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Valuation of Interacting Time-to-Build and Growth Real Options in Infrastructure Investments

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Abstract

This paper argues that real options approach presents a better valuation approach for valuing infrastructure investments when compared to traditional discounted cash flow approach. Managerial flexibilities, in various forms of real options, can be incorporated into infrastructure projects to expand the projects' values. The paper identifies two key types of real options present in infrastructure investments as time-to-build and growth options and extends an earlier developed closed-form option valuation formula to value these options. The paper uses a numerical case of investment in railroad infrastructure project and shows that both types of real options, when embedded in infrastructure projects, add values to the projects. It however shows that the value of growth option is far more than the value of time-to-build option as growth options create opportunities for follow-on investments. It also shows that when the two options are present in an infrastructure investment, the time-to-build real option interacts with the growth option to reduce the latter's value.

Keywords: capital budgeting, infrastructures, real options, time-to-build option, growth option

1. Introduction

The decision about what projects to undertake and which to reject is perhaps the single most important decision that a firm can make (Copeland et al., 2005). Economics defines investment as an act of incurring an immediate cost in the expectation of future rewards (Dixit & Pindyck, 1994). Investment decisions determine what values are added to firms and hence fundamental to how financial objectives of firms are met. Traditional investment appraisal tools such as discounted cash flow (DCF) and payback periods have however been criticised for their inability to incorporate managerial flexibility into the valuation process. Management's flexibility to adapt its future actions in response to altered future market conditions has been shown to expand an investment opportunity value (Trigeorgis, 1993b). These managerial flexibilities are now being valued the same way financial options are valued. An option gives the holder the right, but not the obligation, to buy (a call option) or sell (a put option) an underlying asset on or before the maturity date. The breakthrough in options' valuation came with the formula developed by Black and Scholes (Black & Scholes, 1973) for the valuation of European call option. The payoff to the holder is asymmetric. The holder pays the option price for the downside protection and benefits from the upside potential

With a close analogy to financial options, corporate decision making is naturally discretionary and asymmetric because management has future decision rights about the use of corporate resources, such as financial resources or assets in place (Brosch, 2008). Stewart C. Myers of MIT Sloan School of Management coined the term "Real Options" in 1977 to show the close analogy thus bringing the financial options' valuation approach into valuation of "real" assets or investment projects (Myers, 1977). The starting point in using real options analysis in the valuation of an investment project is the identification of options available to the manager in order to maximize the value of the investment project. Different types of real options have been identified (Trigeorgis, 1993b). The different types of real options identified by the author have been studied by a number of other authors. These include, among others, the options to wait/defer (Titman, 1985; McDonald & Siegel, 1986; Ingersoll & Ross, 1992; Grenadier, et al., 2010), time to build option (Majd & Pindyck, 1987; Pacheco-de-Almeida & Zemsky, 2003), option to alter operating scale (Brennan & Schwartz, 1985; McDonald & Siegel, 1985), option to abandon (Arya & Glover, 2003; Huang & Chou, 2006; Wong, 2011), option to switch

(Margrabe, 1978; Henseler & Roemer, 2013), growth options (Kester, 1984; Tong et al., 2008) and multiple interacting options (Trigeorgis, 1991; Trigeorgis, 1993a; Rose, 1998).

Managerial flexibility embedded in investment projects typically takes the form of a collection of real options (Trigeorgis, 1993a). Investment in infrastructure presents an illustration of managerial flexibility, in form of different types of real options, interacting to expand the value of the investment project. Infrastructure provides a platform and creates the strategic context in which the firm can grow (Smit & Trigeorgis, 2009). Private and public institutions invest in different types of infrastructure such as information & communication technology, roads, rails, ports, land and so on. These investments create other investment opportunities with inherent managerial flexibilities in form of real options. The trend in research in real options has been to identify and value different real options in an investment project including investments in infrastructure. Investment in infrastructure presents an interesting case because it is easy to reject an investment in infrastructure based on their negative returns if managerial flexibility is not considered. This paper uses a real option model to analyse infrastructure investment with a particular application to investment in railroad infrastructure. It thus sets out to identify the managerial flexibilities, in form of real options, which are present in investment in infrastructure and how these multiple interacting real options affect the value of investment in infrastructure. This paper applies real options analysis to infrastructure investments using a numerical case of investment in railroad infrastructure by a national government through public private partnership (PPP). The two interacting options considered in the paper are time-to-build and growth real options.

One of the key issues in real options is identifying the dominating interacting real options in an investment project and using appropriate real option technique to value these options. A number of studies have examined interactions of multiple real options in investment projects and how these interactions affect the values of the investment projects (Trigeorgis, 1993a; Rose, 1998; Trigeorgis, 1991; Huang & Chou, 2006; Wong, 2011) including interactions of multiple real options in infrastructure investment. In all, real options model to value interactions of time-to-build (staged investment) and growth options in infrastructure investment seems to be lacking. This paper thus attempts to develop a real option model that can be used to value railroad infrastructure investments with interacting time-to-build and growth options.

With the general introduction of the paper discussed in this section, the next section covers the review of literature from options to real options evolution and their applications to different types of investment scenarios. The section reviews current topics on real options and their applications to investments in infrastructure including current challenges being faced. The section concludes by looking at how the findings from the paper intend to bridge the gap in real options in general and in valuation of infrastructure investments using real options in particular. Section three discusses different real options techniques available and defends the choice of the technique used in the paper. The section also discusses the development of the real options model and its application to valuation of the railroad infrastructure investment by a national government. The fourth section discusses the findings and their implications to policy makers, the private investors and to the eventual users of the infrastructure investments. It also further discusses the limitations of the developed model and its applications. Section five concludes the paper.

2. Literature Review

2.1 Real Options and Infrastructure Investment

Infrastructure investment opens up valuable follow-on investments and can thus be viewed as springboards generating a portfolio of corporate real options (Smit & Trigeorgis, 2009). A firm' investment in infrastructure can be likened to option price paid to give the firm the right to incur future costs and thus be entitled to streams of future cash flows from the follow-on investments. Investment in land, for example, provides the firms with the right to incur development costs (exercise price) and be entitled to subsequent optimal cash flows in forms of rents depending on unfolding market situations. If the investment in land is not incurred, the investor/firm will not benefit from favourable real estate market. Growth real options thus affect the value of the investment in land. Other real option that affects the value of the land investment is option to wait or option to defer the investment in land.

Other investments in infrastructure by firms such as information & communication technology and oil & gas infrastructure can also be valued using similar real options approach. The dominant type of real options in these infrastructure investments is growth option. Investments in infrastructure usually provide for the firms, opportunities to exercise the options of follow-on investments subject to resolution of market uncertainties. Oil and gas firms lay gas pipelines incurring costs in the process which then give them the right to supply gas to customers under favourable market conditions. Telecommunication firms also invest in laying of telecommunication cables with the options to link customers up to these cables and supply them with voice and data services, receiving revenues in the process. Apart from growth options, other types of real options can also be identified in infrastructure investments depending on the type of infrastructure investments. These real options include option to wait/defer, options to abandon, option to switch inputs/outputs and time-to-build option (staged-investment). While some of these real options have been valued in infrastructure investments in some studies, few attempts have been made to value the interactions of these options in capital investments in general and infrastructure investments in particular. However, of arguably more importance are investments in infrastructure projects such as road, railroad, airport and seaport by government which are now being executed through public-private partnership (PPP) arrangements.

Investments in road and port infrastructure especially through public-private partnership are now becoming common especially in developing countries and are geared towards fast-tracking the development of critical infrastructure which are closely tied to economic growth. Valuations of these infrastructure investments are usually complex with the project contracts having numerous clauses in form of flexibilities especially on the part of private partners to which the concessions of the infrastructure developments are granted. Using traditional DCF techniques to value these investments will not be optimal as this will fail to consider different real options that naturally will add value to the project to make them viable. Otherwise, the infrastructure investment will not be financially viable to the private partner. In addition to the different types of real options found in infrastructure investments, other types of real options that have been identified include minimum revenue guarantee (MRG), option to defer payment of concession fees and options to transfer ownership of the infrastructure back to government before the maturity period.

A common example of PPP infrastructure project is build-own-transfer (BOT) project. Here the private partner builds the infrastructure, operates it and transfers ownership to government at the end of the concession period. Other forms of PPP models include build-transfer-own (BTO), build-own-operate-transfer (BOOT), private finance initiative (PFI), franchise and other forms of management contracts (Quium, 2011). These PPP models incorporate different forms of real options. MRG, for example, are usually provided by government to provide private investors with downside protection in case of sharp drop in revenues. Government guarantees a minimum amount of annual revenues to the private investors in order to make the infrastructure investment financially viable. A real option approach has been used to value an MRG and the option to abandon in a BOT project (Huang & Chou, 2006). Real options analysis has also been used to value the option to defer payment of concession fees in PPP projects and the option to transfer ownership back to government before the end of the concession period (Rose, 1998). Because of the complex nature of infrastructure investments, especially PPP infrastructure projects with their inherent flexibilities, real options present equally rigorous valuation tools helping the partners to take optimal investment decisions. The more real options types in infrastructure investments, the more complicated the real options models to value them. Identifying the dominant real option types is thus necessary to develop a more tractable model which also produces approximate values of the projects with some reliable degree of accuracy.

The paper uses a railroad infrastructure project as a numerical application of the real option models. However the same analysis can be performed on other infrastructure investment projects. The railroad infrastructure project used in this paper is the standard gauge-new rail line passing through nine key cities in an undisclosed country. The feasibility study was concluded in 2014 and bids for development of the route from prospective railway firms and other interested private investors have also been evaluated. Railroad infrastructure investments are characterized by uncertainties both in investment costs and future revenues. Building real options into the project can thus be used to provide downside protection and the opportunities for upside potential. This is a new rail line with great uncertainties in revenues. Staging the investments will thus provide the necessary flexibility needed to enhance the value of the investment. Investment in the first stage of the infrastructure will provide the necessary option to invest in the subsequent stage of the infrastructure project as uncertainties in revenues are resolved. If revenues turn out to be very low and/or investment costs very high, the investments will only be viable if flexibilities such as minimum revenue guarantees had been included in the contract. Other flexibilities in terms of amount and payment terms of concession fees can also be exercised by the investors if included initially in the contract.

This paper sets out to value this time–to-build option in the railroad infrastructure and to see how it interacts with the other type of real option in the project. The paper will also examine how these options and their interactions affect the value of the railroad investment project. Traditionally, infrastructure investments have been known to provide opportunities for follow-on investments or growth options. These opportunities for future investments will also be valued in the railroad infrastructure investment. While few studies have looked at different types of real options in infrastructure projects, including their interactions, study on interactions between time-to-build and growth option seems to be lacking. The railroad infrastructure analyzed in this paper will be valued as a two-stage project: C1 – C2 – C3 – C4 – C5 and C5 – C6 – C7 – C8 – C9 routes where C1 to C9 are the nine cities. The model used can however be extended to value more than two stages of investment. The growth option valued in this paper is the opportunity for extension of the railroad to C10, a major city in the south-eastern part of the country from C5, the fifth city. The initial investment provides opportunity for this follow-on investment with the attendant future revenues. The models value these multiple interacting real options and also examine how they both affect the value of the railroad investment project.

2.2 Real Option Valuation Techniques and Infrastructure Investments

A key assumption from financial option theory to real option is that the values of the underlying assets follow a stochastic process. In real options, the values of the underlying assets are determined by cash flows. The cash flows of capital investments, just like the values of underlying assets in financial options, are assumed to follow a stochastic process. In most cases these cash flows, and hence the values of these assets, depend on the prices of outputs of the capital investment. For examples the value of an oil field will depend on the price of crude oil, the value of a toll road infrastructure will depend on the prices of tolls paid while the value of railroad infrastructure investment will depend on the track access fees paid by rail operators. These prices/fees are uncertain and are modelled as stochastic processes. In some other situations, investment costs are equally uncertain and are equally modelled as stochastic processes. Brownian motion (Geometric or Arithmetic) and mean reverting processes, two key stochastic processes, have been used to model the costs and cash flows of investment projects, including infrastructure investments, in a number of real options studies on real option valuations.

The stochastic processes when analysed mathematically result in partial differential equations. Closed-form solutions have been obtained in a number of studies to value investment projects in general (Margrabe, 1978; McDonald & Siegel, 1985; McDonald & Siegel, 1986; Majd & Pindyck, 1987; Carr, 1988; Pindyck, 1990; Grenadier, et al., 2010; LIU, 2010; Wong, 2011) and infrastructure investments in particular (Huang & Chou, 2006). These studies have produced relatively good estimates of option values. However in some cases where analytical solutions could not be obtained, the resulting partial differential equations have been solved numerically (Brennan & Schwartz, 1985; Paddock et al., 1988; Rose, 1998) while in others the underlying stochastic processes have been approximated directly (Titman, 1985; Trigeorgis, 1991; Trigeorgis, 1993a; Smit, 1997; Panayi & Trigeorgis, 1998; Benninga & Tolkowsky, 2002; Herath & Park, 2002; Sodal et al., 2009). Although numerical solutions that approximate stochastic processes directly are more tractable and less rigorous, real option techniques that approximate the resulting partial differential equations value capital projects to a greater degree of accuracy. Researchers computing a smaller number of option values may prefer the binomial approximation (direct approximation of stochastic process), while practitioners in the business of computing a larger number of option values will generally find that the finite difference (approximation of resulting PDE) approximations are more efficient (Geske & Shastri, 1985). This paper uses the closed-form solutions to the underlying partial differential equations to value the infrastructure investments

2.3 Time-to-Build and Growth Options in Infrastructure Investments

This paper values time-to-build and growth options in infrastructure investment with a numerical application to a proposed railroad investment. Staging investments in infrastructure incorporates managerial flexibility into capital budgeting process. The first stage of investment provides the right but not the obligation to invest in subsequent stage(s). These are quite applicable in infrastructure investments such as tollroad and railroad projects. Staging the investments allows resolution of uncertainties in project benefits and costs. On successful completion of the first stage, the right to invest in subsequent stages can now be exercised. In railroad investments, technical uncertainties in construction affect project costs while at the same time market uncertainties affect project revenues. Incorporating the value of this staged-investment option will thus affect the overall project value. This paper incorporates this option value into the valuation of the railroad project and evaluates its effect on the project value.

Another key characteristic of infrastructure investments is the option for future investment opportunities that they incorporate. Completion of infrastructure projects such as railroad, airport, seaport and tollroad provide opportunities for further investment and entitlement to future cash flows from these follow-on investments. These future investment opportunities are usually not considered when traditional capital budgeting tools are used for infrastructure investment appraisal. It has been shown that the value of this option can be substantial (Kester, 1984) and can make a somewhat negative NPV infrastructure project to be profitable. The paper looks at the growth option inherent in the railroad investment considered and its effect on the overall project value.

Finally the interactions of these options and their effects on the project value are analyzed.

3. Method

It is assumed that pre-construction processes have been concluded in readiness for the commencement of the construction stage. It is also assumed that the private investors are committed to constructing the first stage once it is started and cannot abandon it until the stage is completed. For the staging option, the construction and initial operation of the first stage take place between time $t = t_p$ and $t = t_n$. The investors then decide whether or not to exercise the option to construct and operate the second stage as shown in Figure 1:

Figure 1. 2-Stage rail road investment plan

Let Rs_1 be the operating revenues from the first stage, Rs_2 the operating revenues from the second stage and R the operating revenues from the overall project (stages 1 and 2) including revenues from the possible extension of the route. The value of R is uncertain and is assumed to follow a geometric Brownian motion (GBM) stochastic process:

$$\frac{dR}{R} = \alpha dt + \sigma dz = (\mu - \delta_R)dt + \sigma dz \tag{1}$$

where α is the instantaneous growth rate of the project overall revenues, σ the instantaneous standard deviation of the project revenues and dz is the increment to Gauss Wiener process with expected value of zero and variance dt. δ_R is the shortfall rate between the project discount rate μ and the instantaneous rate of growth in project revenues R.

In the same way, let I_{S1} be stage 1's project cost, I_{S2} the stage 2's project cost and I the overall project costs (including all construction and operating costs). It is assumed that I increases over time but is not random in nature. I can therefore be expressed as

$$\frac{dI}{I} = \varepsilon dt = (\mu - \delta_I)dt \tag{2}$$

ε is the instantaneous growth rate in the project costs and δ_I is the shortfall rate in project costs.

3.1 Valuing Time-to-Build Option in Infrastructure Investment

A fundamental step in using real options in capital budgeting is the identification of the real option type in the capital budgeting project. In staged-investment, the initial stage creates another option for the stage(s) following it. The time-to-build option is viewed as a European call option on option or a compound option on the initial option to invest in the infrastructure project with time to maturity, t_a, the time period between t_n and t_s (as shown in Figure II). At $t = t_p$, the infrastructure investor invests I_{S1} in the first stage receiving revenues, R_{S1} over the concession period from the stage 1 part of the project. At $t = t_n$, the investor decides whether or not to invest I_{S2} in order to receive revenue R_{S2} from stage 2 and overall revenues of R_S as shown below in Figure 2.

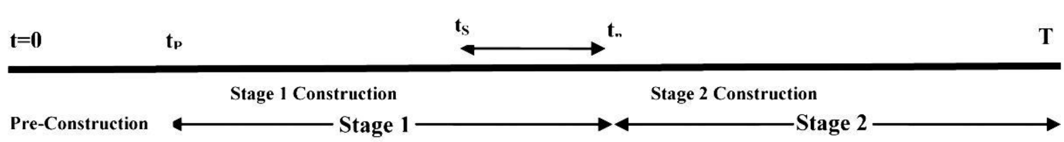

Figure 2. Time-to-build option in rail road investment project

Let F_S be the value of the option to invest for the overall project (the two stages). Then F_S can be denoted as F_S (R_S, I_S, t) where I_S is the overall project costs, including capital and operational costs. From the fundamental option valuation equation (Broadie & Detemple, 2004), the partial differential equation that describes F_S can be represented as:

$$\frac{1}{2}\frac{\partial^2 F_S}{\partial {R_S}^2}\sigma^2 {R_S}^2 + \left(r-\delta_{R_S}\right)R_S\frac{\partial F_S}{\partial R_S} + \left(r-\delta_{I_S}\right)I_S\frac{\partial F_S}{\partial I_S} + \frac{\partial F_S}{\partial t} - rF_S = 0 \tag{3}$$

The above is subject to the boundary condition

$$F_S(R_S, I_S, t_s) = Max\left[C_S - \left(I_S^{t_s} - I_{S2}e^{-t_a}\right), 0\right] \tag{4}$$

C_S is the value of the option on the second stage on the option to invest in the infrastructure, $I_S^{t_s}$ is the value of the overall investment cost I_S at time t = ts and $t_a = t_n - t_s$

C_S is a function of the state variables R_S, I_{S2} and t, hence the partial differential equation that describes C_S can be represented as:

$$\frac{1}{2}\frac{\partial^2 C_S}{\partial {R_S}^2}\sigma^2 {R_S}^2 + \left(r-\delta_{R_S}\right)R_S\frac{\partial C_S}{\partial R_S} + \left(r-\delta_{I_{S2}}\right)I_S\frac{\partial C_S}{\partial I_{S2}} + \frac{\partial C_S}{\partial t} - rF_S = 0 \tag{5}$$

This is subject to the boundary condition

$$C_S(R_S, I_{S2}, t_n) = Max\left[\left(R_S^{t_n} - I_S^{t_n}\right),\quad 0\right] \tag{6}$$

Using the transformation below (Huang & Chou, 2006)

$\varepsilon = R_S / I_S$; $X(\varepsilon, t) = F_S / I_S$; $Y(\varepsilon, t) = C_S / I_S$

and following the authors' solution method and the solution method of compound option (Geske, 1979), the formula for the option to invest is obtained as:

$$F_S(R_S, I_S, 0) = R_S^0 B\left(x, y; \sqrt{\frac{t_a}{t_n}}\right) - I_S^0 B\left(x-\sigma\sqrt{t_a}, y-\sigma\sqrt{t_n}; \sqrt{\frac{t_a}{t_n}}\right) - I_{S2}^0 N\left(x-\sigma\sqrt{t_a}\right) \tag{7}$$

where

$$x = \frac{\ln\left(\frac{R_S^0}{R_S^{*0}}\right) + (r-\delta+\frac{\sigma^2}{2})t_a}{\sigma\sqrt{t_a}}$$

$$y = \frac{\ln\left(\frac{R_S^0}{I_S^0}\right) + (r-\delta+\frac{\sigma^2}{2})t_n}{\sigma\sqrt{t_n}}$$

R_S^{*0} is the present value of the critical revenue from the project at time t = 0. It is the value at which the present value of the project just equals the discounted values of the exercise prices. N (.) and B (.) are the cumulative functions of the univariate and bivariate standard normal distribution respectively.

The price formula for time-to-build option, f_S, is thus given by

$$f_S = F_S - (R^0 - I^0) \tag{8}$$

R_0 is the value of all revenues at time t = 0 from the project without any form of flexibility or real option while I_0 is the value of both capital and operating costs incurred on the project without incorporating real options.

3.2 Valuing Growth Option in Infrastructure Investments

Infrastructure investments offer valuable investment opportunities as they provide the necessary platforms to undertake future investments. However, some of these future investment opportunities may not be appropriated to the party making the infrastructure investment. Infrastructure investments such as railroad and tollroad projects provide future investment opportunities for firms that will make use of these infrastructures for transportation and other purposes. These are also reflected in increase in tolls for tollroad and track access fees paid by operators for railroad infrastructure. However, in the particular case analyzed in this paper, the growth option is viewed as an option to extend the rail route to a key location depending on the successful completion of the overall project and on market situations. Because this option on option can only be exercised on successful completion of the project, it is modelled separately from the time- to- build option.

This growth option is viewed as a European call option on option to invest in the infrastructure project with time to maturity, t_b, the time period between t_g and t_n as shown in Figure 3. On completing the original route at time t

= t_g, the infrastructure investor then has an option to extend the route from Aba, somewhere along the route, to Enugu, another major city in the country. The option expires at time t = t_n. In addition to the upfront cost incurred to build the option into the project in form of cost of terminus and other costs, the investor incurs I_g as construction cost for the extension at time t = t_n.

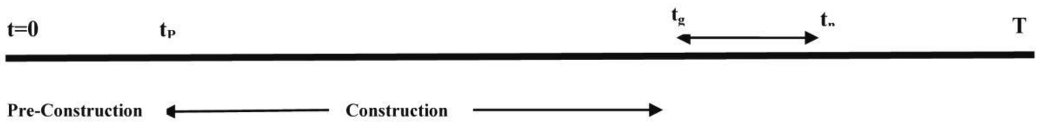

Figure 3. Growth option in rail road investment project

Let F_G be the value of the option to invest on rail road project with only growth option built into it. F_G is denoted as $F_G(R_G, I_G, t)$ where I_G is the overall project costs including incurred upfront costs for later extension of the route. R_G is the overall revenue including expected revenue from the extended part of the route. In the same way as for time-to-build option, the partial differential equation that describes F_G is represented as:

$$\frac{1}{2}\frac{\partial^2 F_G}{\partial {R_G}^2}\sigma^2 {R_G}^2 + \left(r - \delta_{R_G}\right)R_G\frac{\partial F_G}{\partial R_G} + \left(r - \delta_{I_G}\right)I_G\frac{\partial F_G}{\partial I_G} + \frac{\partial F_G}{\partial t} - rF_G = 0 \quad (9)$$

subject to

$$F_G\left(R_G, I_G, t_g\right) = Max\left[C_G - \left(I_G^{t_g} - I_g e^{-t_b}\right),\ \ 0\right] \quad (10)$$

C_G is the value of the growth option to extend the route; $I_G^{t_g}$ is the value of the overall investment cost I_G at time t = t_g; I_g is the total operating and capital costs of the extension and time $t_b = t_n - t_g$, is the time to maturity of the growth option. It is the time at which the option to extend the route is either exercised or allowed to expire unexercised.

The partial differential equation that describes $C_G(R_G, I_g, t)$ is

$$\frac{1}{2}\frac{\partial^2 C_G}{\partial {R_G}^2}\sigma^2 {R_G}^2 + \left(r - \delta_{R_G}\right)R_G\frac{\partial C_G}{\partial R_G} + \left(r - \delta_{I_g}\right)I_g\frac{\partial C_G}{\partial I_g} + \frac{\partial C_G}{\partial t} - rF_G = 0 \quad (11)$$

This is subject to the boundary condition

$$C_G\left(R_G, I_g, t_n\right) = Max\left[\left(R_G^{t_n} - I_G^{t_n}\right),\ \ 0\right] \quad (12)$$

Using similar transformation as above and the solution method of compound option, the price formula for the growth option to invest is

$$F_G(R_G, I_G, 0) = R_G^0 B\left(x, y; \sqrt{\frac{t_b}{t_n}}\right) - I_G^0 B\left(x - \sigma\sqrt{t_b}, y - \sigma\sqrt{t_n}; \sqrt{\frac{t_b}{t_n}}\right) - I_g^0 N\left(x - \sigma\sqrt{t_b}\right) \quad (13)$$

where

$$x = \frac{\ln\left(\frac{R_G^0}{R_G^{*0}}\right) + (r - \delta + \frac{\sigma^2}{2})t_b}{\sigma\sqrt{t_b}}$$

$$y = \frac{\ln\left(\frac{R_G^0}{I_G^0}\right) + (r - \delta + \frac{\sigma^2}{2})t_n}{\sigma\sqrt{t_n}}$$

R_G^{*0} is the present value of the critical revenue from the project at time t = 0. N (.) and B (.) are the cumulative functions of the univariate and bivariate standard normal distribution respectively.

The price formula for the growth option, f_g, to extend the route is given by

$$f_g = F_G - (R^0 - I^0) \quad (14)$$

3.3 Valuing the Interaction of Time-to-Build and Growth Options in Infrastructure Investments

The time-to-build option is the option to build the second stage on the initial option to invest without considering the growth option (the option to extend the route). The growth option is the option to extend the route from a key location on the route to another major city in the country. This growth option does not also consider staging the development of the route. Some cost is incurred to build the growth option into the investment project and it is only exercised at time $t = t_n$. The additional costs incurred are in form of design costs to incorporate option to extend the railroad route subject to evolution of rail passengers' traffic and freights on completion of the initial stage of the project. The two options interact to affect the value of an infrastructure project.

In this section, the two options, the time-to-build and the growth options, are considered simultaneously. For the purpose of this study, the two options are made to interact such that after the completion of the first stage, there are two options thus: option to develop the second stage and option to extend the route to a major city. These two interacting options are constructed as a European call option on the initial option to invest in the infrastructure project. These options can only be exercised at time $t = t_n$ as shown in Figure 4. At $t = t_n$, the investor decides whether or not to invest I_{SG} on both the second stage and on the extension.

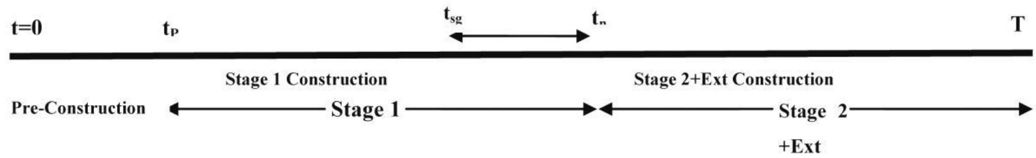

Figure 4. Interacting time-to-build and growth options

Let F be the value of the option to invest with these two built-in options; R is the overall project revenues and I the total operating and capital costs, then F, denoted by F(R, I, t) is described by the following partial differential equation:

$$\frac{1}{2}\frac{\partial^2 F}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial F}{\partial R} + (r-\delta_I)I_S\frac{\partial F}{\partial I} + \frac{\partial F}{\partial t} - rF = 0 \tag{15}$$

The above is subject to the boundary condition

$$F(R, I, t_{sg}) = Max[C - (I^{t_{sg}} - I_{SG}e^{-t_c}),\quad 0] \tag{16}$$

C is the value of the combined staging and growth options on the initial option to invest in the railroad infrastructure project, $I^{t_{sg}}$ is the value of the overall investment cost I at time $t = t_{sg}$, I_{SG} is the combined total operating and capital costs of the second stage and for extending the route and $t_c = t_n - t_{sg}$

C, denoted by $C(R, I_{SG}, t)$ is likewise described by:

$$\frac{1}{2}\frac{\partial^2 C}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial C}{\partial R} + (r-\delta_{I_{SG}})I_{SG}\frac{\partial C}{\partial I_{SG}} + \frac{\partial C}{\partial t} - rF = 0 \tag{17}$$

This is subject to the boundary condition

$$C(R, I_{SG}, t_n) = Max[(R^{t_n} - I^{t_n}),\quad 0] \tag{18}$$

Following the approach as was done for isolated time-to-build and growth options above, the price formula for option to invest is given as:

$$F(R, I, 0) = R^0 B\left(x, y; \sqrt{\frac{t_c}{t_n}}\right) - I^0 B\left(x - \sigma\sqrt{t_c}, y - \sigma\sqrt{t_n}; \sqrt{\frac{t_c}{t_n}}\right) - I_{SG}^0 N\left(x - \sigma\sqrt{t_c}\right) \tag{19}$$

where

$$x = \frac{\ln\left(\frac{R^0}{R^{*0}}\right) + (r - \delta + \frac{\sigma^2}{2})t_c}{\sigma\sqrt{t_c}}$$

$$y = \frac{\ln\left(\frac{R^0}{R^{*0}}\right) + (r - \delta + \frac{\sigma^2}{2})t_n}{\sigma\sqrt{t_n}}$$

R^{*0} is the present value of the critical revenue from the project at time t = 0. N (.) and B (.) are the cumulative functions of the univariate and bivariate standard normal distribution respectively.

The price formula for these combined time-to-build and growth options, f_{SG}, is given by

$$f_{SG} = F - (R^0 - I^0) \quad (20)$$

$R_0 - I_0$ is the NPV of the railroad infrastructure project without the embedded real options.

3.4 Numerical Application

The railroad project C1 – C2 – C3 – C4 – C5 – C6 – C7 – C8 – C9, a distance of about 673 kilometres, is used as a numerical case to apply the real options models above. The price formulae are used to estimate the time-to-build and growth options and also the interactions of the two options in the standard gauge new rail route. The country's government has evaluated bids for the development of the route and ten other rail routes and currently evaluating the procurement options, including the PPP option, for the development of the routes. As the contracts for the construction, operation and maintenance of the route are yet to be written, it is assumed, for the purpose of this paper, that the identified real options types can be incorporated into the rail road project. The financial data from the initial feasibility study are used along with market data from country's Central Bank and her Stock Exchange to estimate the real option parameters. The construction of the first stage is estimated to take 3.5 years and the stage operated for 1.5 years before exercising the option to develop the second stage of the project. The construction period for the whole route (the two stages) is estimated at seven (7) years while the concession of the rail route is expected to be for a maximum period of 30 years.

The development and concession of the rail route will be awarded and granted through a competitive bid tendering. The structure of the PPP will be determined after the amendments of the country's Railway Act. However initial discussions indicate that the government and the concessionaire will bear the construction costs. The concessionaire contributions to the construction costs will be through competitive tendering and is put at XXX10 (XXX is the local currency) for the purpose of this paper. In addition to this, the concessionaire is expected to pay an annual concession fee to the government. The assumed extension of the route will pass through additional three provinces of the federation. The present value of the cost of the first stage (both capital and operating costs) for the whole concession period is estimated at XXX8.3 billion while the present values of both capital and operating costs for the second stage and the assumed extension of the route are respectively XXX7.1 billion and XXX1.8 billion. Full details of the financial data can be obtained from the authors. Other parameters are estimated as follows:

3.4.1 Weighted Average Cost of Capital, WACC

This is the discount rate (or the growth rate α) to be used to discount future cash flows and costs and it is estimated at 13.5 percent in the initial feasibility study. The dividend payout rate, δ, is assumed to be zero.

3.4.2 Risk-Free Rate, r

Monthly Treasury bill rates were obtained from the Central bank from January, 2006 to February, 2014. The rates have the highest value of 15 percent, the lowest value of 1.04 percent and volatility of 3.56. The mean value of the Treasury bill rates is 8.56 percent and is used as the risk-free rate in the price formulae.

3.4.2 Volatility, σ

The infrastructure asset considered in this paper, just like some other real asset studied in real options analysis, is not publicly traded. However Monte Carlo simulations of the project values based on the variables driving the project value are used to estimate the project volatility. These variables are highly uncertain and include freight and passenger traffics, project costs and the uncertain rate of discount used for the project. The estimated minimum and maximum values of the above variables are used to generate random values of the project. The volatility, σ, estimated from 1000 iterations of the project value simulation is 0.4544 and is used in this study

The above estimates and the estimated discounted values of revenues from the rail project as well as present values of accompanying capital and operating costs for the two real option types and for their interactions are as shown in Table 1:

Table 1. Real option parameter values

Time-to-build Option			
$\boldsymbol{R_S^0}$ (XXX million)	16,435	σ	0.4554
$\boldsymbol{R_S^{*0}}$ (XXX million)	9,207	t_a (yrs)	1.5
$\boldsymbol{I_S^0}$ (XXX million)	14,608	t_n (yrs)	6.0
$\boldsymbol{I_{S2}^0}$ (XXX million)	6,888	Risk-free rate, r	8.56%
WACC	13.5%	δ	0
Growth Option			
$\boldsymbol{R_G^0}$ (XXX million)	16,935	σ	0.4554
$\boldsymbol{R_G^{*0}}$ (XXX million)	7,413	t_b (yrs)	1.0
$\boldsymbol{I_G^0}$ (XXX million)	15,623	t_n (yrs)	9.0
$\boldsymbol{I_g^0}$ (XXX million)	1,814	Risk-free rate, r	8.56%
WACC	13.5%	δ	0
Multiple Interacting Staging and Growth Options			
$\boldsymbol{R^0}$ (XXX million)	19,549	σ	0.4554
$\boldsymbol{R^{*0}}$ (XXX million)	11,284	t_c (yrs)	1.5
$\boldsymbol{I^0}$ (XXX million)	16,676	t_n (yrs)	6.0
$\boldsymbol{I_{sg}^0}$ (XXX million)	8,702	Risk-free rate, r	8.56%
WACC	13.5%	δ	0

4. Results and Discussion

The results from the models are as shown in Table 2. The project value, without incorporating any form of flexibility, is XXX0.86 billion. The project values and the corresponding real option values for time-to-build, growth and the multiple interacting options along with their discussions are presented below.

Table 2. Project and real option values

Project Value (Without Flexibility) (XXX Million)	860
Time-to-build Option	
Project Value (XXX Million)	991
Time-to-build Option Value (XXX Million)	131
Growth Option	
Project Value (XXX Million)	5,707
Growth Option Value (XXX Million)	4,847
Multiple Interacting Time-to-Build and Growth Options	
Project Value (XXX Million)	1,182
Multiple Interacting Option Value (XXX Million)	322

4.1 Time-to-Build Option

When only time-to-build option is incorporated into the rail infrastructure project, the project value is estimated at approximately XXX1 billion and hence a time-to-build option value of about XXX0.13 billion. Although staging the construction of the project adds value to the investment project, the real option value, at 15 percent of the project value, is not substantial when considered in relative terms. The staging option incorporates option to wait before developing subsequent stage(s) of the project and is thus expected to add substantial value to the project value. The option to wait can be as much as twice the project value (McDonald & Siegel, 1986). However, a possible explanation for the relatively low value of the time-to-build option in this case may be due to the nature of the stage 2 part of the numerical case considered. While locations in stage 1 include C4, the second richest city in the country, the same cannot be said of locations on stage 2 part of the route. In addition, revenues are also expected to be generated from intra-city passenger services that are only applicable in stage 1. This thus makes the stage 1 part of the entire route to be more attractive than the stage 2 section of the route. The partners can be guided by these findings in the drafting of the contract. The findings will be quite helpful to both parties in determining other terms such as including minimum revenue guarantees by the government especially for the stage 2 part of the project. If the option to stage the development of the route is included in the contract, the private investor should be adequately motivated to develop and operate the stage 2 part of the route.

4.2 Growth Option

The value of the growth option when considered alone is XXX4.8 billion or 564 percent of the project value. This is quite substantial and further reinforces the importance of growth option especially in infrastructure investments. Opportunities for follow-on investments are quite important in infrastructure projects. Usually the exercise price of growth option or the value of follow-on investment is low when compared to the cost of initial investment. This thus magnifies the value of the infrastructure investment especially when there is favourable market condition. This plays out in the numerical case considered in this paper as the present value of follow-on investment on the extension, at XXX1.8 billion, is relatively low and thus magnifies the project value when exercised. There are other opportunities for follow-on investments that are indirectly linked to the rail route operations. Manufacturing firms and other firms will build new plants and/or expand existing plants to further boost their revenues. Jobs are created while other additional revenues accrue to the government. The new rail route also reduces traffic pressures on the main alternative mode of transportation, the road transportation. These direct and indirect opportunities for follow-on investment are key considerations for the government in determining both financial and economic viabilities of the project and in determining the reserve bid amount for the project

4.3 Interactions of Time-to-Build and Growth Options

The project value reduces to XXX1.18 billion when the two options interact, producing a real option value of XXX0.32 billion. The real option value is slightly more than the value of flexibility when the time-to-build option was considered in isolation. The combined real option is however substantially less that the option value for growth option alone. This shows that the staging option interacts with the growth option to reduce the latter's value as the exercise price is increased while at the same time the time to maturity of the growth option is shortened. These findings show that it may be suboptimal for government to package the two options into one contract with a single concessionaire. The government should either have a separate contract for the extended part of the route or increase the concession fees if the concessionaire is to be granted the two identified flexibilities in one single contract.

4.4 Sensitivity Analysis

The real option parameters used in this paper are mostly estimates and it will be worthwhile to see how changes in their values affect the project value and the real option values. The sensitivities of estimated real option values to these parameters are analyzed by increasing and reducing the parameter values by 20 percent. One of the key assumptions in the models used in this paper is that the interest rates are constant over the life of the project. This is generally not practicable. The sensitivities of the estimated real option values to changes in interest rates are analysed by changing the values of discount and risk free rates used in the base case by ±20 percent. As expected, reducing the discount rate greatly increases the project values and the options to invest in the three scenarios discussed in this paper. Increasing the project discount rate, on the other hand greatly reduces the project values turning the hitherto profitable railroad project into a negative NPV project.

However while the effects of discount rate on the project values are significant, the effects are less significant on the values of the three real option cases considered in the paper. For time-to-build option, the analysis shows that the lower the discount rate, the lower the time-to-build option while the higher the discount rate, the higher the time-to-build option. This is also true when the time-to-build option interacts with the growth option. However, the effects are opposite for the isolated growth option. This shows that for the multiple interacting options, the effects of time-to-build option on the interaction are more significant. Also while the discount rate significantly affects the project values, its effects on the option values are less significant. For risk-free rate, the effects are not significant on both the project values and the option values. A 20 percent decrease in risk free rate slightly reduces option values in the three scenarios considered while a 20 percent increase in the base case risk free rate has opposite effects on the option values.

Finally, the effects of volatility on both the project values and on the option values are expectedly significant. A 20 percent decrease in the value of volatility not only reduces the value of time-to-build option, but also turns it into zero. It therefore can not be exercised. This shows that for a relatively low value of volatility, time-to-build option may not add value to the rail road project. The effects are also the same for multiple interacting time-to-build and growth options considered in the paper. The value of the multiple option turns to zero for a 20 percent decrease in volatility. This also confirms the dominant effects of time-to-build option in its interactions with growth option considered in this case. However for the isolated growth option, while the lowered volatility reduces the growth option, the reduction is not as significant as for time-to-build option and the multiple interacting options. On the other hand, a 20 percent increase in volatility greatly increases real option values in the three scenarios considered in the paper. This supports the findings from extant studies on real options that the higher the volatility, the higher the option values.

5. Conclusion

The paper examines the interactions of time-to-build and growth option in infrastructure investment using a numerical case of an investment in rail road project. A real option model is obtained to first value only time-to-build real option in the rail road infrastructure project. The construction of the project is staged into two and real option analysis is used to estimate the effect of the time-to-build flexibility on the value of the project. The paper also considers growth option in form of the opportunity to incur follow-on investment to extend the original route from a location along the route to another major city in the country. This growth option is valued in isolation in the paper and its effect on the overall project value discussed. Finally a real option model to value the interactions of the two real options identified above is obtained.

The results from the models show that both options, the time-to-build or staging and the growth options, add values to the rail road project when each is considered in isolation. However it is shown that growth option value is quite substantial. Infrastructure projects in general usually generate opportunities for follow-on investments. Revenues generated from the follow-on investments are usually substantial when compared to costs incurred on these additional investments. Furthermore the results of the real option model developed to estimate the value of the interactions of the two options show that while the value of the combined real option is greater than the value of the time-to-build option, it is substantially less than the isolated value of the growth option. Thus the time-to-build option interacts with the growth option considered in the rail road project to reduce the latter's value. The time-to-build option interacts with the growth option to increase the exercise price of the growth option while at the same time it reduces its time to maturity.

It may thus be suboptimal to combine the two real options above in a single contract and if they are to be combined, additional revenues, for example increased concession fees, should accrue to the government. It also guides the investor on key flexibilities to negotiate for during the railroad project tendering processes. The models in this paper are however limited in scope in that they only consider two real option types: time-to-build and growth options. Other types of real options that can be incorporated into infrastructure projects include option to wait, option to abandon and option to guarantee minimum revenues among others. While it is assumed in this paper that these flexibilities, if they are present, can be incorporated into the two real options valued, it will be interesting to value all these real option types in a single infrastructure project.

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