Asset Price Volatility, Credit Rationing and Rational Lending Discrimination

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Abstract

Observations of significant differences in access to credit, loan terms and the volume of lending between demographically distinct groups of borrowers are often interpreted as evidence of ethnic, racial or gender discrimination by lenders. The competitive structure of credit markets and the accuracy of measuring individual credit risk render extant models of lending discrimination based on assumptions of credit market inefficiencies, such as adverse selection, increasingly implausible. In stark contrast to existing models of demographic discrimination, we consider a representative credit market, described described for expositional clarity in the context of mortgage lending, in an economy having complete markets, common knowledge and arbitrage-free pricing. Market equilibria in this classical environment may exhibit discrimination even when borrowers, who are distinguished only by observable demographic traits, share an identical measure of individual credit risk. Relatively costlier loan terms, a higher frequency of loan denials, or a complete rationing of credit to a particular demographic class of borrowers may be a value-maximizing strategy when rational lenders share a common perception that one or more such traits are directly related to adverse features of the representative asset securing the loan to borrowers in this class. Omitted from standard statistical underwriting and regulatory review procedures, these features reduce the value of the collateral available to the lender in the event of future default. When loans are secured by such assets and both lenders and borrowers act strategically, discrimination on this basis will be a property of all market equilibria and can be consistent with a second-best efficient allocation of credit.

Keywords: complete markets, credit rationing, efficiency, lending discrimination, perfect Markovian equilibria

1. Introduction

Why do some mortgage borrowers appear to encounter significantly different loan terms than other borrowers to whom they appear identical in their measure of credit risk? Why does the volume of credit seem to be periodically rationed to some borrowers? Evidence of lending discrimination, particularly in regard to residential mortgages, poses a significant anomaly for the classical efficiency of credit markets. (Note 1).

Two different explanations for this evidence most often given, each depending on implicit or explicit situations of market failure. The first, popular among policymakers and the public, asserts that lender preferences over demographic traits of borrowers are the primary cause of discrimination. These preferences either violate the criteria for economic rationality, are biased against certain borrowers owing to demographic traits that appear unrelated to credit risk, or are affected by behavioral limits on cognition which lead to suboptimal loan decisions. (Note 2). Assuming lenders possess market power, such preferences could lead to systematic discrimination between borrowers in several forms, including significant variation in loan terms and even the rationing of credit through systematic denials of loan applications. (Note 3).

Traditional economic explanations, in contrast, involve inefficiencies affecting loan underwriting, due either to exogenous constraints on the accurate pricing of individual credit risk or to the assumption of an asymmetric dispersion of information across lenders and borrowers. (Note 4) Both adverse selection and moral hazard, for example, can produce dispersion in loan terms, denial rates and the rationing of credit across distinct groups of borrowers who differ by unobservable probabilities of default. (Note 5) The competitive structure of most credit markets, the availability of credit histories and the widespread use of increasingly sophisticated statistical tests of

credit risk posed by loan applicants, however, render the realism of these respective explanations suspect.

We offer a novel and fundamentally different explanation for observations of discrimination and rationing in credit markets. We demonstrate that evidence of credit allocations consistent with lending discrimination can arise in efficient credit markets, in contrast to the reliance of traditional explanations on the a priori assumption that credit markets are inefficient. (Note 6).

Our explanation uses a model of a representative credit market embedded in a traditional continuous-time economy exhibiting complete markets and arbitrage-free valuation. We augment this environment by incorporating the strategies chosen by lenders and borrowers in negotiating the terms of a standard secured loan. These strategies include the respective choices of these parties to exercise their options implicit in a loan of this type as a percentage of initial collateral value at the time of loan origination. The strategic timing of exercise by each party will depend, in part, on the properties of the asset(s) serving as collateral. Part or all of this collateral, in the case of a residential mortgage market, is the property being financed. Since each party will choose his optimal strategy in negotiating a loan, including the timing of his option exercise, credit market equilibrium can feature loans exhibiting contrasting terms or credit rationing if borrowers purchase properties which differ in the collateral risk they pose to lenders. If such borrowers also exhibit distinct demographic or other observable qualities, then a common belief among lenders in a relation between the demographic traits of a borrower and the collateral risk of his property will generate a non-cooperative equilibrium in which discrimination will be observed, even if all borrowers appear identical by standard measures of credit risk.

Empirical evidence of systematic discrimination most often arises in the American market for residential mortgages. Such evidence is normally provided by formal or informal tests of whether the loan terms or denial frequencies received by one class of borrowers, relative to another, arise only as a direct or indirect result of such demographic traits as the race, ethnicity or gender of members of this class. (Note 7) Under the condition that lenders all perceive a stable relation, whether objective or subjective in nature, between a given demographic class of borrowers and the properties of the representative asset(s) securing their loans, we show that endogenous discrimination in the allocation of credit across different classes of borrowers will invariably be observed in market equilibrium. (Note 8) Unlike the inefficient equilibria exhibited in earlier approaches involving lender preferences or asymmetric information, however, our equilibrium arises in an economy with complete markets and arbitrage-free pricing. Conditional on this common belief of lenders, moreover, this equilibrium is consistent with an efficient allocation of credit across all borrowers. (Note 9).

The paper is organized as follows. We review the relevant previous research on equilibrium credit discrimination in the next section. Our model is described in Section 3. Analytical and numerical results are presented in Section 4. Section 5 offers a discussion of the implications of our model for both applied economic theory and public policy. Concluding remarks appear in the final section.

2. Literature and Context

The significance of this paper is in its resolution of the anomaly of discriminatory loan terms and credit rationing within a market satisfying the classical assumptions of finance theory. This section compares and contrasts our analysis to previous economic explanations of lending discrimination and credit rationing.

The traditional approach to explaining idiosyncratic lending patterns across different borrowers was pioneered by Hodgman (1960) and Freimer and Gordon (1963), and refined in subsequent models by Jaffee and Modigliani (1969), Barro (1976), Baltensperger (1976), and others. Differential access to credit ultimately arose, in these papers, from an exogenous inability of lenders to accurately price credit risk among individual borrowers. While able to predict lending patterns, which, for a suitable choice of parameters, could approximate empirical evidence of discrimination, these models could not generate endogenous credit market equilibria exhibiting discrimination based on observable borrower characteristics.

In a subsequent approach, Stiglitz and Weiss (1981) analyzed the endogenous decisions of lenders who, owing to the assumed presence of either adverse selection or moral hazard, could price risk to individual borrowers only on the basis of the average degree of credit risk of a group of borrowers. Loan terms, as a result, could vary across groups of borrowers and under extreme circumstances a group could experience the rationing of credit. Bester (1994), Hart (1995), Hart and Moore (1998), Gorton and Kahn (2000), Martin (2008), and Que (2016), among many others, subsequently refined this approach to include the influence of monitoring costs and renegotiation in the design loan contracts.

Models based on asymmetric information are unsuitable, however, in explaining discrimination owing to observable

traits distinguishing borrowers of equal credit risk. Equilibrium allocations of credit in these models necessarily depends on the unobservable characteristics of borrowers directly germane to their individual credit risk. (Note 10) Like those models relying on an exogenous ability to price risk, the market equilibria in asymmetric information models are necessarily inefficient in allocating risk.

Our model and results are distinguished in two fundamental ways from this earlier research. First, we endogenously derive both the existence and magnitude of equilibrium lending discrimination using only the classical assumptions of financial asset-pricing models. Second, we show that, in our model, these assumptions generate equilibria consistent with a efficient allocation of credit risk.

The model used in this paper, more specifically, is devoid of any exogenous source of market failure and instead assumes an economy with complete capital markets, common knowledge of the actuarial risk of default on the part of both borrowers and lenders, and, as a result, arbitrage-free pricing of credit risk. Our model augments these assumptions by explicitly incorporating both the strategic aspects of mortgage loan negotiation and the presence of observable demographic traits which, while independent of the measure of credit risk in standard statistical underwriting methods, may differentiate borrowers in the common perception of risk held by lenders. This allows us to apply the familiar replication techniques of contingent claims analysis to accurately value, at origination and at any time up to and including the maturity date, the mortgage contract to a given lender and any borrower.

Consequently, it also allows us to determine whether systematic variation in loan terms and credit availability across demographically distinct groups of borrowers requires the assumption of market inefficiency or whether they can be explained solely in terms of the strategic exercise of the embedded options each party holds in a standard mortgage loan. In turn, this allows us to conclude whether either idiosyncratic lender preferences or market inefficiency is necessary for lending discrimination to be observed, as asserted by traditional explanations of lending discrimination, or whether discrimination can simply result from the economic incentives of lenders and borrowers inherent in actual mortgage lending.

Our paper is also related to research on the (non-strategic) exercise of the default and prepayment options possessed by mortgage borrowers. Dokko and Edelstein (1991), Kau, Keenan, Muller and Epperson (1995), Keenan and Kau (1995), Crawford and Rosenblatt (1995), Archer, Ling and McGill (1996), and Deng, Quigley and Van Order (2000) have used contingent claims methods to value both fixed-rate and adjustable rate-mortgage contracts. Analogous to the valuation of risky debt by Merton (1974), these option-based models generate predictions for default based on the current value of a property relative to the discounted value of continuing to service its mortgage. Our paper differs from these in analyzing the option to default in the strategic context of loan negotiation between lender and borrower. We consequently expand this traditional option-based literature on mortgage termination to include strategic options, deriving the   endogenous response of the lender's offer of loan terms and supply of mortgage credit to the strategic timing of the decision of the borrower to exercise his default or prepayment options as well as to the characteristics of the property which secures his loan.

Finally, our approach to the design and valuation of mortgage contracts parallels that of the 'strategic debt service' literature, as exemplified by Anderson and Sundaresan (1996) and Archaya et al. (2006). These authors analyze, in a game-theoretic extension of the standard Merton (1973) model of corporate debt, the effect on debt pricing of strategic renegotiation and bankruptcy costs. Our model differs from those, however, in using collateral risk as the basis for loan valuation and in its focus on the degree of lending discrimination as an endogenous function of collateral risk.

3. The Model

3.1 Assumptions

We model the loan contract negotiated by a representative borrower and lender as a stochastic differential game in which the observed terms of that contract emerge from the conditions of a perfect Markov equilibrium in that game. The type of contract we consider is a standard fixed-rate mortgage loan secured by the property being purchased and displaying an arbitrary amortization structure over a finite term to maturity. Specific versions of this general contract include the common mortgage contracts found in virtually all jurisdictions within the United States and elsewhere. (Note 13).

This contract specifies that the lender advances a sum to the borrower, who in return promises to make a scheduled sequence of coupon payments to the lender over the life of the loan. The borrower defaults on the mortgage if he fails to make a payment as agreed and the lender can then foreclose upon the borrower, seizing the property securing the contract and reselling it. The initial market value of that property is common knowledge to both agents but that

value evolves randomly and consequently has, at each moment, an uncertain future value. The borrower, through his equity ownership, receives a measurable flow of (housing) services from the property and the property depreciates at an exogenous rate. (Note 14) The value of housing services, depreciation and any investment by the borrower in the property are all also assumed to be common knowledge.

Two aspects of the mortgage contract, both present in virtually all actual mortgages found in the U.S. and elsewhere, are particularly relevant to our analysis. First, in the event of default, the value of the past flow of housing services cannot be retrieved by the lender. Since the prospect of these flows was capitalized in the initial market value of the property, that value is excluded from the security available to the lender at the time of default. Both borrower and lender, consequently, are aware that an inverse relationship exists between the conditional value of the loan collateral at any time before the loan matures and the magnitude of housing services or, equivalently, the rate at which the property depreciates under the ownership of the borrower. (Note 15) Second, the options embedded in standard mortgage loans, including the borrower's options to default and prepay and the lender's option to foreclose in the event of default, are all components of their respective strategy spaces. As such, both players will choose the exercise of these options in their own interests and, in the non-cooperative market equilibrium, these choices will be each player's optimal response to the choices comprising the strategy of his rival.

The optimal strategies of each player are determined through the valuation functions of each party about their respective claims on the asset serving to secure the loan. Since the economy is assumed to have complete markets, we apply the standard arbitrage-free valuation method to derive a pair of linked partial differential equations and corresponding boundary conditions, which must be satisfied by the values of these contingent claims. Defining the state space of the game to be the support of all asset values and dates relevant to the decisions of the borrower and lender, we employ recursive methods to endogenously derive, through these differential equations, those subsets of this state space which represent the strategy spaces of each of the parties. A numerical solution for the game is obtained by finding those sequential paths in the state space which represent the "best-reply" strategies of the parties and which, in turn, determine market equilibrium. Each choice of parameter values for the market yields a distinct and unique equilibrium. The comparison of the properties of interest exhibited by alternative equilibria are compared and through this comparison we characterize lending discrimination in the form of loan terms and the volume of credit exchanged between lenders and different classes of borrowers.

3.2 Specification

Consider a representative credit market in which participants trade, over dates t, a single risky asset of value $a(t)$. We interpret this asset as a representative residential property, but it can be interpreted as any risky asset suitable to other applications. Participants also have access to the market for a riskless asset with maturity T and return r. The market operates in a continuous time economy, which satisfies the assumptions of the classical asset valuation environment. In particular, this economy has a complete filtered probability space $[\Omega, \Im, P]$, where Ω represents the space of events in this economy, $\Im$ represents the corresponding filtration, a set of sequential sigma algebras $\{\Im t\}_{0 \leq t \leq T}$ representing information available to traders at time t, and P is the actuarial probability measure over asset value and defined on Ω. All market participants observe each $\Im_t$ and base all decisions at each date t, $0 \leq t \leq T$, on this observation. All such observations are also assumed common knowledge among participants. The evolution in value of any other assets in this economy is also assumed to be well defined and adapted in this space. Asset markets are complete with respect to the source of risk arising in our representative credit market and, consequently, the representative market for credit upon which we focus exhibits, arbitrage-free valuation.

Debt contracts in this market will be interpreted as standard residential mortgages which correspond to a standard nonrecourse form under which a lender advances a unit of credit to a borrower at date 0 for which the borrower is obligated to remit both continuous coupon payments at a constant rate c until maturity T and a terminal balance $C(T)$, where $C(t)$ represents the unpaid loan balance at all dates $t \in [0,T]$. (Note 16) The rate premium γ paid by the borrower is determined by the constant coupon rate specified in the contract, relative to the riskless rate r. The unique collateral securing the loan is the property being financed. Since different residential properties exhibit distinct qualities relevant to their market value, we assume that all properties eligible as collateral belong to a suitably defined set Φ, and that the vector of qualities distinguishing different properties are indexed by the vector $\varphi \in \Phi$ (Note 17).   The value of the representative property at date t, $a(t)$, evolves according to the diffusion:

$$a(t) = \alpha(a,t)dt + \sigma a(t)dz(t) \tag{1}$$

where $z(t)$ is a standard Brownian motion, $\alpha(a,t)$ is the expected drift at all t and σ measures the conditional volatility of this value. The value of any specific property φ evolves as a case of (1) distinguished by a unique value σ_φ of the volatility parameter.

Each borrower is assumed to display an exogenous degree of an innate trait, observable by the lender but intrinsically independent of any measure of the borrower's risk of default. We index this trait over some bounded subset Θ of the real line and denote the degree of this trait displayed by an arbitrary borrower by $\theta \in \Theta$. (Note 18) We will, for the purposes of realism in our treatment of discrimination in mortgage lending, interpret these qualities as demographic in nature. In any application of our model intended to generate the same pattern of loan terms as in any particular set of observations of demographic discrimination in an actual mortgage market, these qualities may include ethnicity, gender, race and other traits relevant to these particular observations.

Finally, we choose to model the economic source of discrimination as the perceived volatility σ in the diffusion (1) describing the evolution of the market value of the generic property. If a lender perceives, either objectively or subjectively, a correlation between the observable degree to which a borrower exhibits the common demographic trait, θ, and those characteristics of the specific property this borrower wishes to finance, φ, then under these circumstances, the lender acts as if the effective volatility in the intertemporal value of this property as a function of the borrower's degree, θ. Our specification of this correlation can be written in very general terms as the composition function,

$$\widetilde{\varphi} = \varphi(\theta)$$

and, consequently, the lender's perception of risk or the lender's uncertainty over the future value of the property will be:

$$\sigma = \sigma(\widetilde{\varphi}(\theta)) \tag{2}$$

We will use the notation σ, the volatility parameter in the value (1) in the generic asset, for notational simplicity in what follows but we will use this perceived risk or uncertainty on the part of the lender to solve for the presence and magnitude of lending discrimination below.

The initial value of this asset is common knowledge, as is the evolution of this value in (1). Since these features are exogenous and cannot be influenced by either party to the loan, neither moral hazard nor adverse selection are present in this market. While he services the loan, the borrower receives a continuous flow of housing services, $\pi(a,t)$, measurable in market terms. Foreclosure, following a failure by the borrower to pay c at any date t, results in sale of the asset. The lender receives $max[\ a(t) - b(a,t),\ 0]$ from this sale, where $b(a,t)$ is the liquidation cost incurred by the lender, and the borrower receives any residual funds exceeding the unpaid balance $C(t)$.

Bargaining over the terms of the loan contract is represented by the choice of intertemporal strategies by borrower and lender. Their respective strategy spaces include choice of the timing of any exercise of the range of options normally embedded in the loan covenants. The principal element of the space of the lender is, in the current paper, his specification of the initial amount of credit advanced, $C(0)$, and, contingent on default, the date of foreclosure. This space could, in a more complex version of the model, be augmented to include several additional options, including the right to call the loan. The principal strategic elements chosen by the borrower are the timing of both his exercise of the option to default or to prepay the existing loan balance. The strategy of the lender and of the borrower are each selected to maximize, subject to the strategy of his counterparty, the value of his contingent claim on the asset collateralizing the loan.

Denote by $L(a,t)$ the values to the lender of payments received from the borrower and from his option to foreclose and denote by $B(a,t)$ the value of the borrower's position in the contract, also inclusive of his options to default or prepay. Consistent with the traditional features of our economy, application of standard arbitrage pricing methods yield the respective functions $L(a,t)$ and $B(a,t)$, generating these values for all possible combinations (a,t). Solutions for these value functions, under each choice of parameters, represent the respective values of the debt and equity claims on the asset in a perfect Markovian equilibrium.

Solutions to these functions satisfy a pair of partial differential equations, linked by the best-reply strategies selected by each party and by the respective boundary conditions for each equation. This pair of equations is

$$rL = (^1/_2)(a\sigma)^2 L_{aa} + (ra - \pi)L_a + c + L_t \tag{3}$$

$$rB = \left((^1/_2)(a\sigma)^2 (a\sigma)^2 B_{aa} + (ra - \pi)B_a + c + B_t \right) \tag{4}$$

with the corresponding boundary conditions

$$L(\breve{a},t) = max\{0, \breve{a} - \eta(\breve{a},t)\} \tag{5}$$

and

$$L^*(\hat{a},t) = C(t) \tag{7}$$

$$\pi(\hat{a}, t) - c + E_t B^*(\hat{a}, t) = 0 \qquad (8)$$

The term $E_t(\bullet)$ is the expectations operator under the unique equivalent martingale measure induced by our assumption of complete markets and $B^*(a, t) = (e^r dt)B(a + da, t + dt)$ and $L^*(a, t) = (e^r dt)L(a + da, t + dt)$ are the respective risk-adjusted values of the claims of the borrower and lender, discounted at the riskless interest rate. Denoting by $\bar{\bar{B}}(a, t)$ and $\bar{L}(a, t)$ the respective values of the parties' claims if the loan terminates through the exercise of an option by either party, the terms $\breve{a}$ and $\hat{a}$ are the respective asset values triggering default and prepayment b the borrower. At these values the functions $B(a,t)$ and $L(a,t)$ satisfy the value-matching and smooth-pasting criteria. The value-matching condition requires the borrower's value function to be continuous at the respective asset value inducing him to default at date t, $\breve{a}$, or to prepay at date t, $\hat{a}$, as defined respectively by

$$B(\breve{a}, t) = \bar{\bar{B}}(\breve{a}, t) \qquad (9)$$

$$B(\hat{a}, t) = \bar{\bar{B}}(\hat{a}, t) \qquad (10)$$

while the smooth-pasting condition requires the first derivatives of $B(\bullet)$ and $\bar{\bar{B}}(\bullet)$ to be continuous at these same points. The lender's value function is required to satisfy analogous criteria at those distinct points where he would exercise his option to foreclose or, if the game allows this option, to call.

3.3 Numerical Solution Procedure

Since the finite maturity of the loan precludes an analytical solution, we characterize market equilibrium through numerical solutions for the valuation equations and boundary conditions (3)−(8). (Note 19) We use a recursive finite difference procedure to obtain these solutions. (Note 20) This requires representation of the respective strategies of the lender and borrower in terms of subsets of the underlying state space of our game. This state space is defined by all specific pairs of exogenous values a of the asset and corresponding dates t relevant to the respective strategy choices made by the lender and borrower. If the set of all asset values is denoted by A and the compact set of all dates relevant to the loan contract by T, then the state space of the game is A x T, which is the support of the continuum of all possible states (a,t). It is within A x T that the strategies chosen by the lender and borrower are nested.

Any strategy chosen by either of these parties, consequently, can, for the purposes of a numerical solution for equilibrium, be represented as regions (subsets) of A x T. These sets comprising each feasible strategy of the lender and borrower are defined by the property that any realization of the state (a,t) within them triggers the exercise of an option by one of these respective parties.

One element of every strategy of the lender, for example, consists of his choice of the initial balance $C(0) \in A$ at the origination of the loan. A second element is his choice of a date, contingent on default by the borrower, at which he chooses to foreclose. Since, in the current model, the lender will always choose to foreclose immediately at default, the lender's strategy space can be entirely represented by the closed set A x $\{0\}$. Similarly, the strategy of the borrower can be represented in terms of two closed regions of AxT. The first is D, which is that subset of A xT containing all states at which the borrower chooses to default by ceasing the coupon payment. The analogous region, in which the borrower will prepay the outstanding loan balance, should his asset be sufficiently valuable, can be denoted by $P \subset AxT$. The strategy selected by the borrower is the union of these respective regions, $D \cup P$.

The endogenous derivation of numerical solutions for $L(a,t)$ and $B(a,t)$ proceeds by using the values of the claims of each party. Representing A xT as a discrete rectangular grid of a and t values, a solution *for* equilibrium of our game proceeds by calculating the respective values for $L(\bullet)$ and $B(\bullet)$ at each grid point (a,t). These values are determined using the Crank-Nicholson method of calculating the discrete approximation for the partial derivatives in equations (2) and (3). The regions comprising the strategy of each of the parties are then found by calculating, from every point at maturity T, the respective values of the loan to each party. This calculation is repeated at T - 1 and, in general, at each prior grid point $t-1$ accessible from each of the analogous points at t. These values indicate whether, at each such grid point, either party could do better by exercising his options and terminating the loan. A numerical solution for the unique equilibrium of the game, given the choice of parameter values describing the exogenous features of the market, is then obtained by using these values to find the unique 'path,' for each node at T, of (a, t) values through our discrete approximation of the state space A xT for which the subsets we derived constitute best-reply strategies for the parties. The characterization of discrimination in the market is then obtained by comparing the terms and volume of credit for the loan contract in the equilibria generated by each of our choices of alternative parameter values.

4. Results

Analytical solutions for the valuation equations (3)−(4), conditional on the conditions (5)-(8) delineating the

termination regions *(a, t)* in which the respective options held by borrower and lender are optimally exercised, cannot be obtained when these boundaries vary with the term remaining on the loan. The boundaries pertaining to virtually all mortgage loans, for example, will vary in this way owing to their finite maturity. Numerical solutions can be applied in these cases, permitting us to consider the nature of the lending equilibrium in realistic parametric cases and providing additional insight into factors affecting the flow of mortgage credit.

When lenders perceive a direct relation between the degree of one or more demographic traits displayed by borrowers with the salient characteristics of the property securing the loan, best-reply strategies and equilibrium loan contract values to market participants will be conditional on the degree to which a given borrower displays one or more of the observable demographic traits assumed. (Note 21) Since empirical data on mortgage lending discrimination restricts borrowers to belong to one of a very small number of different demographic classes, we will present our results in terms of the restriction that exactly two such classes exist or, equivalently, that Θ has only two elements. (Note 22) We will also, only for purposes of simplicity, also assume that the respective members of each such class wish to finance a property with volatility σ, the quality distinguishing properties in (1)-(2), common to the property of every other member of their class but distinct from those financed by members of the other class. These assumptions are simultaneously reflected in equation (2) for each of the two values of θ.

The presence and magnitude of equilibrium discrimination, as a consequence, can be deduced directly through the comparing the equilibrium loan terms for each of the two classes of borrowers. Selection of alternative sets of values of the parameters determining the stochastic evolution of the property common to the choice of class and those representing the institutional features of the market allow us to compare the properties of the strategies and debt and equity values in the equilibrium arising for each class. Such comparisons allow us to measure the relative difference between loan terms and the volume of credit exchanged in each such equilibrium. We can, therefore, numerically assess the existence and magnitude of systematic lending discrimination in these comparisons.

Since the features most frequently cited in empirical evidence of lending discrimination, such as in analyses of HMDA data in the U.S., are comparisons of the amount of credit obtained at loan origination and selected loan terms, we focus the presentation of our results on these two properties of credit market equilibrium. (Note 23) We use the initial loan balance, which is also the current value of the loan to the lender $C (0)$, to measure the amount of credit and the rate premium π to represent the terms of the loan and compare these values across the equilibria corresponding to alternative sets of parameters describing the exogenous features of the credit market. The existence of a perceived correlation between classes of loan borrowers distinguished by traits unrelated to credit risk and the parametric characteristics of the diffusion process describing the value of the representative asset securing the equilibrium loan contract for each class yields different loan terms and balances offered to the members of each such class.

The exogenous features of the market we consider include the instantaneous mean $\alpha(a,t)$ and, conditional on the borrower's type, θ, the volatility $\sigma(\theta)$ exhibited over time by the collateral asset; the net flow of value $\pi(a,t)$ accruing to equity in the asset; the cost $b(a,t)$ incurred by the lender in liquidating the asset in the event of foreclosure; and any cost $f(C(t))$ incurred by the borrower should he prepay the loan. We assume, for simplicity in the presentation and discussion of the parametric cases below, that the net revenue flow from the asset $\pi(a, t)$, liquidation costs $b(a, t)$ and prepayment costs $f(C(t))$ are all independent of time and homogeneous in their arguments. These allow us to represent conveniently the initial loan balance as a percentage of the initial value of the asset. We interpret as annual the per-period values for the riskless rate and rate premium, the asset volatility and the maturity of the loan as a convenience for the reader. Finally, we note that the equilibrium value of the rate premium on a given mortgage, γ, is determined through the no-arbitrage condition for all debt of the risk corresponding to this mortgage.

We present our results as equilibrium values of the initial loan balance $C(0)$, per corresponding rate premium γ, for alternative values of a chosen exogenous variable, holding constant all other parameters at benchmark values. The benchmark values underlying the results presented below are, respectively, a riskless annualized interest rate of $r = .03$, an annualized proportional volatility (standard deviation) in the value of the collateral asset of $\sigma = .2$; an instantaneous flow of value to equity π of ten basis points, a maturity T of five periods, a one basis point flow of coupon payments c; and both liquidation $b(a,t)$ and refinancing $f(C(t))$ costs of zero. We also specify six gridpoints, or equivalently five 'periods,' over the state space A xT for our calculations of the numerical solutions to equations (2)–(7).

We first consider the influence of liquidation costs on the equilibrium amounts and terms of credit and show that, above a certain threshold rate premium, the lender will rationally ration credit. Table 1 illustrates this by depicting how loan balances $C(0)$ vary with successive two percentage point increases in the corresponding equilibrium rate

premium γ , as the costs b of asset liquidation in foreclosure increase from a 'low value' (10%) to a 'high' value (30%). Higher balances correspond to higher rates in each case, but, as expected, higher liquidation costs reduce the initial loan balance at each rate premium. Averaged over the 12-percentage point range of rates, initial balances are 86.34% and 78.37% at the respective low and high liquidation costs, which is approximately an eight-percentage point difference. Balances also increase, at a decreasing rate, as rates rise from 2% to 8%, but with a lower average increase as liquidation costs increase. Note, in particular, that, as liquidation costs become sufficiently high, rate increases beyond a threshold point (8% in this example) elicit virtually no increases in initial balances. Credit, after this threshold, is strictly rationed.

Table 1. Loan terms: effects of liquidation costs

RATE PREMIUM	CASE ONE ($b = 10\%$)	CASE TWO ($b = 30\%$)
π	$C(0)$	$C(0)$
0.02	71.70	57.60
0.04	82.70	65.00
0.06	89.00	68.00
0.08	93.00	70.70
0.10	95.50	71.80
0.12	97.20	71.90
0.14	98.30	71.90

Note. Table1 illustrates the influence of liquidation costs on equilibrium lending terms. Case One shows that, for relatively low costs (b=10%) of liquidating the asset at default, the initial loan balance is 71.7% at a rate premium of 2%. This balance increases monotonically with this rate until it reaches 98.3% of the asset value at a 14% rate. Case Two illustrates, however, that higher liquidation costs reduce both the amount of credit available at each rate premium as well as rate at which that amount increases at each successively higher rate. Liquidation costs of 30% reduce the initial loan balance to only 57.6% at a two percent premium while the increases in the balance for equal rate increases steadily decline until, at a rate premium of 10%, the balance reaches only 71.90%. True credit rationing occurs after this point, with subsequent rate increases producing no increase in credit. Higher rates provide no increase in value for the lender because liquidation costs imply that the increased risk of default they induce outweighs any increase in the value of higher coupon payments.

Table 2. Loan terms: effects of asset price volatility

RATE PREMIUM	CASE ONE ($\sigma = 15\%$)	CASE TWO ($\sigma = 30\%$)
π	$C(0)$	$C(0)$
0.02	78.10	58.10
0.04	88.70	73.70
0.06	93.60	82.10
0.08	96.20	87.50
0.10	97.70	91.10
0.12	98.70	93.70
0.14	99.20	94.40

Note. Analogously, Table 2 illustrates the influence of annual volatility in asset value on equilibrium lending terms. Case One shows that, for relatively low volatility (σ =10%), the initial loan balance is 78.10% at a rate premium of 2%. Credit available to the borrower increases monotonically with this premium until it reaches 99.2% of the asset value at a 14% rate. This increase in credit, however, exhibits a sharply decreasing rate of increase, from approximately a 14% growth in initial balance as the premium rises from a value of 2% to 4%, to only 1% growth as the premium rises from a value of 12% to 14%. Higher volatility ($\sigma = 30\%$), as expected, reduces the amount of credit available, at each corresponding value of the rate premium, to an average balance of 83% relative to an average of 93.4% at the lower volatility.

Table 2 illustrates the influence of price volatility on credit available at each rate. When volatility is relatively 'low' (σ =15%), for each increase in rates balances increase, but at a rate that decreases sharply from 14% at a 2% premium to 1% at a 12% premium. Doubling the volatility, as in Case Two, reduces the amount of credit available at each corresponding rate at an average decline of 10.2 percentage points relative to the lower volatility. Successive rate increases accompany balances increasing, again at a decreasing rate. Note, however, that in contrast to the analogous comparison involving liquidation costs, the average successive increase in balances is greater when volatility is high than when it is low.

We now illustrate, in the last set of our selected results, the effects on the availability of credit from simultaneous variation in the volatility of asset value and costs of liquidating that asset at foreclosure. Table 3 measures these effects through four parametric combinations corresponding to those in Tables 1 and 2. In the first case, loan terms are shown for 'low' (b=10%) and 'high' (b=30%) liquidation costs, conditional on the 'low' value (15%) of price volatility. The second case repeats this same increase in liquidation costs, but now at the 'high' volatility value (30%).

Table 3. Loan terms: combined effects of liquidation costs and asset price volatility

	VOLATILITY (15%)			VOLATILITY (30%)	
	b = .10	b = .30		b = .10	b = .30
γ	C(0)	C(0)	γ	C(0)	C(0)
0.0200	0.7690	0.7050	0.0200	0.5230	0.4990
0.0400	0.8400	0.7660	0.0400	0.6610	0.5600
0.0600	0.8700	0.7910	0.0600	0.7400	0.6030
0.0800	0.8840	0.7980	0.0800	0.7890	0.6340
0.1000	0.8900	0.8040	0.1000	0.8220	0.6540
0.1200	0.8940	0.8090	0.1200	0.8440	0.6480
0.1400	0.8970	0.8130	0.1400	0.8590	0.6420

Note. The effects on equilibrium loan terms of simultaneous variation in the volatility of asset value and costs of liquidating that asset at foreclosure are illustrated in Table 3. Loan terms, again represented by the rate premium and corresponding initial loan balance, are shown for four combinations of parameter values. In the first case, loan terms are shown for the same 'low' (b=10%) and 'high' (b=30%) liquidations costs as considered in Table 1, conditional on the annual price volatility of 15% used for Case One in Table 2. The second case shows these same loan terms for the same two values of liquidation cost, but now conditional on the annual price volatility of 30% used for Case Two in Table 2.

Three aspects of our results are particularly notable. First, the data in Tables 1-2 clearly illustrate, as expected, that loan balances at any given rate are considerably lower for borrowers when lenders incur higher liquidation costs at constant price volatility or when price volatility increases at constant costs of liquidation. Table 3 illustrates, in addition, that the adverse impact on credit caused by a given increase in liquidation costs or volatility is significantly worsened by a respective increase in volatility or liquidation costs. Consider, for example, that the approximately 6 percentage point decline in average balances caused by a given rise in liquidation costs from 10% to 30% at an annual volatility of 15% increases to an approximately 22 percentage point decline in average balances for the same increase in liquidation costs at an annual volatility of 30%, a difference of 16 percentage points. A similar difference in average balance decline caused by a doubling of volatility occurs when the costs of liquidation increase.

Second, the response of balances to small variations in loan rates also differs in these same situations. Our results in Table 3 suggest that a given increase in liquidation costs will have a significantly smaller effect on the rate balances grow, per unit increase in loan rates, when price volatility is relatively low than when it is high. When annual volatility is 15%, for example, an increase in liquidation costs from 10% to 30% corresponds to a 20 basis point decrease in the average growth of balances per unit rate increase, but the same increase in liquidation costs at an annual volatility of 30% corresponds to a 454 basis point decrease. The opposite is true, however, for the case of a given doubling of price volatility at low, relative to high, liquidation costs.

The third, and most striking, feature of our results is that not only can the 'rationing' of credit occur to different borrowers, based on the characteristics of the assets used to collateralize their respective loans, but, for possible, if extreme, parameter values in the market, increases in the rate premiums that rationed borrowers are willing to pay can actually reduce the loan balance they receive. A comparison of Tables 1–3 illustrates that, as rate premiums rise, the successive increases in balances decline at any combination of liquidation costs and price volatility. This declining responsiveness of balances can appear as incipient or actual credit rationing.   However, when liquidation costs and price volatility are both high, Table 3 demonstrates that, above a threshold rate of 10%, each successive increase of two percentage points in rates actually causes credit to decline by 60 basis points.

5. Discussion

Set in a classical economy devoid of market failures and idiosyncratic preferences, our results prove that lending discrimination on a demographic basis can arise when rational lenders risk-price credit to maximize returns on

portfolios of existing and new loans in the presence of complete markets, arbitrage-free pricing and common knowledge of market parameters. Owing to the strategic exercise by both borrowers and lenders of the options embedded in a typical loan, discrimination will normally be a feature of credit market equilibrium if all lenders perceive a common and positive correlation between those borrowers distinguished only by an observable demographic trait and the relative collateral risk posed to lenders by the properties securing the loans of these borrowers. Equilibrium discrimination will occur if this perception is common to all lenders, regardless of whether it is based on objective or subjective considerations, despite the identical standard measures of credit risk exhibited by all borrowers. (Note 24).

Such discrimination takes the form of loan terms that feature costs of borrowing that increase as the risk to the value of the properties being financed by these demographically distinct borrowers increases. If the properties securing loans to minority borrowers, for example, are anticipated to be less valuable to the lender at the time of a possible default than the properties of other borrowers, then discrimination, in the form of disparate treatment, will be observed.

This result of our model poses a subtle but significant difficulty for explanations of observed discrimination that attribute such discrimination to the preferences, irrationality or cognitive limitations of lenders, or to the presence of such market failures as adverse selection or moral hazard. It also casts some doubt on the robustness of evidence of discrimination when this evidence is based on econometric tests that fail to include those property characteristics identified by our model as influences on the costs of borrowing.

Assume that observers external to a given credit market have, whether by subjective methods or by more formal means, found a repeated pattern of discrimination by lenders correlated with a certain demographic feature exhibited by borrowers bearing no intrinsic relationship to individual credit risk. This behavior by lenders will appear to these observers as inconsistent with the purely economic incentives lenders would face in a competitive and efficient credit market. Under these circumstances, observers will conclude that this market is either allocationally inefficient or inequitable with respect to the welfare of disadvantaged borrowers, or both. (Note 25).

When asked the reasons why disparate treatment of borrowers has occurred in any particular case under investigation, most observers and analysts will respond on the basis of one or more of the traditional explanations described earlier. They might cite potential causes specific to the preferences and abilities of lenders, including demographic prejudice, irrationality, behavioral limits on cognition leading to systematic errors in the decisions made by lenders or other such characteristics. (Note 25) They might alternatively cite reasons based on possible flaws in the credit market itself, such as a common inability of lenders to accurately underwrite loans, perhaps owing to the use of technology inadequate for the accurate measure of credit risk or to the presence of adverse selection or moral hazard on the part of borrowers, or to distortions to lender incentives arising from secondary markets, or to some other type of market failure asserted to be present in this market and possibly in most or all credit markets.

These explanations appear entirely reasonable in most cases. If two borrowers are distinguished, say, by ethnicity, but standard underwriting methods yield identical measures of credit risk for each of them, why would return-maximizing lenders competing in an efficient market offer loan terms which make credit to one qualified borrower more costly than to the other? (Note 26).

Regardless of any potential demographic bias or other idiosyncrasies in their preferences, rational lenders in the given market will protest that they are only responding to market incentives. They will likely contest the accuracy of the finding of disparate treatment or other types of demographic discrimination in their lending practices. When pressed, they may use several common counterarguments, including ones based on technical considerations. These could, for example, include sampling distortions and other flaws in data collection, incomplete or misleading definitions of the variables for which data is collected, bias in the analytical methods used, or a number of other issues. (Note 27) Although extremely unlikely in most situations, one or more of these objections may in fact be plausible in some particular case under investigation.

None of the features underlying these explanations, arguments and counterarguments exist in our model. Yet, under plausible parametric values, the model can generate lending data similar to the real data used as evidence of lending discrimination in this market. How can this be?

Equilibrium discrimination, in its most general form, arises in our model for two reasons. First, lenders and borrowers both behave strategically in order to maximize the value accruing to them from their position in the loan contract. This strategic behavior includes decisions by each party whether and at what time to exercise the options, embedded in the loan contract, over which they exert control. (Note 28) Either party to the contract will choose a

strategy, which includes these exercise decisions, in his own interest and to the detriment of his counterparty. (Note 29) The counterparty, of course, foresees his opponent's incentives and takes them into account when selecting his own strategy. A promise by either party to refrain from actions, such as the exercise of one of his options, that are costly to the counterparty and to act instead in their mutual interest is not credible and is treated as such in the selection of strategies by each party.

Second, both parties are aware that the intertemporal flow of housing services to the borrower and ongoing property depreciation constitute sources of economic value. The borrower, as equityholder, receives this flow of services as long as he services his loan. The lender, however, cannot retrieve the values of past service flows in the event of default. While the prospect of this flow was capitalized, in the classical environment of our model, into the market value of the property at the time of loan origination, the value of this flow is unavailable as security to the lender at the time of a default. Consequently, the collateral value of the borrower's property is diminished to the extent that this flow is higher or the likelihood of its loss through default is greater.

These two features guarantee that those borrowers, even with a standard measure of credit risk identical to other borrowers, who finance properties which pose relatively greater collateral risk to the lender will bear correspondingly higher costs of borrowing in equilibrium. As an example, a lender would prefer that a borrower who defaults does so at the last moment the value of the liquidated property securing his loan fully covers the unpaid loan balance. Since property values can always appreciate, however, the borrower will always choose to delay his irreversible exercise of this option beyond that time. (Note 30) This delay could be lengthy, causing a significant loss to the lender, such as when the value to the borrower of property services exceeds his costs of servicing the mortgage. The probability of this loss is, obviously, increased when the volatility of this property's price is greater. Similar, the lender will foresee a greater magnitude of loss when the flow of these services or his cost of property liquidation is higher.

The case in which discrimination is observed to occur on purely demographic grounds can now be seen to arise immediately when lenders in this market also share a common belief in a correlation between one or more demographic traits distinguishing a borrower and any characteristics of the property that borrower wishes to finance that pose, relative to the properties of other borrowers, a higher risk of loss to the lender in the event of default. (Note 31) These characteristics include greater degrees of price volatility, higher foreclosure and liquidation costs, and rates of depreciation and property services, as featured in our model, as well as any other features of the disadvantaged borrower's property that increases this risk.

Regardless of whether this common perception among lenders arises as a subjective phenomenon or is objectively based on loan data possessed by lenders, when the correlation between collateral risk and the properties purchased by members of a particular demographic group is higher than that for the members of a distinct group, borrowers from this latter group will receive more favorable loan terms than borrowers from the former group. If these features include ethnicity, race or gender identifying the 'minority' group of borrowers, for example, then lenders acting only on the economic incentives they perceive will rationally discriminate in the provision of credit to the minority group, relative to other groups of borrowers. Whether using current econometric procedures or more subjective methods to assess lending data, external observers will see a `pattern and practice' of disparate treatment in regard to the costs of borrowing borne by minority borrowers. When lenders believe that the collateral available to them in the event of a default by these borrowers is significantly less than that of other borrowers, per dollar of property value at the time of loan origination, then our results show that these lenders might actually be rationed in their ability to obtain credit on any terms. (Note 32) Rationing might occur, for example, if, in comparison to other borrowers, lenders believe that foreclosing upon these borrowers in the event of default is more costly as a percentage of property value, or that that these borrowers consume property services at a greater percentage rate, or that their properties exhibit a higher depreciation rate, relative to property value. Indeed, as such costs increase beyond certain endogenous threshold levels, these borrowers could encounter a situation in which lenders offer them initial balances which decline as the loan rate offered by these borrowers increases!

The most extensive documentation of lending discrimination on the basis of differences in racial or ethnic characteristics occurs in the American market for residential mortgage loans. Consider an example in which the neighborhoods of a city are segregated on the basis of one or more features of a minority group and a majority group. This segregation could have occurred from a preference by the members of one or both groups for various locational qualities, such as geographical proximity to other members of their class relative to the other class, or be caused from sources entirely exogenous to the mortgage market, such as the differential treatment of these two groups in regard to the provision by local government of public or private services in their neighborhoods, or from a variety of other

reasons beyond the influence of lenders. If lenders believe that depreciation rates are higher in a given minority neighborhood than in a majority neighborhood, perhaps owing to property crime arising from the relative scarcity of police services in the former neighborhood, or are risk-averse and more uncertain about the appreciation of property prices or foreclosure and liquidation costs in the former neighborhood, perhaps owing to relatively fewer sales or costlier, less frequent or less accurate property appraisals in that neighborhood, or for other reasons, then lenders will perceive that the collateral value of the representative property will be less in that neighborhood than in a majority neighborhood. (Note 33) As a consequence, even if they share similar measures of credit risk with their majority counterparts, prospective homebuyers or current propertyowners in the minority neighborhood will only be able to obtain mortgages (and similar loans) that have significantly less attractive terms.

Whether lenders commonly perceive that properties in the minority neighborhood carry a greater relative risk in their collateral value on the basis of objective evidence or because of an erroneous belief that one or more demographic traits can be used to infer property risk, such differential loan terms will be the outcome of market equilibria in which the personal preferences of lenders are independent of the demographic features of borrowers and act only to maximize the returns to loan portfolios. Moreover, conditional on this common belief, the equilibrium allocation of credit among borrowers will be consistent with an efficient allocation of risk. (Note 34).

6. Concluding Remarks

The objective of this paper is to provide an explanation for the extensive evidence of demographic discrimination in mortgage and other credit markets using contingent-claims valuation and based entirely on the classical assumptions of financial economics. Such an explanation, if successful, would show whether any of the alternative presumptions of preferences marked by demographic prejudice or irrationality, cognitive bounds on decision-making, or market failures and allocational inefficiency in credit markets, underlying existing explanations are prerequisites to explain lending discrimination. It would also, by implication, determine whether current financial regulations or other public policy measures are necessarily useful in enhancing economic efficiency in these markets.

The model in this paper is the first example of such an explanation. Based on an economy exhibiting complete markets, arbitrage-free pricing and common knowledge of all relevant parameters, the model incorporates an explicit consideration of the respective choice of strategies by a representative lender and borrower, including the strategic exercise of the options embedded in the generic form of loan contract observed in actual mortgage markets. When lenders share a common perception that an observable demographic trait distinguishing one group of borrowers from groups is directly related to the relative degree of risk posed to lenders by the characteristics of the properties which secure the mortgages of these borrowers, then, in the presence of rational lenders and absent any source of market failure, our results demonstrate that credit market equilibria will invariably exhibit discrimination in the loan terms and cost of credit to the members of this group. Such discrimination occurs even when, under standard underwriting procedures, the representative members of different groups have similar measures of individual credit risk.

While this conclusion is of real significance to the theory and modeling of mortgage and other credit markets, it also has significant implications for empirical applications of financial economics based on current data and for the design of incentive-based regulations and other public policies intended to mitigate both the efficiency and welfare inequities of such discrimination. These implications concern both the reasons for lender behavior and their response to incentives and the robustness of evidence of lending discrimination based on statistical inferences drawn from actual lending data.

It is important to note that our results in no way deny the existence or possible ubiquity of various types of lending discrimination, such as the disparate treatment of minority mortgage applicants. Neither does it deny that such discrimination can and likely does often arise from demographic bias in lender preferences, errors in lending decisions arising from behavioral limitations or from various sources of market failure. What our results do, however, is demonstrate that these qualities are only sufficient, rather than necessary, conditions for lending discrimination to occur. Evidence of lending discrimination, as a result, cannot imply that lenders are irrational, prejudiced, systematically err in their lending decisions or otherwise perversely respond, relative to standard economic predictions, to market incentives. Nor can such evidence imply the presence of any efficiency failures in credit markets.

Our results also demonstrate the potential fragility of evidence of discrimination based on current econometric analyses of HMDA or similar data. HMDA data, for example, fails to include observations of any features of those properties or other assets securing mortgage loans relevant to the collateral risk to which borrowers are exposed. If the design of econometric tests requires holding constant or equal the credit risk of given pairs or groups of sample borrowers, then the omission of observations or variables measuring the collateral risk from those features could

mean a failure to properly control for the full measure of the credit risk posed to a lender from an individual borrower.

Current statistical evidence of lending discrimination, as a consequence of both results, is subject to misinterpretation and is, in and of itself, an insufficient basis on which to design or adopt financial regulations intended to enhance the economic efficiency of residential mortgage lending. The results in this paper also encourage caution in implementing policies to mitigate inequities in the relative welfare of disadvantaged borrowers. Using only traditional economic models and empirical evidence which may be less than robust in ascertaining the nature of the incentives to which lenders respond in actual mortgage markets, redistributive policies may be effective in their purpose or may alternatively reduce the welfare of all borrowers through a distortion of the incentives to lend and borrow.

More generally, this paper also demonstrates the feasibility of applying the contingent claims method of valuation to the analysis and explanation of differences in credit available to borrowers based on factors other than the conventional statistical measures of individual credit risk used in underwriting mortgage loans. The advantages of using a complete market and complete information-common knowledge environment are compelling in attaining a benchmark measure of the efficiency of lending and the allocation of credit based on the use of standard mortgage and other debt contracts. It permits one to describe equilibrium option exercise strategies by both borrowers and lenders that are independent of risk attitudes, endowments and divergent expectations about the future value of the assets securing these contracts. It enforces consistency between collateral characteristics such as dividend-like flows of services from these assets and no-arbitrage variations in their market value. It offers the promise of an operational method of calculating the respective values of a particular mortgage loan, at any moment, to both lender and borrower under empirically plausible conditions. Finally, it could also, as a consequence, provide the means of accurately measuring both the credit risk ultimately borne by both borrowers and lenders and the disparities in the costs of credit to borrowers belonging to different demographic groups. This has the potential to enhance both the accuracy of underwriting methods and the efficacy of public policy in reducing any existing inefficiencies in credit allocation and in reducing or eliminating inequities in the relative welfare of affected   borrowers.

Acknowledgements

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Notes

Note 1. Discrimination in credit markets occurs when borrowers of one type receive different terms or volume of available credit than other types of borrowers, holding all else equal, on the basis of one or more qualities distinguishing borrower types. Discrimination on the basis of observable demographic traits which appear unrelated to credit risk is most often classified as either 'disparate treatment,' in which a borrower receives relatively unattractive loan terms owing directly to is membership in an identifiable demographic cohort (usually defined by race, ethnicity or gender), and `disparate impact,' in which borrowers of different such cohorts transact under criteria, imposed by the lender, that disadvantage members of a particular cohort. Descriptions of different forms of lending discrimination can be found in A.B.A. (2012) and an overview of current U.S. fair lending regulations and statutes can be found in Federal Reserve Board (2005).

Note 2. Campbell, Jackson, Madrian and Tufano (2011) provide a survey of these explanations.

Note 3. Discrimination in lending can take various forms, including differential costs of loan origination, different loan terms and balances, loan covenants differing over classes of borrowers, overages charged at origination, or systematic denials of loan applications and even the possible rationing of credit offered to certain types of borrowers. The seminal empirical study on discrimination in underwriting and loan approval rates was Munnell, Tootell, Browne and McEneaney (1996) while Courchane and Nickerson (1997) provided the original analysis of discrimination through mortgage overages. Comprehensive surveys of both empirical and economic explanations of various forms of mortgage lending discrimination appear in Ladd, (1998), LaCour-Little (1999), and Turner and Skidmore (2000).

Note 4. Ladd (1998), LaCour-Little (1999) and Turner and Skidmore (2000), among others, provide comprehensive surveys of the empirical evidence of lending discrimination in mortgage markets and the corresponding economic explanations of it.

Note 5. Neither approach is, however, fully satisfactory. The first approach implies that management of financial institutions systematically fails to maximize the value of shareholder equity while the second does not apply to discrimination between borrowers which is based on borrower qualities that are both observable and independent of credit risk.

Note 6. The primary data used in providing statistical evidence of lending discrimination in the U.S. is compiled annually by the Federal Reserve Board under the auspices of the he 1975 Home Mortgage Disclosure Act (HMDA) and is known as HMDA data. This data consists of observations of the number and dollar amount of both home mortgage and home improvement loans, by census tract or county, and more recently of the race, gender and income of loan applicants, from by mortgage lending institutions with assets of $10,000,000.00 or more. A description of HMDA data can be found in Federal Reserve Board (2015).

Note 7. Econometric evidence attributing discrimination, usually in terms of disparate treatment, to differences in demographic traits requires that such tests control for differences in borrower risk measured through standard statistical underwriting and regulatory review procedures. Avery, Brevoort and Canner (2007) and Lubin (2008) survey alternative econometric methods of testing for demographic discrimination in mortgage lending as well as the properties of corresponding lending data.

Note 8. Our model, it should be noted, is quite general and applies to any credit market featuring standard debt contracts with finite maturities, fixed terms and rates and limited contingent covenants, such as found in a wide variety of corporate and public fixed-income liabilities.

Note 9. This condition of efficiency is, of course, conditional on the properties of the standard form of mortgage loan contract and similar credit products that appear in the U.S. and elsewhere.

Note 10. This remains true even when such contracts are designed to screen individual loan applicants on the basis of their individual credit risk.

Note 11. Bresson and Priuli (2006), Josa-Fombellida and Rincon-Zapatero (2008), Ewald and Xiao (2009), and Bensoussan, Siu, Yam, and Yang (2014) analyze perfect Markov equilibria in stochastic differential games with properties similar to those of our model.

Note 12. Our mortgage contract subsumes any fixed-rate mortgage with a finite maturity and with any realistic provisions for foreclosure and partial or full collateralization, inclusive of the property being financed. For example, although we treat the mortgage contract in our model as non-recourse, our results can easily be applied to partial or full recourse mortgage loans by reinterpreting the asset securing the loan to include portions of the borrower's portfolio in addition to the property being financed. Campbell (2012) provides an excellent overview and analysis of contractual structures for mortgage debt, while similarities and differences between American and European mortgage contracts are surveyed by the European Mortgage Federation (2016), Harris and Meir (2016), Coles and Hardt (2000) and many others. Courchane and Giles (2002) provide a comprehensive comparison of mortgage contracts in the United States and Canada.

Note 13. This flow of property services is, consequently, exactly analogous to the flow of dividends on a share of corporate stock.

Note 14. These properties of the standard form of American mortgage contracts are documented in Keyles (1995).

Note 15. Although realistic to model depreciation as endogenous and, immediately prior to default, large, we will regard the rate of depreciation as exogenous, both for simplicity in describing the dimensionality of the borrower's strategy space and because we assume borrower preferences are devoid of enmity toward lenders. Observe that an endogenous depreciation rate, or equivalently, the endogeneity of the flow of housing services, would exacerbate losses to the lender arising from the standard form of residential mortgage contracts and, consequently, result in harsher loan terms to borrowers.

Note 16. Our model and results could, as easily, apply to mortgage contracts with recourse through the reinterpretation of a as the portfolio of assets, the value of which is given by (1), serving as collateral for the mortgage loan. Note also that we focus our analysis on standard mortgage contracts, which, in the spirit of Townsend (1979), Lacker (2000), Hvide and Leite (2009) and others, are motivated by implicit considerations of the costs to state verification. Although our model could apply to more elaborate debt contracts, which in theory could be

indexed or made contingent, at each moment, on the random value of collateral, we presume in this paper that such alternative arrangements are infeasible because of regulation, index risk, uncertain legal enforceability, or simply, as described by Keyles (1996), the actual difficulties in negotiating fully--contingent debt contracts. See Bolton and Dewatripont (2005) for a comprehensive review of costly state verification and the costs of contingent financial contracts

Note 17. The index φ could be interpreted as a vector but for simplicity (without loss of generality) we will treat φ as a scalar. Analogously, the set Φ is, again for simplicity, assumed to be discrete but could equally well be interpreted as a continuum.

Note 18. Analogously to Φ, the set Θ could, in general, be interpreted as a continuum of vector valued θ, but again for simplicity we treat the set Θ as discrete and the index θ as a scalar.

Note 19. The perfect equilibrium of the lending game is described by the linked partial differential equations (6) and (7), combined with the free boundary conditions (9) and (10), characterizing lender and borrower strategies. Although these differential equations cannot, in general, be solved analytically, an analytical solution, following Merton (1976), can be derived when the mortgage loan has no finite term to maturity, or equivalently is an infinitely-lived, fixed-rate, interest-only mortgage. Since such a mortgage is of little empirical interest, the solution is omitted from this paper, instead focusing on numerical solutions for realistic choices of market and contract parameters. The solution and its proof is available on demand from the author.

Note 20. A standard reference to the implementation of the finite-difference method we use is Trottenberg, Oosterlee, and Schuller (2000).

Note 21. The two fundamental sources from which such a perception arises are location and the value of services flowing to owners from the features of a representative property. See below for specific examples.

Note 22. This is done without loss of generality. The binary values of Θ can be interpreted to designate 'majority' and 'minority' borrowers, 'white' and 'black' borrowers, 'male' and 'female' borrowers, and so on, depending upon the nature of the data to which the model is applied.

Note 23. Prominent examples of such analysis include Avery, Beeson and Calem (1997), Avery, Brevoort and Canner (2007).

Note 24. A common perception based on subjective impressions of collateral risk and demographic characteristics is distinct from the usual concept of demographic prejudice, since the representative lender's preferences have no necessary ranking over demographic traits. An objective perception may, for example, take the form of statistical evidence of correlation between these two variables based on internal loan data similar across lenders or possibly from herding behavior among rational and prejudice-free lenders, a phenomenon observed among equity traders and empirically confirmed in Kallinterakis and Andronikidi (2010) and Economoua, Gavriilidis, Goyald and Kallinterakis (2014), among others.

Note 25. Nickerson, Nebhut and Courchane (2000) and Clarke, Roy and Courchane (2009), among others, examine the robustness of findings of disparate treatment obtained from various econometric tests applied to both public data and proprietary underwriting data from private banks.

Note 26. Owing to the assumptions in our model, variation in loan terms cannot occur through economic price discrimination, search or other transactions costs generating price dispersion when the market in question conforms to the market in our model. See Avery, Brevoort and Canner (2012) for analysis of whether statistical underwriting methods could produce disparate treatment on grounds other than these.

Note 27. See Turner and Skidmore for details.

Note 28. Note that the reasons we cite are ultimately the result of the generic, non-contingent form of loan contract used in most real credit markets.

Note 29. The 'zero-sum' nature of these actions assumes that the value of each participant's position in the loan contract cannot be affected by binding side-payments or other transactions involving credible commitment or cooperation.

Note 30. Renegotiation of the terms of the loan or legal provisions allowing loan redemption by the borrower are qualitatively irrelevant to this conclusion if the lender bears foreclosure costs.

Note 31. Observe, from equations (7)-(10), that, since the composition function (2) describing housing price volatility is continuous, decreasing values of this correlation commensurately reduce the costs of a disparity in loan

terms received by demographically distinct borrowers. .

Note 32. This, of course, is the path-breaking result of Stiglitz and Weiss (1981), but our results prove it can occur even in a market with common knowledge of the salient parameters by all market participants.

Note 33. While the example here pertains to a situation in which neighborhood location and the demographic features of the residents of that neighborhood influence discrimination through their combination, lending discrimination based on location only has been studied in models incorporating asymmetric information, such as in Lang and Nakamura (1993). Evidence of the occurrence of this combination is given by Feinberg and Nickerson (2002), who provide an empirical analysis of the relationship between neighborhood crime rates and property values for all fifty American states, and by Seitzer (2008) and Leonard, Jha, and Zhang (2015) in regard to foreclosure costs. Other studies of relationships between property location and lending discrimination include van Order, Westin and Zorn (1993), Galster (2003), Deng, Ross and Wachter (2003), Wyly, Atia and Hammel (2004) and Li, Hossain and Ross (2010).

Note 34. Obviously the magnitude or frequency of the difference in loan terms would be reduced or eliminated if the underlying reasons for this common belief among lenders were partially or fully eliminated. If this belief is based on an objective analysis of existing data, then, for example, any reduction to the difference in the respective risk to collateral values in new data would mitigate the difference in loan terms, as would dispersion in the degree to which lenders hold this belief in common.

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