On the Design Method of Economically Optimal Size of Projects

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Abstract

This paper analyzed the most disputed issue on the economically optimal size design of the size-variable projects. The paper confirms that the concept that the size at which the NPV of the size-variable project getting its maximum value is the economically optimal size is incorrect and that the size is the maximum size rather than the economically optimal one. The economically optimal size can be only restricted within the range in which NPV & NPVR get their maximum value respectively as its upper and lower limits, and the economically optimal size within the range can only be worked out according to the concrete case. Two kinds of different models optimizing the size of the size-variable projects are set up in the paper.

Keywords: Economically optimal Size, NPV (net present value), NPVR (rate of net present value)

1. Introduction

One kind of projects such as dams, buildings, roads and so on the size (such as their height, width, etc.) of which can be varied are called size-variable projects. The others are called size-fixed projects, just as its name implying, the size of size-fixed project is fixed, they cannot be segmented, either the whole or nothing. On the other hand, the sizes of the size-variable projects are variable, and then how large the size of the project should be? How to determine the suitable size of a size-variable project? Most of the sizes of the size-variable projects can be determined by their economical efficiency criterion. But how to design a veritable optimum size of a size-variable project still remains a much disputable topic in engineering economics.

Generally, when an optimum size of a size-variable project is designed, the NPV of the size-variable project is often used as the objective function, such as the famous Weingartner Optimal Model. It means that the size of the project will be the optimum one when its NPV gets the maximum value.

Is it always the case no matter what the conditions may be? In other words, is the maximum value of NPV of the size-variable project the sufficient and necessary condition for it to get its economical optimum size? If so, what is the reason?   If not, why? And how to work out the economical optimum size of a size-variable project?

2. Optimization-model development

A common production function can be expressed as follows:

Q=f (C, L,……)

Where C, the capital

L, the labors

… …

Q stands for the maximum output of a certain amount of production factors such as capital, labors etc. combined together.

In order to make the problem much simpler, it is assumed that the capital, C, is the only variable factor among the production factors which makes the output, Q, changed. That means Q is the function of the only variable C, the capital, i.e., Q =f(C).

Generally, with the capital increasing, the production will increase rapidly at the beginning. When the capital is increased to some degree, the production will increase slowly, and even will decrease when the investment capital comes to a certain amount (Figure 1).

Insert Figure 1 Here

2.1 The defining of the three different regions and the maximum NPV

The relationship between NPV and C behaves in the same way as that between Q and C, it was studied previously both

in textbook(Fu, 1996, pp.59-62) and in magazine(Hu,1995,pp.92-96),   and therefore the curve of NPV-C can be drawn as Figure 2.

Just as shown in Figure 2, NPV will be increased fleetly at the beginning and the acceleration is becoming smaller and smaller with the capital increase, and even decreases after the capital is increased up to some degree.

NPVR (the Rate of Net Present Value) is the NPV of per unit investment capital, consequently, it can be calculated as follows:

NPVR=NPV/C

Where NPVR, the rate of NPV.

C, the capital invested in the project.

The difference between NPV and NPVR is that NPVR emphasizes the importance on the efficiency of capital used, and it shows how much NPV the project can gain per unit capital used. It is a relative criterion and represents the efficiency of capital. While NPV is an absolute one on the other hand, it just shows how much NPV the project may get without mentioning any amount of capital involved. Therefore, the same quantity of NPV with quite different quantity of capital involved may have quite different meaning, but it will make no difference if only judged by NPV.

Based on the above reasons, many textbooks and papers concerning the comparison among mutual exclusive projects make it clear that the project with higher NPV will be the better one under the condition that the capital of investment is the same amount, otherwise they should be further evaluated with criterion NPVR. Hu(1995, pp 32-37) made comparison analysis about the evaluation criterion.

Insert Figure 2 Here

The curve of NPVR-C can also be drawn based on the curve of NPV-C (Figure 2), for NPVR is NPV divided by the capital, C. Figure   2 shows that NPVR and NPV get their maximum values where the amount of capital used is P_a and P_b respectively. In this way, there are three different zones divided by the two maximum values. In zone I, with the capital increasing, both NPV and NPVR are increased, therefore the increase of investment capital should not stop in this zone and should keep on increasing until it comes into zone II. In zone III, both NPV and NPVR are decreased with the capital increasing. It shows that the capital is a bit too great, and should be reduced in order to make NPV & NPVR much greater. From the above analysis, it can be seen that it is the range of zone II that the optimum size of the size-variable project must reside, for both the increase in capital from zone I and the decrease from zone III will all fall into zone II. In zone II, with the capital increasing, the NPV is keeping on increasing while the NPVR is going down all the way through. At point P_b, the NPV of the size-variable project gets its maximum value, but its NPVR is the smallest within the zone in the meantime.

Is the point, P_b, where NPV gets its maximum value, the optimum size?

Above all, it is necessary to make clear what characteristics point P_b has. Firstly, its NPV reaches the maximum value, in other words, the marginal increment of NPV is equal to zero at this point, i.e., dNPV/dC=0. Secondly, its NPVR is the smallest within zone II, and it is really much small in practice.

In order to prove the statement whether the size where the NPV of the size-variable project get its maximum value is the economically optimal size or not, another size of investment capital, P_d, is chosen randomly within zone II (Figure 2). Which size is better, size P_b or size P_d (to be convenient, the project with a size of P_b is called size-P_b-project, so is the size- P_d-project)?

On the one hand, there is:

$NPV_{Pb} > NPV_{Pd}$

While, on the other hand, there is:

$NPVR_{Pb} < NPVR_{Pd}$

From the above comparison, the conclusion that size-P_b-project is absolutely better than size- P_d-project can not be drawn, because the investment capital involved is quite different between the two projects. The following example can also confirm the above statement.

Hu(1995, p.95) quoted such an example as following. Suppose there are two projects, namely A and B. Project A needs the capital of about 208,000 Yuan, and will get a net income of about 31,000 Yuan annually for 20 years; at the end of the project remains will be valued at 53,900 Yuan. While project B may yield the net income of 11,000 Yuan annually for 20 years and the remaining value will be 1,000 Yuan, the capital needed is 30,000 Yuan at the beginning of the project. Which is the better one (the supposed discount rate is 10%)?

Project A:

$NPV_A = -208,000 + 31,000(P/A, 10\%, 20) + 53,900(P/F, 10\%, 20) = 63,943$ (Yuan)

$NPVR_A = NPV_A/C_A = 63,943/208,000 = 0.307$

Project B:

$NPV_B = -30,000 + 11,000(P/A, 10\%, 20) + 1,000(P/F, 10\%, 20)$

$= 63,803$ (Yuan)

$NPVR_B = NPV_B/C_B = 63,803/30,000 = 2.127$

$NPV_A > NPV_B$

$NPVR_A < NPVR_B$

The example shows that even though project A's NPV is larger than that of project B, but the difference is only 140 Yuan, which is too little compared with the difference of their investments, 178,000 Yuan. Suppose there is a project with an investment of 178,000 Yuan just gets a NPV of 140 Yuan in return after 20 years. The project is feasible for its NPV is 140 Yuan, which is greater than zero, but it is by no means a better one compared with the existing projects, A and B. In other words, it means that project A is not better than project B even though its NPV is a bit greater.

From the above example, it can be seen that the size-P_b-project will not be always better than the size-P_d-project if the concrete conditions concerned is taken into account. Besides, if the capital margin, P_c ($P_c = P_b - P_d$), is invested in a new project instead of leaving it alone as the above example, the project portfolio composing of the size-P_d-project and the newly invested project which is invested by the capital margin, P_c, will be better than the size-P_b-project itself with the same amount of capital being consumed. As it is shown in Figure 2, the NPVR at size P_b is the smallest one within zone II, its actual value is quite small, so it is not difficult to find out potential projects with a higher NPVR than that of the size-P_b-project (a fact meeting in practice will give a detail explanation about it in the following part "Model two"). If so, the project portfolio composing of one of the new potential projects and the size-P_d-project is proved to be better than the size-P_b-project (Refer to the appendix for demonstration). For size-P_d-project is chosen randomly, it confirms that the case occurs universally.

2.2 The project portfolio

In the above demonstration, it is assumed that the amount of investment capital of the new project P_c whose NPVR is larger than that of the size-P_b-project is the capital margin between size P_b and P_d (Figure 2), therefore the amount of investment capital needed for the new project should not larger than the difference between size P_b and size P_d. But according to the un-segmentation theory of the size-fixed project, the amount of investment need for size-fixed project cannot be changed at will. That means the investment capital required for the new project may not always equal to the capital margin between size P_b and P_d, in other words, the spare capital P_c may not always be made full use of. In fact, it is unnecessary for the size-fixed projects to be segmented for the above project portfolio, for the different size-fixed projects can be accommodated (instead of being segmented) by different size of size-P_d-project to form a project portfolio, because the size-P_d-project is just a size-variable project, whose size is just needed to be fixed.

Just as it is said above, the size of size-fixed project can not be changed, but the size of size-variable project that seeks for the economically optimal size is variable, and the size-P_d-project is the very size-P_b-project by reducing its size from size P_b to size P_d(Figure 2). Therefore, the project portfolio can be formed in this way that by adjusting the size of size-variable project to accommodate one of the size-fixed potential projects under the condition of a certain amount of capital involved. Because the efficiency of capital of the size-P_b-project is the lowest one within zone II, therefore, there may be many potential projects that are economically feasible available, they may be size-variable projects or size-fixed ones. Combining one of these potential projects with the size-variable project to form a series of the project portfolios whose investment capital are limited to the amount P_b. All of these project portfolios formed in this way can be compared with one another only by the criterion NPV, for they all have the same amount of investment capital involved, and the best one selected in this way will surely have a greater NPV than the maximum NPV of the size-variable project itself for the former projects have higher NPVR than the latter one.

In fact, the size where NPV gets its maximum value is not the optimum size but the maximum size, because it is just at the point where NPV gets its maximum value that the marginal increment of NPV comes to zero, it means that the size is the largest, it should not be increased any more, otherwise its NPV will decrease instead of increase.

2.3 The economically optimal size

Where is the optimum size of a size-variable project then? And how to get the optimum size of a given size-variable project ?

From the analysis made above, it shows that the reason why the size where the NPV of a size-variable project gets its maximum value is not the optimum size is that the efficiency of capital, NPVR, is so low at the very size that there may be many potential projects of which the efficiency of capital, NPVR, will be greater than it, if a new investment is made

with the reduced capital by reducing the size of the size-P_b-project to some degree, the two projects combined with each other to form a new project portfolio will yield a much greater NPV, for the efficiency of the capital of the two new projects are both greater than that of the size-P_b-project itself.

The paper will provide two kinds of methods to set up the optimum models based on the above analysis according to the different conditions.

2.4 Model one

In order to get the veriest maximum value of NPV, it is recommended that when the size of size-variable projects is need to be designed, both the size-variable project and the potential projects which are feasible in that condition should be taken into account. In other words, it is the project portfolio consisting of both the size-variable project and the other potential projects not the only size-variable project itself should be taken as the objective function when an optimization model for economically optimal size is set in order to get the veriest maximum NPV value.

If a size-variable project needs to be optimized its economical size, the NPV made by the given amount of capital is taken as the objective function. To be simple, supposed that there may be n+1 projects, one of them is the size-variable project whose size needed to be determined, the other n projects are all supposed to be size-fixed ones, which means their $NPVR_J$ (J=1, 2, n) are fixed. The total amount of capital can be used is C_p.

Therefore, the model can be expressed as following (fig 3).

Objective function:

$$MAX[NPV(C_p)] = \int_0^{x_1} [NPV_I(X)/X]dx + \Sigma_{J=1}^n NPVR_J \times X_J$$

Constraint condition:

$$X_I + \Sigma_{J=1}^n X_J \leq C_P$$

$$P_a \leq X_I \leq P_b$$

$$0 \leq X_J \leq C_P$$

Insert Figure 3 Here

The optimum model has such features as followings:

Both the size-variable project and the potential projects are taken into account in the model, in this way, the efficiency of the investment capital can be made as high as possible.

Both NPV and NPVR made in this way will be at least equal to, if not larger than, that one obtained from the model in which only the NPV of the size-variable project itself is taken as the objective function.

2.5 Model two

2.5.1 Introduction to Model two

The second model will be set up upon a practical case. In this case, the NPV of the size-variable project is taken as the objective function as it is done usually, but the difference is that the constraint of the efficient use of resource such as capital, land, etc. should be applied to the objective function in order to make full use of resource. A practical example of well-drilling projects during oil-field development in oil industry will be introduced to explain the principle of this kind of model.

During oil-field development in oil industry, there are two kinds of well-drilling technology used widely, i.e.: vertical well drilling and horizontal well drilling. Generally, the oil reservoirs lie underground horizontally, a traditional vertical well is to drill a vertical well hole and hit the oil reservoir vertically, only a certain part of oil reservoir can be covered and a certain amount of oil is taken out in this way. On the contrary, a horizontal well can drill a horizontal well hole within the oil reservoir, therefore a horizontal well can get more oil out of the hole than a vertical well. Besides, the longer a horizontal well bore drilled within the oil reservoir, the more oil will be taken out. In the meantime, the longer a horizontal well bore is drilled, the more cost will be spent on it, besides cost, there are lots of factors affect the oil production. The amount of oil produced by a horizontal well is not proportional to the length of horizontal well section being drilled, and their relationship is just as that of Q vs. C (fig. 1), which is explained above. In general, with the horizontal well bore being drilled on, the NPV of this horizontal well will be increased fleetly at the beginning, later on, the rate of increase will become a bit slower, when the length of horizontal well bore is increased to a certain distance, the NPV of this horizontal well will come to the maximum value, from then on, if the length of horizontal well bore keeps on drilling, the NPV of this horizontal well will begin to drop.

2.5.2 The Relationship among NPV, NPVR & dNPV/dL

According to the above analysis, the curve between the horizontal well bore length and the NPV of the horizontal well can be drawn as Figure 4. It is quite similar to the curve of NPV—C, the only difference is the horizontal axis, it is the

reservoir length a horizontal well covers rather than the capital it takes, in this case, the capital invested in this kind of projects is supposed to be limitless, the only variable is the length of the horizontal well within the reservoir. In Figure 4, the curve of NPV—L is the above one, the others below it are the curves of NPVR—L and dNPV/dL—L of horizontal well respectively, and the straight line is the curve of NPVR—L & dNPV/dL—L of a vertical well. As it is said above that a vertical well is a well drilled vertically through the oil reservoir, and covers a certain area however it is drilled. It means that a vertical well consumes a certain amount of oil reservoir and products a certain amount of oil. In other words, a vertical well has a fixed value of NPV, besides, the capital and the reservoir it taking is also fixed, therefore, the NPVR and dNPV/dL of a vertical well are equal to each other and unchangeable, it can therefore be drawn as a straight line, VV (Figure 4). Contrary to the nature of a vertical well, a horizontal well is quite changeable, both its NPV & NPVR will be quite different with the different horizontal well bore length. At the beginning, when the horizontal well has not drilled its horizontal section long enough, the cost it take is much more than that of a vertical well, but the production it get is not great enough to make up for the cost it takes compared with a vertical well, therefore, the NPV and NPVR of horizontal well are both smaller than those of a vertical well at the very beginning. With the horizontal section being drilled longer and longer, its advantage appears, the oil-production increase rapidly compared with that of cost. Figure 4 shows that the curve NPV and NPVR of the horizontal well increase very fast at the beginning and its dNPV/dL is therefore much higher then. With the horizontal well bore being drilled on, its dNPV/dL first come to the maximum value, and then followed by the curve of NPVR, while the NPV of the horizontal well is still increasing but at a low rate. In year 2000, Hu(2000,pp,81-82) published his study on the relationship among NPV, NPVR & dNPV/dL between the horizontal well and vertical well.

Insert Figure 4 Here

With the horizontal well bore prolonging, the curve of dNPV/dL—L of the horizontal well comes across the straight line, the curve of dNPV/dL—L of the vertical well, at point B. From then on, keeping on drilling the horizontal well bore will get less NPV than stopping to drill a vertical well even though the NPV of the horizontal well is still increasing, for the value of dNPV/dL of the horizontal well falls below that of the vertical well after the point B (Figure 4). Therefore, it is at the point B that an optimal horizontal well bore should be stopped in order to get the veriest maximum value of NPV of the project portfolio. If the horizontal well bore continue to drill to the point F, where NPV of the horizontal well itself gets its maximum value. The size of the project or the length of the horizontal well bore at the point F is not the optimum size but the maximum one, because it is just at the point where NPV gets its maximum value that the marginal increment of NPV comes to zero, it means that the size is the largest, it should not be increased any more, otherwise its NPV will decrease instead of increase. Although the marginal NPV of the horizontal well is below that of the vertical well, NPVR of the horizontal well is still larger than that of the vertical well before the point ,C, therefore the whole NPV of the horizontal well is greater than that of the vertical wells with the same amount of resource being consumed (Figure   4), It means that the oil-field developed by horizontal well can still get a greater NPV than developed by vertical wells as long as the length of the horizontal well is not drilled to exceed the point ,C, eventhough its length overpass the point, F. If the horizontal well bore keep on drilling overpass point C, it is not only the marginal increment of NPV but also the NPVR of the horizontal well are both smaller than those of the vertical well. It means that if the length of horizontal well bore is drilled surpass the point C, a oilfield developed by drilling vertical wells will get more NPV than by the horizontal well.

2.5.3 The Optimal Length of a Horizontal Well

As it is said above, in order to get more NPV from a certain amount of resource, the length of the horizontal well bore should be stopped at the point B (Figure 4), where the marginal increment of NPV of the horizontal well is equal to that of the vertical well, rather than keep on drilling till the point F, where the NPV of the horizontal well comes to its maximum value, let alone the point C.

The above analysis shows a very important conclusion that the size where the NPV of the size-variable project gets its maximum value is by no means the economically optimal size but an upper limit. Because the marginal increment of NPV of the size-variable project is equal to zero when its maximum value of NPV arrives, if the length is keeping on increasing from that point on, the value of NPV will decrease rather than increase, therefore, that point is an upper limit, the maximum size would never surpass it.

Because the point at which the NPV of the size-variable project get its maximum value is the upper limit point, the point at which the marginal increment of NPV of the horizontal well is equal to that of the vertical well is the interesting point for the optimal horizontal well bore length design. The dNPV/dL of the vertical well must be used as the special constraint when the model for optimal horizontal well bore length is set up, without which the horizontal well bore length may be designed much too long to be the optimal length any more.

2.5.4 Model two

Supposed the length of resource covered by one vertical well is ∆L, the horizontal well bore length of a horizontal well

should be at least L_{min}, otherwise the income gained from a horizontal well could not make up for the cost spent on a horizontal well, the largest length is named as L_{max} which may be limited by the size of oil reservoir, the capacity of the rig used, the technology of drilling, etc.. In order to enlarge the NPV of the project with a certain amount of resource, the drilling of this horizontal well bore should be stopped as soon as its dNPV/dL is no more greater than that of a vertical one, i.e.:

$NPV_H (L+\Delta L) - NPV_H (L) \geq NPV_V$

Here, NPV_H & NPV_V (NPV_V is a constant) stands for the NPV of a horizontal well and that of a vertical well respectively.

Therefore, the model can be expressed as following:

Objective function:

$MAX[NPV(L)]= MAX\{\Sigma_{I=1}^{m}[CI(L)-CO(L)]_t/(1+i)^t\}$

Constraint condition:

$NPV_H(L+\Delta L)-NPV_H(L)\geq NPV_V$ (while, $d^2\{NPV_H (L)\}/dL^2<0$)

$L_{max}\geq L\geq L_{min}$

Here, CI(L), CO(L) stands for the input & output of the crash flow of a horizontal well with L meters horizontal well bore length every year respectively, and m is the economic life of a horizontal well.

2.5.5 Example

There is an oilfield of which length is about 4000 meters, and its width is wide enough to place one row of wells only. A vertical well covers about 400 meters in length, and yields the value of NPV about 23×10^5 dollars, therefore, the NPV per meter of a vertical well can be calculated as $23\times10^5\div400$ =5750 dollars per meters. If it is developed by horizontal wells, how long will the horizontal well bore be drilled?

If an optimal model is set by taking NPV of a horizontal well as an objective function, the length of a horizontal well can be worked out, it is about 3280 meters, and the NPV of that length is about 240×10^5 dollars. Is it the optimal length for a horizontal well in this region? For the NPV per meter of a vertical well is 5750 dollars per meters. If the dNPV/dL of the vertical well is imposed on the design model, another result can be made, the optimal length is about 2806 meters, and the NPV of that length is about 222×10^5 dollars.

Which is the better one then? It is the one that can gain more NPV with the same amount of resource being consumed. For the total amount of resource is about 4000 meters, a horizontal well with 3280 meters takes 3280 meters, and there are about 4000-3280=720 meters left. For a vertical well covers about 400 meters in length and yields the value of NPV about 23×10^5 dollars, there are fewer than two vertical wells can be placed for the spare space left by the horizontal well with a horizontal well bore length of 3280 meters. Therefore, the total NPV gained by this project portfolio(project B) is not larger than 286×105(240×105+2×23×105=286×105) dollars. The average NPV per meter is 7264.1$/m。 While the other project portfolio(project A) consists one horizontal well with a 2806 meters horizontal well bore length and three vertical wells. because the spare length left by the horizontal well with a 2806 meters horizontal well bore length may allow three (i.e. :(4000-2806)/400=2.98) vertical wells to be placed, and it can yield a total NPV of 291×105(222×105+3×23×105=291×105) dollars. The average NPV per meter is 7009.8$/m。 The results are listed in the following table:

Insert Table 1 Here

From the example, it shows that if the length of a horizontal well bore is designed just by taking the NPV of the horizontal well as the objective function without the any constraint on the efficient use of resource, the length of a horizontal well bore will be designed too long to be the optimal one any more. Therefore, the constraint on the efficient use of resource should be imposed in order to get the veriest optimal size if the NPV of the size-variable project is taken as the objective function.

3. Conclusion

The size at which the NPV of the size-variable project gets its maximum value is not the economically optimal size but the maximum one, for the size can not be increased any longer.

If NPV of the size-variable project is taken as the objective function when an optimization model is set up, the efficiency of resource consumed should be imposed as the constraint upon the objective function. Otherwise, the size made in this way will yield too large a size to be the optimal one any more.

The economically optimal size is the size at which the NPV of the project portfolio comprising both the size-variable project and the potential projects get the maximum value not the size at which the NPV of the size-variable project itself gets the maximum value, for the former NPV is always greater than the latter one under the condition of the same

amount of resource being consumed.

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Appendix

There are two projects, size-P_b-project and size-P_d-project, the capital invested on them are P_b and P_d respectively (Figure 2), the NPVR of size-P_d-project is greater than that of size-P_b-project, i.e.: $NPVRp_d > NPVRp_b$.

Supposed another new size-P_c-project, of which capital is P_c ($P_c = P_b - P_d$), has higher capital efficiency than that of size-P_b-project (for the capital efficiency of size-P_b-project is so low that it is easy to find projects of which capital efficiency is higher than that of size-P_b-project), i.e.: $NPVRp_c > NPVRp_b$.

Project O is a project portfolio comprising of size-P_d-project and size-P_c-project, and it comsuming the same amount capital as project P_b.

∵　$NPVRp_d > NPVRp_b$

　　$NPVRp_c > NPVRp_b$

∴　$NPVp_o = NPVp_d + NPVp_c$

　　　　$= NPVRp_d \times P_d + NPVRp_c \times P_c$

　　　　$> NPVRp_b \times P_d + NPVRp_b \times P_c$

　　　　$= NPVRp_b \times (P_d + P_c)$

　　　　$= NPVRp_b \times P_b$

　　　　$= NPVp_b$

And,

$NPVRp_o = NPVp_o / (P_d + P_c)$

　　　　$> NPVp_b / (P_d + P_c)$

　　　　$= NPVp_b / P_b$

　　　　$= NPVRp_b$

i.e.: $NPVp_o > NPVp_b$

　　$NPVRp_o > NPVRp_b$

Conclusion:

Project portfolio O is better than size-P_b-project with the same amount resource being consumed.

Table 1. Results of the Example

The objective function	Special constraint	horizontal well bore length (m)	well numbers*	Consumed resource (m)	Total NPV ($)	NPV per meter ($/m)
(Vertical well)	--	--	10V.	4000	230×10^5	5750
Project A MAX{NPV(L)}	$dNPV/dL \gtreqqless 5750$	2806	1H+3V	4006	291×10^5	7264.1
Project B MAX{NPV(L)}	--	3280	1H+2V	4080	286×10^5	7009.8

Note: "well numbers" here means the number of well being drilled within a certain area. For example, "1H+2V" means one horizontal well and two vertical wells being drilled within 4080 meters in length.
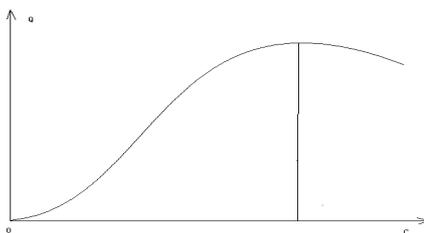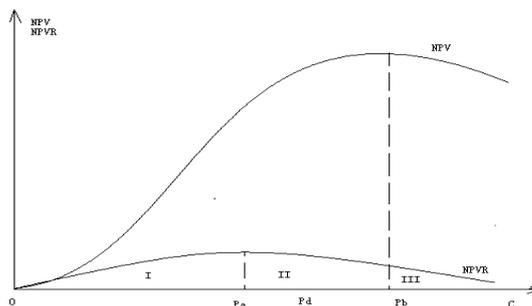
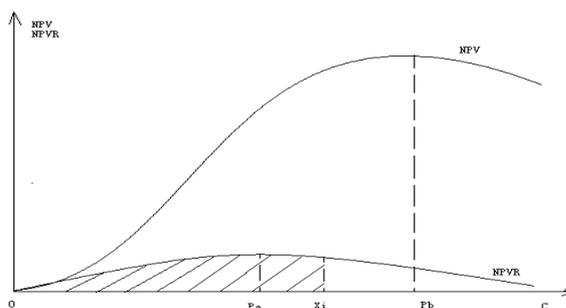Figure 1. Output (Q) vs. Capital (C)

Figure 2. NPV/NPVR vs. Capital (C)

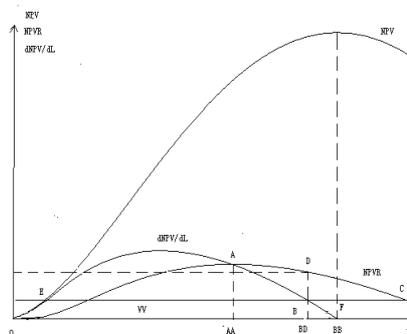Figure 3. Curve of Size-Variable Project for Model One

Figure 4. Relationship among NPV, NPVR & dNPV/dL between a Horizontal Well & a Vertical Well