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Jump Volatility Estimates of High Frequency Data and Analysis Based on HHT

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Abstract

As the global financial market risk increases, countries stress more onthe management and prevention of financial risks. These financial risks come from the volatility of the market, and thus we can build more comprehensive understanding of financial markets by analyzing the composition and the law of the financial volatility in different frequency. Based on Hilbert Huang Transform, the realized volatility analysis model is establishedto decompose the volatility into various signal in dissimilar frequency. First of all, the realized leap volatility is obtained through the previous research findings and Capital Asset Pricing Model. Then, considering thenonlinearity and instability of the volatility, we use the Hilbert Huang Transform to decompose the volatility and obtain IMFs in different frequencies and trend functions.

Keywords: capital asset pricing model, realized leap volatility, hilbert huang transform

1. Introduction

Modern scientific and technological progress as well asthe progress of financial innovation enhance the liquidity and volatility of product in the financial market. The fluctuation in the modern economy is severely destructive and infectious so that it is difficult to guard against. Furthermore, the volatility in the modern economic and financial market is instant and interactive. Therefore, the management of volatility and prevention of risk are of great importance.

The volatility of asset returns has always been of great importance in many financial applications. The framework of mean-variance analysis of Markowitz (1952) is considered to be the cornerstone of modern securities analysis theory. This method is widely used in the optimal decision of asset allocation. Based on volatility behavior of the BS model in 1973 (Tsay, 2002), value at risk model has been established. Then, in terms of volatility estimations, Parkinson (1980) has used the highest and lowest price to estimate volatility. Garman and Klass (1980), Rogers and Satchell has calculated volatility through highest, lowest, opening and closing price, setting the foundation of volatility research by Rogers and Satchell (1991).

The jump behavior is studied inmany documents using statistical t-test in order to obtain realized volatility. However, considering that it can only be judged whether jump behavior exists and how many times jump behavior occur, to solve this problem, the realized volatility analysis model is established based on Hilbert Huang Transform as a means of the signal analysis method.

2. The Modeling

2.1 Volatility and Asset Prices

Firstly, we determine the object of the research. There are three general types of volatilities: instant, implied and historical volatility. Instant volatility refers to changing fluctuation in the general continuous time model, which is very difficult to measure. Implied volatility generally refers to volatility parameters used to determine the option value in the BS option pricing model. Historical volatility refers to the actual range of pricing fluctuation in a certain time period of the past. This paper studies historical volatility.

2.1.1 Estimated Method of Dynamic

As for historical volatility, we determine the sample intervalfrequencyto estimate daily volatility in the first place. In general, the higher the frequency is, the more accurate estimated volatility is. According to the foundations of theoretical research in realized volatility, Rogers and Satchell (1991) has developed a more precise calculation method of volatility on the condition that drift of the volatility is not equal to zero. It can be described as follows:

$$\sigma^2{}_{full\_d} = \frac{1}{N}\sum_{i=1}^{N}(\ln(\frac{H_i}{C_i})\ln(\frac{H_i}{O_i}) + \ln(\frac{L_i}{C_i})\ln(\frac{L_i}{O_i})) \tag{1}$$

Where, H_i is the highest price, L_i is the lowest price, O_i is the opening price and C_i is the closing price.

Next, we estimate the volatility dynamically by using the RiskMetrics exponential smoothing estimation method. This method overcomes the drawback of saltation in volatility estimationand is also a form of the *GARCH* model.

$$\hat{\sigma}_t{}^2 = \lambda\hat{\sigma}_{t-1}^2 + (1-\lambda)\sigma^2{}_t \tag{2}$$

Where, $\hat{\sigma}_t{}^2$ is the estimated value on the t[th] day, $\sigma^2{}_t$ is the actual value on the t[th] day, λ is the model parameter.

2.1.2 Capital Asset Pricing Model

In this paper, we use the data per 5 min to estimate the daily volatility. This is the basis of realized volatility.

1) The composition of asset prices

In the framework ofmodel with jump behaviors, asset prices usually consist of two parts:

$$p(t) = p_1(t) + p_2(t) \tag{3}$$

Where $p_1(t)$ is martingale process with a continuous sample path.

$$p_1(t) = p_0 + \int u_t dt + \int \sigma_t dW_t \tag{4}$$

Where, u_t is the drift process with continuous and locally bounded variation. σ_t is the stochastic volatility process, $\int \sigma_t dB_t$ is thestandard Brownian motion process.

2) $p_2(t)$ is a stochastic point process independent of $p_1(t)$.

$$p_2(t) = \int \kappa_t dq_t \tag{5}$$

Where, q_t is the standardized counting process. If a jump behavior happens at the moment of t, then $dq_t = 1$. Otherwise, $dq_t = 0$. κ_t is a random variable of jumping behavior intensity at the moment of t.

3) The determination of continuous time model containing jump behavior

$$p(t) = p_0 + \int u_t dt + \int \sigma_t dW_t + \int \kappa_t dq_t \tag{6}$$

Then, we use the geometric levy model of Ma Hucheng, Wang Xianzhen (2009) to make accurate estimation of σ_t and κ_t, thus obtaining the dynamic model.

2.1.3 The Leap Model Based on Realized Volatility

In continuous jump-diffusion model, QV is equal to the sum of variance of continuous sample path (IV) and the variance of the jumping portion (JV). It can be described as follows:

$$QV = IV + JV = \int_0^1 \sigma_s{}^2 dB_s + \sum_{s=0}^{1} \kappa_s{}^2 \tag{7}$$

Where, σ_s is the stochastic volatility process; κ_s is the random jump process; B_s is the Brownian motion.

To study whether the price jumps and what is the intensity of the jumping, the estimation of QV and IV should be fulfilled accurately respectively. Thus, the deduction to variance of jumping behavior (JV) to determine whether jumping behavior has occurred.

We discretize the continuous time, which is to transform the equation from $QV = IV + JV$ in continuous time to $RV = BV + JV$ in discrete time. And we calculate RV and BV using the method of Eric Poon (Barndorff-Nielsen & Shephard, 2004) and Rockinger (2007). The calculation can be described as follows:

$$QV = \int_0^1 \sigma_s^{\,2} dB_s + \sum_{s=0}^{1} \kappa_s^{\,2} \longrightarrow RV = \sum_{j=1}^{I=1/\Delta} (p_{j\Delta} - p_{(j-1)\Delta})^2 \tag{8}$$

$$IV = \int_0^1 \sigma_s^{\,2} dB_s \longrightarrow BV = (\sqrt{2/\pi})^{-2} \sum_{j=1}^{I=1/\Delta} | p_{j\Delta} - p_{(j-1)\Delta} || p_{(j+1)\Delta} - p_{j\Delta} | \tag{9}$$

$$JV = RV - BV \longrightarrow \sum_{s=0}^{1} \kappa_s^{\,2} \tag{10}$$

In order to guarantee the credibility of practical estimator of JV , this paper uses the approach of significant test defined by Andersen, Bollerslev and Diebold to carry on the test of significance (Bollerslev & Diebold, 2007). Then we use the method of realized jumping estimation defined by Tauchen and Zhou (2006) to study the jumping behavior (Tauchen & Zhou, 2006).

2.2 The Decompositionmodel of Volatility Based on Hilbert Huang Transform

Analysis of the stock market volatility can be likened to the process of signal processing and analysis. Signal processing methods include Fourier transform, wavelet analysis and Hilbert Huang transform adopted in this paper. This section introduces the outline of Hilbert Huang transform, the reason why we choose it and its improved method.

2.2.1 Introduction of Hilbert Huang Transform

The Hilbert-Huang Transform (HHT) is an empirically based data-analysis method. Its basis of expansion is adaptive, so that it can produce physically meaningful representations of data from nonlinear and non-stationary processes.

The advantage of being adaptive has a price: the difficulty of laying a firm theoretical foundation.

The HHT consists of two parts: empirical mode decomposition (EMD) and Hilbert spectral analysis (HAS). This method is potentially viable for nonlinear and nonstationary data analysis, especially for time- frequency- energy representations.

The development of the HHT was motivated by the need to describe nonlinear distorted waves in detail, along with the variations of these signals that naturally occur in non-stationary processes.

Hilbert Huang Transform is proposed by Norden E. Huang (fellow of Academia Sinica) in 1998. In Hilbert Huang Transform, the data is decomposed into intrinsic mode functions (IMF), and the decomposition process is known as Empirical mode Decomposition (EMD). This method can correctly get the instantaneous frequency of the data.

IMF has two characteristics. Firstly, the extremum is followed by zero-point. Secondly, the average value of the upper envelope defined by local maximumsand lower envelope defined by the local minimumsis close to zero. IMF is similar to the wave (sinusoid-like), but the cycle and amplitude of these portions similar to the wave are notfixed.

Processing steps are as follows:

For $X(t)$, we describe it in Hilbert Huang Transformation:

$$\hat{X}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau = X(t) * h(t) \tag{11}$$

Toform a complex conjugate, we consider $X(t)$ and $\hat{X}(t)$ as the real part and imaginary part respectively to reconstruct the signal as $Z(t)$.

$$Z(t) = X(t) + i\hat{X}(t) = a(t)e^{i\theta(t)} \tag{12}$$

The instantaneous amplitude $a(t)$ and instantaneous phase $\theta(t)$ are:

$$a(t) = \sqrt{X(t)^2 + \hat{X}(t)^2} \tag{13}$$

$$\tan\theta(t) = \frac{\hat{X}(t)}{X(t)} \tag{14}$$

Instantaneous phase $\theta(t)$ is the derivative of instantaneous frequency $\omega(t)$:

$$\omega(t) = \frac{d\theta(t)}{dt} \tag{15}$$

These analytic signal, known as frequency, can be obtained by the Hilbert transform from original signal. Furthermore, we could extracted such characteristic parameters as signal instantaneous amplitude, instantaneous phase and instantaneous frequency. Therefore, several stationary IMFs areobtained.

The result is as below:

$$s(t) = \sum_{k=1}^{n} c_k(t) + r_n(t) \tag{16}$$

Where, $c_k(t)$ is the k[th] stable IMF. $r_n(t)$ is the trend function.

2.2.2 The Reason to Choose Hilbert Huang Transform

Table 1. Comparison of Fourier, Wavelet and Hilbert

	Fourier	Wavelet	Hilbert
Basis	a priori	a priori	adaptive
Frequency	convolution: global uncertainty	convolution: regional uncertainty	differentiation: local, certainty
Presentation	energy-frequency	energy-time-frequency	energy-time-frequency
Nonlinear	no	no	yes
Non-stationary	no	yes	yes
Feature Extraction	no	discrete: no; continuous: yes	yes
Theoretical base	theory complete	theory complete	empirical

1) Adaptively

The base of Fourier transform is the trigonometric functions, and the base of Wavelet transform is the wavelet base meeting capacitive conditions. Choosing the “base” correctly is very difficult, but considering the self-adaptation of Hilbert Huang transform， it is unnecessary to consider to choose the right base.

2) Nonlinear and non-stationary

Hilbert Huang transform is different from traditional methods becauseit completely prevent the issues of linearity and stability, so that HHT is a suitable approach to analyze nonlinear and nonstationary signals.

3) Impregnability from Heisenberg’s uncertainty principle

Fourier transform and wavelet transform are subject to the limitation of the Heisenberg’s uncertainty principle, which requires that the product of the time window and frequency window is a constant which means the time accuracy and precision of frequency cannot achieved simultaneously. Nevertheless, this problem could behandled by HHT, thus this situation makes it very suitable to analyze mutational volatility in real financial market.

Due to those three reasons above, Hilbert Huang transform is the most suitable method for the analysis of the volatility of the stock comparing to others.

2.2.3 Improved Hilbert Huang Model

In the process of Empirical Mode Decomposition (EMD), there will be a mixed mode problem. Mixed mode problem is that there are mixed signals in different scales in the same intrinsic mode function, or signals in the same scale appear in different intrinsic modal functions.

To solve the problem of mixed mode, we employ the overall empirical mode decomposition method, which is the noise-assisted data analysis. In the beginning, white noise is added into signals. Then the signal is handled by empirical mode decomposition, and the two steps should be repeated for several times in order to obtain several groups of intrinsic modal functions. Finally we average Intrinsic Modal Functions (IMF) in each group to offset the impact of the noise.

3. Empirical Study of Volatility Model in China

In recent years, with the increasing fluctuation of international financial market, countries around the world face common issues about how to prevent the volatility risk from financial markets. Firstly, by taking five stocks from China securities market as the research object, we use the realized volatility signature diagram to determine the optimal sampling frequency, which is the time interval from each sample. Then, we calculate the variance of jumping volatility by the concept of realized volatility. Afterwards, we obtain the IMFs from different frequencies and trend functions after decomposing the volatility by Hilbert Huang transform. Finally, we make empirical analysis of realized jumping volatility by ways of the duration of influence, the degree of influence and the correlation of influence respectively.

3.1 Selection Rules of Data

3.1.1 The Determination of Selected Stock Data

To reflect the comprehensiveness, we select five stocks which could demonstrate China Financial Markets.

From macroscopic aspect, two type of index are selected including Shanghai Composite Index and Shenzhen Component Index which contain almost all the stocks in China Stock Markets. These indexes embody the trend of China Financial Markets for its integrity and wide influence. From microscopic aspect, according to the rank of circulation market value from Tidal Wave 1000 Index, Vanke (SZ000002), from top 200, are regarded as the large-cap stock and Dongfeng Motor (SH600006), from rank 201 to 500, are considered as the middle-cap stock while Yangtze river (SH600119), from rank 501 to 1000, refers to the small-cap stock.

3.1.2 The Determination of Time Scales of Data

For appropriate interval frequency, we should determine time interval of data by Realized volatility Signature Figure Method.

Realized volatility signature figure is a common tool to determine optimal interval frequency. The basic idea is that we could draw a signature figure by calculating the variance from different interval frequency of Realized volatility, therefore determine the optimal interval frequency whose variance is the average of the whole variance. As a result, the optimal frequency is characterized by the stability of sample average because it is the result after correcting the noise from market microstructure. Here is the formula of realized volatility signature figure:

$$JV_k^{mean} = \frac{1}{N}\sum_{t=1}^{N} JV_{t,k}^{(sparse)} \tag{17}$$

Where, $JV_{t,k}^{(sparse)}$ is the variance of realized volatilityfrom different frequency per k min. JV_k^{mean} is the mean among JV with the frequency of k minute.

Realized jumping variance signature figure is depicted according to the Shanghai composite index (SH1S0001) data from January 2, 2008 to June 30, 2009.

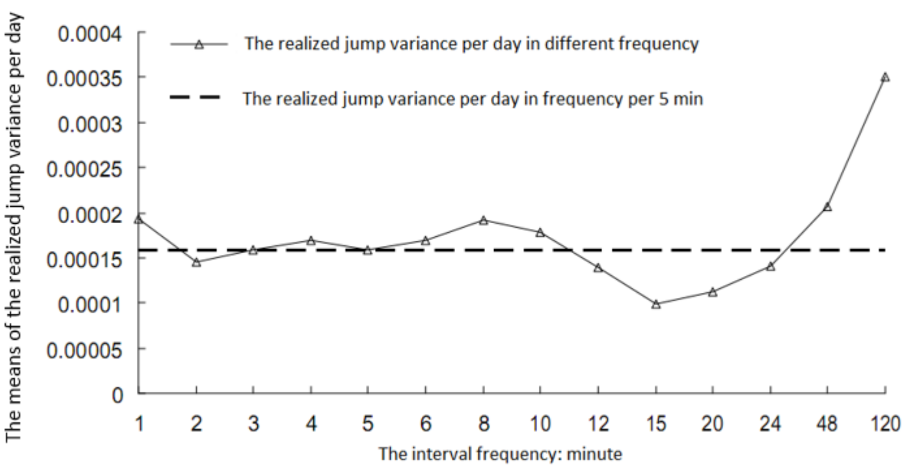

Figure 1. Signature figure of realized jumping variance

From the Figure 1, the means of realized jump variance reach to the relatively stability states in the interval frequency of 5 minutes. Plus for the sake of convenience, this paper selects frequency per 5 min as sampling interval frequency, whose scaleis from January 2, 2001 to August 1, 2008

3.2 The Calculation of Volatility

Rogers and Satchell (1991) developed a calculation method to estimate the volatility when the drift is not equal to zero, and estimate the volatility per day (Rogers & Satchell, 1991).

According to the discrete form of realized leap assets model, we compute realized volatility thus obtain the components of continuous sample and the Jump volatility.

On the basis of measurement of quadratic power variation, we have profound research on the jump behavior of realized volatility in terms of Shanghai Composite Index. The realized volatility is decomposed into the variance of continuous sample path and discrete jump variance so that we could research the statistical characteristics of jumping variance sequence and predict realized volatility from Shanghai Composite Index with HAR-RV-CJ model.

The results demonstrate that almost all kinds of the predictability of realized volatility, such as per day, per week, and per month frequency in data, stem from path variance of continuous sample. It shows that the determinants of predictable realized volatility in China stock market is continuous sample path in the quadratic power variation.

$$\begin{cases} RV = \sum_{j=1}^{I=1/\Delta} (p_{j\Delta} - p_{(j-1)\Delta})^2 \\ BV = (\sqrt{2/\pi})^{-2} \sum_{j=1}^{I=1/\Delta} | p_{j\Delta} - p_{(j-1)\Delta} || p_{(j+1)\Delta} - p_{j\Delta} | \\ JV = RV - BV \end{cases} \tag{18}$$

Taking the Shenzhen Component Index as an example, we demonstrate the data and as below. Other stocks are processed by the same methods.

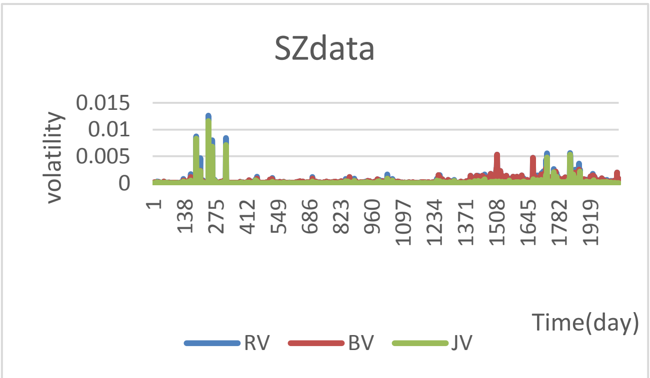

Figure 2. Jump behavior from realized volatility in Shenzhen Component Index

By preliminary analysis, the jumping times from different stocks are described in statistics Table 2.

Table 2. Frequency statistics of realized jumping volatility

	Shenzhen	Shanghai	Wanke	Dongfeng	Changjiang
Data Bulk	2046	2019	1974	1916	1897
Jump times	529	205	654	800	849

We could conclude that large-cap stock jumps not too often, and a small-cap stock jumps more frequently.

Furthermore, the fact that the jumping ranges from small-cap stock is much wider than counter part indicates that the microscopic market, in line with the actual situation, is more active.

In order to undertake an in-depth study on the jump behavior of realized volatility, decomposition on datum is conducted by Hilbert Huang Method.

3.3 Hilbert Huang Decomposition

HHT is adopted to decompose RV , BV and JV respectively to handle the problem of nonlinear and non-stability of volatility. Here is the result:

$$s(t) = \sum_{k=1}^{n} c_k(t) + r_n(t) \tag{19}$$

Where, $c_k(t)$ is the k^{th} stable IMF , $r_n(t)$ is the trend function. Then we analysis volatility though instantaneous frequency.

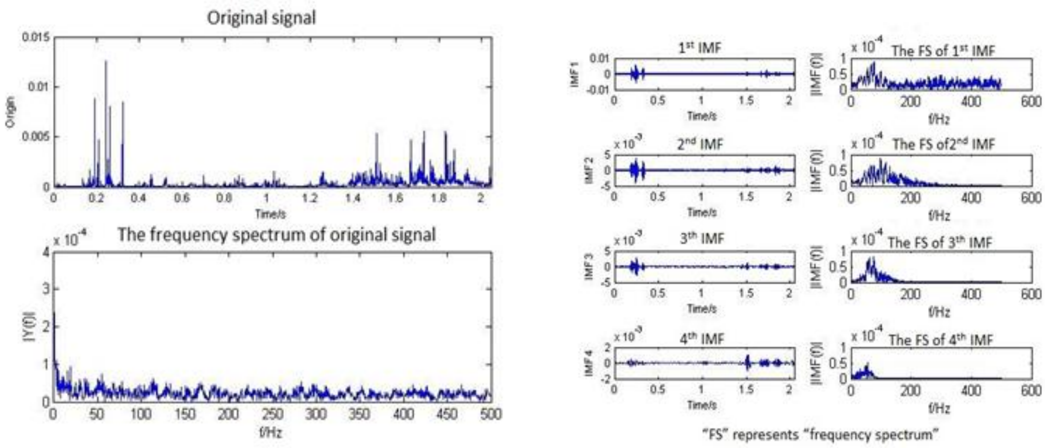

Figure 3. Hilbert Huang decompositionon total volatility (RV) in Shenzhen Component Index

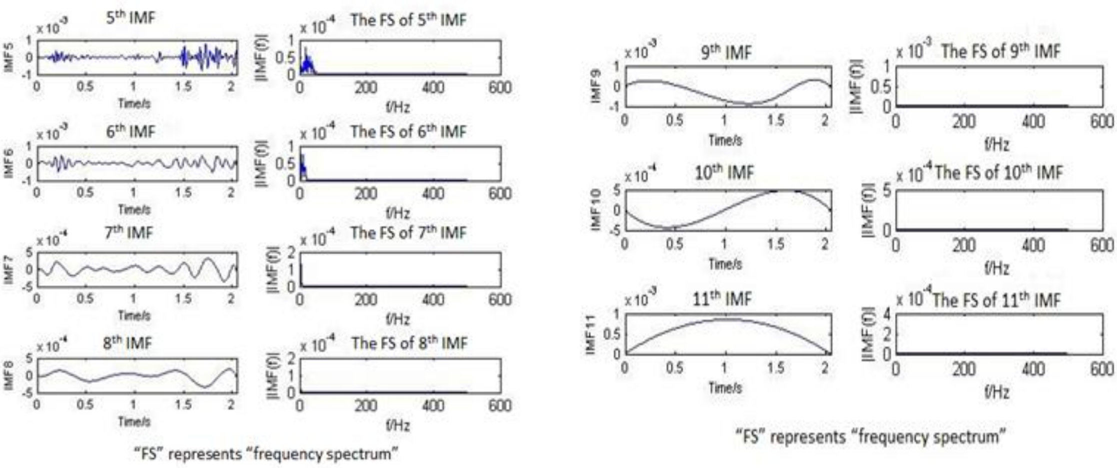

Figure 4. Hilbert Huang decompositionon total volatility (RV) in Shenzhen Component Index

From the Figure 3 and Figure 4, we can tell that because of the self-adaptability of HHT, it determine the number of IMF automatically. The results show that RV , BV and JV in five stocks happened to be decomposed into 9, 10 or 11, which partly reflect the similarity of volatility.

From the perspective of extent of volatility,by analyzing all the stock volatilities, we found that the dot product of IMF1 is the largest which means that the most affected volatility frequency is IMF1.

From the perspective of frequency of volatility. The frequencies of first several IMFs are extremely high, and its influence extent is relatively high so that such IMF, to some extent, could be regarded as main interference.

Above two ways, former IMF contribute more to the influence because of their high frequency and high wave range while later IMFs contribute less for the low frequency and low wave range so that former IMFs, to some extent, could represent macroscopic market while later IMFs represent microscopic market.

Decomposition on volatility is only the first step to analysis. With the presence of linear and stable volatility, it is easier for researchers to conduct characteristics analysis, risk management, risk prediction and so on.

4. Conclusion

Based on Hilbert Huang transform analysis, we establish realized volatility model and make an empirical analysis to China's financial market.

First of all, according to the previous research of volatility, five stocks in China's stock market, representing large-cap, mid-cap and small-cap stocks respectively, are utilized to calculate realized jump volatility and to analyze its jump behavior. Then, in order to solve problems of nonlinear and non-stability status of datum, the decomposition of volatility is conducted by using the Hilbert Huang transform to form different stationary frequent volatilities (IMF) and trend functions. According to the frequency and amplitude from IMF, we obtain different meanings from IMFs. For example, to some extent, some frequency IMFs represent the microcosmic noise.

Through the decomposition of volatility, we obtain different kinds of stable and linear volatility (IMF), which provides a technology preparation for volatility characteristics analysis, and provides a new perspective into the research of volatility as a means of signal analysis.

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