Stock Exchange of Singapore All Share Index, the Taiwan Stock Exchange Weighted Price Index, the Stock Exchange of Thailand Index, the Tokyo Stock Price Index, and the Standard and Poor's 500 Index. The results of his study indicated that four of six Pacific Basin Region stock exchanges are influenced less than 10% weekly from a change in the returns of Japan and USA stock markets.

Lee (2004) in his study researches the transition mechanism of the stock market return via wavelet analysis. He used the wavelet analysis instead of CARCH models, claiming that his analysis researches the potential and possible interactions of international stock markets. Using daily data from the USA and Korean stock exchange he resulted that developed markets volatility one-way influence the developing ones.

Trang Nha Le and Makoto Kakinaka (2010) researched volatility and secondary effects in three major stock markets, such as these of Japan, USA and China, as well as two emerging ones of Indonesia and Malaysia from the years 2005 to 2007. Using CARCH models they found that there are significant mean spillover effects from the three major markets to the two emerging markets. Furthermore they found that the size of the USA stock market influence upon the emerging ones is greater than those of Japan and China. Lastly, the results of their study indicated that the USA stock market influence upon the Indonesian market is greater than in the Malaysian.

Prashant (2014) researched the return and volatility among the indexes BSE and DJIA of India and US Stock Markets respectively. To do this he used model GARCH-BEKK from January 2, 2012 to April 4, 2014. His study results indicated that DJIA index exercises more influence on BSE in terms of shocks and volatility transmission . Furthermore, he proved that total volatility is greater in the USA stock exchange.

Thenmozh and Chand (2016) in their study showed that forecasting returns based on global reserves attributed better in day trading both in emerging and major markets. Forecasting the returns, the researchers used vector regression for six stock markets, namely those of USA (Dow Jones, S&P500), UK (FTSE-100), India (NSE), Singapore (SGX), Hong Kong (Hang Seng) and China (Shanghai stock exchange) for the period 1999-2011. The empirical analysis shows that models with other global market price information outperform forecast models based merely on auto-regressive past lags and technical indicators.

3. Methodology

Uncertainty plays an essential role in economic analysis and is usually measured with volatility**.** There are time-series, mainly financial, who display periods of mass volatility. These time-series experience periods with dramatic increases and decreases, during which their variance is varying over time. Therefore, researchers can test the variance of this particular time-series in the varying period, namely the conditional variance. Hence, we can describe the time-series models with conditional variance as conditional heteroscedastic models. Engle (1982) suggested that the varying variance can be described through an autoregressive model depending on its former values. Specifically this model is described as Autoregressive Conditional Heteroscedastic Model, known as ARCH model.

Therefore, based on a structured model, variance can be measured and forecasted. Variance forecasting is crucial to pricing and risk management. In various studies, several variance models have been suggested to be able to include the features of financial time-series efficiency or an asset. The features of an asset's efficiency that researchers acknowledge are as follows:

- The variance of an asset develops with time, in a constant way.
- Periods of great movement in prices, alternate with periods where prices don't move. This feature is known as volatility clustering.
- The variance of an asset does not tend to infinity.
- There is an asymmetric movement in variance.
- Usually, extreme kurtosis or fat-tailedness is observed in variance.

3.1 ARCH-GARCH Varying Models

ARCH-GARCH varying models are consisted of two equations. The first one (mean equation) describes the data over another variable (if present) adding the standard error. The second one (covariance equation) define the evolution of conditional covariance of the error, from the mean equation as a variance of the past conditional covariance of the lagged error.

The mean equation in ARCH – GARCH models is depicted as:

$$R_t = \mu + \varepsilon_t \text{ (ARCH model)} \quad (4)$$

The error term εt in a simple mean equation is linear unrelated, but not time-independent.

$$R_t = \mu + \beta\sigma_t^2 + \varepsilon_t \text{ (GARCH model)} \quad (5)$$

Covariance equation in ARCH – GARCH models is depicted as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} a_i\, \varepsilon_{t-i}^2 \text{ (ARCH model)} \quad (6)$$

The non-linear dependency that the error term εt depicts can be described through the use of squared lagged errors Parameters ω, α_1, α_2,...,α_q are unknown and because covariance is a positive number, meaning that the positive terms are in place, where ω>0 and $\alpha_1,\alpha_2,\ldots,\alpha_q \geq 0$.

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \text{ (GARCH model)} \quad (7)$$

We hypothesize that, for p≥0 and q>0, the parameters are unknown and because of covariance being a positive number the following conditions are in place ω≥0 and $\alpha_i \geq 0$ for i=1,…,q and $\beta_i \geq 0$ for j=1,…,p

ARCH(1,1) model is depicted as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon^2_{t-1} \quad (8)$$

where ω≥0 and $\alpha_i \geq 0$ for the positive number of σ_t^2

GARCH(1,1) model is depicted as:

$$\sigma^2_t = \omega + \alpha_1 \varepsilon^2_{t-1} + \beta_1 \sigma^2_{t-1} \quad (9)$$

where ω≥0, $\alpha_i \geq 0$ and $\beta_i \geq 0$ for the positive number of σ_t^2

3.3 ARCH- GARCH Models Features

ARCH – CARCH model according to Engle(1982) is based on the two following hypothesis:

- The error term ε_t in a simple mean equation ($y_t = \mu_t + \varepsilon_t$) is linear unrelated, but not time-independent.
- The non-linear dependency that the error term ε_t depicts can be described through the use of squared lagged errors

3.4 ARCH-GARCH Models Test and Evaluation

Evaluation of ARCH-GARCH models is possible through the use of maximum likelihood method. The logged equation of maximum likelihood is depicted as:

$$lnL(\theta) = -(1/2)\sum_{t=1}^{n}\left[ln(2\pi) + ln\left(\sigma_t^2(\theta)\right) + z_t^2(\theta)\right] \quad (10)$$

where

θ is the vector of parameters ($\mu, \omega, \alpha, \beta$) estimated that maximize the objective function $\ln L(\theta)$,

z_t represents the standardized residual calculated as $(\Delta y_t - \mu)/\sqrt{\sigma_t^2}$.

3.5 Diagnostic Checking of the Model ARCH-GARCH

There are plenty diagnostic tests for the analysis of ARCH- GARCH modeling. The residual correlogram is used to test the residual autocorrelation, while the squared residual correlogram is used to test the autocorrelation of the conditional heteroscedasticity of residuals. To define if the time-series presents autocorrelation or heteroscedasticity we use the Ljung and Box (Q-statistics) (1978). This statistics are depicted as:

$$Q_m = n(n+2)\sum_{k=1}^{m} e_k^2/(n-2) \quad (11)$$

where:

e_k is the residual autocorrelation in lag k.

n is the residual number.

m is the time-lags number tested.

The model is appropriate when the probability of Ljung and Box Q-statistics is higher than 5%.

3.6 The GARCH-M Model

Engle, Lilien, and Robins (1987), build on the ARCH(q) methodology for the purpose of conditional covariance influencing the mean order. In other words we could say that Engle, Lilien, and Robins (1987) restructured the conditional variance model in order that the mean of a sequence to be depended on the conditional covariance. These models are named ARCH-M and are adequately adjusted to the study of the return of financial products. The model that Engle, Lilien and Robins built is the following:

$$R_t = \mu + other\ terms + \delta\sigma^2_{t-1} + \varepsilon_t \text{ (mean equation)} \quad (12)$$

$$\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \text{ (covariance equation)} \quad (13)$$

where:

μ is the conditional mean of the stock market return in time t.

σ^2_t is the conditional covariance that reflects the risk premium.

ω is a constant term.

ε_t is the error term in time t.

μ, δ, ω, α_i and β_j are parameters for evaluation.

In this study the model GARCH-M (1,1) is used, considering the study of Bollerslev (1986), where he claims that the length of the time lag of the squared error and the conditional variance is enough for the stock market return model.

GARCH-M (1,1) model equation can be depicted as:

$$R_t = \mu + other\ terms + \delta\sigma^2_{t-1} + \varepsilon_t \text{ (mean equation)} \quad (14)$$

$$\sigma^2_t = \omega + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1} \text{ (covariance equation)} \quad (15)$$

3.7 Forecasting Performance

ARCH-GARCH models are used for forecasting the variance return. Forecasting in ARCH-GARCH models is estimated both in-sample, and out-of-sample. The best forecasting price is given from the mean squared error. Furthermore, other indexes that are usually used for forecast return are the Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE) and the inequality index of Theil (U-Theil) (1967).

These indexes are depicted as follows:

$$MSE = (1/T)\sum_{t=1}^{T}(\widehat{Y}_t - Y_t)^2 \quad (16)$$

$$MAE = (1/T)\sum_{t=1}^{T}|\widehat{Y}_t - Y_t|^2 \quad (17)$$

$$RMSE = \sqrt{(1/T)\sum_{t=1}^{T}(\widehat{Y}_t - Y_t)^2} \quad (18)$$

$$MAPE = (1/T)\sum_{t=1}^{T}|(\widehat{Y}_t - Y_t)/Y_t|^2 \quad (19)$$

the inequality index of Theil is given as:

$$U = \left(\sqrt{(1/T)\sum_{t=1}^{T}(\widehat{Y}_t - Y_t)^2}\right) / \left(\sqrt{(1/T)\sum_{t=1}^{T}(\widehat{Y}_t)^2} + \sqrt{(1/T)\sum_{t=1}^{T}(Y_t)^2}\right) \quad 0 \leq U \leq 1 \quad (20)$$

where:

Y_t: Actual value of endogenous variable Y at time t.

$\widehat{Y}_t$: Redacted value of endogenous variable Y at time t.

T: Number of observations in the simulations (of the sample).

If the inequality index of Theil U=0, then the actual prices of the time-series, would equal the predicted ones $Y_t = \widehat{Y}_t$ for all t, therefore in this case we can say that there is a "perfect fit" between actual and predicted data. Otherwise if variable U=1, there is no right forecast for the studied model. Consequently the individual indexes of Theil are presented, known as inequality proportions, and are depicted as:

- Bias proportion: indicates the systematic differences in actual and forecasted values.

$$UM = ((\bar{\widehat{Y}} - \bar{Y})^2)/((1/T)\sum_{t=1}^{T}(\widehat{Y}_t - Y_t)^2) \quad (21)$$

where:

$\bar{\widehat{Y}}$ and $\bar{Y}$ are the time-series mean of $\widehat{Y}_t$ and Y_t correspondingly. Bias proportion counts the distance between the mean of the simulated series and the mean of the actual one.

- Variance proportion: indicates unequal variances of actual and forecasted values.

$$US = ((\hat{S}_{\hat{Y}} - S_Y)^2)/ ((1/T)\sum_{t=1}^{T}(\widehat{Y}_t - Y_t)^2) \quad (22)$$

where:

$\hat{S}_{\hat{Y}}$ and S_Y are the standard deviations of series $\hat{Y}_t$ and Y_t correspondingly. Variance proportion counts the distance between the variance of the simulated series and the variance of the real one.

- Covariance proportion: indicates the correlation between the actual and forecasted values (zero=perfect correlation between actual and forecasted values).

$$UC = (2(1\text{-}\rho)\, \hat{S}_{\hat{Y}} S_Y) / \left((1/T)\sum_{t=1}^{T}(\hat{Y}_t - Y_t)^2\right) \quad (23)$$

where:

ρ is the correlation variable between $\hat{Y}_t$ and Y_t. Covariance proportion counts the balance of the non-systematic error of simulation.

The forecasting ability of a model is sufficient, when the bias and covariance proportions are low.

The relationship between the above proportions is:

$$UM+US+UC=1 \quad (24)$$

4. Data and Descriptive Statistics

The data for this study was collected from the websites www.nasdaqomxnordic.com for the OMX Copenhagen 20, OMX Stockholm 30 and OMX Helsinki 25 indexes and from the website www.oslobors.no for the AXLT index of Oslo Børs. The data covers the period from January 3rd 1983 to April 7th 2016 for the Norwegian Index and contains 8347 observations, the period from October 10th 1996 to May 11th 2016 for the Danish Index and contains 4893 observations, the period from September 30th 1986 to May 11th 2016 for the Swedish Index and contains 7434 observations and lastly the period from September 3rd 2001 to May 11th 2016 for the Finnish Index and contains 3687 observations.

The daily return of stock markets is calculated as:

$$R_t = \ln(X_t/X_{t-1}) * 100 = (\ln X_t - \ln X_{t-1})*100 \quad (25)$$

where,

X_t is the daily closing price of stock market at time t,

R_t is the daily return of stock market.

The daily closing prices of the AXLT, OMX Copenhagen 20, OMX Stockholm 30 and OMX Helsinki 25 Indexes and their returns are presented at Figures 1 and 2, correspondingly.

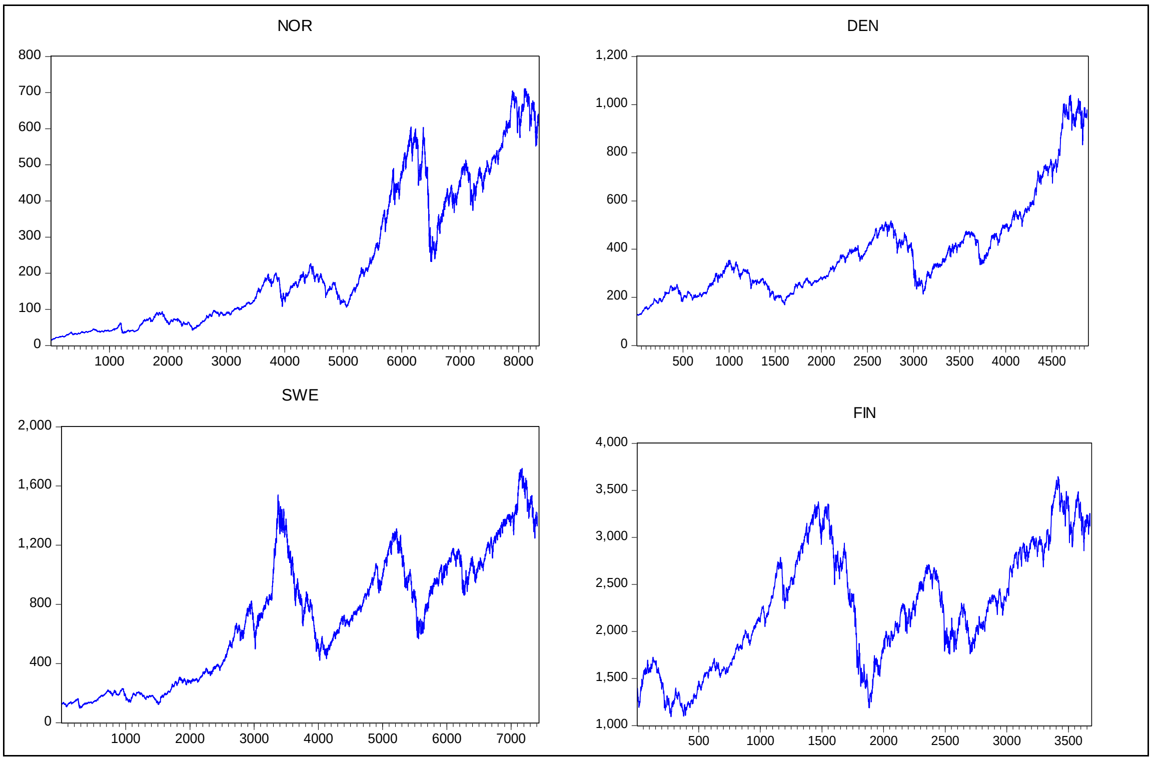

Figure 1. Daily closing prices of the Norwegian, Danish, Swedish and Finnish stock markets

From Figure 1 we can assume that the daily closing prices of all stock markets display a random walk.

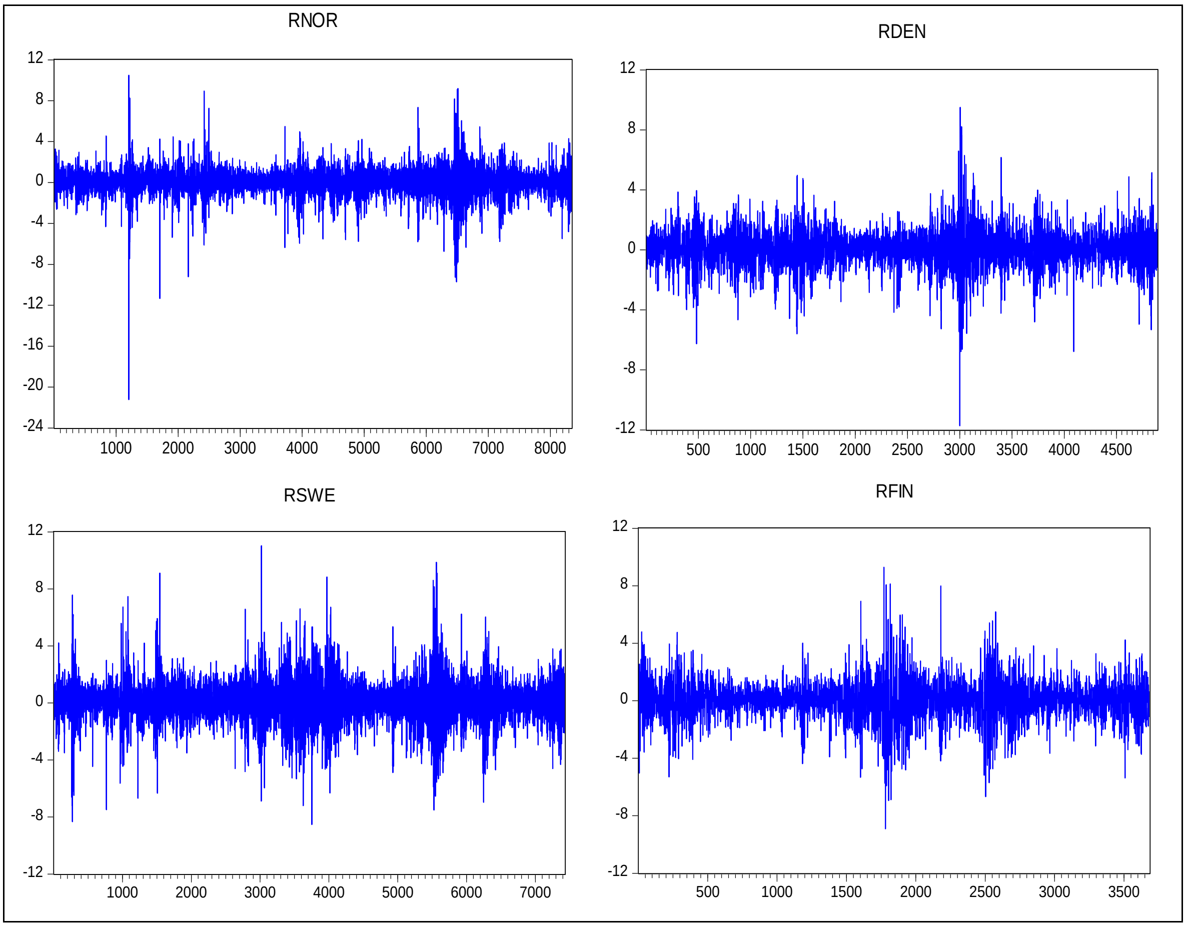

Figure 2. Daily returns of the Norwegian, Danish, Swedish and Finnish stock markets

From Figure 2 we can assume that the daily returns of all the stock markets are stationary. Consequently we can move on to Tables 1 and 2 of correlograms and check if autocorrelation exists in the daily returns of stock markets, as well as the form of autocorrelation in the correlograms of the daily squared returns.

Table 1. Correlogram of the daily return of the Norwegian, Danish, Swedish and Finnish stock markets

Sample: 1 8347
Included observations: 8346

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.061	0.061	30.978	0.000
		2	-0.012	-0.016	32.165	0.000
		3	-0.024	-0.023	37.144	0.000
		4	0.009	0.012	37.886	0.000
		5	-0.022	-0.024	41.951	0.000
		6	-0.005	-0.002	42.121	0.000
		7	0.051	0.051	63.734	0.000
		8	0.019	0.011	66.617	0.000
		9	0.028	0.029	73.401	0.000
		10	0.015	0.014	75.356	0.000
		11	0.011	0.009	76.281	0.000
		12	0.005	0.008	76.519	0.000
		13	0.024	0.025	81.423	0.000
		14	0.027	0.023	87.594	0.000
		15	0.037	0.035	99.291	0.000
		16	0.020	0.016	102.76	0.000
		17	-0.003	-0.005	102.82	0.000
		18	-0.017	-0.016	105.27	0.000
		19	-0.004	-0.003	105.44	0.000
		20	0.001	-0.001	105.44	0.000
		21	0.010	0.007	106.30	0.000
		22	0.000	-0.007	106.30	0.000
		23	0.014	0.009	107.88	0.000
		24	0.001	-0.004	107.89	0.000
		25	0.036	0.035	118.59	0.000
		26	0.014	0.010	120.23	0.000
		27	-0.009	-0.010	120.85	0.000
		28	-0.009	-0.008	121.51	0.000
		29	0.007	0.005	121.87	0.000
		30	0.002	-0.001	121.90	0.000
		31	-0.022	-0.022	125.96	0.000
		32	0.014	0.014	127.58	0.000
		33	-0.013	-0.017	128.98	0.000
		34	0.002	0.002	129.00	0.000
		35	-0.019	-0.020	132.09	0.000
		36	0.017	0.016	134.64	0.000

Sample: 1 4893
Included observations: 4892

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.041	0.041	8.3529	0.004
		2	-0.030	-0.032	12.689	0.002
		3	-0.024	-0.021	15.428	0.001
		4	0.022	0.024	17.906	0.001
		5	-0.033	-0.036	23.150	0.000
		6	-0.021	-0.017	25.245	0.000
		7	-0.009	-0.008	25.606	0.001
		8	-0.004	-0.006	25.667	0.001
		9	0.014	0.015	26.671	0.002
		10	0.011	0.009	27.256	0.002
		11	0.034	0.033	32.798	0.001
		12	0.000	-0.002	32.798	0.001
		13	-0.001	0.001	32.800	0.002
		14	0.026	0.028	36.192	0.001
		15	0.001	-0.002	36.194	0.002
		16	0.014	0.018	37.121	0.002
		17	0.001	0.003	37.127	0.003
		18	-0.012	-0.012	37.830	0.004
		19	0.005	0.009	37.956	0.006
		20	0.007	0.005	38.178	0.008
		21	-0.014	-0.013	39.077	0.010
		22	0.009	0.011	39.465	0.012
		23	0.000	-0.003	39.465	0.018
		24	-0.013	-0.014	40.315	0.020
		25	0.034	0.034	45.909	0.007
		26	-0.021	-0.027	48.128	0.005
		27	-0.015	-0.012	49.193	0.006
		28	-0.001	0.001	49.195	0.008
		29	0.038	0.034	56.186	0.002
		30	0.005	0.004	56.319	0.003
		31	-0.019	-0.018	58.176	0.002
		32	-0.004	-0.000	58.256	0.003
		33	-0.004	-0.007	58.319	0.004
		34	-0.003	-0.003	58.373	0.006
		35	-0.011	-0.007	58.931	0.007
		36	0.001	-0.002	58.936	0.009

Sample: 1 7432
Included observations: 7431

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.022	0.022	3.6006	0.058
		2	-0.028	-0.028	9.2403	0.010
		3	-0.034	-0.033	17.700	0.001
		4	0.001	0.001	17.702	0.001
		5	-0.008	-0.010	18.156	0.003
		6	-0.035	-0.036	27.373	0.000
		7	0.008	0.009	27.826	0.000
		8	0.015	0.012	29.462	0.000
		9	0.019	0.017	32.156	0.000
		10	-0.019	-0.019	34.838	0.000
		11	0.015	0.017	36.479	0.000
		12	0.022	0.021	40.223	0.000
		13	0.014	0.013	41.606	0.000
		14	0.011	0.014	42.582	0.000
		15	0.023	0.026	46.588	0.000
		16	0.022	0.022	50.319	0.000
		17	0.013	0.015	51.497	0.000
		18	-0.026	-0.022	56.644	0.000
		19	-0.037	-0.033	66.698	0.000
		20	-0.001	0.000	66.700	0.000
		21	-0.003	-0.005	66.764	0.000
		22	-0.001	-0.002	66.768	0.000
		23	0.032	0.031	74.418	0.000
		24	0.021	0.015	77.665	0.000
		25	0.059	0.057	103.30	0.000
		26	0.011	0.012	104.20	0.000
		27	-0.005	-0.001	104.42	0.000
		28	0.001	0.004	104.42	0.000
		29	0.024	0.025	108.81	0.000
		30	0.007	0.008	109.17	0.000
		31	-0.022	-0.018	112.93	0.000
		32	-0.016	-0.015	114.82	0.000
		33	-0.001	-0.001	114.83	0.000
		34	-0.011	-0.014	115.80	0.000
		35	-0.023	-0.020	119.78	0.000
		36	0.028	0.025	125.43	0.000

Sample: 1 3687
Included observations: 3686

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.031	0.031	3.4375	0.064
		2	-0.012	-0.013	3.9425	0.139
		3	-0.025	-0.024	6.2420	0.100
		4	0.000	0.002	6.2422	0.182
		5	-0.062	-0.062	20.220	0.001
		6	-0.021	-0.017	21.785	0.001
		7	0.013	0.013	22.414	0.002
		8	0.025	0.021	24.684	0.002
		9	-0.020	-0.022	26.122	0.002
		10	-0.021	-0.023	27.823	0.002
		11	0.017	0.016	28.847	0.002
		12	-0.009	-0.010	29.122	0.004
		13	-0.012	-0.009	29.698	0.005
		14	0.015	0.014	30.495	0.007
		15	0.007	0.001	30.668	0.010
		16	0.038	0.039	35.947	0.003
		17	0.033	0.033	39.957	0.001
		18	-0.043	-0.046	46.777	0.000
		19	-0.011	-0.005	47.188	0.000
		20	0.003	0.006	47.220	0.001
		21	0.008	0.011	47.454	0.001
		22	0.018	0.021	48.646	0.001
		23	0.001	-0.005	48.651	0.001
		24	-0.030	-0.033	51.947	0.001
		25	0.053	0.057	62.244	0.000
		26	0.019	0.022	63.629	0.000
		27	-0.004	-0.005	63.701	0.000
		28	-0.011	-0.010	64.136	0.000
		29	0.049	0.049	72.905	0.000
		30	0.018	0.019	74.093	0.000
		31	-0.039	-0.036	79.654	0.000
		32	-0.015	-0.010	80.459	0.000
		33	0.023	0.016	82.432	0.000
		34	-0.043	-0.038	89.171	0.000
		35	-0.020	-0.005	90.611	0.000
		36	0.038	0.031	96.008	0.000

Ljung and Box (Q-statistics) indicate that a serial correlation exists for all time lags of the autocorrelation function.

Table 2. Correlogram of the squared daily return of the Norwegian, Danish, Swedish and Finnish stock markets.

Sample: 1 8347
Included observations: 8346

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.307	0.307	787.39	0.000
		2	0.148	0.059	969.55	0.000
		3	0.143	0.091	1139.3	0.000
		4	0.162	0.099	1357.8	0.000
		5	0.153	0.073	1553.3	0.000
		6	0.137	0.056	1710.2	0.000
		7	0.157	0.082	1916.5	0.000
		8	0.150	0.057	2104.5	0.000
		9	0.143	0.051	2275.1	0.000
		10	0.120	0.025	2394.8	0.000
		11	0.133	0.049	2542.8	0.000
		12	0.154	0.063	2741.8	0.000
		13	0.137	0.033	2899.7	0.000
		14	0.167	0.077	3132.7	0.000
		15	0.147	0.033	3314.0	0.000
		16	0.168	0.068	3551.0	0.000
		17	0.127	0.006	3686.2	0.000
		18	0.112	0.010	3791.2	0.000
		19	0.111	0.010	3894.2	0.000
		20	0.073	-0.035	3938.4	0.000
		21	0.104	0.024	4029.2	0.000
		22	0.100	0.003	4112.7	0.000
		23	0.109	0.019	4213.1	0.000
		24	0.069	-0.029	4252.5	0.000
		25	0.096	0.025	4329.6	0.000
		26	0.079	-0.014	4382.2	0.000
		27	0.082	0.007	4438.8	0.000
		28	0.088	0.006	4503.2	0.000
		29	0.078	-0.001	4554.7	0.000
		30	0.072	-0.010	4598.0	0.000
		31	0.065	-0.004	4633.6	0.000
		32	0.069	0.003	4673.6	0.000
		33	0.068	0.003	4712.8	0.000
		34	0.057	-0.005	4740.0	0.000
		35	0.074	0.019	4786.0	0.000
		36	0.055	-0.007	4811.0	0.000

Sample: 1 4893
Included observations: 4892

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.216	0.216	228.55	0.000
		2	0.291	0.257	644.53	0.000
		3	0.224	0.137	890.82	0.000
		4	0.237	0.127	1166.0	0.000
		5	0.375	0.280	1856.3	0.000
		6	0.181	0.013	2016.2	0.000
		7	0.252	0.071	2328.5	0.000
		8	0.187	0.029	2500.4	0.000
		9	0.248	0.083	2800.9	0.000
		10	0.214	0.016	3025.4	0.000
		11	0.184	0.024	3190.7	0.000
		12	0.262	0.105	3527.5	0.000
		13	0.161	-0.005	3654.7	0.000
		14	0.197	-0.001	3845.7	0.000
		15	0.170	0.018	3987.5	0.000
		16	0.193	0.038	4171.1	0.000
		17	0.222	0.048	4413.9	0.000
		18	0.155	0.004	4532.7	0.000
		19	0.159	-0.014	4656.3	0.000
		20	0.100	-0.059	4705.8	0.000
		21	0.217	0.076	4937.2	0.000
		22	0.169	0.026	5078.2	0.000
		23	0.168	0.025	5217.2	0.000
		24	0.096	-0.065	5263.0	0.000
		25	0.099	-0.022	5310.8	0.000
		26	0.128	-0.031	5391.0	0.000
		27	0.113	-0.006	5453.5	0.000
		28	0.157	0.043	5575.2	0.000
		29	0.068	-0.033	5597.8	0.000
		30	0.085	-0.033	5633.6	0.000
		31	0.077	-0.018	5662.5	0.000
		32	0.110	0.036	5722.4	0.000
		33	0.122	0.014	5796.2	0.000
		34	0.076	-0.001	5824.9	0.000
		35	0.075	-0.014	5852.7	0.000
		36	0.075	0.016	5880.4	0.000

Sample: 1 7432
Included observations: 7431

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.218	0.218	351.74	0.000
		2	0.217	0.178	701.84	0.000
		3	0.234	0.170	1108.9	0.000
		4	0.185	0.091	1364.7	0.000
		5	0.210	0.112	1691.1	0.000
		6	0.174	0.060	1915.9	0.000
		7	0.192	0.083	2191.5	0.000
		8	0.159	0.035	2378.9	0.000
		9	0.175	0.060	2605.6	0.000
		10	0.184	0.064	2856.3	0.000
		11	0.179	0.059	3094.1	0.000
		12	0.158	0.027	3279.3	0.000
		13	0.161	0.035	3471.4	0.000
		14	0.143	0.013	3624.7	0.000
		15	0.145	0.022	3781.6	0.000
		16	0.152	0.031	3954.5	0.000
		17	0.135	0.013	4089.9	0.000
		18	0.152	0.035	4263.2	0.000
		19	0.126	0.005	4382.3	0.000
		20	0.105	-0.017	4465.1	0.000
		21	0.147	0.038	4626.0	0.000
		22	0.132	0.022	4756.4	0.000
		23	0.106	-0.009	4840.7	0.000
		24	0.114	0.005	4938.2	0.000
		25	0.144	0.045	5092.5	0.000
		26	0.096	-0.016	5161.3	0.000
		27	0.114	0.012	5259.0	0.000
		28	0.123	0.020	5372.3	0.000
		29	0.082	-0.023	5422.0	0.000
		30	0.114	0.020	5519.2	0.000
		31	0.116	0.023	5619.8	0.000
		32	0.098	-0.001	5690.9	0.000
		33	0.100	0.006	5764.9	0.000
		34	0.103	0.010	5843.8	0.000
		35	0.112	0.021	5936.8	0.000
		36	0.094	0.002	6003.3	0.000

Sample: 1 3687
Included observations: 3686

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.139	0.139	71.500	0.000
		2	0.227	0.212	262.16	0.000
		3	0.212	0.168	427.55	0.000
		4	0.156	0.080	517.32	0.000
		5	0.303	0.229	855.51	0.000
		6	0.190	0.097	989.57	0.000
		7	0.179	0.049	1108.3	0.000
		8	0.145	0.005	1186.5	0.000
		9	0.198	0.087	1331.3	0.000
		10	0.178	0.042	1448.0	0.000
		11	0.228	0.106	1639.6	0.000
		12	0.188	0.061	1771.1	0.000
		13	0.173	0.047	1881.8	0.000
		14	0.174	0.023	1993.3	0.000
		15	0.130	-0.017	2055.6	0.000
		16	0.209	0.059	2217.2	0.000
		17	0.196	0.068	2359.2	0.000
		18	0.175	0.037	2473.2	0.000
		19	0.168	0.025	2578.0	0.000
		20	0.123	-0.018	2633.9	0.000
		21	0.199	0.054	2781.2	0.000
		22	0.175	0.027	2894.7	0.000
		23	0.184	0.038	3021.0	0.000
		24	0.118	-0.035	3072.4	0.000
		25	0.165	0.036	3173.8	0.000
		26	0.143	-0.000	3249.5	0.000
		27	0.133	-0.017	3315.7	0.000
		28	0.184	0.033	3441.2	0.000
		29	0.096	-0.030	3475.1	0.000
		30	0.147	0.000	3555.2	0.000
		31	0.113	-0.012	3602.6	0.000
		32	0.144	0.021	3679.7	0.000
		33	0.147	0.014	3760.7	0.000
		34	0.140	0.020	3834.0	0.000
		35	0.182	0.066	3957.9	0.000
		36	0.119	0.007	4011.1	0.000

Results of Table 2 indicate that Ljung and Box (Q-statistics) for all time lags is statistical significant, therefore an ARCH effect exists.

Table 3. Descriptive statistics of the daily return of the Norwegian, Danish, Swedish and Finnish stock markets

	NOR	DEN	SWE	FIN
Mean	0.045110	0.041417	0.031782	0.021683
Median	0.092587	0.085747	0.069711	0.063065
Maximum	10.48099	9.496355	11.02284	9.285563
Minimum	-21.21879	-11.72319	-8.526937	-8.905445
Std. Dev.	*1.320211*	1.289933	1.462234	1.444397
Skewness	-1.012962	-0.268218	0.025068	-0.036580
Kurtosis	17.89832	7.847888	7.250442	6.317618
Jarque-Bera	78613.88	4849.151	5594.544	1691.251
Probability	0.000000	0.000000	0.000000	0.000000
Sum	376.4856	202.6100	236.1706	79.92401
Sum Sq.Dev.	14544.98	8138.264	15886.28	7687.956
Observations	8346	4892	7431	3686

Table 4. Stationarity test of the daily return of the Norwegian, Danish, Swedish and Finnish stock markets

Variable	ADF		P-P	
	C	C,T	C	C,T
RNOR	-85.93(0)*	-85.95(0)*	-86.51[25]*	-86.50[25]*
R SWE	-84.31(0)*	-84.31(0)*	-84.32[15]*	-84.32[15]*
RFIN	-58.86(0)*	-58.85(0)*	-58.87[11]*	-58.86[11]*
RDEN	-67.09(0)*	-67.08(0)*	-67.03[14]*	-67.02[14]*

Note. 1. *, ** and *** show significant at 1%, 5% and 10% levels respectively.

2. The numbers within parentheses followed by ADF statistics represent the lag length of the dependent variable used to obtain white noise residuals.

3. The lag lengths for ADF equation were selected using Schwarz Information Criterion (SIC).

4. Mackinnon (1996) critical value for rejection of hypothesis of unit root applied.

5. The numbers within brackets followed by PP statistics represent the bandwidth selected based on Newey West (1994) method using Bartlett Kernel.

6. C=Constant, T=Trend..

After the stationarity detection with tests Dickey-Fuller (1979, 1981) and Phillips-Perron (1988) of all time-series we can define the form of model ARMA (p, q) from the correlogram of Table 1. Parameters p and q can be defined from the partial autocorrelation and correlation variable, correspondingly, comparing them with the critical value $\pm 2 / \sqrt{n} = \pm 2 / \sqrt{8347} = \pm 0.022$ for the Norwegian stock exchange, ± 0.028 for the Danish stock exchange, ± 0.023 for the Swedish stock exchange and ± 0.032 for the Finnish stock exchange.

Therefore for Norway and Finland p value will be 0<p <1 and q value will be 0<q<1, for Denmark p value will be 0<p<2 and q value will be 0<q<2 and lastly for Sweden p value will be 0<p<3 and q value will be 0<q<3.

Consequently we can create Table 5 as such:

Table 5. Comparison of models within the range of exploration using AIC, SIC and HQ

ARIMA model	AIC	SC	HQ
RNOR			
(1,0,0)	3.3903	3.3928	3.3912
(0,0,1)	**3.3902**	**3.3927**	**3.3911**
(1,0,1)	3.3904	3.3938	3.3915
R DEN			
(0,0,1)	**3.3462**	**3.3502**	**3.3476**
(0,0,2)	3.3469	3.3512	3.3478
(1,0,0)	3.3463	3.3503	3.3477
(1,0,1)	3.3463	3.3516	3.3482
(1,0,2)	3.3468	3.3524	3.3481
(2,0,0)	3.3462	3.3510	3.3476
(2,0,1)	3.3463	3.3524	3.3481
(2,0,2)	3.3462	3.3532	3.3480

R SWE			
(0,0,1)	3.5979	3.6017	3.5989
(0,0,2)	3.5975	3.6012	3.5988
(0,0,3)	**3.5965**	**3.6012**	**3.5981**
(1,0,0)	3.5979	3.6017	3.5989
(1,0,1)	3.5980	3.6017	3.5993
(1,0,2)	3.5968	3.6015	3.5984
(1,0,3)	3.5968	3.6024	3.5987
(2,0,0)	3.5974	3.6012	3.5987
(2,0,1)	3.5968	3.6014	3.5983
(2,0,2)	3.5969	3.6025	3.5989
(2,0,3)	3.5970	3.6035	3.5993
(3,0,0)	3.5966	3.6013	3.5982
(3,0,1)	3.5969	3.6025	3.5988
(3,0,2)	3.5971	3.6036	3.5993
(3,0,3)	3.5965	3.6039	3.5990
R FIN			
(1,0,0)	3.57368	3.57874	3.57548
(0,0,1)	**3.57366**	**3.57872**	**3.57546**
(1,0,1)	3.5741	3.5809	3.5765

The results from Table 5 indicate that according to the criteria of Akaike (AIC), Schwartz (SIC) and Hannan-Quinn (HQ) model ARIMA (0,0,1) is the fittest for the Norwegian, Finnish and Danish stock while model ARIMA (0,0,3) for the Swedish one.

Table 6. ARMA models estimation

Dependent Variable: RNOR
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 09/08/16 Time: 10:04
Sample: 2 8347
Included observations: 8346
Convergence achieved after 65 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.045099	0.016066	2.807182	0.0050
MA(1)	0.062339	0.004407	14.14640	0.0000
SIGMASQ	1.736120	0.009588	181.0735	0.0000

R-squared	0.003804	Mean dependent var	0.045110
Adjusted R-squared	0.003565	S.D. dependent var	1.320211
S.E. of regression	1.317856	Akaike info criterion	3.390249
Sum squared resid	14489.65	Schwarz criterion	3.392776
Log likelihood	-14144.51	Hannan-Quinn criter.	3.391112
F-statistic	15.92906	Durbin-Watson stat	2.001003
Prob(F-statistic)	0.000000		

Inverted MA Roots	-.06

Dependent Variable: RDEN
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 09/20/16 Time: 13:27
Sample: 2 4893
Included observations: 4892
Convergence achieved after 19 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.041419	0.019522	2.121716	0.0339
MA(1)	0.043848	0.009314	4.707982	0.0000
SIGMASQ	1.660569	0.018323	90.62963	0.0000

R-squared	0.001814	Mean dependent var	0.041417
Adjusted R-squared	0.001406	S.D. dependent var	1.289933
S.E. of regression	1.289026	Akaike info criterion	3.346264
Sum squared resid	8123.502	Schwarz criterion	3.350247
Log likelihood	-8181.962	Hannan-Quinn criter.	3.347662
F-statistic	4.442121	Durbin-Watson stat	2.002324
Prob(F-statistic)	0.011818		

Inverted MA Roots	-.04

Dependent Variable: RSWE
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 09/28/16 Time: 13:31
Sample: 2 7432
Included observations: 7431
Convergence achieved after 23 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.031768	0.016839	1.886558	0.0593
MA(1)	0.021677	0.007570	2.863561	0.0042
MA(2)	-0.027264	0.007630	-3.573291	0.0004
MA(3)	-0.036572	0.007491	-4.882066	0.0000
SIGMASQ	2.132627	0.020084	106.1845	0.0000

R-squared	0.002438	Mean dependent var	0.031782
Adjusted R-squared	0.001901	S.D. dependent var	1.462234
S.E. of regression	1.460843	Akaike info criterion	3.596578
Sum squared resid	15847.55	Schwarz criterion	3.601230
Log likelihood	-13358.09	Hannan-Quinn criter.	3.598176
F-statistic	4.537761	Durbin-Watson stat	1.999909
Prob(F-statistic)	0.001163		

Inverted MA Roots	.35	-.19+.26i	-.19-.26i

Dependent Variable: RFIN
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 09/20/16 Time: 13:29
Sample: 2 3687
Included observations: 3686
Convergence achieved after 15 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.021689	0.024894	0.871258	0.3837
MA(1)	0.031205	0.012778	2.442078	0.0146
SIGMASQ	2.083729	0.029878	69.74029	0.0000

R-squared	0.000954	Mean dependent var	0.021683
Adjusted R-squared	0.000411	S.D. dependent var	1.444397
S.E. of regression	1.444100	Akaike info criterion	3.573664
Sum squared resid	7680.624	Schwarz criterion	3.578720
Log likelihood	-6583.263	Hannan-Quinn criter.	3.575464
F-statistic	1.757709	Durbin-Watson stat	2.000585
Prob(F-statistic)	0.172584		

Inverted MA Roots	-.03

After the model estimation we test the existence of conditional heteroscedasticity (ARCH(q) test), from the squared residuals of the last model. Table 7 presents us with the results.

Table 7. ARCH(q) effect test

Sample: 1 8347
Included observations: 8346

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.340	0.340	963.11	0.000
		2	0.156	0.046	1165.9	0.000
		3	0.147	0.091	1345.8	0.000
		4	0.164	0.095	1569.3	0.000
		5	0.160	0.075	1783.0	0.000
		6	0.135	0.046	1936.1	0.000
		7	0.159	0.084	2148.1	0.000
		8	0.152	0.052	2341.2	0.000
		9	0.145	0.051	2517.6	0.000
		10	0.127	0.029	2652.1	0.000
		11	0.133	0.045	2799.9	0.000
		12	0.149	0.055	2985.3	0.000
		13	0.139	0.034	3146.2	0.000
		14	0.172	0.080	3392.5	0.000
		15	0.148	0.027	3575.6	0.000
		16	0.172	0.073	3823.3	0.000
		17	0.121	-0.009	3946.3	0.000
		18	0.116	0.020	4059.0	0.000
		19	0.111	0.006	4162.5	0.000
		20	0.074	-0.033	4207.9	0.000
		21	0.104	0.026	4298.0	0.000
		22	0.102	0.005	4385.4	0.000
		23	0.111	0.019	4487.7	0.000
		24	0.071	-0.028	4530.2	0.000
		25	0.094	0.025	4603.5	0.000
		26	0.080	-0.014	4657.3	0.000
		27	0.085	0.011	4717.3	0.000
		28	0.091	0.008	4786.2	0.000
		29	0.078	-0.003	4836.8	0.000
		30	0.073	-0.009	4881.5	0.000
		31	0.067	-0.000	4919.7	0.000
		32	0.068	-0.001	4958.6	0.000
		33	0.070	0.007	5000.0	0.000
		34	0.053	-0.012	5023.6	0.000
		35	0.079	0.027	5075.7	0.000
		36	0.055	-0.011	5101.4	0.000

Sample: 1 4893
Included observations: 4892

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.210	0.210	215.02	0.000
		2	0.286	0.254	616.67	0.000
		3	0.219	0.135	851.11	0.000
		4	0.232	0.125	1114.2	0.000
		5	0.382	0.292	1828.1	0.000
		6	0.169	0.003	1968.0	0.000
		7	0.252	0.072	2278.2	0.000
		8	0.178	0.024	2432.8	0.000
		9	0.243	0.082	2722.2	0.000
		10	0.210	0.013	2938.2	0.000
		11	0.174	0.023	3087.3	0.000
		12	0.260	0.106	3420.2	0.000
		13	0.150	-0.009	3530.4	0.000
		14	0.194	-0.000	3716.0	0.000
		15	0.163	0.018	3846.1	0.000
		16	0.189	0.039	4021.1	0.000
		17	0.220	0.048	4258.1	0.000
		18	0.152	0.010	4371.4	0.000
		19	0.154	-0.017	4487.2	0.000
		20	0.098	-0.054	4534.3	0.000
		21	0.212	0.073	4754.4	0.000
		22	0.169	0.029	4895.2	0.000
		23	0.166	0.028	5031.2	0.000
		24	0.094	-0.064	5074.7	0.000
		25	0.094	-0.023	5118.1	0.000
		26	0.126	-0.031	5196.6	0.000
		27	0.108	-0.010	5253.6	0.000
		28	0.154	0.041	5369.7	0.000
		29	0.065	-0.031	5390.2	0.000
		30	0.084	-0.030	5424.8	0.000
		31	0.074	-0.017	5452.2	0.000
		32	0.105	0.033	5506.8	0.000
		33	0.124	0.018	5581.9	0.000
		34	0.072	-0.002	5607.3	0.000
		35	0.076	-0.013	5635.7	0.000
		36	0.072	0.018	5661.4	0.000

Sample: 1 7432
Included observations: 7431

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.225	0.225	376.24	0.000
		2	0.224	0.183	749.71	0.000
		3	0.240	0.172	1178.3	0.000
		4	0.189	0.089	1443.1	0.000
		5	0.213	0.111	1779.9	0.000
		6	0.177	0.058	2012.1	0.000
		7	0.194	0.081	2293.0	0.000
		8	0.160	0.033	2483.4	0.000
		9	0.173	0.055	2705.7	0.000
		10	0.184	0.063	2956.8	0.000
		11	0.174	0.053	3182.7	0.000
		12	0.155	0.023	3360.9	0.000
		13	0.157	0.030	3543.4	0.000
		14	0.143	0.015	3695.4	0.000
		15	0.147	0.026	3856.4	0.000
		16	0.149	0.028	4021.6	0.000
		17	0.133	0.013	4154.1	0.000
		18	0.148	0.032	4317.4	0.000
		19	0.127	0.007	4436.7	0.000
		20	0.109	-0.011	4524.8	0.000
		21	0.146	0.039	4683.1	0.000
		22	0.129	0.019	4806.6	0.000
		23	0.105	-0.009	4888.1	0.000
		24	0.115	0.008	4986.7	0.000
		25	0.144	0.046	5140.8	0.000
		26	0.096	-0.015	5209.7	0.000
		27	0.113	0.012	5305.5	0.000
		28	0.118	0.015	5408.8	0.000
		29	0.082	-0.020	5459.0	0.000
		30	0.114	0.021	5555.3	0.000
		31	0.121	0.030	5664.1	0.000
		32	0.096	-0.003	5733.0	0.000
		33	0.101	0.008	5809.1	0.000
		34	0.100	0.006	5883.1	0.000
		35	0.112	0.022	5976.0	0.000
		36	0.094	0.001	6042.3	0.000

Sample: 1 3687
Included observations: 3686

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.138	0.138	70.711	0.000
		2	0.225	0.210	257.54	0.000
		3	0.211	0.167	421.10	0.000
		4	0.154	0.079	508.55	0.000
		5	0.304	0.232	850.28	0.000
		6	0.186	0.092	977.46	0.000
		7	0.179	0.050	1095.3	0.000
		8	0.143	0.004	1170.5	0.000
		9	0.198	0.089	1315.5	0.000
		10	0.179	0.044	1433.8	0.000
		11	0.226	0.106	1622.4	0.000
		12	0.186	0.059	1750.6	0.000
		13	0.166	0.041	1852.8	0.000
		14	0.172	0.022	1962.7	0.000
		15	0.127	-0.018	2022.1	0.000
		16	0.208	0.059	2181.9	0.000
		17	0.191	0.065	2316.7	0.000
		18	0.176	0.042	2431.8	0.000
		19	0.163	0.022	2530.2	0.000
		20	0.123	-0.016	2586.1	0.000
		21	0.196	0.051	2728.7	0.000
		22	0.171	0.026	2836.9	0.000
		23	0.183	0.037	2960.6	0.000
		24	0.117	-0.031	3011.2	0.000
		25	0.162	0.034	3108.1	0.000
		26	0.138	-0.003	3179.3	0.000
		27	0.132	-0.016	3243.9	0.000
		28	0.182	0.034	3366.6	0.000
		29	0.092	-0.031	3398.2	0.000
		30	0.146	0.002	3477.1	0.000
		31	0.112	-0.009	3523.8	0.000
		32	0.139	0.018	3596.1	0.000
		33	0.149	0.017	3678.7	0.000
		34	0.139	0.023	3750.6	0.000
		35	0.183	0.069	3875.3	0.000
		36	0.119	0.009	3928.3	0.000

Table 7 results indicate that the autocorrelation and partial autocorrelation variables are statistically significant. Therefore the null hypothesis for the absence of ARCH or GARCH effect is rejected.

5. Empirical Results

Keeping in mind that the ARCH effect exists in the returns of the stock markets of Norway, Denmark, Sweden and Finland we can move on to the evaluation of the models ARCH(q), GARCH(p, q) and GARCH-M(p, q). The parameters are evaluated with the BHHH algorithm (Berndt et al., 1974). The evaluation of models ARCH-GARCH-GARCH-M is presented in Tables 8a, 8b, 8c and 8d using three distributions for every country.

Table 8a. Estimated ARCH-GARCH-GARCH-M models for the daily returns of Norway

	ARMA(0,1)-ARCH (1,0)		
Parameter	Normal	t-Student	GED
ω	1.049(0.000)	1.058(0.000)	1.018(0.000)
α_1	0.401(0.000)	0.408(0.000)	0.387(0.000)
		D.O.F=4.226(0.000)	PAR=1.142(0.000)
LL	-13400.04	**-12834.64**	-12896.37
Jarque-Bera	11940.87(0.000)	13041.28(0.000)	13068.85(0.000)
ARCH(1)	1.364(0.242)	1.753(0.1855)	1.616(0.204)
Q^2(1)	1.365(0.243)	1.754(0.185)	1.615(0.2037)
	ARMA(0,1)-GARCH(1,1)		
Parameter	Normal	t-Student	GED
ω	0.056(0.000)	0.036(0.000)	0.043(0.000)
α_1	0.162(0.000)	0.126(0.000)	0.138(0.000)
β_1	0.808(0.000)	0.852(0.000)	0.837(0.000)
		D.O.F=7.686(0.000)	PAR=1.405(0.000)
LL	-12591.19	**-12360.71**	-12410.64
Jarque-Bera	10895.61(0.000)	1428.65(0.000)	12761.06(0.000)
ARCH(30)	23.758(0.783)	30.736(0.428)	27.712(0.586)
Q^2(30)	23.390(0.799)	30.490(0.441)	28.059(0.567)
	ARMA(0,1)-GARCH-M (1,1)		
Parameter	Normal	t-Student	GED
ω	0.056(0.000)	0.036(0.000)	0.043(0.000)
α_1	0.162(0.000)	0.126(0.000)	0.138(0.000)
β_1	0.809(0.000)	0.852(0.000)	0.837(0.000)
		D.O.F=7.686(0.000)	PAR=1.405(0.000)
LL	-12591.18	**-12360.70**	-12410.60
Jarque-Bera	10899.43	14212.21(0.000)	12742.25(0.000)
ARCH(30)	23.821(0.780)	30.697(0.430)	27.914(0.575)
Q^2(30)	23.453(0.796)	30.449(0.0443)	27.564(0.594)

Note. 1.Values in parentheses denote the *p*-values. 2. LL is the value of the log-likelihood.

Table 8b. Estimated ARCH-GARCH-GARCH-M models for the daily returns of Denmark

	ARMA(0,1)-ARCH (1,0)		
Parameter	Normal	t-Student	GED
ω	1.154(0.000)	1.179(0.000)	1.138(0.000)
α_1	0.315(0.000)	0.319(0.000)	0.311(0.000)
		D.O.F=4.888(0.000)	PAR=1.243(0.000)
LL	-7952.25	-7757.382	-7771.147
Jarque-Bera	1703.559(0.000)	1723.106(0.000)	1719.885(0.000)
ARCH(1)	5.540(0.018)	5.242(0.022)	5.437(0.019)
Q^2(1)	5.544(0.019)	5.246(0.022)	5.441(0.020)
	ARMA(0,1)-GARCH(1,1)		
Parameter	Normal	t-Student	GED
ω	0.050(0.000)	0.044(0.000)	0.047(0.000)
α_1	0.111(0.000)	0.117(0.000)	0.115(0.000)
β_1	0.856(0.000)	0.856(0.000)	0.855(0.000)
		D.O.F=9.752(0.000)	PAR=1.523(0.000)
LL	-7557.238	-7499.126	-7507.439
Jarque-Bera	556.6304(0.000)	611.3892(0.000)	584.5145(0.000)
ARCH(30)	24.802(0.734)	26.952(0.625)	25704(0.690)
Q^2(30)	24.540(0.747)	26.284(0.661)	25274(0.712)
	ARMA(0,1)-GARCH-M (1,1)		
Parameter	Normal	t-Student	GED
ω	0.050(0.000)	0.044(0.000)	0.047(0.000)
α_1	0.111(0.000)	0.117(0.000)	0.115(0.000)
β_1	0.856(0.000)	0.856(0.000)	0.855(0.000)
		D.O.F=9.745(0.000)	PAR=1.523(0.000)
LL	-7557.204	**-7499.085**	-7507.436
Jarque-Bera	553.5764(0.000)	615.2196(0.000)	585.4903(0.000)
ARCH(30)	24883(.0.730)	26.859(0.630)	25.678(0.691)
Q^2(30)	24.623(0.743)	26.190(0.665)	25.249(0.713)

Note. 1.Values in parentheses denote the *p*-values. 2. LL is the value of the log-likelihood.

Table 8c. Estimated ARCH-GARCH-GARCH-M models for the daily returns of Sweden

ARMA(0,3)-ARCH (1,0)			
Parameter	Normal	t-Student	GED
ω	1.4888(0.000)	1.515(0.000)	1.452(0.000)
α_1	0.317(0.000)	0.349(0.000)	0.323(0.000)
		D.O.F=4.377(0.000)	PAR=1.186(0.000)
LL	-13013.86	-12645.75	-12674.25
Jarque-Bera	2991.354(0.000)	3154.959(0.000)	3118.569(0.000)
ARCH(1)	3.749(0.052)	5.957(0.014)	5.037(0.024)
Q^2(1)	3.751(0.053)	5.960(0.015)	5.040(0.025)
ARMA(0,3)-GARCH(1,1)			
Parameter	Normal	t-Student	GED
ω	0.038(0.000)	0.025(0.000)	0.030(0.000)
α_1	0.097(0.000)	0.095(0.000)	0.096(0.000)
β_1	0.884(0.000)	0.894(0.000)	0.890(0.000)
		D.O.F=9.038(0.000)	PAR=1.499(0.000)
LL	-12242.42	-12109.21	-12145.77
Jarque-Bera	2594.030(0.000)	3443.449(0.000)	3004.152(0.000)
ARCH(30)	11.953(0.998)	16.746(0.975)	14.541(0.992)
Q^2(30)	11.986(0.999)	16.301(0.980)	14.378(0.993)
ARMA(0,3)-GARCH-M (1,1)			
Parameter	Normal	t-Student	GED
ω	0.038(0.000)	0.025(0.000)	0.030(0.000)
α_1	0.097(0.000)	0.095(0.000)	0.096(0.000)
β_1	0.883(0.000)	0.894(0.000)	0.890(0.000)
		D.O.F=9.016(0.000)	PAR=1.498(0.000)
LL	-12242.21	**-12109.09**	-12145.69
Jarque-Bera	2567.730(0.000)	3465.321(0.000)	3020.694(0.000)
ARCH(30)	11.895(0.998)	16.785(0.975)	14.581(0.991)
Q^2(30)	11.929(0.999)	16.345(0.980)	14.420(0.993)

Note. 1.Values in parentheses denote the *p*-values. 2. LL is the value of the log-likelihood.

Table 8d. Estimated ARCH-GARCH-GARCH-M models for the daily returns of Finland

ARMA(0,1)-ARCH (1,0)			
Parameter	Normal	t-Student	GED
ω	1.586(0.000)	1.673(0.000)	1.565(0.000)
α_1	0.262(0.000)	0.275(0.000)	0.261(0.000)
		D.O.F=4.206(0.000)	PAR=1.161(0.000)
LL	-6490.810	-6317.607	-6318.631
Jarque-Bera	1053.977(0.000)	1051.303(0.000)	1050.031(0.000)
ARCH(1)	4.779(0.028)	4.711(0.030)	4.949(0.026)
Q^2(1)	4.784(0.029)	4.716(0.030)	4.954(0.026)
ARMA(0,1)-GARCH(1,1)			
Parameter	Normal	t-Student	GED
ω	0.019(0.000)	0.016(0.000)	0.018(0.000)
α_1	0.078(0.000)	0.078(0.000)	0.078(0.000)
β_1	0.911(0.000)	0.913(0.000)	0.912(0.000)
		D.O.F=11.939(0.000)	PAR=1.598(0.000)
LL	-6043.635	-6023.131	-6021.973
Jarque-Bera	80.132(0.000)	83.137(0.000)	81.579(0.000)
ARCH(1)	1.387(0.238)	1.228(0.267)	1.303(0.253)
Q^2(1)	1.389(0.239)	1.230(0.267)	1.304(0.253)
ARMA(0,1)-GARCH-M (1,1)			
Parameter	Normal	t-Student	GED
ω	0.019(0.000)	0.016(0.000)	0.018(0.000)
α_1	0.079(0.000)	0.078(0.000)	0.078(0.000)
β_1	0.911(0.000)	0.913(0.000)	0.912(0.000)
		D.O.F=11.966(0.000)	PAR=1.599(0.000)
LL	-6043.226	**-6023.118**	-6023.968
Jarque-Bera	76.780(0.000)	82.487(0.000)	81.195(0.000)
ARCH(1)	1.452(0.228)	1.240(0.265)	1.311(0.252)
Q^2(1)	1.453(0.228)	1.242(0.265)	1.312(0.252)

Note. 1.Values in parentheses denote the *p*-values. 2. LL is the value of the log-likelihood.

The last tables presented us the evaluations and the standard errors for the parameters with the value of log-likelihood function, as well as the tests of normality, autocorrelation and conditional heteroscedasticity. The result of each table revealed the statistical significance of coefficients, in every country, model and distribution. Also, no autocorrelation or conditional heteroscedasticity issue is found. Furthermore, models ARMA(0,1)-GARCH-M(1,1) models, and model ARMA(0,3)-GARCH-M(1,1) in case of Sweden, are the ones with the max Log Likelihood value using the t-Student distribution. Therefore, these models can be used for forecasting.

6. Forecasting

In forecasting the Nordic stock markets using the models ARMA(0,1)-GARCH-M(1,1) and ARMA(0,3)-GARCH-M(1,1) we applied both the dynamic and static process. The dynamic process forecasts the time periods after the first time period of the sample, using the last forecast values from the lags of the dependent variable and the ARMA terms. This process is known as n-step ahead forecasts. The static process uses real, and not forecasted values, of the dependent variable. This process is known as one step- ahead forecast.

In Figures 3, 4, 5 and 5 we present the criterions of the evaluation of stock markets forecasting using the dynamic and static process, respectively.

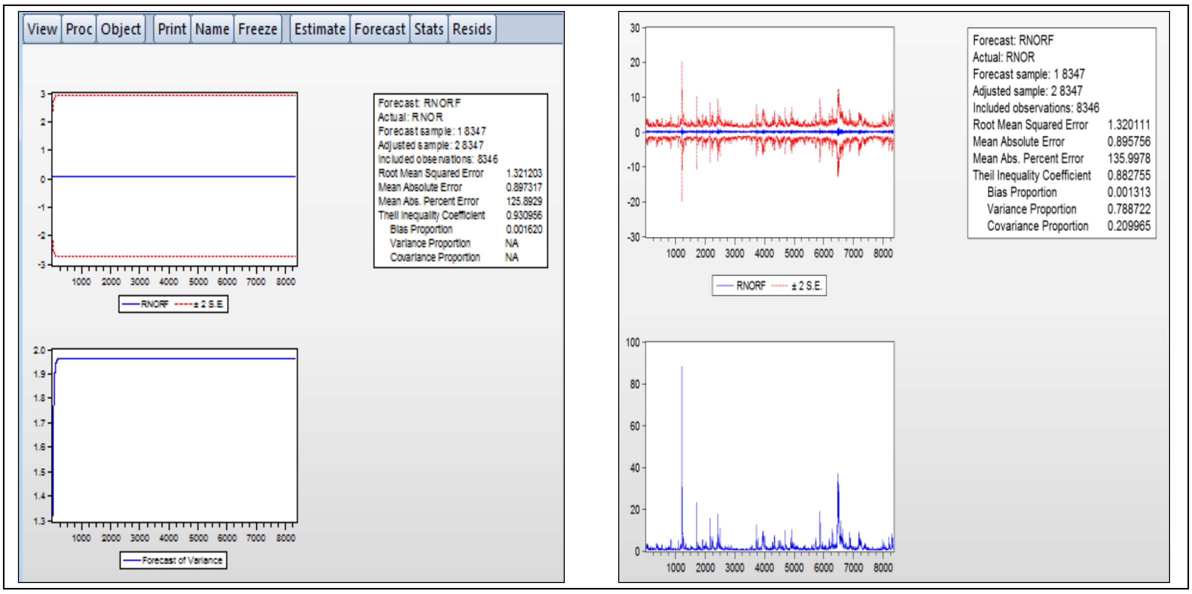

Figure 3. Dynamic and static forecast of the daily returns of Norwegian stock

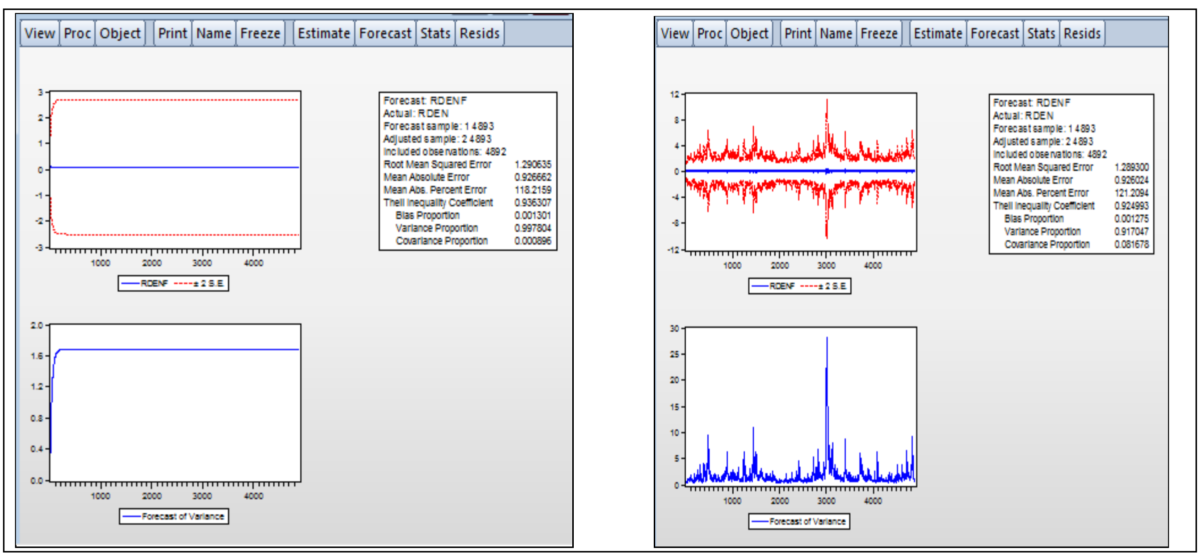

Figure 4. Dynamic and static forecast of the daily returns of Danish stock

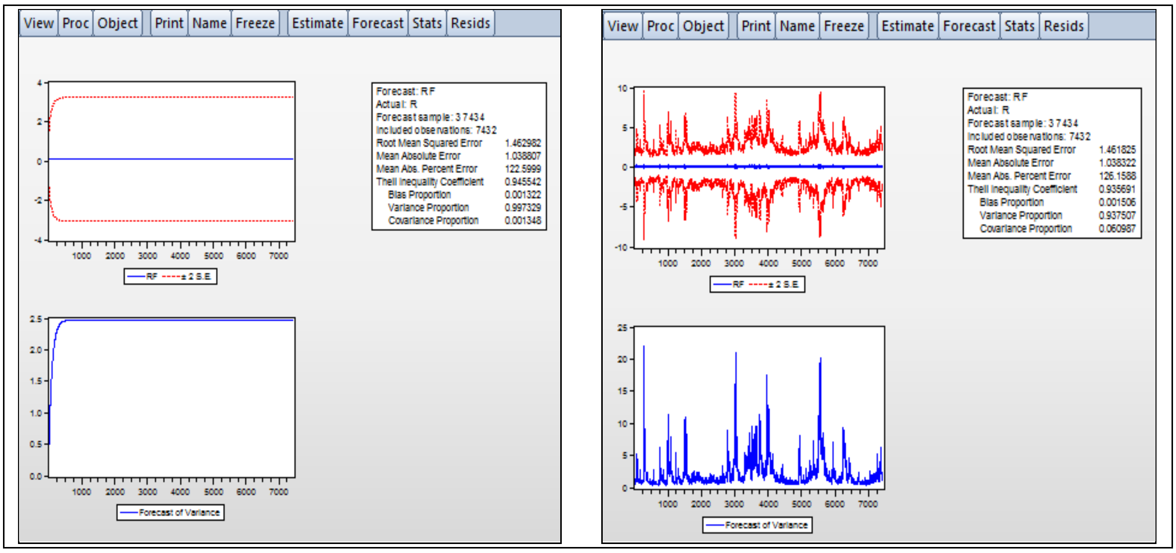

Figure 5. Dynamic and static forecast of the daily returns of Swedish stock

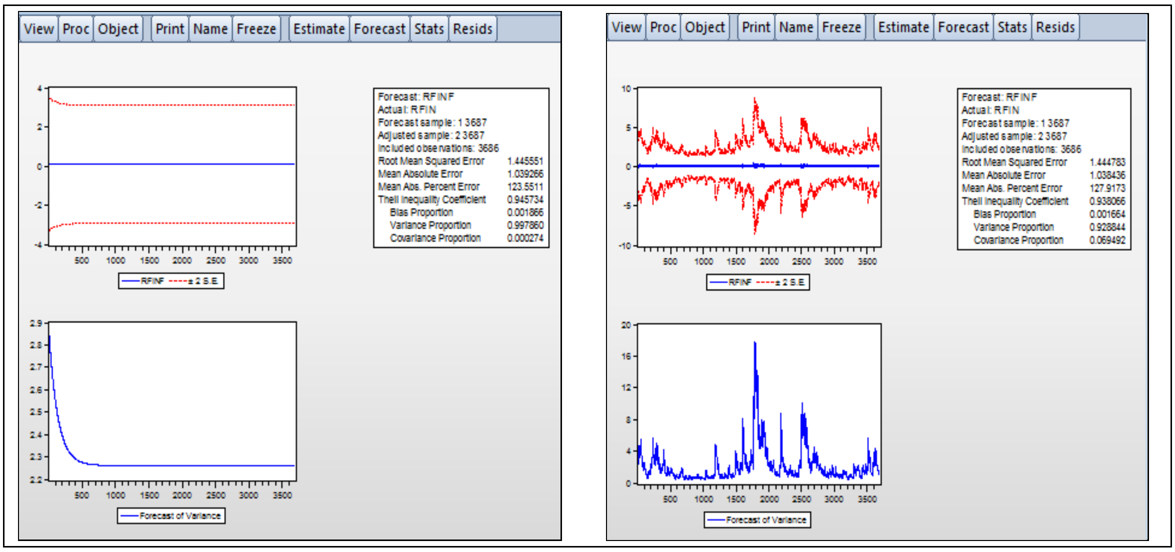

Figure 6. Dynamic and static forecast of the daily returns of Finnish stock

Based on the diagrams above we observe that static process gives better results than the dynamic one in each of the researched stock market (Theil index is lower in static process). But the fact that the Theil index is close to number one indicates that there is no appropriate forecast for the researched model.

7. Discussion and Conclusion

This study aims at modeling the volatility and the secondary effects at the four Nordic stock exchanges. The results indicated that each of the markets is well described by an ARMA(0,1)-GARCH-M(1,1) model except that of Sweden which model ARMA(0,3)-GARCH-M(1,1) describes it best. Each market's returns and volatilities are strongly dependent on their own past values. Linear dependence is probably due to the presence of a time-varying risk premium or in a form of market ineffectiveness. Volatility in each one indicated that the ill news is stronger than good news. Research of volatility and returns of each market, is motivated by the ongoing discussion related to the enactment of common Nordic stock market among Nordic countries. This merging will create the fourth biggest stock exchange market in Europe, after London, Paris and Frankfurt.

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