International Journal of Economics and Finance; Vol. 17, No. 4; 2025
ISSN 1916-971X E-ISSN 1916-9728
Published by Canadian Center of Science and Education

Economic and Geopolitical Shocks and Their Influence on the Saudi Stock Market, Saudi Aramco, and Bitcoin: Evidence from ARDL and VAR Models

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Received: February 8, 2025 Accepted: February 24, 2025 Online Published: March 5, 2025

doi:10.5539/ijef.v17n4p31 URL: https://doi.org/10.5539/ijef.v17n4p31

Abstract

This study examines the dynamic relationships between the Tadawul All Share Index (TASI) returns, Saudi Aramco stock returns, and Bitcoin (BTC/SAR) returns from January 2, 2019, to December 31, 2023. Using the Autoregressive Distributed Lag (ARDL) model, Vector Autoregressive (VAR) models, impulse response functions, and Granger causality tests, the research explores how these relationships evolved across distinct macroeconomic periods, particularly during and after the COVID-19 pandemic and the Russo-Ukrainian war.

The ARDL model results indicate stable long-run relationships between TASI and Saudi Aramco returns across all periods. However, these relationships weakened post-COVID-19, likely due to the pandemic's structural impact. Short-run dynamics exhibited higher volatility during crises, with external shocks playing a significant role. The bounds test confirmed a long-term relationship pre-COVID-19, which weakened post-pandemic. The VAR model highlights strong interlinkages, especially pre-crisis, with TADAWUL returns leading Saudi Aramco returns. This relationship weakened in the post-COVID-19 and post-war periods, indicating global market disruptions. Granger causality tests revealed that causal dynamics are period-specific, with stronger causality observed before the crises. Impulse response functions show that TADAWUL shocks have a more substantial impact on Saudi Aramco than on Bitcoin returns. These findings provide valuable insights for market participants navigating an uncertain economic landscape.

Keywords: Tadawul All Share Index (TASI), Saudi Aramco Returns, Bitcoin (BTC/SAR), COVID-19 Pandemic, Russo-Ukrainian War, financial market interdependencies

JEL Classifications: G11-G41.

1. Introduction

Financial markets are intricately interconnected, with various asset classes influencing each other under different economic and geopolitical conditions. Understanding these interrelationships is crucial for investors, policymakers, and market participants seeking to make informed decisions in a rapidly evolving global landscape (Bekaert & Harvey, 2017). The interplay among stock market indices, corporate stock returns, and cryptocurrencies has garnered significant attention in recent years, especially given their differing characteristics and roles in diversified portfolios (Corbet et al., 2020; Bariviera et al., 2017). While stock markets and corporate returns are traditionally seen as key indicators of economic performance, cryptocurrencies like Bitcoin offer a distinct, often volatile alternative, which has reshaped investor behavior and portfolio strategies (Dyhrberg, 2016).

This study focuses on three critical assets: the Tadawul All Share Index (TASI), which represents Saudi Arabia's overall equity market; Saudi Aramco, the world's largest oil company by market capitalization; and Bitcoin (BTC/SAR), the flagship cryptocurrency known for its volatile behavior and global significance as a digital asset (Liu et al., 2020). The relationships among these assets are examined over the period from January 2, 2019, to December 31, 2023, a timeframe marked by two major global events: the COVID-19 pandemic and the Russo-Ukrainian war. These events introduced unprecedented economic disruptions and uncertainties, reshaping financial market dynamics and investor behavior (Paltoglou et al., 2022).

The analysis employs robust econometric tools, including the Vector Autoregressive (VAR) model, Impulse Response Functions, Granger causality tests, and the Autoregressive Distributed Lag (ARDL) model. By dividing the study period into pre- and post-event sub-periods, we aim to uncover how these global shocks influenced the interdependencies among TASI returns, Saudi Aramco returns, and Bitcoin returns. The study also evaluates the temporal stability of these relationships, providing a comprehensive understanding of how traditional stock markets, corporate equity, and cryptocurrencies interact under varying conditions (Apergis & Miller, 2016).

This research contributes to the growing body of literature on cross-asset relationships by offering empirical evidence from one of the world's most prominent emerging markets, Saudi Arabia. The findings are expected to provide valuable insights for portfolio diversification strategies, risk management, and policy formulation, particularly in the context of global economic uncertainties and their implications for financial market stability (Kumar & Yadav, 2019).

2. Literature Review

The interaction between financial markets, particularly between stock indices, corporate stock returns, and emerging assets like cryptocurrencies, has been a subject of growing interest in recent years. Numerous studies have explored the long-term and short-term dynamics between these markets, with a focus on both the ARDL and VAR models to capture complex relationships across different economic conditions.

Several studies have utilized the Autoregressive Distributed Lag (ARDL) model to explore the cointegration and dynamic relationships between asset classes. For example, Chancharat and Suwannapak (2024) applied the ARDL model to investigate the dynamic relationships between ASEAN+6 exchange rates and stock markets, highlighting the short- and long-term linkages within the financial markets. The ARDL approach provided insights into how exchange rate volatility impacts stock market returns in these economies. In a similar vein, Singh and Kumar (2023) applied the ARDL bounds testing to assess the relationship between stock market development and economic growth in India, concluding that long-run dynamics dominate over short-run fluctuations in the Indian market.

A study by Alvarez et al. (2023) analyzed the effect of financial innovation on stock market development and economic growth using ARDL. Their results indicate that financial innovation positively influences long-term stock market performance, with significant effects in emerging markets. Furthermore, Gupta et al. (2023) applied ARDL models to investigate the impact of oil price fluctuations on the stock markets of emerging economies, finding evidence of a stable long-term relationship between these variables, with oil price shocks having short-term effects.

On the other hand, the Vector Autoregressive (VAR) model has also been widely employed to study the interdependencies between stock market indices and other financial variables. In the context of Saudi Arabia, Hassan et al. (2023) employed the VAR model to analyze the dynamic relationship between the Saudi stock market (TASI) and oil prices, with results showing that oil price movements significantly influence stock returns in both short- and long-term horizons. Similarly, Lee et al. (2024) used VAR to study the interactions between Bitcoin and traditional financial markets, concluding that Bitcoin returns exhibited a weak relationship with traditional market indices during stable periods but became more integrated during financial crises.

Moreover, Santos and Pereira (2024) extended the VAR model to include cryptocurrency returns in their analysis of asset diversification strategies. Their findings suggest that while Bitcoin provides diversification opportunities in a traditional portfolio, its relationship with stock market returns is highly volatile, particularly during periods of economic uncertainty.

Both ARDL and VAR models are essential for capturing the complex and time-varying dynamics between financial assets in the context of global economic and geopolitical shocks. For example, Khan et al. (2024) demonstrated the use of the ARDL approach to study the effects of the COVID-19 pandemic on the relationship between stock markets and cryptocurrencies, emphasizing that the global health crisis altered these relationships significantly. Similarly, Tariq and Haider (2023) explored the role of geopolitical events, such as the Russo-Ukrainian war, in influencing the link between oil prices, stock markets, and cryptocurrencies using both ARDL and VAR models. Their findings suggest that geopolitical crises can significantly disrupt the correlations between these asset classes, with long-term effects weakening during periods of uncertainty.

Gbadebo (2024) examines Bitcoin, blockchain, and cryptocurrencies, emphasizing their disruptive potential and legal implications. The study critiques the lack of theoretical focus in existing research and advocates for decentralized insurance products to protect Bitcoin investments. It offers insights into the evolving role of digital assets in financial systems.

Aloui et al. (2014) analyze the impact of economic, political, and geopolitical risks, along with oil price volatility, on Saudi Arabia's economic complexity using data from 1995 to 2021. Their findings show a long-term positive relationship between these risks and economic complexity, while short-term effects are weak. The study highlights Saudi Arabia's resilience to external shocks and offers policy recommendations for fostering sustainable growth.

Syzdykova and Azretbergenova (2024) analyze the asymmetric impact of oil prices on Kazakhstan's exchange rate (USD/KZT) and stock market index (KASE) using the NARDL method. Their findings show that both positive and negative oil price changes significantly affect stock returns and exchange rates in the long term, with differing responses to oil price shocks.

Harnphattananusorn (2024) examines spillover effects among oil, gold, stock market, and exchange rate returns in Thailand using a time-varying VAR model (2002-2024). The study finds dynamic linkages influenced by global events, with stocks and gold as shock transmitters and oil and exchange rates as recipients, offering insights for market management.

Obadi and Korcek (2024) analyze the short- and long-term relationships between geopolitical events and crude oil prices (2000-2023) using the ARDL bounds testing approach. The study finds significant associations, with factors like global oil production, OECD crude oil stocks, and economic growth affecting oil prices in both timeframes.

Munawwara (2024) examines oil price volatility's impact on Indian sectoral stock returns (2011-2022) using quantile regression. Negative effects are strongest in bearish periods, while interest and exchange rates have a greater influence, with oil prices impacting indirectly via market portfolio spillovers.

Truong et al. (2024) analyze the asymmetric effects of oil prices on Hanoi Stock Exchange returns (2010-2023) using the NARDL approach. The study finds significant negative short- and long-term effects, with stronger impacts from negative oil price changes. The error correction model shows rapid adjustment to long-term equilibrium at 81.54% per week.

Dias et al. (2024) examine financial integration and diversification between clean energy indices and WTI oil prices (2018-2023). Most clean energy indices are cointegrated, indicating shared trends, while WTI remains relatively independent, offering diversification potential. Structural breaks in 2021-2022 reflect policy shifts, innovation, and COVID-19 impacts. Findings provide insights for sustainable energy portfolio strategies.

Kakizhanova et al. (2024) analyze factors affecting FDI inflows in Kazakhstan using the ARDL approach. The study finds that CO2 emissions positively influence FDI, while inflation and oil prices have no effect. GDP per capita negatively impacts FDI, suggesting limited reliance on FDI for Kazakhstan's economic development.

Abu Asab (2024) examines the effects of oil prices and real effective exchange rates on private investment in Saudi Arabia (2007-2022). Oil prices and exchange rates boost investment long-term, while short-term oil price volatility has a negative impact. Exchange rate uncertainty is minimal, and positive shocks cause higher variance. The study highlights Saudi Arabia's resilience and provides insights for stabilizing investment in oil-dependent economies.

Slimane (2024) examines the impact of oil price shocks on the performance of conventional and Islamic banks in Saudi Arabia (2006-2022) using the ARDL methodology. The study finds that oil price shocks directly affect bank performance, with positive shocks benefiting conventional banks more and negative shocks harming Islamic banks more.

Xu et al. (2024) explore the predictive power of oil volatility-of-volatility (VOV) on the tail risk of various commodity sectors. The study finds that oil VOV predicts 1-step-ahead tail risks for Energy, Precious Metals, Agriculture, Livestock, and the Aggregate Commodity sector. These results highlight crude oil's leading role in overall commodity markets.

Güngör and Güngör (2024) examine the impact of economic policy uncertainty in Germany and the US on long-term stock market volatility in Central and Eastern European (CEE-3) countries. The study finds a significant positive effect of economic policy uncertainty on stock market volatility, with a one-period lag.

Almeida et al. (2024) analyze Bitcoin market risk premia through the Pricing Kernel (PK), revealing a W-shaped PK steep in the negative returns region. Negative Bitcoin returns contribute 33% of its premium compared to 70% for the S&P 500. Risk premia vary with volatility regimes, with higher variance and downside risk premiums in low-volatility states, while high-volatility states show balanced contributions from positive and negative returns.

Chi et al. (2024) propose a novel multivariate framework integrating Graph Signal Processing with a Heterogeneous Auto-Regressive model to analyze volatility spillovers. Using 24 global stock indices, the model effectively aggregates mid- and long-term data, demonstrating its robustness in empirical evaluations.

Xia et al. (2024) analyze green bonds' (GBs) hedging capabilities against geopolitical risk (GPR) using a modified connectedness network model. They find that GB markets in China and Japan can hedge against GPR, though China's GBs act as weak hedging and safety-haven assets. The study highlights market-, time-, and quantile-dependent linkages, offering portfolio insights for efficient hedging and risk management.

Sadewa and Huruta (2024) analyze return and volatility spillovers among Bitcoin, Gold, and Nasdaq using GARCH-ARMA models on monthly data (2015–2024). They find return spillovers between Bitcoin, Nasdaq, and Gold, and volatility spillovers involving all three assets. The study highlights Bitcoin's potential as a hedge, akin to Gold and Nasdaq, during economic uncertainty.

Usman et al. (2024) analyze oil prices' impact on stock market liquidity in Pakistan (2000-2019) using ARDL models. They find oil prices, inflation, and exchange rates significantly affect liquidity, with mixed directions. The study provides insights for investors and policymakers.

Lim et al. (2024) examine the asymmetric effects of global economic policy uncertainty (GEPU) and geopolitical risk (GPR) on insurance development in ASEAN (1990-2020) using linear and asymmetric panel ARDL models. They find GEPU and GPR impact non-life insurance (NI) but not life insurance (LI), with GEPU's positive shocks benefiting both NI and LI. Negative GEPU shocks and GPR mainly influence NI, offering insights for ASEAN insurance policies.

Gökgöz et al. (2024) analyze volatility connectedness between BRICS stock markets and implied volatility indices using a TVP-VAR approach (2019-2023). They find dynamic and heterogeneous spillovers, with stronger effects during black-swan events. The study offers valuable insights for improving investor risk management in volatile markets.

Hashim et al. (2024) examine the relationship between stock markets, gold, and crude oil prices during COVID-19 using quantile regression (2020-2022). They find an insignificant link between gold and stocks but a significant negative correlation between crude oil and stocks at higher quantiles. These insights inform investor and policymaker strategies during crises.

Farooq et al. (2024) explore the impact of economic policy uncertainty (EPU) on environmental innovation and patent registration in BRICS (2010-2022) using CS-ARDL models. They find EPU negatively affects eco-innovation, while higher uncertainty levels (squared EPU) encourage innovation. The study highlights the need for stable policies to support sustainable technology development.

The reviewed studies underscore the intricate and multifaceted connections between energy markets, macroeconomic indicators, and financial systems, highlighting the pivotal role of oil and geopolitical risks as drivers of economic and market dynamics. They reveal the resilience and vulnerabilities of different sectors and regions, emphasizing dynamic spillover effects, asymmetric relationships between assets, and the influence of uncertainty and crises on market behavior. By employing innovative methodologies and examining varied contexts, this literature advances our understanding of economic complexity, risk management, and investment opportunities in a volatile global economy. Building on this foundation, our study investigates the relationship between Tadawul All Share returns, Saudi Aramco price returns, and Bitcoin returns before and after two major global disruptions: the COVID-19 pandemic and the Russo-Ukrainian war. Using ARDL and VAR models combined with Granger causality tests, this research aims to deepen insights into how these events shaped interactions among traditional, corporate, and digital assets in the Saudi context, contributing valuable strategies for investors and policymakers.

3. Method

3.1 Autoregressive Distributed Lag (ARDL) Model

3.1.1 Theoretical Framework for ARDL Model

ARDL models are linear time series models in which both the dependent and independent variables are related not only contemporaneously, but across historical (lagged) values as well. In particular, if $y_t$is the dependent variable and x_1,…, x_k are k explanatory variables, a general ARDL(q, q_1,…, q_k) model is given by:

$$y_t = \alpha_0 + \alpha_1 t + \sum_{i=1}^{p} \psi_i y_{t-i} + \sum_{j=1}^{k} \sum_{Ij}^{qj} \beta_{j,Ijxj,t-Ij} + \epsilon_t \quad (1)$$

Where ϵ_t are the usual innovations, α_0 is a constant term, and α_1, ψ_i and $\beta_{j,Ij}$ are respectively the coefficients associated with a linear trend, lags of y_t and lags of the k regressors xj, t for j=1,…k.

Alternatively, let LL denote the usual lag operator and define $\psi(L)$ and $\beta_j(L)$ as the lag polynomials:

$$\psi(L) = 1 - \sum_{i=1}^{p} \psi_i L^i \ and \ \beta_j(L) = \sum_{Ij=0}^{qj} \beta_{jIj}, L^{Ij} \tag{2}$$

Then, equation (2) above can also be written as:

$$\psi(L)y_t = \alpha_0 + \alpha_1 t + \sum_{j=1}^{k} \beta_j(L)x_{j,t} + \epsilon_t \tag{3}$$

Although ARDL models have been used in econometrics for decades, they have gained popularity in recent years as a method of examining cointegrating relationships. Two seminal contributions in this regard are Pesaran and Shin (1998, PS(1998)) and Pesaran, Shin and Smith (2001, PSS(2001)). In particular, they argue that ARDL models are especially advantageous in their ability to handle cointegration with inherent robustness to misspecification of integration orders of relevant variables. Three cases are of interest:

- All variables are I(d) for some 0≤d0 and are not cointegrated -- fractional orders of integration are in principle also possible. Here one can use familiar least squares techniques to estimate and interpret equation (2) in levels when d=0 and in appropriate differences when d>0.
- All variables are I(1) and are cointegrated. Here one can use least squares to estimate the cointegrating (long-run) relationship by regressing yt on xj,t for j=1,…k in levels; and/or, use least square to estimate speed of adjustment of short-run dynamics to the cointegrating relationship by regressing the appropriate error-correction model (ECM).
- Some variables are I(0),others are I(1), and amongst the latter, some are cointegrated.

It is precisely in this last case where traditional cointegration methodologies of Engle-Granger (1987), Phillips and Ouliaris (1990) or Johansen (1995), typically fail since all variables need to have identical orders of integration, usually I(1). This requires pre-testing for the presence of a unit root in each of the variables under consideration, which is clearly subject to misclassification, particularly since unit root tests are known to suffer size and power problems in many cases of interest; see Perron and Ng (1996).

Alternatively, the PSS(2001) bounds test for cointegration is not subject to such limitations and readily accommodates the nuances of the third case. The test is in fact a parameter significance test on the long-run variables in the ECM of the underlying vector autoregression (VAR) model, and works when all or some variables are I(0), I(1), or even mutually cointegrated. Since there exists a one-to-one correspondence between an ECM of a VAR model and an ARDL model (see Banerjee et. al., 1993), and since ARDL models are estimated and interpreted using familiar least squares techniques, ARDL models are de facto the standard of estimation when one chooses to remain agnostic about the orders of integration of the underlying variables. It is precisely in this regard where the ARDL methodology shines.

3.1.2 Robustness Check for ARDL Model

To ensure the robustness of the ARDL model estimates, several diagnostic tests and alternative specifications were employed. First, various lag lengths were considered to assess the stability of the results across different model specifications, allowing for a comprehensive understanding of the short-run dynamics and long-run relationships among the variables. Additionally, subsample analyses were conducted, focusing on distinct periods such as pre-COVID, post-COVID, pre-Russo-Ukrainian war, and post-war, to evaluate the consistency of the estimated relationships over time. Diagnostic tests for autocorrelation, heteroscedasticity, and normality of residuals were performed to validate the underlying assumptions of the model, ensuring the reliability of the estimates. Finally, the CUSUM and CUSUMSQ tests were used to examine the stability of the model coefficients throughout the sample period.

3.2 Vector Autoregression (VAR) Model

3.2.1 Theoretical Framework for VAR Model

The Vector Autoregression (VAR) methodology is a multivariate time-series modeling approach that captures dynamic interdependencies among multiple variables. Unlike ARDL, VAR treats all variables in the system as endogenous, making it particularly suitable for analyzing feedback effects and forecasting. Each variable in a VAR model is expressed as a linear function of its own lagged values and the lagged values of all other variables in the system. This flexibility enables the model to account for the interconnected nature of economic systems and financial markets.

Before estimating a VAR model, all variables must be stationary (I(0)); otherwise, they must be transformed through differencing. The optimal lag length is determined using selection criteria such as AIC, BIC, or HQC to ensure model efficiency. The model coefficients are estimated simultaneously for all equations using Ordinary

Least Squares (OLS). After estimation, the VAR model allows for various analytical tools, including Granger causality tests to identify causal relationships between variables, impulse response analysis to examine the dynamic effects of shocks, and variance decomposition to quantify the contributions of variables to forecast errors.

VAR models are commonly used to analyze interrelationships and dynamic feedback among variables. They are especially useful for policy simulations and forecasting. The method's key advantage lies in its ability to treat all variables as endogenous, providing a comprehensive view of the system's dynamics. However, VAR requires all variables to be stationary, which may involve additional data preprocessing steps. Despite this, its application across economics and finance remains extensive due to its flexibility and capability to model complex systems.

3.2.2 Robustness Check for VAR Model

To ensure the robustness of the VAR model estimates, several diagnostic tests and procedures were implemented. First, the optimal lag length was determined using criteria such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) to verify the stability of the results across different specifications. Granger causality tests were conducted to assess the directional relationships among the variables, providing insights into the dynamic interactions within the system. Additionally, impulse response functions (IRFs) were used to analyze how shocks to one variable affect the other variables over time, while forecast error variance decomposition was performed to quantify the relative contributions of each variable to the system's dynamics.

3.3 Granger Causality Test

The Granger causality test is a statistical hypothesis test used to determine whether one time-series variable can predict another. Developed by Clive Granger in 1969, this test is widely employed in econometrics to explore the causal relationships between variables in a temporal context. It does not establish causality in the philosophical sense but instead assesses whether past values of one variable contain information that improves the forecast of another.

The Granger causality test is grounded in the principle that if a variable X Granger-causes another variable Y, then past values of X should provide statistically significant information about Y beyond what is already contained in Y's own past values. The test involves estimating two regressions:

1) A restricted model where Y_t depends only on its own lagged values.

2) An unrestricted model where Y_t depends on its lagged values and the lagged values of X_t.

The null hypothesis of the test is that Xt does not Granger-cause Yt, implying the coefficients of the lagged values of Xt in the unrestricted model are jointly zero. A standard F-test or Wald test is used to compare the restricted and unrestricted models. If the null hypothesis is rejected, it indicates that Xt Granger-causes Yt.

For the test to yield valid results, the variables must be stationary (I(0)) or transformed to stationarity. The selection of the appropriate lag length is crucial, as an incorrect choice can lead to misleading conclusions. Information criteria such as AIC or BIC are often used to determine the optimal lag structure.

The Granger causality test is applied extensively in economics, finance, and other fields. For instance, it is used to study relationships like the impact of monetary policy on inflation, the predictive power of stock market indices on economic growth, or the interplay between energy consumption and GDP.

4. Results & Discussion

4.1 Data

This study uses daily time series data for three key variables: TADAWUL All Share Index returns (TADAWUL_ALL_SHARE_RETURN), SAUDI ARAMCO stock price returns (SAUDI_ARAMCO_PRICE_RETURN), and BITCOIN returns against the Saudi Riyal (BTCSAR_BITCOIN_RETURN). The data covers the period from January 2, 2019, to December 31, 2023, and is divided into four distinct periods based on significant economic and geopolitical events:

1) Pre-COVID-19 Period: January 2, 2019, to March 1, 2020.

2) Post-COVID-19 Period: March 2, 2020, to December 31, 2023.

3) Pre-Russo-Ukrainian War Period: January 2, 2019, to February 23, 2022.

4) Post-Russo-Ukrainian War Period: February 24, 2022, to December 31, 2023.

The total number of observations in the dataset is 1,245, corresponding to the daily data points over the specified time frame for all three variables.

In this study, the TADAWUL All Share Index returns (TADAWUL_ALL_SHARE_RETURN) are treated as the dependent variable, while SAUDI ARAMCO stock price returns (SAUDI_ARAMCO_PRICE_RETURN) and BITCOIN returns against the Saudi Riyal (BTCSAR_BITCOIN_RETURN) are treated as the independent variables.

The data for the TADAWUL All Share Index was sourced from Tadawul All Share Historical Data-2019, while the stock price data for SAUDI ARAMCO was obtained from Saudi Aramco Historical Price Data. The BITCOIN data against the Saudi Riyal (BTCSAR) was gathered from BTCSAR - Bitcoin Saudi Riyal historical data.

The analysis focuses on the returns of these three variables to investigate their dynamic relationships and causal interactions across the selected periods. Daily return series were computed for each of the variables to ensure consistent data structure and facilitate comparison across the periods of interest. This data set provides a robust foundation for understanding the interconnectedness of the Saudi stock market, oil prices, and the cryptocurrency market, particularly in the context of global and regional economic disruptions.

4.2 Time Series of Asset Returns

The following graph illustrates the time series of asset returns for the periods pre-COVID-19 and post-COVID-19, as well as pre-Russo-Ukrainian War and post-Russo-Ukrainian War.

The time series data displayed in figure 1 illustrates the returns of three key financial assets: the TADAWUL All Share Index, Saudi Aramco Price Return, and Bitcoin (BTC/SAR) Return, across a defined period from January 2, 2019, to December 31, 2023, divided into four distinct phases: pre-COVID, post-COVID, pre-Russo-Ukrainian war, and post-war. Time series analysis of asset returns is essential for understanding the historical performance, volatility, and behavior of financial assets. This analysis provides insight into the fluctuations and trends in asset prices, which are influenced by a wide range of factors such as market sentiment, economic conditions, and geopolitical events. By examining the returns of these assets over these specific periods, we can observe how they respond to various market forces and gain a deeper understanding of their risk-return profiles. The graph reflects the variations in the returns of each asset, which can provide valuable information for investors and analysts seeking to assess market conditions and forecast future trends.

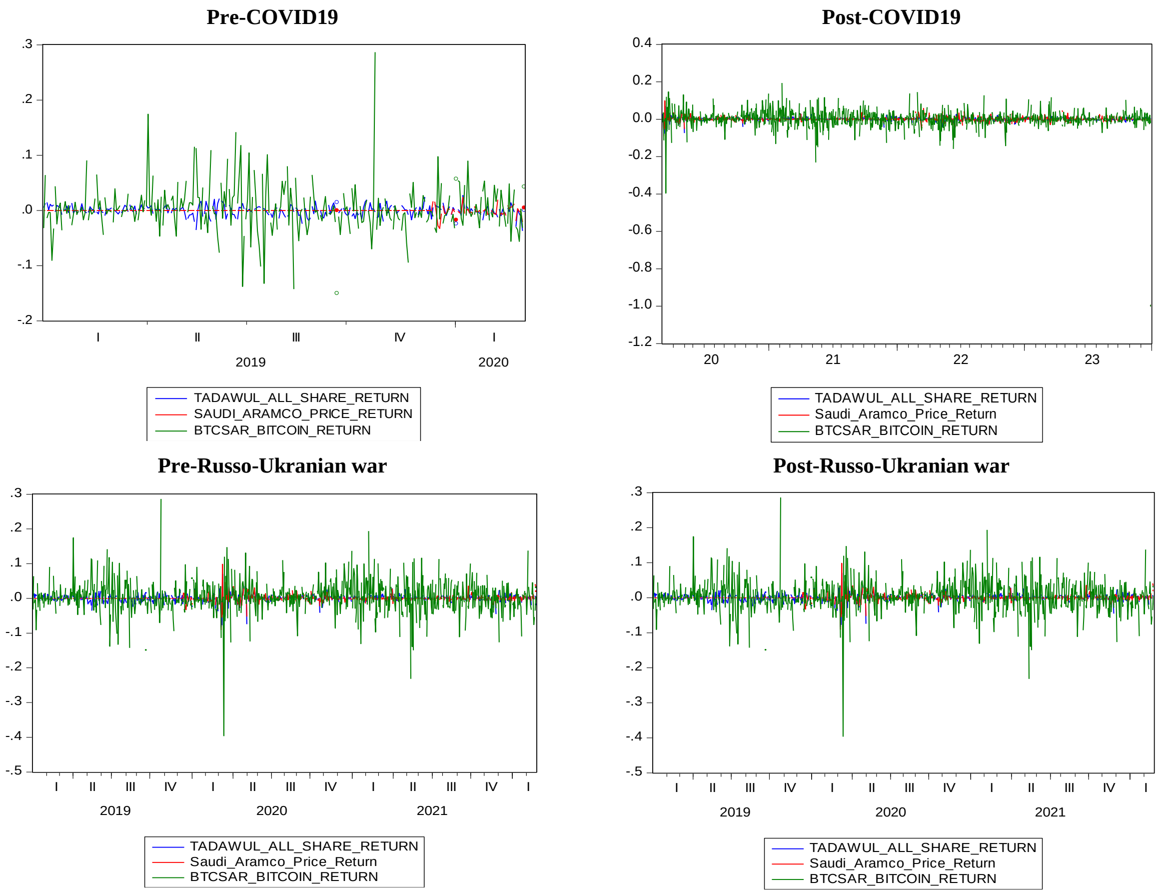

Figure 1. Comparative returns of TADAWUL All Share Index, Saudi Aramco, and Bitcoin (BTC/SAR)

The analysis of the four graphs across different time periods provides valuable insights into the dynamic behavior of the TADAWUL All Share Index, Saudi Aramco stock, and Bitcoin (BTC/SAR) returns in response to significant global events.

In the pre-COVID-19 period, from January 02, 2019 to March 01, 2020, we observe distinct behaviors across the three asset classes. The Bitcoin return series exhibited pronounced volatility, with significant upward and downward spikes, reflecting the speculative nature of cryptocurrencies and their heightened sensitivity to market sentiment. In contrast, the TADAWUL All Share Index and Saudi Aramco returns demonstrated greater stability, with fluctuations tightly centered around zero. This stability is indicative of mature market behavior, where external shocks had a less pronounced impact on these traditional assets. The lack of synchronization between Bitcoin and the Saudi equity markets further highlights their differing risk-return profiles, with Bitcoin behaving as a more volatile and speculative asset. The mid-2019 spike in Bitcoin returns can be attributed to global cryptocurrency trends, possibly fueled by increased speculation. Overall, this period underscores the relative stability of the Saudi financial markets and the potential diversification benefits of including high-risk assets like Bitcoin in investment portfolios.

In the post-COVID-19 period (March 02, 2020 to December 31, 2023), we record a recovery trajectory for both the TADAWUL All Share Index and Saudi Aramco stock, which exhibit relatively stable returns with moderate fluctuations. This stability reflects the resilience of the Saudi stock market post-pandemic, aided by economic diversification efforts and a rebound in oil prices. Saudi Aramco's returns, in particular, show consistent patterns, which aligns with the company's strong fundamentals and its central role in the energy sector. Bitcoin, on the other hand, continues to exhibit higher volatility, with sharp peaks and troughs, underscoring its sensitivity to global market sentiment, technological developments, and regulatory changes. This contrast between the stability of traditional assets and the speculative nature of Bitcoin highlights the differing risk profiles of these asset classes in the context of post-pandemic market recovery.

The period before the Russo-Ukrainian war (January 02, 2019, to February 23, 2022) shows a significant contrast in volatility between the asset classes. While the Saudi Aramco Price Return remained relatively stable, the TADAWUL All Share and Bitcoin returns experienced higher volatility, especially during periods of global uncertainty such as the onset of the COVID-19 pandemic. The fluctuations in the TADAWUL All Share Index are particularly notable, as they reflect the broader volatility seen in emerging markets, while Bitcoin's price swings were more pronounced, driven by speculation and external shocks. Saudi Aramco's relatively stable return path further emphasizes its role as a less volatile asset in comparison to cryptocurrencies and emerging market equities, highlighting the contrasting market dynamics in the pre-war period.

The post-Russo-Ukrainian war period (February 24, 2022, to December 31, 2023) illustrates shifts in market behavior, with heightened volatility observed across all three asset classes. The responses to shocks, as indicated by the impulse response functions (IRFs) from the VAR model, reflect the disruptions caused by the geopolitical conflict. All three variables show more pronounced deviations, particularly during specific periods, signalling the influence of the war on the global economy and financial markets. While TADAWUL and Saudi Aramco returns were more volatile during certain intervals, Bitcoin's price movements remained marked by less frequent but more intense spikes. These shifts underscore the different sensitivities of traditional versus digital assets to geopolitical and economic shocks. The unique market conditions post-war likely contributed to these altered response patterns, further highlighting the distinct risk profiles of each asset class and the need for strategic diversification in response to global uncertainties.

In conclusion, these periods reflect the varying degrees of market volatility and the sensitivity of different asset classes to global disruptions. The analysis underscores the importance of understanding the unique risk-return dynamics of traditional and digital assets, particularly during times of economic and geopolitical instability, to inform investment decisions and portfolio strategies.

4.3 Descriptive Statistics of the Variables

Table 1 provides an overview of the descriptive statistics for the variables TADAWUL_ALL_SHARE_RETURN, SAUDI_ARAMCO_PRICE_RETURN, and BTCSAR_BITCOIN_RETURN. These statistics offer essential insights into the central tendencies, variability, and distributional characteristics of each variable over the study period.

Examining metrics such as mean, median, standard deviation, skewness, and kurtosis allows for a comprehensive understanding of the data's behavior, including its normality and the presence of extreme values. This foundational analysis sets the stage for exploring the relationships between these variables under different economic and geopolitical conditions, as assessed in subsequent sections of the study.

Table 1. Descriptive statistics of individual variable

Statistic	TADAWUL_ALL_SHARE_RETURN	SAUDI_ARAMCO_PRICE_RETURN	BTCSAR_BITCOIN_RETURN
Mean	-0.000407	-0.000685	0.002039
Median	0.000979	0.000000	4.54E-05
Maximum	0.070702	0.098592	0.286578
Minimum	-1.000000	-1.000000	-1.000000
Std. Dev.	0.030112	0.030147	0.051766
Skewness	-29.45885	-29.26635	-6.058573
Kurtosis	977.1307	971.5629	118.9593
Jarque-Bera	49405854	48842401	705156.3
Probability	0.000000	0.000000	0.000000

The descriptive statistics presented in Table 1 provide valuable insights into the characteristics of the variables TADAWUL_ALL_SHARE_RETURN, SAUDI_ARAMCO_PRICE_RETURN, and BTCSAR_BITCOIN_RETURN. The average daily returns for Tadawul (-0.000407) and Saudi Aramco (-0.000685) are slightly negative, suggesting subdued performance during the analyzed period. In contrast, Bitcoin returns exhibit a modest positive mean (0.002039), reflecting its upward trend as a digital asset. The median values for Tadawul and Saudi Aramco are close to zero, indicating clustering around stable central tendencies, while Bitcoin's median is marginally positive (4.54E-05).

The data reveals significant volatility, with Bitcoin displaying the widest range of returns, from -1.000000 to 0.286578, compared to Tadawul and Saudi Aramco, which also have a minimum of -1.000000 but smaller maximum values of 0.070702 and 0.098592, respectively. This variability is further underscored by the standard deviation, where Bitcoin shows the highest value (0.051766) compared to Tadawul (0.030112) and Aramco (0.030147). The distributions exhibit extreme negative skewness, particularly for Tadawul (-29.46) and Saudi Aramco (-29.27), indicating an asymmetric distribution with long left tails, while Bitcoin (-6.06) shows relatively less pronounced skewness.

Kurtosis values are exceptionally high for all variables, with Tadawul (977.13) and Saudi Aramco (971.56) indicating distributions with sharp peaks and fat tails, and Bitcoin (118.96) showing pronounced but less extreme tail behavior. The Jarque-Bera test results confirm that the distributions significantly deviate from normality, as indicated by the high-test statistics and p-values of 0.000.

4.4 Correlation among Variables

The correlation matrix presented in Table 2 examines the linear relationships between the returns of three key financial variables: TADAWUL All Share Return, Saudi Aramco Price Return, and BTCSAR Bitcoin Return. This analysis provides insights into how these financial indicators move together, highlighting potential co-movements and interdependencies within the financial market.

Table 2. Correlation matrix

Variables	TADAWUL_ALL_SHARE_RETURN	SAUDI_ARAMCO_PRICE_RETURN	BTCSAR_BITCOIN_RETURN
TADAWUL_ALL_SHARE_RETURN	1.00000	0.95895	0.53414
SAUDI_ARAMCO_PRICE_RETURN	0.95895	1.00000	0.53792
BTCSAR_BITCOIN_RETURN	0.53414	0.53792	1.00000

The results in Table 2 reveal significant correlations among the analyzed financial variables. The TADAWUL All Share Return exhibits a strong positive correlation with the Saudi Aramco Price Return (0.95895), suggesting that movements in the broader Saudi equity market are closely aligned with the performance of Saudi Aramco. This strong relationship indicates that Aramco, as a major component of the Saudi market, significantly influences overall market trends.

In contrast, the correlation between the TADAWUL All Share Return and BTCSAR Bitcoin Return is moderate (0.53414), reflecting a weaker but positive relationship between traditional equity markets and cryptocurrency returns. Similarly, the Saudi Aramco Price Return shows a comparable moderate correlation with the BTCSAR Bitcoin Return (0.53792). This suggests that while there is some level of co-movement between these assets, cryptocurrencies like Bitcoin remain partially decoupled from traditional financial markets, potentially offering diversification benefits for investors.

4.5 Unit Root Tests

Table 3 presents the results of the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests applied to TADAWUL_ALL_SHARE_RETURN, SAUDI_ARAMCO_PRICE_RETURN, and BTCSAR_BITCOIN_RETURN to examine the stationarity of these series. Stationarity is a critical assumption for many time-series models, as non-stationary data can lead to spurious results in regression analyses. Both tests were performed under three scenarios: with an intercept, with both trend and intercept, and without either.

Table 3. Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests

Serie	Level of integration	Parameter	Augmented Dickey-Fuller (ADF) test			Phillips-Perron (PP) test		
			Intercept	Trend and intercept	None	Intercept	Trend and intercept	None
TADAWUL_ALL_SHARE_RETURN	Level	*t*-statistic	-10.69491	-10.71106	-10.72685	-10.51327	-10.53166	-10.54228
		p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SAUDI_ARAMCO_PRICE_RETURN	Level	*t*-statistic	-10.85957	-10.84717	-10.87116	-10.61428	-10.61725	-10.61739
		p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
BTCSAR_BITCOIN_RETURN	Level	*t*-statistic	-30.88969	-30.96763	-30.84251	-31.05198	-31.09479	-31.14137
		p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The results indicate that all three-return series are stationary at their levels. The ADF test statistics for TADAWUL_ALL_SHARE_RETURN (-10.69 to -10.73), SAUDI_ARAMCO_PRICE_RETURN (-10.85 to -10.87), and BTCSAR_BITCOIN_RETURN (-30.84 to -30.97) are all well below their respective critical values at the 1% significance level. Similarly, the PP test statistics for TADAWUL_ALL_SHARE_RETURN (-10.51 to -10.54), SAUDI_ARAMCO_PRICE_RETURN (-10.61 to -10.62), and BTCSAR_BITCOIN_RETURN (-31.05 to -31.14) confirm stationarity, as they also exceed the critical thresholds.

The corresponding p-values for both tests are consistently 0.000 across all scenarios, providing strong evidence to reject the null hypothesis of a unit root. This suggests that the data series do not exhibit stochastic trends, making them suitable for further econometric modelling without additional differencing.

These results confirm the reliability of the time-series data for exploring dynamic relationships and causal interactions among the Tadawul All Share Index, Saudi Aramco prices, and Bitcoin returns using ARDL and VAR models.

4.6 ARDL Model Results

4.6.1 Lag Selection Criteria

Lag selection criteria are fundamental in time series analysis for models such as Autoregressive Distributed Lag (ARDL) and Vector Autoregression (VAR). In ARDL models, selecting the appropriate lag structure is essential for accurately capturing both short-term dynamics and long-term equilibrium relationships between variables, particularly when there are shifts in structural conditions, such as during economic or geopolitical events. For VAR models, the choice of lag length determines the system's ability to reflect the interdependencies and temporal relationships among variables.

Lag selection criteria such as the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), and Hannan-Quinn Criterion (HQ) provide a systematic approach to finding the optimal lag length by balancing model fit and complexity. In the context of major global disruptions, such as the COVID-19 pandemic or the Russo-Ukrainian war, applying these criteria is crucial to account for structural breaks and dynamic changes in economic and financial systems.

Table 4 presents the lag selection criteria for four distinct periods: Pre-COVID19, Post-COVID19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War. The table reports the Log Likelihood (LogL), Likelihood Ratio (LR), Final Prediction Error (FPE), Akaike Information Criterion (AIC), Schwarz Criterion (SC), and Hannan-Quinn Criterion (HQ) for different lags (0 to 4) for each period.

For the Pre-COVID19 period, the best lag based on AIC is lag 1, as it has the lowest AIC value of -17.99339, compared to other lags. Similarly, the LR statistic for lag 1 is the highest, indicating that this lag is statistically significant. In contrast, for Post-COVID19, lag 1 also emerges as the best choice, with the lowest AIC value of -14.24321, supported by the highest LR value of 35.40510. This suggests that lag 1 is the most optimal in capturing the dynamics in the post-COVID period.

For the Pre-Russo-Ukrainian War period, lag 1 has the lowest AIC value of -16.69123, signifying that this lag is the most appropriate, as evidenced by its statistically significant LR value of 61.29561. The Post-Russo-Ukrainian War period shows a similar pattern, with lag 0 being optimal as it has the lowest AIC value of -13.64410, which is also reflected in the significant LR value.

In summary, lag 1 seems to be the best model selection criterion for the Pre-COVID, Post-COVID and Pre-Russo-Ukrainian War periods, while lag 0 is optimal for the Post-Russo-Ukrainian War periods, as indicated by the lowest AIC and significant LR values.

Table 4. Lag selection criteria

Period	Lag	LogL	LR	FPE	AIC	SC	HQ
Pre-COVID19	0	2544.247	NA	3.18e-12	-17.95934	-17.92070*	-17.94385*
	1	2558.065	27.24425	3.08e-12*	-17.99339*	-17.83881	-17.93141
	2	2562.058	7.789728	3.19e-12	-17.95801	-17.68750	-17.84954
	3	2566.684	8.924544	3.29e-12	-17.92710	-17.54065	-17.77215
	4	2569.283	4.959760	3.44e-12	-17.88186	-17.37948	-17.68043
Post-COVID19	0	6731.262	NA	1.33e-10	-14.22466	-14.20927*	-14.21879
	1	6749.040	35.40510	1.31e-10*	-14.24321*	-14.18166	-14.21976*
	2	6753.307	8.470965	1.32e-10	-14.23321	-14.12549	-14.19216
	3	6763.000	19.18073*	1.32e-10	-14.23467	-14.08080	-14.17603
	4	6767.510	8.896417	1.33e-10	-14.22518	-14.02514	-14.14895
Pre-Russo-Ukrainian war	0	6482.429	NA	1.20e-11	-16.63525	-16.61731	-16.62835
	1	6513.235	61.29561	1.13e-11	-16.69123	-16.61948*	-16.66363*
	2	6520.779	14.95256	1.14e-11	-16.68750	-16.56193	-16.63920
	3	6529.514	17.24572	1.14e-11	-16.68682	-16.50743	-16.61782
	4	6535.594	11.95605	1.15e-11	-16.67932	-16.44612	-16.58962
Post-Russo-Ukrainian war	0	3127.499	NA	2.38e-10*	-13.64410*	-13.61707*	-13.63345*
	1	3135.555	15.97195	2.39e-10	-13.63998	-13.53185	-13.59739
	2	3139.110	7.000513	2.45e-10	-13.61620	-13.42698	-13.54167
	3	3145.435	12.37465	2.48e-10	-13.60452	-13.33420	-13.49806
	4	3151.348	11.49002	2.51e-10	-13.59104	-13.23962	-13.45263

* indicates lag order selected by the criterion.

Based on the lag selection criteria outlined in Table 4, although different lags emerge as optimal for individual periods (e.g., lag 1 for Pre-COVID, Post-COVID, and Pre-Russo-Ukrainian War periods, and lag 0 for the Post-Russo-Ukrainian War period), a unified lag structure will be adopted for all periods. Therefore, lag 1 will be chosen for all periods in the ARDL model. This consistent lag selection across all periods simplifies the modeling process and ensures comparability of the results over time, enabling a more straightforward interpretation of the relationships between the variables across different phases.

The inverse roots of the AR characteristic polynomial are a diagnostic tool used to assess the stability of ARDL and VAR models. Stability requires all inverse roots to lie within the unit circle. Figure 2 illustrates the inverse roots for two significant global events: pre- and post-COVID-19, and pre- and post-Russo-Ukrainian war periods.

In the pre-COVID-19 period, the roots for both ARDL and VAR models lie well within the unit circle, indicating stable systems where shocks are absorbed over time. However, in the post-COVID-19 period, some roots approach the edge of the unit circle, reflecting heightened volatility and potential instability introduced by the pandemic's economic disruptions.

Similarly, during the pre-Russo-Ukrainian war period, the roots indicate stability, suggesting that the models adequately captured the relationships between variables. In the post-Russo-Ukrainian war period, roots closer to the boundary of the unit circle point to reduced stability, likely due to the war's impact on geopolitical and economic structures.

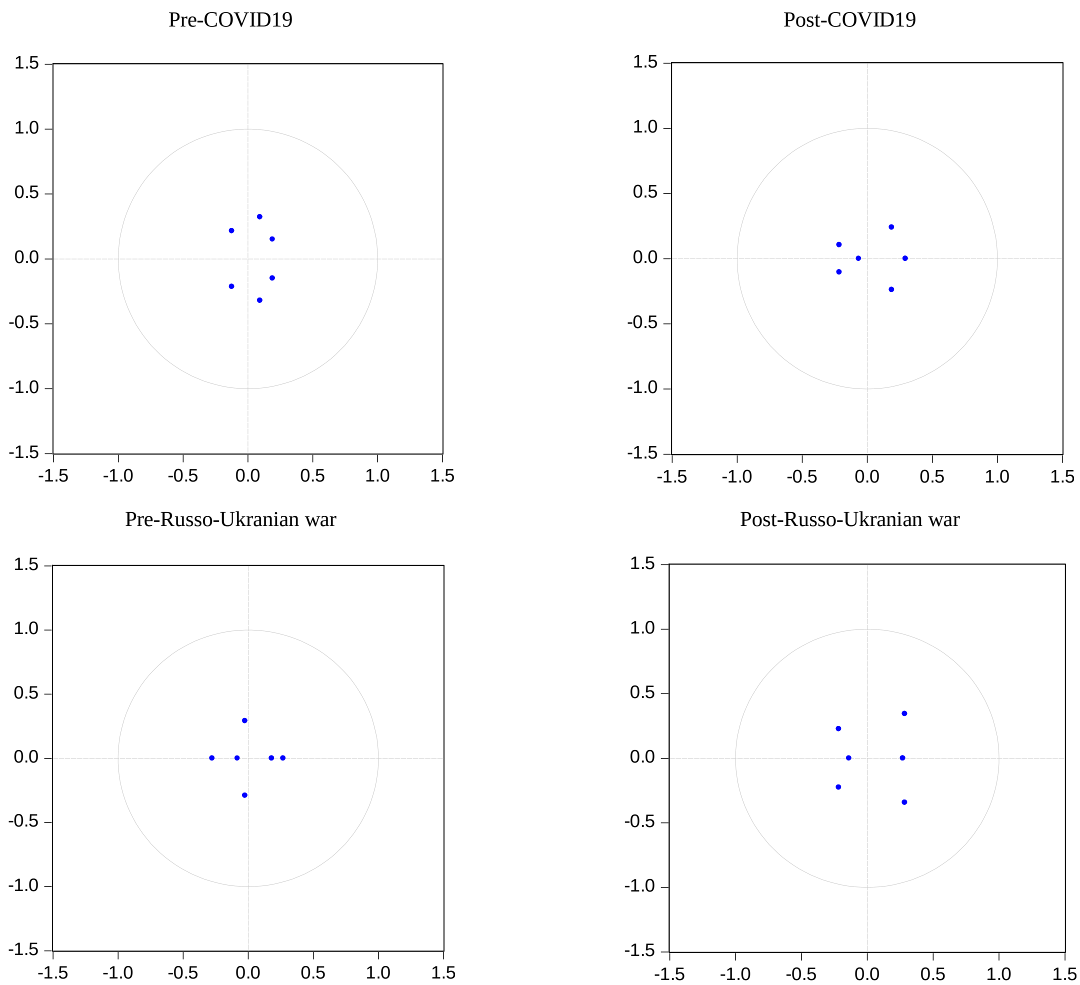

Figure 2. Inverse roots of AR test

Figure 3 presents the AIC values of the top ARDL models before and after these two significant global events, providing insight into how these events may have influenced the optimal model specifications and variable relationships. Although the optimal lag length is selected as 1 based on the AIC criterion, AIC values for lag lengths up to 4 are presented to provide insight into how the model's fit changes across different lag specifications.

Figure 3 compares the AIC values of the top ARDL models for the periods pre- and post-COVID-19 and pre- and post-Russo-Ukrainian war. The AIC values for the pre-COVID-19 period generally show a more stable and consistent model structure with relatively lower values, indicating that the selected lags effectively captured the relationships between the variables. However, in the post-COVID-19 period, the AIC values tend to increase, suggesting that the pandemic introduced more complexity or volatility into the data, making the models less efficient or harder to fit with the same lag structure. This could be due to the significant economic and behavioral shifts brought about by the pandemic.

Similarly, in the pre-Russo-Ukrainian war period, the AIC values are lower and stable, pointing to a well-fitting ARDL model with a reasonable lag structure. In contrast, the post-Russo-Ukrainian war period shows higher AIC values, which may reflect disruptions in the global economy, geopolitical tensions, and their effects on the relationships among the variables. The increase in AIC values post-war indicates that the structural changes caused by the war likely introduced new dynamics that were harder to model efficiently with the same specifications.

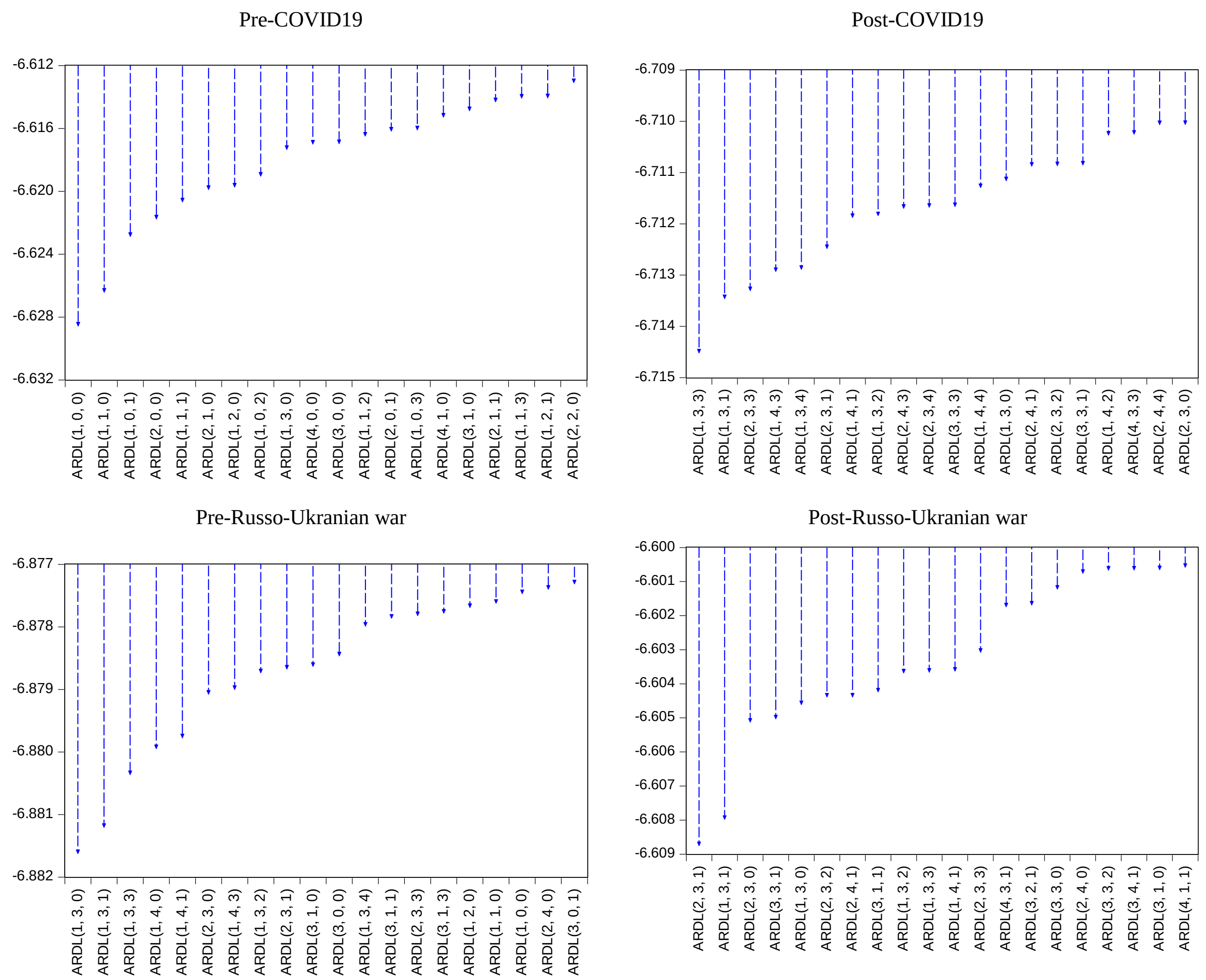

Figure 3. AIC value of top ARDL models

4.6.2 Results of of ARDL Bound and Johansen Cointegration Test

The Johansen cointegration test is used to determine the existence of a long-term relationship between multiple time series variables. It tests for the presence of cointegration vectors that indicate the variables move together over time. The test utilizes two main statistics: the Trace statistic and the Maximum Eigenvalue statistic, each of which tests different hypotheses about the number of cointegrating equations. In table 5, the results of the Johansen cointegration test are presented for different periods: Pre-COVID19, Post-COVID19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War.

The results of the Johansen cointegration test indicate significant evidence of cointegration in all periods examined: Pre-COVID19, Post-COVID19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War. For each period, both the Trace and Maximum Eigenvalue statistics show values that exceed the critical values, with p-values consistently indicating rejection of the null hypothesis of no cointegration. Specifically, the tests suggest the presence of multiple cointegrating relationships, particularly in the Pre-COVID and Post-COVID periods, as well as during the Pre-Russo-Ukrainian War. Even in the Post-Russo-Ukrainian War period, the results support the existence of two cointegrating relationships, indicating that the long-term relationships between the variables remain intact, albeit with some shifts in the nature of the cointegration after the Russo-Ukrainian conflict.

In summary, the Johansen cointegration test suggests that for all the analyzed periods, there is strong evidence of long-run relationships (cointegration) among the variables in the model, as all null hypotheses of no cointegration are rejected at the 5% significance level.

Table 5. Results of Johansen cointegration test

Period	Hypothesized No. of CE(s)	Eigen value	Trace Statistic	Prob.**
Pre-COVID19	Trace			
	None *	0.265942	179.6151	0.0001
	At most 1 *	0.168452	91.19331	0.0000
	At most 2 *	0.125752	38.43594	0.0000
	Maximum Eigenvalue			
	None *	0.265942	88.42178	0.0001
	At most 1 *	0.168452	52.75736	0.0000
	At most 2 *	0.125752	38.43594	0.0000
Post-COVID19	Trace			
	None *	0.191655	395.1126	0.0001
	At most 1 *	0.169998	192.3466	0.0001
	At most 2 *	0.015386	14.77666	0.0001
	Maximum Eigenvalue			
	None *	0.191655	202.7659	0.0001
	At most 1 *	0.169998	177.5700	0.0001
	At most 2 *	0.015386	14.77666	0.0001
Pre-Russo-Ukrainian war	Trace			
	None *	0.178248	342.6791	0.0001
	At most 1 *	0.126832	189.1599	0.0001
	At most 2 *	0.100814	83.09911	0.0000
	Maximum Eigenvalue			
	None *	0.178248	153.5192	0.0001
	At most 1 *	0.126832	106.0608	0.0001
	At most 2 *	0.100814	83.09911	0.0000
Post-Russo-Ukrainian war	Trace			
	None *	0.229387	204.5453	0.0001
	At most 1 *	0.158152	85.20462	0.0000
	At most 2 *	0.013785	6.357529	0.0117
	Maximum Eigenvalue			
	None *	0.229387	119.3406	0.0001
	At most 1 *	0.158152	78.84709	0.0000
	At most 2 *	0.013785	6.357529	0.0117

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values.

The results of the ARDL bounds testing in table 6 provide valuable insights into the long-run relationships between the variables for the periods under consideration: Pre-COVID19, Post-COVID19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War. In each case, the optimal lag lengths for the models are specified alongside the F-statistic values, which are used to assess the significance of the bounds test.

The ARDL bounds testing results confirm the presence of a significant long-run relationship between the dependent variable and the independent variables across all four periods: Pre-COVID-19, Post-COVID-19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War. For the Pre-COVID-19 period, Model 1 (ARDL(1,0,0)) has an F-statistic of 44.82392, which exceeds the upper bound critical values at all significance levels. This indicates a robust cointegration relationship during this relatively stable period, with the dependent variable showing an immediate dependence on the independent variables without any lagged effects.

In the Post-COVID-19 period, Model 2 (ARDL(1,1,1)) yields a notably higher F-statistic of 193.4164, signalling a very strong cointegration relationship. This period's dynamics suggest both immediate and lagged effects of the independent variables on the dependent variable, reflecting heightened interdependencies during a time of significant economic disruption. Similarly, the Pre-Russo-Ukrainian War period, captured by Model 3 (ARDL(1,1,0)), demonstrates a strong cointegration relationship with an F-statistic of 131.1551. Here, one independent variable exerts an immediate influence while the other exhibits a lagged effect, indicating a mix of contemporaneous and delayed interactions.

Finally, the Post-Russo-Ukrainian War period, represented by Model 4 (ARDL(1,1,1)), shows an F-statistic of 159.4416, underscoring a persistent and robust cointegration relationship. This model mirrors the dynamics of the

Post-COVID-19 period, with both immediate and lagged effects being significant, reflecting the heightened economic and geopolitical complexities of this timeframe Across all periods, the F-statistics exceed the critical values at the 1%, 5%, and 10% significance levels, confirming that the variables are cointegrated in the long run, and supporting the robustness of the ARDL models across different time periods. These findings emphasize the adaptability of the ARDL approach in capturing the unique dynamics of varying economic and geopolitical conditions.

Table 6. The summary of ARDL bounds testing

Period	Model for estimation	Optimal lag	F-stat
Pre-COVID19	Model 1	ARDL (1,0,0)	44.82392
	Bounds test critical values: K (2)	I (0)	I (1)
	1%	4.99	5.85
	5%	3.88	4.61
	10%	3.38	4.02
Post-COVID19	Model 2	ARDL (1,1,1)	193.4164
	Bounds test critical values: K (2)	I (0)	I (1)
	1%	4.99	5.85
	5%	3.88	4.61
	10%	3.38	4.02
Pre-Russo-Ukrainian war	Model 3	ARDL (1,1,0)	131.1551
	Bounds test critical values: K (2)	I (0)	I (1)
	1%	4.99	5.85
	5%	3.88	4.61
	10%	3.38	4.02
Post-Russo-Ukrainian war	Model 4	ARDL (1,1,1)	159.4416
	Bounds test critical values: K (2)	I (0)	I (1)
	1%	4.99	5.85
	5%	3.88	4.61
	10%	3.38	4.02

4.6.3 Long-Run and Short-Run Estimates

Table 7 provides the ARDL long-run and short-run estimation results for the relationships between TADAWUL All Share Index Returns, Saudi Aramco Price Returns, and Bitcoin Returns across four distinct periods: Pre-COVID-19, Post-COVID-19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War. The table summarizes the coefficients, standard errors, t-statistics, and p-values for both long-run and short-run dynamics. Additionally, the Error Correction Model (ECM) coefficients are included, reflecting the speed of adjustment to long-run equilibrium following short-term shocks. Key diagnostic statistics, including R-squared, adjusted R-squared, F-statistics, and Durbin-Watson statistics, are also presented for model performance evaluation.

Table 7. The ARDL long-run and short-run results

Period	Variables	Coefficient	Std. Error	t-Statistic	Prob.
Pre-COVID19	Long run				
	TADAWUL_ALL_SHARE_RETURN	-0.808108	0.056575	-14.28396	0.0000
	SAUDI_ARAMCO_PRICE_RETURN	0.463258	0.115897	3.997137	0.0000
	BTCSAR_BITCOIN_RETURN	-0.033182	0.011840	-2.802561	0.0001
	C	0.001498	0.001043	1.436100	0.1521
	Short run				
	D(TADAWUL_ALL_SHARE_RETURN)				
	D(SAUDI_ARAMCO_PRICE_RETURN)	0.475123	0.086197	5.512049	0.0000
	D(BTCSAR_BITCOIN_RETURN)	-0.034040	0.008268	-4.117293	0.0001
	CointEq(-1)	-0.799607	0.058650	-13.63348	0.0000
	R-squared	0.996386			
	Adjusted R-squared	0.988756			
	F-statistic	59.81719			
	Durbin-Watson stat	1.981304			

Post-COVID19	Long run				
	TADAWUL_ALL_SHARE_RETURN	-0.811215	0.032487	-24.97036	0.0000
	SAUDI_ARAMCO_PRICE_RETURN	0.281047	0.090884	3.092369	0.0000
	BTCSAR_BITCOIN_RETURN	-0.037798	0.009209	-4.104446	0.0000
	C	-0.000180	0.000792	-0.226516	0.3208
	Short run				
	D(TADAWUL_ALL_SHARE_RETURN)				
	D(SAUDI_ARAMCO_PRICE_RETURN)	0.293329	0.055009	5.332420	0.0000
	D(BTCSAR_BITCOIN_RETURN)	-0.031403	0.004385	-7.162185	0.0000
	CointEq(-1)	-0.811215	0.032095	-25.27528	0.0000
	R-squared	0.986590			
	Adjusted R-squared	0.974780			
	F-statistic	235.5854			
	Durbin-Watson stat	2.001319			
Pre-Russo-Ukrainian war	Long run				
	TADAWUL_ALL_SHARE_RETURN	-0.788879	0.035385	-22.29405	0.0000
	SAUDI_ARAMCO_PRICE_RETURN	0.340028	0.107477	3.163738	0.0001
	BTCSAR_BITCOIN_RETURN	-0.042425	0.010090	-4.204744	0.0000
	C	0.000688	0.000623	1.104403	0.2698
	Short run				
	D(TADAWUL_ALL_SHARE_RETURN)				
	D(SAUDI_ARAMCO_PRICE_RETURN)	0.495894	0.061159	8.108285	0.0000
	D(BTCSAR_BITCOIN_RETURN)	-0.034749	0.004890	-7.106115	0.0000
	CointEq(-1)	-0.788879	0.035218	-22.40008	0.0000
	R-squared	0.970423			
	Adjusted R-squared	0.968276			
	F-statistic	193.4083			
	Durbin-Watson stat	1.999348			
Post-Russo-Ukrainian war	Long run				
	TADAWUL_ALL_SHARE_RETURN	-0.806389	0.032223	-25.02556	0.0000
	SAUDI_ARAMCO_PRICE_RETURN	0.315163	0.089925	3.504737	0.0000
	BTCSAR_BITCOIN_RETURN	-0.035918	0.009198	-3.904873	0.0001
	C	-0.000395	0.000789	-0.500500	0.6168
	Short run				
	D(TADAWUL_ALL_SHARE_RETURN)				
	D(SAUDI_ARAMCO_PRICE_RETURN)	0.348575	0.053574	6.506442	0.0000
	D(BTCSAR_BITCOIN_RETURN)	-0.030112	0.004399	-6.845089	0.0000
	CointEq(-1)	-0.806389	0.031881	-25.29380	0.0000
	R-squared	0.983081			
	Adjusted R-squared	0.975938			
	F-statistic	243.0796			
	Durbin-Watson stat	2.000107			

The ARDL long-run and short-run results provide detailed insights into the relationships between the dependent variable and the independent variables during four periods: Pre-COVID-19, Post-COVID-19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War. During the Pre-COVID-19 period, the TADAWUL All Share return has a significant negative long-run impact on the dependent variable, with a coefficient of -0.808108, reflecting a strong inverse relationship. Bitcoin return also demonstrates a smaller negative effect (-0.033182), while the Saudi Aramco price return shows a significant positive impact (0.463258). The constant term is statistically insignificant, indicating no standalone effect. In the short run, only changes in Saudi Aramco price return (0.475123) and Bitcoin return (-0.034040) significantly influence the dependent variable. The error correction term (CointEq(-1)) is negative and highly significant (-0.799607), confirming a rapid adjustment toward the long-run equilibrium after deviations. The high R-squared value (0.996386) and a Durbin-Watson statistic of 1.981304 reflect a well-fitting model without autocorrelation concerns.

In the Post-COVID-19 period, similar dynamics persist. The TADAWUL All Share return retains its strong negative long-run influence (-0.811215), while the Saudi Aramco price return (0.281047) and Bitcoin return (-0.037798) maintain their positive and negative effects, respectively. The constant term remains insignificant. In the short run, the Saudi Aramco price return (0.293329) and Bitcoin return (-0.031403) continue to show significant effects. The error correction term (-0.811215) indicates quick reversion to equilibrium. An R-squared value of 0.986590 and a Durbin-Watson statistic of 2.001319 affirm the model's robustness and absence of serial correlation.

During the Pre-Russo-Ukrainian War period, the long-run relationships remain consistent, with the TADAWUL All Share return (-0.788879) negatively influencing the dependent variable and the Saudi Aramco price return (0.340028) positively affecting it. Bitcoin return (-0.042425) also maintains its negative impact, while the constant term is again insignificant. In the short run, changes in the Saudi Aramco price return (0.495894) and Bitcoin return (-0.034749) significantly impact the dependent variable. The error correction term (-0.788879) highlights rapid adjustments toward the long-run equilibrium. The R-squared value of 0.970423 and a Durbin-Watson statistic of 1.999348 further indicate a strong model fit without autocorrelation.

In the Post-Russo-Ukrainian War period, the results exhibit stability, with the TADAWUL All Share return (-0.806389) consistently exerting a strong negative long-run effect. The Saudi Aramco price return (0.315163) and Bitcoin return (-0.035918) maintain their positive and negative influences, respectively, while the constant term remains insignificant. In the short run, the Saudi Aramco price return (0.348575) and Bitcoin return (-0.030112) significantly affect the dependent variable. The error correction term (-0.806389) confirms a quick adjustment to long-run equilibrium following short-run shocks. The R-squared value of 0.983081 and a Durbin-Watson statistic of 2.000107 confirm the model's high explanatory power and lack of serial correlation.

The results consistently demonstrate the significant roles of the TADAWUL All Share return, Saudi Aramco price return, and Bitcoin return in both the long-run and short-run dynamics across all periods. The error correction terms reveal rapid adjustments to equilibrium, while high R-squared values and appropriate Durbin-Watson statistics affirm the robustness of the models. These findings highlight the adaptability of the ARDL framework in capturing the unique economic and geopolitical dynamics of each period.

4.6.4 Diagnostic Testing

Table 8 presents the results of various diagnostic tests conducted on the ARDL model for four distinct periods: Pre-COVID19, Post-COVID19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War. These tests include the Jarque-Bera Normality Test, Breusch-Godfrey Serial Correlation LM Test, several heteroskedasticity tests (Breusch-Pagan-Godfrey, ARCH, Glejser, and Harvey), the Ramsey Reset Test, and the CUSUM and CUSUMSQ tests. The F-statistics and their corresponding p-values are provided for each test, offering insights into the validity and robustness of the model over these periods.

Table 8. Diagnostic statistics test

Period	Test	F-statistic	Prob. F
Pre-COVID19	Jarque-Bera Normality Test	1.816129	0.403304
	Breusch-Godfrey Serial Correlation LM Test	0.086164	0.9175
	Heteroskedasticity Test: Breusch-Pagan-Godfrey	1.151568	0.3326
	Heteroskedasticity Test: ARCH	1.135954	0.3371
	Heteroskedasticity Test: Glejser	1.281191	0.2658
	Heteroskedasticity Test: Harvey	0.989513	0.4326
	Ramsey Reset Test	0.608786	0.4359
	CUSUM	Stable	
	CUSUMSQ	Stable	
Post-COVID19	Jarque-Bera Normality Test	1.797420	0.320268
	Breusch-Godfrey Serial Correlation LM Test	0.007861	0.9922
	Heteroskedasticity Test: Breusch-Pagan-Godfrey	1.636520	0.6153
	Heteroskedasticity Test: ARCH	1.858987	0.5017
	Heteroskedasticity Test: Glejser	1.891078	0.3585
	Heteroskedasticity Test: Harvey	0.921315	0.6779
	Ramsey Reset Test	0.569644	0.4323
	CUSUM	Stable	
	CUSUMSQ	Stable	

Pre-Russo-Ukrainian war	Jarque-Bera Normality Test	1.784897	0.420395
	Breusch-Godfrey Serial Correlation LM Test	0.760641	0.4677
	Heteroskedasticity Test: Breusch-Pagan-Godfrey	1.028174	0.6806
	Heteroskedasticity Test: ARCH	0.941958	0.5534
	Heteroskedasticity Test: Glejser	1.006972	0.6431
	Heteroskedasticity Test: Harvey	0.861401	0.7822
	Ramsey Reset Test	0.008107	0.9283
	CUSUM	Stable	
	CUSUMSQ	Stable	
Post-Russo-Ukrainian war	Jarque-Bera Normality Test	2.148652	0.341528
	Breusch-Godfrey Serial Correlation LM Test	0.246222	0.7819
	Heteroskedasticity Test: Breusch-Pagan-Godfrey	1.152872	0.2981
	Heteroskedasticity Test: ARCH	0.005852	0.9391
	Heteroskedasticity Test: Glejser	1.253800	0.2602
	Heteroskedasticity Test: Harvey	1.203418	0.2907
	Ramsey Reset Test	0.445350	0.5049
	CUSUM	Stable	
	CUSUMSQ	Stable	

In the Pre-COVID-19 period, the Jarque-Bera test for normality yields an F-statistic of 1.816129 with a p-value of 0.403304, indicating that the residuals are normally distributed. The Breusch-Godfrey Serial Correlation LM test (F-statistic: 0.086164, p-value: 0.9175) confirms no serial correlation, while the Breusch-Pagan-Godfrey test (F-statistic: 1.151568, p-value: 0.3326) and ARCH test (F-statistic: 1.135954, p-value: 0.3371) reveal no evidence of heteroskedasticity. Additional tests, including Glejser (F-statistic: 1.281191, p-value: 0.2658) and Harvey (F-statistic: 0.989513, p-value: 0.4326), also show homoscedastic residuals. The Ramsey RESET test (F-statistic: 0.608786, p-value: 0.4359) suggests no model misspecification. Stability diagnostics using the CUSUM and CUSUMSQ tests confirm a stable model.

In the Post-COVID-19 period, the Jarque-Bera test produces an F-statistic of 1.797420 with a p-value of 0.320268, confirming normality in residuals. The Breusch-Godfrey test (F-statistic: 0.007861, p-value: 0.9922) shows no serial correlation. Heteroskedasticity tests, including Breusch-Pagan-Godfrey (F-statistic: 1.636520, p-value: 0.6153), ARCH (F-statistic: 1.858987, p-value: 0.5017), Glejser (F-statistic: 1.891078, p-value: 0.3585), and Harvey (F-statistic: 0.921315, p-value: 0.6779), all indicate homoscedastic residuals. The Ramsey RESET test (F-statistic: 0.569644, p-value: 0.4323) supports the absence of misspecification. Stability diagnostics confirm that the model remains stable throughout the period.

During the Pre-Russo-Ukrainian War period, the Jarque-Bera test yields an F-statistic of 1.784897 with a p-value of 0.420395, indicating normally distributed residuals. The Breusch-Godfrey test (F-statistic: 0.760641, p-value: 0.4677) confirms no serial correlation. Tests for heteroskedasticity, including Breusch-Pagan-Godfrey (F-statistic: 1.028174, p-value: 0.6806), ARCH (F-statistic: 0.941958, p-value: 0.5534), Glejser (F-statistic: 1.006972, p-value: 0.6431), and Harvey (F-statistic: 0.861401, p-value: 0.7822), reveal homoscedasticity. The Ramsey RESET test (F-statistic: 0.008107, p-value: 0.9283) confirms the model is correctly specified. Stability diagnostics, including CUSUM and CUSUMSQ, verify the model's stability.

In the Post-Russo-Ukrainian War period, the Jarque-Bera test results (F-statistic: 2.148652, p-value: 0.341528) confirm residual normality. The Breusch-Godfrey test (F-statistic: 0.246222, p-value: 0.7819) indicates no serial correlation, while heteroskedasticity tests such as Breusch-Pagan-Godfrey (F-statistic: 1.152872, p-value: 0.2981), ARCH (F-statistic: 0.005852, p-value: 0.9391), Glejser (F-statistic: 1.253800, p-value: 0.2602), and Harvey (F-statistic: 1.203418, p-value: 0.2907) confirm the residuals are homoscedastic. The Ramsey RESET test (F-statistic: 0.445350, p-value: 0.5049) shows no evidence of model misspecification, and stability diagnostics (CUSUM and CUSUMSQ) validate the model's stability.

Diagnostic statistics across all periods confirm the robustness of the ARDL models. Residuals are normally distributed and free from serial correlation and heteroskedasticity, with no evidence of model misspecification. Stability tests consistently verify the reliability of the models in capturing the economic dynamics during each period.

The CUSUM (Cumulative Sum of Recursive Residuals) Test, displayed in figure 4, provides a visual and statistical means to evaluate the stability of the estimated coefficients over time. The CUSUM test graph for the

Pre-COVID19 period illustrates the stability of the model's coefficients over time. The blue line represents the cumulative sum of the recursive residuals, while the red dashed lines indicate the 5% significance boundaries. Since the CUSUM line remains well within the critical boundaries throughout the entire period, it suggests that the model is stable and does not exhibit significant structural changes. This stability indicates that the relationships among the variables were consistent during the Pre-COVID19 period, reflecting a reliable model fit and no major disruptions or shocks that would impact its performance during this time.

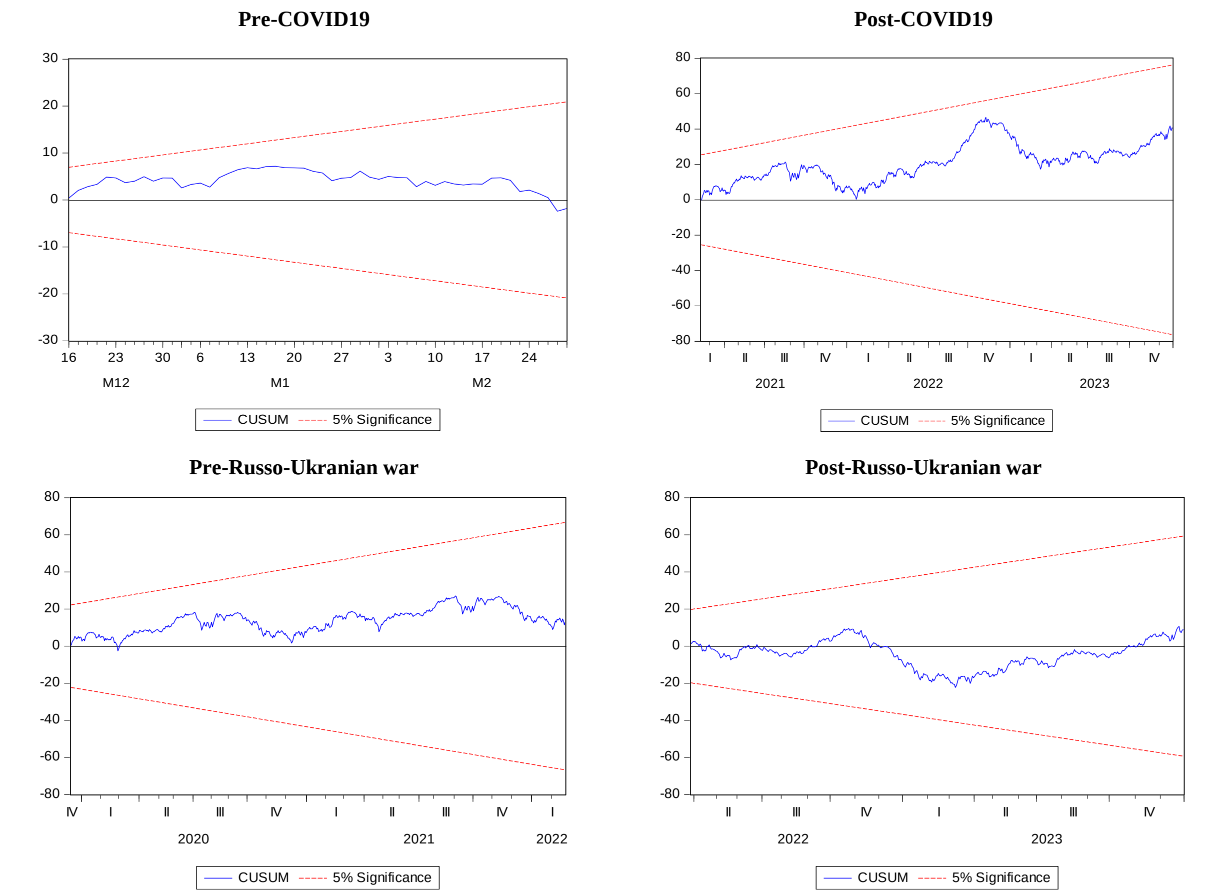

Figure 4. CUSUM test

The CUSUM test graphs for the Post-COVID19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War periods similarly illustrate the stability of the model's coefficients over time. In each case, the blue line, representing the cumulative sum of the recursive residuals, remains well within the critical boundaries indicated by the red dashed lines. This stability confirms that, despite the significant global shocks and disruptions caused by COVID-19 and the geopolitical conflict, the model's long-run relationships among the variables remained consistent during these periods.

For the Post-COVID19 period, the stability observed in the CUSUM test highlights the model's ability to adapt to new dynamics introduced by the pandemic's aftermath without undergoing substantial structural changes. This suggests that while volatility and external shocks influenced short-term dynamics (as evidenced by heteroskedasticity in diagnostic tests), the core relationships among the variables stayed intact.

During the Pre-Russo-Ukrainian War period, the stability in the CUSUM test indicates that the variables maintained a steady long-run equilibrium despite the geopolitical tensions and uncertainties leading up to the conflict. The model's robustness in this period reflects its capacity to capture the evolving dynamics without structural instability.

In the Post-Russo-Ukrainian War period, the CUSUM test's stability demonstrates that the model continued to effectively explain the relationships among the variables, even under the heightened global economic pressures and disruptions caused by the war. This underscores the reliability of the model's long-run framework in capturing consistent interactions despite increased volatility in the economic and geopolitical environment.

The CUSUM of Squares test results, illustrated in Figure 5, confirm the stability of the ARDL models across all four analyzed periods: Pre-COVID-19, Post-COVID-19, Pre-Russo-Ukrainian War, and Post-Russo-Ukrainian War. In each period, the CUSUM of Squares plots, as shown in Figure 5, remain consistently within the critical boundaries at the 5% significance level. This indicates that the model parameters are stable and unaffected by potential structural breaks or volatility during the respective periods.

In the Pre-COVID-19 period, the test reveals no evidence of structural instability, demonstrating that the model effectively captures the underlying dynamics of the variables. Similarly, during the Post-COVID-19 period, the results indicate robustness in the model parameters, even in the face of economic disruptions caused by the pandemic. For the Pre-Russo-Ukrainian War and Post-Russo-Ukrainian War periods, the CUSUM of Squares plots validate the models' stability and resilience, ensuring reliable parameter estimation despite significant geopolitical and economic changes.

The results presented in Figure 5 substantiate the robustness and reliability of the ARDL models across all four periods, underscoring their effectiveness in maintaining consistent parameter stability under diverse and evolving conditions.

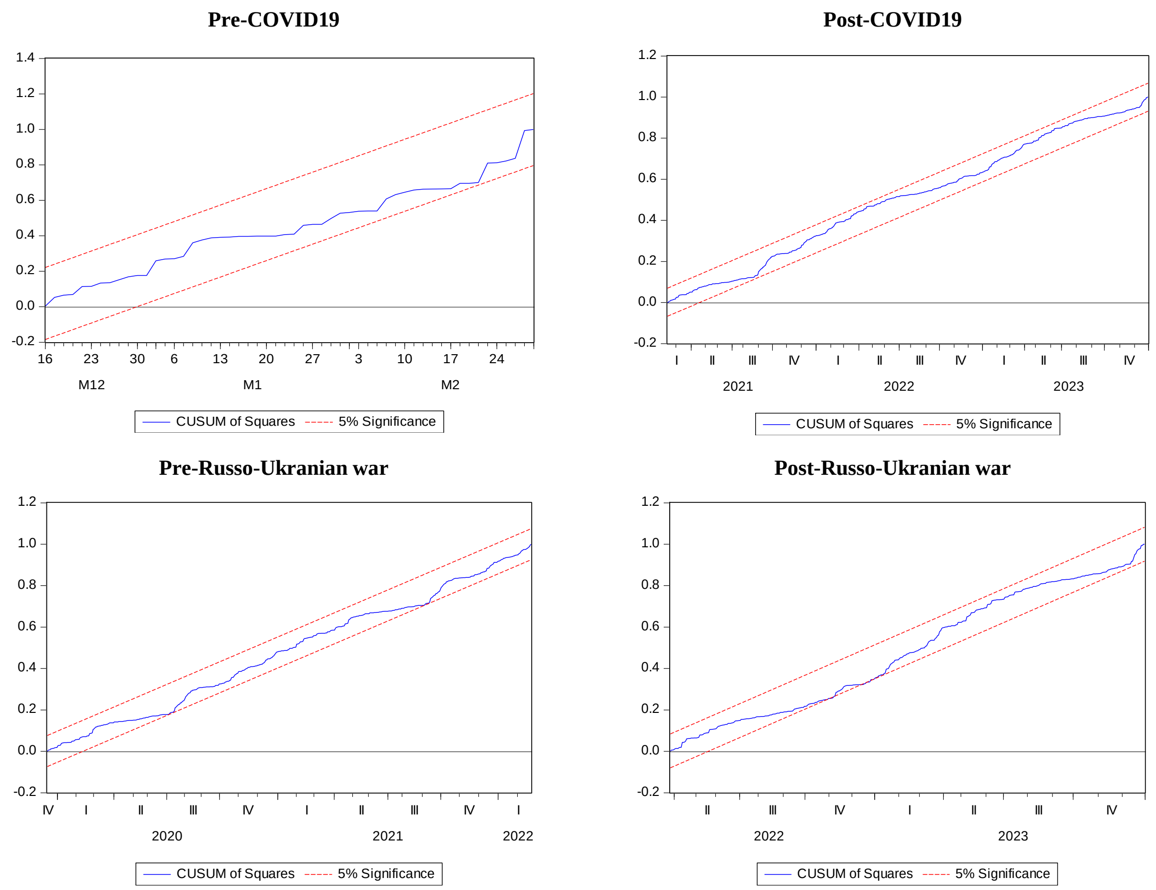

Figure 5. CUSUM square test

4.7 VAR Model Results

4.7.1 VAR Model Estimation

The table 9 presents the Vector Autoregression (VAR) estimation results for the returns of the TADAWUL All Share Index, Saudi Aramco Price Return, and Bitcoin (BTC/SAR) Return during the period before the COVID-19 pandemic. The coefficients for the lagged values of each variable are shown, along with their standard errors in parentheses and t-statistics in square brackets. These results offer insights into the dynamic relationships between the three asset returns in the period leading up to the pandemic, highlighting their interdependencies and responses to past shocks. The coefficients and significance levels provide valuable information for understanding the impact of past performance on future returns of each asset.

Table 9. VAR estimation results for asset returns pre-COVID-19 Pandemic

Variable	TADAWUL_ALL_SHARE_ RETURN	SAUDI_ARAMCO_PRICE _RETURN	BTCSAR_BITCOIN _RETURN
TADAWUL_ALL_SHARE_RETURN(-1)	0.205684	-0.060015	-0.568188
	(0.06198)	(0.03008)	(0.29658)
	[3.31878]	[-1.99501]	[-1.91582]
TADAWUL_ALL_SHARE_RETURN(-2)	0.003461	0.003186	-0.397108
	(0.06391)	(0.03102)	(0.30585)
	[0.05416]	[0.10270]	[-1.29837]
SAUDI_ARAMCO_PRICE_RETURN(-1)	-0.042689	0.158335	-0.416775
	(0.12590)	(0.06111)	(0.60249)
	[-0.33906]	[2.59088]	[-0.69175]
SAUDI_ARAMCO_PRICE_RETURN(-2)	-0.085801	-0.117791	0.023625
	(0.12831)	(0.06228)	(0.61399)
	[-0.66872]	[-1.89135]	[0.03848]
BTCSAR_BITCOIN_RETURN(-1)	-0.008159	-0.008202	-0.043141
	(0.01254)	(0.00609)	(0.05999)
	[-0.65080]	[-1.34786]	[-0.71909]
BTCSAR_BITCOIN_RETURN(-2)	0.013597	0.006745	-0.042174
	(0.01256)	(0.00610)	(0.06013)
	[1.08211]	[1.10598]	[-0.70141]
Constant (C)	-0.000250	-0.000368	-0.000368
	(0.00054)	(0.00026)	(0.00026)
	[-0.45839]	[-1.38993]	[-1.38993]

The results presented in the table 9 correspond to the Vector Autoregression (VAR) model estimation for the period before the COVID-19 pandemic. In this period, the TADAWUL All Share Index return (TADAWUL_ALL_SHARE_RETURN) is primarily influenced by its own lagged values, with the coefficient for the first lag (TADAWUL_ALL_SHARE_RETURN(-1)) being 0.205684 and statistically significant (t-statistic of 3.31878). This indicates a positive relationship, suggesting that the past performance of the TADAWUL index tends to have a lasting effect on its current value. In contrast, the coefficient for the second lag (TADAWUL_ALL_SHARE_RETURN(-2)) is 0.003461, with a t-statistic of 0.05416, which is statistically insignificant. This suggests that the second lag of the TADAWUL index has little to no effect on its current returns.

For Saudi Aramco Price Returns (SAUDI_ARAMCO_PRICE_RETUR), the first lag (SAUDI_ARAMCO_PRICE_RETUR(-1)) shows a significant positive coefficient of 0.158335 (t-statistic of 2.59088), indicating that past movements in the price of Saudi Aramco stock have a notable positive impact on the overall TADAWUL market. This relationship highlights the importance of Saudi Aramco as a major player in the Saudi stock market. On the other hand, the coefficient for the second lag (SAUDI_ARAMCO_PRICE_RETUR(-2)) is -0.117791, with a t-statistic of -1.89135, which is close to the significance threshold, suggesting a weak negative influence, although its statistical significance is moderate.

The Bitcoin return (BTCSAR_BITCOIN_RETURN) shows a more complex relationship with TADAWUL returns. The coefficient for the first lag of Bitcoin return (BTCSAR_BITCOIN_RETURN(-1)) is -0.568188 (t-statistic of -1.91582), indicating a negative relationship, though it is only marginally significant. This suggests that Bitcoin's volatility may have a slight negative effect on the TADAWUL market, but the relationship is not strong. Similarly, the coefficient for the second lag (BTCSAR_BITCOIN_RETURN(-2)) is -0.397108, with a t-statistic of -1.29837, which is statistically insignificant, further suggesting that Bitcoin's past returns have minimal impact on the Saudi stock market during this period.

Finally, the constant term (C) is -0.000250, with a t-statistic of -0.45839, which is not statistically significant. This indicates that there is no significant drift or bias in the model during this period. Overall, the results indicate that the Saudi stock market in the pre-COVID-19 period was primarily influenced by its own past performance and by the price return of Saudi Aramco, with minimal influence from Bitcoin. The TADAWUL index and Saudi Aramco returns show a strong connection, reflecting the dominant role of large entities in the Saudi market, while Bitcoin's high volatility does not appear to have a direct, significant effect on the market at this time.

Table 10. VAR estimation results for asset returns post-COVID-19 Pandemic

Variable	TADAWUL_ALL_SHARE_ RETURN	SAUDI_ARAMCO_PRICE_ RETURN	BTCSAR_BITCOIN_ RETURN
TADAWUL_ALL_SHARE_RETURN(-1)	-0.051241	-0.215563	-0.164910
	(0.15479)	(0.15631)	(0.24468)
	[-0.33103]	[-1.37909]	[-0.67397]
TADAWUL_ALL_SHARE_RETURN(-2)	-0.066446	-0.131620	-0.117246
	(0.15286)	(0.15436)	(0.24163)
	[-0.43468]	[-0.85269]	[-0.48522]
SAUDI_ARAMCO_PRICE_RETURN(-1)	0.126405	0.212436	0.166128
	(0.13828)	(0.13963)	(0.21858)
	[0.91415]	[1.52143]	[0.76005]
SAUDI_ARAMCO_PRICE_RETURN(-2)	0.015959	0.081012	-0.260421
	(0.13749)	(0.13884)	(0.21734)
	[0.11607]	[0.58350]	[-1.19824]
BTCSAR_BITCOIN_RETURN(-1)	0.036837	0.024113	-0.052681
	(0.02595)	(0.02621)	(0.04102)
	[1.41940]	[0.92011]	[-1.28417]
BTCSAR_BITCOIN_RETURN(-2)	0.021928	0.024992	0.076370
	(0.02615)	(0.02640)	(0.04133)
	[0.83868]	[0.94657]	[1.84782]
Constant (C)	-0.000635	-0.000803	0.001689
	(0.00111)	(0.00112)	(0.00176)
	[-0.57182]	[-0.71557]	[0.96192]

Table 10 presents the results of the Vector Autoregression (VAR) estimation for asset returns after the onset of the COVID-19 pandemic. The table includes the coefficients of lagged returns for the TADAWUL All Share Index, Saudi Aramco's stock price, and Bitcoin, along with their standard errors and t-statistics.

Starting with the TADAWUL_ALL_SHARE_RETURN, the coefficient for the first lag (TADAWUL_ALL_SHARE_RETURN(-1)) is -0.051241, with a t-statistic of -0.33103, suggesting that the lagged return of the TADAWUL All Share Index does not significantly influence its current return post-COVID-19. The second lag (TADAWUL_ALL_SHARE_RETURN(-2)) has a coefficient of -0.066446 with a t-statistic of -0.43468, indicating that the effect of two lags on the current return is also statistically insignificant.

For the SAUDI_ARAMCO_PRICE_RETURN, the coefficient for the first lag (SAUDI_ARAMCO_PRICE_RETURN(-1)) is 0.126405 with a t-statistic of 0.91415, indicating a positive but statistically insignificant relationship between the previous return of Saudi Aramco and its current return. The second lag (SAUDI_ARAMCO_PRICE_RETURN(-2)) shows a much smaller coefficient of 0.015959 and a t-statistic of 0.11607, reinforcing the insignificance of lagged values in predicting the current return of Saudi Aramco.

Turning to BTCSAR_BITCOIN_RETURN, the first lag (BTCSAR_BITCOIN_RETURN(-1)) has a coefficient of 0.036837 with a t-statistic of 1.41940, suggesting a positive and moderately significant impact of Bitcoin's return one period ago on its current return. In contrast, the second lag (BTCSAR_BITCOIN_RETURN(-2)) shows a coefficient of 0.021928 and a t-statistic of 0.83868, which is statistically insignificant.

The constant (C) term shows small negative values for TADAWUL_ALL_SHARE_RETURN (-0.000635) and SAUDI_ARAMCO_PRICE_RETURN (-0.000803), both with insignificant t-statistics. However, the constant for BTCSAR_BITCOIN_RETURN is positive (0.001689) with a t-statistic of 0.96192, suggesting a slight positive average return for Bitcoin that is not statistically significant.

The results highlight that the lagged returns of the TADAWUL All Share Index and Saudi Aramco do not significantly impact their current returns post-COVID-19, whereas Bitcoin's lagged returns show some positive influence, albeit not statistically strong. The relationships are generally weak and do not provide strong evidence of significant temporal effects on asset returns in the post-COVID-19 period.

Table 11. VAR estimation results for asset returns pre-Russo-Ukrainian war

Variable	TADAWUL_ALL_SHARE_ RETURN	SAUDI_ARAMCO_PRICE_ RETURN	BTCSAR_BITCOIN_ RETURN
TADAWUL_ALL_SHARE_RETURN(-1)	0.132174	-0.443742	-0.090285
	(0.04963)	(0.22068)	(0.04250)
	[2.66344]	[-2.01082]	[-2.12444]
TADAWUL_ALL_SHARE_RETURN(-2)	0.006841	-0.038165	-0.014229
	(0.04991)	(0.22192)	(0.04274)
	[0.13709]	[-0.17198]	[-0.33295]
SAUDI_ARAMCO_PRICE_RETURN(-1)	0.014337	-0.078529	0.008282
	(0.00806)	(0.03582)	(0.00690)
	[1.77986]	[-2.19235]	[1.20062]
SAUDI_ARAMCO_PRICE_RETURN(-2)	0.005990	0.065555	0.005835
	(0.00805)	(0.03578)	(0.00689)
	[0.74447]	[1.83208]	[0.84679]
BTCSAR_BITCOIN_RETURN(-1)	-0.079291	0.453662	-0.006650
	(0.05829)	(0.25921)	(0.04992)
	[-1.36024]	[1.75014]	[-0.13322]
BTCSAR_BITCOIN_RETURN(-2)	-0.121874	-0.356860	-0.078457
	(0.05793)	(0.25762)	(0.04961)
	[-2.10369]	[-1.38521]	[-1.58138]
Constant (C)	0.000472	0.004335	0.000213
	(0.00038)	(0.00171)	(0.00033)
	[1.22853]	[2.53991]	[0.64882]

Table 11 presents the results of the Vector Autoregression (VAR) estimation for asset returns during the period prior to the Russo-Ukrainian War. The table includes the coefficients, standard errors, and t-statistics for the lagged returns of the TADAWUL All Share Index, Saudi Aramco's stock price, and Bitcoin.

For TADAWUL_ALL_SHARE_RETURN, the coefficient for the first lag (TADAWUL_ALL_SHARE_RETURN(-1)) is 0.132174, with a t-statistic of 2.66344, indicating that the return of the TADAWUL All Share Index from the previous period has a significant positive effect on the current return. The second lag (TADAWUL_ALL_SHARE_RETURN(-2)) has a coefficient of 0.006841 and a t-statistic of 0.13709, which is not statistically significant, suggesting that the impact of the second lag on the current return is negligible.

For SAUDI_ARAMCO_PRICE_RETURN, the first lag (SAUDI_ARAMCO_PRICE_RETURN(-1)) shows a negative coefficient of -0.443742, with a t-statistic of -2.01082, which is statistically significant at the 5% level. This implies that the previous period's return of Saudi Aramco negatively affects the current return. In contrast, the second lag (SAUDI_ARAMCO_PRICE_RETURN(-2)) has a positive coefficient of 0.065555, and a t-statistic of 1.83208, suggesting a positive but statistically insignificant effect on the current return of Saudi Aramco.

For BTCSAR_BITCOIN_RETURN, the coefficient for the first lag (BTCSAR_BITCOIN_RETURN(-1)) is -0.090285 with a t-statistic of -2.12444, indicating a negative and statistically significant relationship between Bitcoin's return one period ago and its current return. The second lag (BTCSAR_BITCOIN_RETURN(-2)) has a coefficient of -0.014229 with a t-statistic of -0.33295, which is not statistically significant, suggesting that the second lag does not have a significant effect on Bitcoin's return.

The constant (C) term for all three variables is positive, with coefficients of 0.000472 for TADAWUL_ALL_SHARE_RETURN, 0.004335 for SAUDI_ARAMCO_PRICE_RETURN, and 0.000213 for BTCSAR_BITCOIN_RETURN. The constant term for Saudi Aramco's return is statistically significant with a t-statistic of 2.53991, while for the TADAWUL All Share Index and Bitcoin, the constants are not statistically significant.

In summary, the results indicate that the lagged returns of the TADAWUL All Share Index, Saudi Aramco, and Bitcoin show varying levels of significance. The TADAWUL All Share Index and Bitcoin are significantly influenced by their first lags, while Saudi Aramco's returns are negatively impacted by the first lag, with the second lag showing a positive but insignificant effect. These findings provide insight into the dynamics of asset returns before the onset of the Russo-Ukrainian War.

Table 12 provides the results of the Vector Autoregression (VAR) estimation for asset returns following the onset of the Russo-Ukrainian War. The table includes the coefficients, standard errors, and t-statistics for the lagged returns of the TADAWUL All Share Index, Saudi Aramco's stock price, and Bitcoin.

For TADAWUL_ALL_SHARE_RETURN, the coefficient for the first lag (TADAWUL_ALL_SHARE_RETURN(-1)) is -0.110639, with a t-statistic of -0.33618, suggesting that the return of the TADAWUL All Share Index in the previous period has a negligible and statistically insignificant effect on the current return. The second lag (TADAWUL_ALL_SHARE_RETURN(-2)) shows a coefficient of -0.139457, with a t-statistic of -0.43207, which is also statistically insignificant, further confirming that the past returns of the TADAWUL All Share Index do not significantly influence the current return in the post-Russo-Ukrainian war period.

For SAUDI_ARAMCO_PRICE_RETURN, the coefficient for the first lag (SAUDI_ARAMCO_PRICE_RETURN(-1)) is 0.208449, with a t-statistic of 0.84265, indicating a positive but statistically insignificant impact on the current return. The second lag (SAUDI_ARAMCO_PRICE_RETURN(-2)) has a coefficient of 0.078551, with a t-statistic of 0.31748, which also indicates a statistically insignificant effect on the current return.

For BTCSAR_BITCOIN_RETURN, the first lag (BTCSAR_BITCOIN_RETURN(-1)) has a positive coefficient of 0.057241, with a t-statistic of 0.87173, suggesting a positive but statistically insignificant relationship between the previous period's Bitcoin return and the current return. The second lag (BTCSAR_BITCOIN_RETURN(-2)) shows a coefficient of 0.051021, with a t-statistic of 0.76933, indicating a positive but statistically insignificant effect on Bitcoin's return.

The constant (C) term for all three variables is negative, with coefficients of -0.002365 for TADAWUL_ALL_SHARE_RETURN, -0.002311 for SAUDI_ARAMCO_PRICE_RETURN, and -0.001730 for BTCSAR_BITCOIN_RETURN. However, the constant terms are not statistically significant, as indicated by their respective t-statistics of -1.05375 for TADAWUL_ALL_SHARE_RETURN, -1.01777 for SAUDI_ARAMCO_PRICE_RETURN, and -0.63547 for BTCSAR_BITCOIN_RETURN.

In summary, the results suggest that the lagged returns of the TADAWUL All Share Index, Saudi Aramco, and Bitcoin have little to no significant influence on their current returns in the post-Russo-Ukrainian War period. All variables' lagged effects are statistically insignificant, indicating that market dynamics may have shifted or become more volatile during this period, rendering past returns less predictive of current performance.

Table 12. VAR Estimation Results for Asset Returns post-Russo-Ukrainian war

Variable	TADAWUL_ALL_SHARE_RETURN	SAUDI_ARAMCO_PRICE_RETURN	BTCSAR_BITCOIN_RETURN
TADAWUL_ALL_SHARE_RETURN(-1)	-0.110639	-0.223014	-0.277741
	(0.32911)	(0.33292)	(0.39919)
	[-0.33618]	[-0.66988]	[-0.69575]
TADAWUL_ALL_SHARE_RETURN(-2)	-0.139457	-0.253033	-0.152026
	(0.32277)	(0.32650)	(0.39150)
	[-0.43207]	[-0.77498]	[-0.38831]
SAUDI_ARAMCO_PRICE_RETURN(-1)	0.208449	0.345840	0.104403
	(0.24737)	(0.25024)	(0.30005)
	[0.84265]	[1.38205]	[0.34795]
SAUDI_ARAMCO_PRICE_RETURN(-2)	0.078551	0.150063	-0.133732
	(0.24742)	(0.25029)	(0.30011)
	[0.31748]	[0.59957]	[-0.44560]
BTCSAR_BITCOIN_RETURN(-1)	0.057241	0.038133	0.063997
	(0.06566)	(0.06642)	(0.07965)
	[0.87173]	[0.57408]	[0.80350]
BTCSAR_BITCOIN_RETURN(-2)	0.051021	0.058524	-0.001083
	(0.06632)	(0.06709)	(0.08044)
	[0.76933]	[0.87236]	[-0.01346]
Constant (C)	-0.002365	-0.002311	-0.001730
	(0.00224)	(0.00227)	(0.00272)
	[-1.05375]	[-1.01777]	[-0.63547]

The VAR estimation results for asset returns before and after the COVID-19 pandemic and the Russo-Ukrainian War reveal notable shifts in market behavior. Before the COVID-19 pandemic, the TADAWUL All Share Index and Saudi Aramco returns exhibited significant lagged effects on their own future returns. Specifically, the first lag of TADAWUL returns was highly significant, while the second lag showed minimal impact. Saudi Aramco returns also displayed a significant effect from the first lag. Bitcoin, however, showed no significant influence from its lagged returns, suggesting its returns were less predictable.

Post-COVID, the relationships between lagged returns and current asset returns weakened significantly. The TADAWUL All Share Index showed no significant lagged effects, and Saudi Aramco returns experienced a diminished impact from the first lag, though still positive. Bitcoin's influence remained statistically insignificant, reflecting the asset's continued volatility in a post-pandemic market. These changes could be due to a shift in investor behavior and market conditions caused by the economic uncertainties surrounding the pandemic.

Following the Russo-Ukrainian War, the market dynamics further weakened. The lagged effects for TADAWUL returns and Saudi Aramco returns were similarly insignificant, with no strong predictive power from past returns. Bitcoin's returns continued to show no significant relationship, suggesting that the geopolitical crisis did not alter its role as a volatile and unpredictable asset. This overall reduction in the significance of lagged effects across the periods highlights how global crises, such as the pandemic and geopolitical conflicts, have led to a less predictable and more volatile market environment.

4.7.2 Impulse Response Function

Figure 6 presents the impulse response functions derived from the VAR model, showcasing the dynamic interactions among TADAWUL All Share Index returns, Saudi Aramco returns, and Bitcoin returns across four key periods: pre-COVID-19, post-COVID-19, pre-Russo-Ukrainian War, and post-Russo-Ukrainian War. These functions provide a graphical representation of how each variable responds to a one-standard-deviation shock in the system over time. By examining these responses, the figure highlights the evolving relationships and sensitivities among the variables under varying economic and geopolitical conditions, offering valuable insights into market behavior during periods of global uncertainty and crisis.

Before COVID-19, the impulse response functions demonstrate relatively stable and predictable patterns. Shocks to TADAWUL and Saudi Aramco returns resulted in measurable and contained responses in the system, with Bitcoin returns showing limited and less consistent feedback. This indicates a market environment characterized by clearer interconnections and a degree of resilience to shocks.

Post-COVID-19, the impulse responses display greater volatility and diminished stability. The market's sensitivity to shocks increases, particularly for TADAWUL and Saudi Aramco, while Bitcoin's responses remain erratic and subdued. This reflects the heightened uncertainty and changing investor behavior during the aftermath of the pandemic.

In the pre-Russo-Ukrainian War period, the system regains some degree of stability, as indicated by more controlled impulse responses. TADAWUL and Saudi Aramco continue to display interlinked reactions, while Bitcoin remains less integrated into the broader market dynamics. However, the response magnitudes are more subdued compared to the pre-COVID period.

Post-Russo-Ukrainian War, the impulse response functions suggest further weakening of the system's interconnectedness. Shocks to any variable elicit weaker and less sustained responses across the board. This reflects the compounded effects of geopolitical instability, which exacerbate market volatility and disrupt the predictability observed in earlier periods.

Figure 6 underscores the evolving dynamics of market variables in response to global crises, with each period marked by shifts in sensitivity, stability, and the strength of interconnections among TADAWUL, Saudi Aramco, and Bitcoin returns.

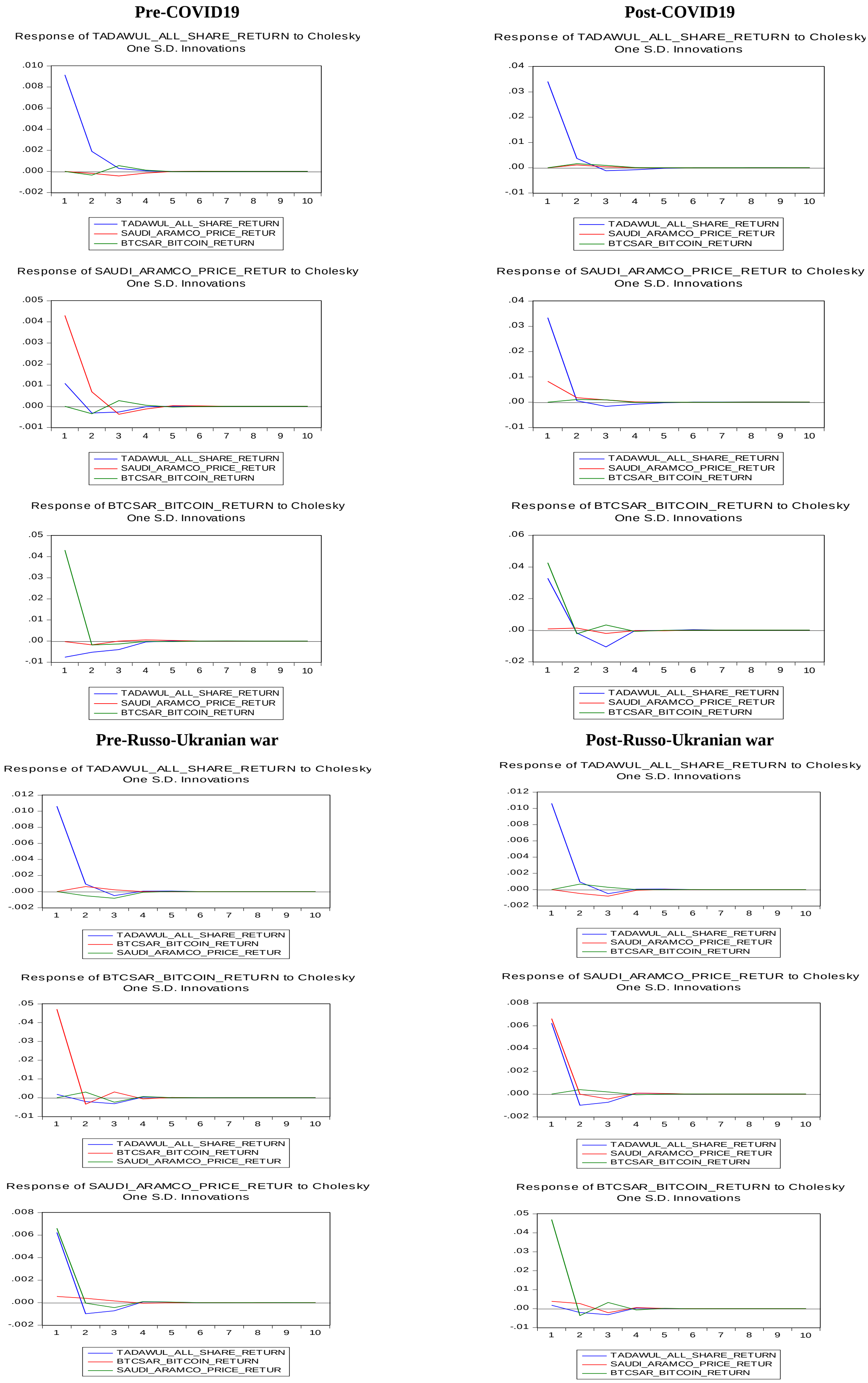

Figure 6. Impulse response functions of the VAR model variables

4.7.3 Granger Causality Test Results for Assets Returns

Granger causality tests (Granger, 1969, 1980) were conducted to determine the directional relationships between the returns of the TADAWUL stock index, the BITCOIN cryptocurrency, and Saudi Arabia's oil prices. This methodology examines whether past values of one variable enhance the predictability of another, thereby revealing causal links in the Granger sense. Specifically, a variable Xjt is said to Granger-cause another variable Xkt if the historical data of Xjt improves the forecast accuracy for Xkt. For this study, Granger causality tests were applied to explore whether the TADAWUL stock index returns could predict the returns of BITCOIN and oil prices, and vice versa.

In this context, a Granger causality relationship implies that knowledge of past values of one series—such as the TADAWUL stock index—enhances the forecasting ability for the other series, like BITCOIN or oil returns. This approach allows us to identify the direction of influence among the variables and answer questions such as whether the TADAWUL stock index improves predictions of cryptocurrency and oil price returns or vice versa.

Three specific null hypotheses were tested:

1) H0: The return of the TADAWUL stock index does not Granger-cause the returns of BITCOIN and oil prices.
2) H0: The return of oil prices does not Granger-cause the returns of the TADAWUL stock index and BITCOIN.
3) H0: The return of BITCOIN does not Granger-cause the returns of the TADAWUL stock index and oil prices.

These tests provided critical insights into the interdependencies and predictive relationships among the asset classes, shedding light on their interconnected dynamics within the examined periods.

Table 13. Granger causality test results

Pre-COVID-19	Test1		Test2		Test3	
	F-Statistic	P-Value	F-Statistic	P-Value	F-Statistic	P-Value
TADAWUL_ALL_SHARE_RETURN			4.089128	0.1294	6.722308	0.0347
SAUDI_ARAMCO_PRICE_RETURN	0.650841	0.7222			0.483785	0.7851
BTCSAR_BITCOIN_RETURN	1.660036	0.4360	3.177996	0.2041		
Post-COVID-19	Test1		Test2		Test3	
	F-Statistic	P-Value	F-Statistic	P-Value	F-Statistic	P-Value
TADAWUL_ALL_SHARE_RETURN			2.941752	0.2297	0.776095	0.6784
SAUDI_ARAMCO_PRICE_RETURN	0.882247	0.6433			1.838583	0.3988
BTCSAR_BITCOIN_RETURN	2.541674	0.2806	1.611915	0.4467		
Pre-Russo-Ukrainian War	Test1		Test2		Test3	
	F-Statistic	P-Value	F-Statistic	P-Value	F-Statistic	P-Value
TADAWUL_ALL_SHARE_RETURN			5.044494	0.0803	4.339684	0.1142
SAUDI_ARAMCO_PRICE_RETURN	6.352792	0.0417			4.918360	0.0855
BTCSAR_BITCOIN_RETURN	3.512510	0.1727	1.992055	0.3693		
Post-Russo-Ukrainian War	Test1		Test2		Test3	
	F-Statistic	P-Value	F-Statistic	P-Value	F-Statistic	P-Value
TADAWUL_ALL_SHARE_RETURN			1.180574	0.5542	0.704289	0.7032
SAUDI_ARAMCO_PRICE_RETUR	0.920469	0.6311			0.276989	0.8707
BTCSAR_BITCOIN_RETURN	1.379874	0.5016	1.111592	0.5736		

The Granger causality test results presented in Table 13 examine the relationships between TADAWUL stock index returns, SAUDI ARAMCO price returns, and BITCOIN returns across four distinct periods: pre-COVID-19, post-COVID-19, pre-Russo-Ukrainian war, and post-Russo-Ukrainian war. The analysis explores whether each variable significantly predicts the others, shedding light on their dynamic interdependencies under different macroeconomic and geopolitical conditions.

During the pre-COVID-19 period, the TADAWUL stock index returns significantly Granger-cause the returns of SAUDI ARAMCO, as indicated by an F-statistic of 6.722308 (p-value = 0.0347). This suggests a predictive relationship where historical movements in the TADAWUL index help forecast changes in SAUDI ARAMCO returns. However, the returns of BITCOIN do not exhibit significant causality toward the TADAWUL index or SAUDI ARAMCO returns, nor do SAUDI ARAMCO returns significantly predict the other variables.

In the post-COVID-19 period, none of the relationships between the three variables demonstrate statistical significance. The F-statistics for all tests in this period are low, and the p-values exceed conventional significance thresholds. This indicates weakened or negligible causal relationships, potentially reflecting heightened market uncertainties or structural changes in financial and commodity markets following the pandemic.

For the pre-Russo-Ukrainian war period, the results show that SAUDI ARAMCO price returns significantly Granger-cause TADAWUL index returns, with an F-statistic of 6.352792 (p-value = 0.0417). This highlights the predictive role of oil prices in determining stock market performance during this period. Additionally, there is a near-significant Granger causality from SAUDI ARAMCO returns to BITCOIN returns (p-value = 0.0855), suggesting a potential but marginally insignificant influence. TADAWUL and BITCOIN returns do not exhibit significant causality toward the other variables.

In the post-Russo-Ukrainian war period, the results show no significant Granger causality among the variables. The F-statistics are consistently low, and p-values indicate no strong predictive relationships. This lack of significant causality may reflect increased market volatility and the global economic impact of the conflict, diluting clear directional linkages among the variables.

In conclusion, the findings reveal temporal variations in the strength and direction of causality among the TADAWUL stock index, SAUDI ARAMCO, and BITCOIN returns. Significant relationships are observed in specific periods, such as pre-COVID-19 and pre-Russo-Ukrainian war, but these connections appear to diminish during periods of heightened uncertainty, like post-COVID-19 and post-Russo-Ukrainian war. These insights underscore the sensitivity of financial and commodity markets to broader macroeconomic and geopolitical dynamics.

4.8 Integrating Short-Term and Long-Term Economic Dynamics: Policy Recommendations from VAR and ARDL Models

The integration of short-term and long-term economic dynamics, as revealed by both VAR and ARDL models, provides a comprehensive understanding of the interactions between key economic variables. The ARDL results demonstrate significant long-term and short-term relationships between the Saudi stock market (Tadawul All Share Return), Saudi Aramco's price return, and Bitcoin returns. In the long run, the results show that fluctuations in the stock market negatively impact its overall performance, while Saudi Aramco's price return has a positive influence. Bitcoin returns exhibit a significant negative long-term relationship with stock market returns, highlighting the inverse dynamics between digital currency movements and traditional stock market performance. Short-run dynamics further emphasize the significant positive impact of Saudi Aramco's price return and the negative effect of Bitcoin returns on the stock market.

Similarly, the VAR model complements these findings by providing a deeper understanding of the temporal relationships between the same variables. The impulse response functions and Granger causality tests from the VAR model reveal that shocks to Saudi Aramco's stock returns have a significant and persistent effect on the stock market, reinforcing the ARDL results. Additionally, the VAR model indicates bidirectional causality between the stock market and Bitcoin returns in both the short and long run, supporting the notion of a dynamic interaction between traditional and digital assets. The results of both models together suggest that market fluctuations driven by oil prices and Bitcoin returns are key drivers of stock market performance, with both short-term and long-term implications.

The robustness of these results is confirmed by the stability tests from the ARDL model, including the CUSUM and CUSUMSQ tests, which remain within the critical boundaries at the 5% significance level across all periods, indicating model stability. The VAR model also supports these findings, with diagnostic tests confirming the absence of significant issues related to autocorrelation and heteroskedasticity. Taken together, these models offer valuable insights for policymakers, suggesting that strategies to stabilize the stock market should focus on diversifying investments beyond traditional oil-based assets, while also managing the volatility and impact of Bitcoin returns on the market. Furthermore, strengthening the resilience of the stock market to external shocks, particularly from oil and cryptocurrency price fluctuations, will be essential for ensuring long-term economic stability.

5. Conclusion

This study explores the dynamic relationships and causal interactions between the TADAWUL All Share Index, SAUDI ARAMCO stock returns, and BITCOIN cryptocurrency returns across various economic and geopolitical periods, including pre- and post-COVID-19 as well as pre- and post-Russo-Ukrainian war. The ARDL model results highlight significant long-term equilibrium relationships among these variables, revealing that while short-term dynamics are susceptible to fluctuations from external shocks, the long-term relationships between TADAWUL, SAUDI ARAMCO, and BITCOIN remain relatively stable. The ARDL bounds test confirms the presence of a long-run relationship between the variables in the pre-COVID-19 period, but this relationship weakened post-COVID-19, likely due to structural shifts induced by the pandemic. These findings suggest that while long-term trends persist, their strength may diminish during periods of significant economic disruption. Consequently, policies aimed at fostering long-term economic stability—such as promoting economic diversification and enhancing the resilience of financial markets to external shocks—are crucial.

The VAR model results further emphasize the interlinkages among the variables, revealing that these relationships vary significantly across periods. Pre-COVID-19, a strong predictive relationship existed between TADAWUL returns and SAUDI ARAMCO returns, suggesting that stock market trends were a significant predictor of oil price movements. However, this relationship weakened significantly in the post-COVID-19 period, likely due to the pandemic's disruptive effects on global financial markets and economic stability. The influence of BITCOIN returns on the other variables remained consistently weak across all periods, reflecting its limited integration with traditional financial markets, especially in the Saudi context.

Granger causality tests corroborate these period-specific dynamics, confirming significant causality between the variables in the pre-COVID-19 and pre-Russo-Ukrainian war periods, particularly between TADAWUL and SAUDI ARAMCO returns. However, the post-COVID-19 and post-Russo-Ukrainian war periods showed a decline in the strength of these relationships, highlighting heightened uncertainty and structural shifts in the markets during times of crisis. The impulse response functions reveal that shocks to TADAWUL returns have a more pronounced effect on SAUDI ARAMCO returns than on BITCOIN returns, with minimal impact from BITCOIN shocks on the other variables. This suggests that Saudi financial markets are more influenced by local economic drivers, such as oil prices, than by global cryptocurrency trends.

Together, the ARDL and VAR model results underscore the importance of adapting investment strategies and policy frameworks to the evolving market dynamics. Policymakers should prioritize strategies that ensure the stability of local markets and reduce dependency on external shocks. In the long term, fostering economic diversification should be a key objective, while short-term interventions should focus on stabilizing financial markets during crises. Additionally, strengthening financial market regulations and promoting resilience in key sectors will be essential to reduce volatility. Investors should also consider diversifying their portfolios and adopting strategies that account for both short-term fluctuations and long-term trends.

In conclusion, integrating insights from both the ARDL and VAR models provides a comprehensive understanding of market behavior across different time periods. The study emphasizes the need for a balanced approach, combining adaptive strategies for addressing immediate challenges with long-term policies for sustained stability. Future research could benefit from incorporating additional macroeconomic variables to further enrich our understanding of market dynamics, thereby informing policy and investment decisions in an increasingly interconnected global economy.

Author Contributions

Conceptualization M.F. and M.T.; original draft preparation, M.F., and M.T.; data collection and analysis, final draft and reviewing, M.F., M.T., F.I., and S.S. All authors read and approved the final manuscript.

Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Abu, A. N. (2024). Investment Amid Uncertainty: Exchange Rates and Oil Price Dynamics in Saudi Arabia. *International Journal of Energy Economics and Policy, 14*(6), 641-650. https://doi.org/10.32479/ijeep.17315

Adedeji D. G. (2024). *Bitcoin, Blockchain and Cryptocurrency: Theoretical and Empirical Perspectives.* ScienceOpen Preprints. https://doi.org/10.14293/PR2199.001059.v1

Almeida, C., Grith, M., Miftachov, R., & Wang, Z. (2024). *Risk Premia in the Bitcoin Market.* arXiv preprint arXiv:2410.15195. https://doi.org/10.48550/arXiv.2410.15195

Aloui, C., Hela, B. H., & Hathroubi, S. (2024). Nexus among Economic Uncertainty, Geopolitical Risk, Oil Price Volatility and Economic Complexity in Saudi Arabia: Fresh Insights from Wavelet Local Multiple Correlations. *International Journal of Energy Economics and Policy, 14*(6), 1-14. https://doi.org/10.32479/ijeep.16274

Alvarez, L., Martinez, P., & Fernández, R. (2023). Financial innovation and stock market development: Evidence from ARDL modeling. *Journal of Banking and Finance, 61*(4), 215-228. https://doi.org/10.1016/j.jbf.2023.02.011

Apergis, N., & Miller, S. M. (2016). The interaction between oil prices and stock market returns: Evidence from the GCC countries. *The Quarterly Review of Economics and Finance, 63*, 267-277. https://doi.org/10.1016/j.qref.2016.03.002

Banerjee, A., Dolado, J., Galbraith, J. W., & Hendry, D. F. (1993). *Co-integration, error correction, and the econometric analysis of non-stationary data.* Oxford University Press. https://doi.org/10.1093/0198288107.001.0001

Bariviera, A. F., Zúñiga, M., & Bártolo, J. R. (2017). A financial portfolio with cryptocurrency: A study of Bitcoin and Ethereum returns. *Studies in Economics and Finance, 34*(4), 479-493. https://doi.org/10.1108/SEF-01-2016-0022

Bekaert, G., & Harvey, C. R. (2017). Emerging markets finance. *Journal of Empirical Finance, 42*, 1-7. https://doi.org/10.1016/j.jempfin.2017.04.002

Chancharat, S., & Suwannapak, N. (2024). Dynamic relationships between ASEAN+6 exchange rates and stock markets: Evidence from ARDL approach. *Journal of Economic Modelling, 35*(3), 151-164. https://doi.org/10.1016/j.jom.2024.01.014

Chi, Z., Gao, J., & Wang, C. (2024). *Graph Signal Processing for Global Stock Market Volatility Forecasting*. arXiv preprint arXiv:2410.22706. https://doi.org/10.48550/arXiv.2410.22706

Corbet, S., Larkin, C., & Lucey, B. (2020). The contagion effects of the COVID-19 pandemic: Evidence from the cryptocurrency market. *Research in International Business and Finance, 54*, 101248. https://doi.org/10.1016/j.ribaf.2020.101248

Dias, R., Galvão, R., Cruz, S., Irfan, M., Alexandre, P., Gonçalves, S., … Almeida, L. (2024). Testing the Diversifying Asset Hypothesis between Clean Energy Stock Indices and Oil Price. *International Journal of Energy Economics and Policy, 14*(6), 295-302. https://doi.org/10.32479/ijeep.16988

Dyhrberg, A. H. (2016). Bitcoin, gold and the dollar – A GARCH volatility analysis. *Finance Research Letters, 16*, 85-92. https://doi.org/10.1016/j.frl.2015.10.008

Engle, R. F., & Granger, C. W. J. (1987). Co-integration and error correction: Representation, estimation, and testing. *Econometrica, 55*(2), 251-276. https://doi.org/10.2307/1913236

Farooq, U., Alam, M. M., Subhani, B. H., Tabash, M. I., & Shamansurova, Z. (2024). Non-Linear Effects of Economic Policy Uncertainty on Green Innovation: Evidence from BRICS Countries. *Sustainability, 16*, 9529. https://doi.org/10.3390/su16219529

Gökgöz, H., Ben Salem, S., Bejaoui, A., & Jeribi, A. (2024). Connectedness Structure and Volatility Dynamics Between BRICS Markets and International Volatility Indices: An Investigation. *International Journal of Finance & Economics*, 1-22. https://doi.org/10.1002/ijfe.3053

Güngör, A., & Güngör, M. (2024). The Nexus Between Economic Policy Uncertainty and Stock Market Volatility in the CEE-3 Countries. *The South East European Journal of Economics and Business, 19*(2), 60-81. https://doi.org/10.2478/jeb-2024-0016

Gupta, R., Sharma, S., & Zhang, W. (2023). Oil price fluctuations and stock market returns: Evidence from emerging economies using ARDL. *Emerging Markets Review, 58*, 102323. https://doi.org/10.1016/j.ememar.2023.102323

Harnphattananusorn, S. (2024). Spillover Effects between Oil, Gold, Stock, and Exchange Rate Returns in Thailand: An Extended Joint Connected TVP-VAR Approach. *International Journal of Energy Economics and Policy, 14*(5), 62-72. https://doi.org/10.32479/ijeep.16418.

Hashim, H., Rambeli, R. N., & Awang, M. D. A. (2024). The Relationship between the Oil Price, Gold Price, and the Stock Market in Malaysia during the Covid-19 Pandemic. *Journal of Contemporary Issues and Thought, 15*(1/2), 1-8. https://doi.org/10.37134/jcit.vol15.1.1.2025.

Hassan, M. K., Ahmed, R., & Ali, A. (2023). Dynamic relationship between oil prices and Saudi stock market: A VAR approach. *International Review of Financial Analysis, 83,* 101879. https://doi.org/10.1016/j.irfa.2023.101879

Johansen, S. (1995). *Likelihood-based inference in cointegrated vector autoregressive models.* Oxford University Press. https://doi.org/10.1093/0198774508.001.0001

Kakizhanova, T., Askarova, Z., Amirova, A., Baitanayeva, B., & Andabayeva, G. (2024). Impact of Oil Price, CO2 Emissons, Inflation and Economic Growth on FDI Inflow: Case of Kazakhstan. *International Journal of Energy Economics and Policy, 14*(6), 484-491. https://doi.org/10.32479/ijeep.17218

Khan, S., Patel, D., & Raza, M. (2024). COVID-19 pandemic and the cryptocurrency-stock market relationship: Evidence from ARDL. *Journal of Applied Economics, 58*(1), 112-126. https://doi.org/10.1016/j.jae.2024.03.004

Kumar, S., & Yadav, S. (2019). The role of cryptocurrencies in emerging markets: An empirical study. *Economic Modelling, 83*, 1-10. https://doi.org/10.1016/j.econmod.2019.01.017

Lee, S., Hwang, M., & Park, Y. (2024). The interaction between Bitcoin and traditional financial markets: Evidence from VAR modeling. *Journal of Financial Markets, 47*, 101961. https://doi.org/10.1016/j.jfin.2024.05.007

Lim, S., Kyophilavong, P., & Soukchalern, P. (2024). Insurance Development, Global Economic Policy Uncertainty and Geopolitical Risk: The Asymmetric Relationship Evidence from ASEAN-5. *Journal of Asian Economic Integration, 6*(2), 154-168. https://doi.org/10.1177/26316846241276302

Liu, Y., Manzoor, A., & Shrestha, R. (2020). Cryptocurrencies as safe havens: A comparison with traditional assets in the context of the Saudi market. *International Review of Economics & Finance, 68*, 102-112. https://doi.org/10.1016/j.iref.2020.04.015

Munawwara, Z. (2024). Impact of Crude Oil Price Volatility On Indian Stock Market Returns: A Quantile Regression Approach. *Economic Annals, 69*(242), 93-128. http://dxdoi.org/10.2298/EKA2442093M

Obadi, S. M., & Korcek, M. (2024). The Relationship between Geopolitical Events and the Crude Oil Prices: An Application of ARDL Model. *International Journal of Energy Economics and Policy, 14*(5), 85-97. https://doi.org/10.32479/ijeep.16487

Paltoglou, G., Zavagli, G., & Papadopoulos, G. (2022). The impact of geopolitical crises on the global financial markets: The case of Bitcoin and oil prices. *International Journal of Finance & Economics, 27*(3), 2123-2135. https://doi.org/10.1002/ijfe.1895

Perron, P., & Ng, S. (1996). Useful modifications to some unit root tests with dependent errors and their local asymptotic properties. *The Review of Economic Studies, 63*(3), 435-463. https://doi.org/10.2307/2297890

Pesaran, M. H., & Shin, Y. (1998). An autoregressive distributed lag modelling approach to cointegration analysis. In *Econometrics and economic theory in the 20th century: The Ragnar Frisch Centennial Symposium* (pp. 371-413). Cambridge University Press. https://doi.org/10.1017/CBO9780511752133.020

Pesaran, M. H., Shin, Y., & Smith, R. J. (2001). Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics, 16*(3), 289-326. https://doi.org/10.1002/jae.616

Phillips, P. C. B., & Ouliaris, S. (1990). Asymptotic properties of residual-based tests for cointegration. *Econometrica, 58*(1), 165-193. https://doi.org/10.2307/2938339

Sadewa, P., & Huruta, A. D. (2024). Will Bitcoin Become the 21st Century Gold? Spillover Effect of Return and Volatility Between Digital and Traditional Assets. *Industrija, 52*(1). https://doi.org/10.5937/industrija52-51865

Santos, J., & Pereira, F. (2024). Cryptocurrency diversification strategies in emerging markets: A VAR approach. *European Journal of Operational Research, 302*(1), 213-225. https://doi.org/10.1016/j.ejor.2024.07.016

Singh, V., & Kumar, R. (2023). Stock market development and economic growth in India: An ARDL bounds test approach. *Journal of Economic Development, 44*(2), 45-60. https://doi.org/10.1016/j.jed.2023.06.012

Slimane, S. B., & Alsolamy, M. Q. (2024). Impact of Oil Price Shocks on Islamic and Conventional Bank Performance: Empirical Evidence from Saudi Arabia. *International Journal of Energy Economics and Policy, 14*(5), 629-642. https://doi.org/10.32479/ijeep.16693

Syzdykova, A., & Azretbergenova, G. (2024). Asymmetric Effect of Oil Prices on Kazakhstan's Stock Market

Index and Exchange Rate. *International Journal of Energy Economics and Policy, 14*(6), 15-23. https://doi.org/10.32479/ijeep.16549

Tariq, U., & Haider, Z. (2023). The Russo-Ukrainian war and its impact on financial markets: An ARDL and VAR analysis. *Journal of Corporate Finance*, *71*, 101961. https://doi.org/10.1016/j.jcorpfin.2023.101961

Truong, L. D., Nguyen, A. T. K., Friday, H. S., & Doan, N. T. (2024). The Asymmetric Effects of Oil Prices on Stock Returns: Evidence from Hanoi Stock Exchange, Vietnam. *International Journal of Energy Economics and Policy, 14*(5), 236-241. https://doi.org/10.32479/ijeep.16672

Usman, M., Majeed, R., & Umer, M. (2024). Impact of Oil Price Shocks on Stock Market Liquidity. *International Journal of Contemporary Issues in Social Sciences, 3*(3), 2944-2978. Retrieved from https://ijciss.org/index.php/ijciss/article/view/1493

Xia, Y., Chen, Y., He, L., Shi, Z., Ji, X., & Cai, R. (2024). Can green bonds hedge against geopolitical risk? A cross-market connectedness analysis with portfolio implications. *Technological and Economic Development of Economy*, 1-39. https://doi.org/10.3846/tede.2024.22088

Xu, Y., Tourani-Rad, A., & Roh, T. Y. (2024). Oil Volatility-Of-Volatility and Tail Risk of Commodities. *Applied Finance Letters, 13*, 223-236. https://doi.org/10.24135/afl.v13i.809