International Journal of Economics and Finance; Vol. 17, No. 2; 2025
ISSN 1916-971X E-ISSN 1916-9728
Published by Canadian Center of Science and Education

Persistence and Pervasiveness of Tax Evasion: An Evolutionary Analytical Framework

Jaylson Jair da Silveira[1], Gilberto Tadeu Lima[2] & Leonardo Barros Torres[3]

[1] Department of Economics and International Relations, Federal University of Santa Catarina, Florianópolis, SC, Brazil

[2] Department of Economics, University of São Paulo, São Paulo, SP, Brazil

[3] School of Economics, University of Surrey, Guildford, Surrey, UK

Correspondence: Gilberto Tadeu Lima, University of São Paulo, Av. Prof. Luciano Gualberto 908, 05508-010 São Paulo, SP, Brazil. Tel: 55-11-3091-5907. E-mail: giltadeu@usp.br

Received: November 12, 2024 Accepted: December 29, 2024 Online Published: January 14, 2025

doi:10.5539/ijef.v17n2p87 URL: https://doi.org/10.5539/ijef.v17n2p87

Abstract

There is considerable evidence that heterogeneity in tax compliance behavior is persistent and pervasive. This paper develops an evolutionary analytical framework in which taxpayers periodically choose between to comply or not to comply with their tax obligations. Aggregate demand formation arising from private and public expenditures depends on the frequency distribution of tax compliance behavior across taxpayers, so that the macrodynamic of the rates of capacity utilization and output growth is coevolutionarily coupled to the microdynamic of tax compliance across individuals. The analytical framework set forth here replicates several pieces of empirical evidence on tax evasion. First, the proportion of non-complying taxpayers (and hence the volume of tax evasion) depends on the tax rate and the expected cost of tax evasion. Second, heterogeneity in tax compliance behavior across taxpayers is evolutionarily persistent instead of temporary. Third, the immediate impact of a change in the proportion of tax evading individuals on the rates of capacity utilization and output growth is non-linear. Fourth, the proportion of non-complying taxpayers and the rates of capacity utilization and output growth vary positively with the tax rate in the evolutionary equilibrium.

Keywords: tax evasion, evolutionary dynamics, heterogeneous behavior, capacity utilization, economic growth

1. Introduction

Government expenditures on goods and services typically constitute an important component of aggregate demand formation and hence play a significant role in the determination of the level of macroeconomic activity. However, the volume of tax revenues from different sources very often becomes a binding constraint to the sustainable expansion of government expenditures, especially when the possibility of public debt accumulation is limited. Moreover, different forms of tax evasion represent a leakage of tax revenues, so that actual tax collection is usually lower than potential tax collection.

The estimated level of tax evasion varies across countries, but it is typically non-negligible and sometimes relatively high. In the US economy, for example, the estimated tax evasion represents a considerable loss of revenues for the federal government, as shown in Table 1. For the individual income tax, for example, the gross tax gap (percentage of total true tax liabilities not paid voluntarily and timely) and the net tax gap (taxes not paid voluntarily and timely minus enforced and late payment) for the 2014-2016 period are estimated to be about 21.5% and 17.6%, respectively.

Table 1. US net and gross tax gap for 2014-2016, in USD billions

	Total true tax liabilities	Tax paid voluntarily and timely		Gross tax gap	
Overall tax gap	3,307	2,811	(85%)	496	
lndividual income tax	1,740	1,383	(79.5%)	357	
Corporate income tax	354	313	(88.4%)	41	
Employment tax	1,131	1,038	(91.8%)	93	
Estate tax	22	17	(77.3%)	5	
	Enforced and late payment	**Net compliance rate (%)**		**Net tax gap**	**Estimated tax evasion (%)**
Overall tax gap	68	87.1		428	12.9
lndividual income tax	51	82.4		306	17.6
Corporate income tax	8	90.5		34	9.5
Employment tax	6	92.3		87	7.7
Estate tax	3	90.9		2	9.1

Source: IRS (2022).

Against this backdrop, this paper develops an analytical framework in which income tax evasion may arise and be persistent, in keeping with the empirical evidence (Slemrod, 2007; Buehn & Schneider, 2016; Alm, 2019). The actual amount of government revenues (income taxes voluntarily paid by complying individuals and fines paid by tax evading individuals who are detected in the random auditing procedure performed by the government) is endogenously time-varying driven by an evolutionary dynamic. Interestingly, the macrodynamic of the level of economic activity, measured by the rates of capacity utilization and output growth, is coevolutionarily coupled to the microdynamic of tax compliance across individuals.

As remarked by Alm (2019), there are several that seem likely to affect the individual tax compliance behavior, yet for tractability theoretical models can incorporate only a few of them. The analytical framework set forth in this paper follows closely the recommendation advanced by Alm (2019) that fruitful research on the subject should recognize that a 'theory' of taxpayer compliance behavior must consist of a 'full house' of theories and models, each explaining the behavior of different individuals at different times and places (and having different two-way interactions with the macroeconomic environment, we would add).

The existing literature on tax compliance has also been addressing the similarly relevant issue of the implications of tax evasion for the level of economic activity, finding mostly mixed results. To some extent due to the complex nature of the two-way dynamic interactions between tax evasion and the income-generation process, it is not surprising that ambiguous results have often been found for whether tax evasion impacts positively or negatively on different measures of the level of economic activity.

Theoretical analyses have found that the impact of tax evasion on the output growth rate can be either positive or negative depending on the transmission channels that are considered and the relative strength of the effects at play (Chen, 2003; Cerqueti & Coppier, 2011; Ivanyna et al., 2016; Varvarigos, 2017). Caballé and Panadés (1997), for example, argue that when public spending can raise the marginal productivity of private capital and stimulate saving, tax evasion may be harmful to economic growth. In contrast, Célimène et al. (2016) argue that tax evasion can be beneficial to economic growth when there are attractive alternatives for tax evaders to invest their illegal proceeds, such as equity markets. Mixed qualitative have also been found for the effect of tax evasion on the level of aggregate output (Varvarigos, 2017; Bethencourt & Kunze, 2019, 2020). Meanwhile, Vasilopoulou and Thomakos (2017) found that higher incidence of tax evasion is associated to lower output growth, a result that may arise in our analytical framework.

The analytical framework developed in this paper replicates several pieces of empirical evidence on tax evasion. First, the proportion of non-complying taxpayers (and hence the volume of tax evasion) depends on the tax rate and the expected cost of tax evasion. Second, heterogeneity in tax compliance behavior across taxpayers is persistent instead of temporary. Third, under such heterogeneity, the immediate (temporary equilibrium) impact of a change in the proportion of tax evading individuals on the rates of capacity utilization and output growth is non-linear. Fourth, the proportion of non-complying taxpayers and the rates of capacity utilization and output growth vary positively with the tax rate in the evolutionary equilibrium.

The remainder of this paper is organized as follows. Section 2 introduces the analytical framework and explores the behavior of the economy in the temporary equilibrium. Section 3 specifies and explores the evolutionary dynamics driving the frequency distribution of tax compliance strategies across individuals in the private sector. This section also offers a discussion of the substance of the main results regarding the behavior of the economy in the evolutionary equilibrium. Section 4 concludes the paper.

2. Macroeconomic Setting and Temporary Equilibrium

The analytical framework therein features a closed economy producing a single good usable for both consumption and investment purposes. The government holds a balanced budget, using all collected tax revenues (including fines levied on tax evaders who are detected) to cover its expenditures. Two homogeneous factors of production are used in the production of the single good of the economy, capital and labor. These inputs are combined by many imperfectly competitive firms by means of a homogeneous fixed-coefficient technology:

$$X = \min\{vK, aL\}, \tag{1}$$

where X is the level of output, K is the stock of capital, L is the level of employment, v is the full-capacity output to capital ratio, and a is the output to labor ratio. These technical coefficients are exogenously given constants. As the technical coefficient v is normalized to one, we measure the rate of capacity utilization, u, by the output to capital ratio, X/K.

Firms produce according to aggregate demand, which is however insufficient to bring forth full capacity utilization. Labor is in excess supply and the level of employment is determined by production, while the price of the single good is set as a markup factor over unit labor costs, both of which do not vary over time. Thus, this analytical framework is cast in real terms, with any increase in aggregate demand being fully accomodated by an increase in output (and hence, given the capital stock, which is predetermined at each point in time, by an increase in the rate of capacity utilization).

Equilibrium in the product market at each point in time is characterized by:

$$\frac{X}{K} = u = \frac{C}{K} + \frac{I}{K} + \frac{G}{K}, \tag{2}$$

where C is private consumption, I is private investment, and G denotes public expenditures. We will refer to this equilibrium in the product market as temporary equilibrium, given that the resulting equilibrium capacity utilization will be parameterized by the frequency distribution of tax-evading and tax-complying individuals in the private sector. This frequency distribution will be predetermined at each point in time, but it will vary over time driven by an evolutionary dynamic, as described in the next section.

At each point in time, therefore, aggregate demand depends, *inter alia*, on the predetermined frequency distribution of tax evading and tax complying individuals in the private sector. In fact, private consumption and government expenditures depend directly (and investment expenditures indirectly) on such a distribution, as described shortly.

The tax rate on income is given by $\tau \in (0,1) \subset \mathbb{R}$. An individual in the private sector periodically chooses between two available and mutually exclusive strategies with respect to the legally due tax obligations on her income: she chooses either to comply (*tax complying strategy*) or not to comply (*tax evading strategy*). At each point in time there is a proportion $\lambda \in [0,1] \subset \mathbb{R}$ of tax evading individuals, while the remaining proportion, $1-\lambda$, is composed of tax complying individuals. It is common knowledge that not all taxpayers are audited. However, all noncomplying individuals who are audited are certainly caught (and have the exact amount of taxes evaded found out) and are unable to avoid either being penalized with a fine or, once penalized, paying in full the resulting fine.

Despite the risks associated with evading taxation, some individuals may choose to do so. The considered tax compliance choice is treated by individuals as an all-or-nothing choice: due income taxes are either paid in full or fully evaded. The probability with which a tax evading individual is caught and hence penalized with a fine is given by $\varepsilon \in (0,1) \subset \mathbb{R}$. The auditing of individuals in the private sector conducted by the government for tax purposes has a fixed cost that is a component of its flow of expenditures, and we assume that such a cost is always covered by the actual (or disposable) government revenues (actual tax collection plus fines levied on tax evading individuals who are caught and penalized). When a tax evading individual is caught, she must pay a fine corresponding to a penalty rate, $\gamma \in (\tau,1) \subset \mathbb{R}$, over her gross income, which is the income she ended up with after having tried to evade her tax obligations. The assumption that $\gamma > \tau$ ensures that a tax evading individual who is caught forfeits a larger proportion of her gross income than would have been the case if she had complied with her tax obligations. The assumption that $\gamma < 1$, in turn, ensures that the fine levied on a tax evading individual who is caught does not leave her without any income left after paying the respective fine. For simplicity and interpretability, we define $\rho \equiv \varepsilon\gamma \in (0,1) \subset \mathbb{R}$, the expected penalty for evading taxation, the endogenous nature of which will be described shortly. Assuming that the aggregate income X is uniformly distributed across individuals, the amount of income of the subpopulation of tax evading individuals is given by λX, whereas $(1-\lambda)X$ is the amount of income of the subpopulation of tax complying individuals. Therefore, the actual (or disposable) government revenues can be written as:

$$X_G^d = [\rho\lambda + \tau(1-\lambda)]X \ . \tag{3}$$

For given values of the tax rate, the government sets the expected penalty for evading taxation in a given point in time in a reactive way. This reaction protocol is implemented as follows. Using their knowledge of the pre-tax or gross aggregate income, the tax authorities can compute the maximum amount of taxes to be collected should all individuals follow the tax complying strategy. If the actual tax collection falls short of that maximum, it does so by the total amount of taxes evaded by the subpopulation of non-complying individuals and hence the gross tax gap. As the gross aggregate income is uniformly distributed across individuals, the tax authorities can compute the amount of taxes to be paid by each individual and uses it together with the gross tax gap to form an estimate of the number (measure) of tax evading individuals. This estimate is made under the premise (that turns out to be correct) that due income taxes are either paid in full or fully evaded, so that it is given by the ratio of the gross tax gap to the amount of taxes to be paid by each individual. This estimate of the number (measure) of tax evading individuals is then used together with the number (measure) of taxpayers to (correctly) estimate the proportion of tax evading individuals, which is given by λ. Before auditing, therefore, although the tax authorities are not able to identify who, if any, the tax evading individuals are, they can indirectly learn the proportion of non-complying individuals in the population of taxpayers. We will assume that the expected penalty set by tax authorities and the pervasiveness of tax evasion, measured by λ, move in the same direction. Formally, we consider the expected penalty as a function $\rho(\lambda)$, with the following properties: $\rho(0) > 0$, $\rho(1) \le \gamma$, $\rho'(\lambda) > 0$, and $\rho''(\lambda) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$.

Meanwhile, the total disposable income of the subpopulation of tax evading individuals and the total disposable income of the subpopulation of tax complying individuals are, respectively, given by:

$$X_e^d = \varepsilon(1-\gamma)\lambda X + (1-\varepsilon)\lambda X = (1-\rho(\lambda))\lambda X \quad \text{and} \quad X_c^d = (1-\tau)(1-\lambda)X \ , \tag{4}$$

so that the total disposable income of the private sector is given by:

$$X^d = X_e^d + X_c^d = [(1-\rho(\lambda))\lambda + (1-\tau)(1-\lambda)]X \equiv X^d(\lambda) \, . \tag{5}$$

It follows that:

$$\frac{\partial X^d(\lambda)}{\partial \lambda} = \left[-\rho'(\lambda)\lambda + \left(1-\rho(\lambda)\right) - (1-\tau)\right]X = \left[\frac{\tau}{\rho(\lambda)} - \left(1+\xi(\lambda)\right)\right]\rho(\lambda)X \ , \tag{6}$$

where $\xi(\lambda) \equiv \rho'(\lambda)\lambda/\rho(\lambda) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$ is the elasticity of expected penalty with respect to proportion of tax evaders.

The sign of derivative in (6) has a turning point at the value of the proportion of tax evading individuals represented by $\bar{\lambda}$, as shown in the top panel in Figure 1, assuming that $\tau/\rho(1) < 1+\xi(\bar{\lambda}) < \tau/\rho(0)$. This critical mass of tax evading individuals is the one at which the expression within bracktes in (6) is null; that is, at point $\bar{\lambda}$ it follows that $\tau/\rho(\bar{\lambda}) = 1+\xi(\bar{\lambda}) > 1$, such that $\tau - \rho(\bar{\lambda}) = \xi(\bar{\lambda})\rho(\bar{\lambda}) > 0$. At point $\bar{\lambda}$, therefore, the pecuniary benefit per unit of gross income associated with evading taxation, given by the tax rate τ, is sufficiently greater than the expected pecuniary cost per unit of gross income associated with evading taxation, which is represented by the expected penalty $\rho(\bar{\lambda})$, such that the net pecuniary benefit of the tax evading strategy, $\tau - \rho(\bar{\lambda})$, is strictly positive. Actually, this net pecuniary benefit of tax evading strategy compensates the negative impact on the disposable income of tax evading individuals, given by $-\xi(\bar{\lambda})\rho(\bar{\lambda})$. As $\rho \equiv \rho(\lambda)$ is such that $\rho'(\lambda) > 0$ and $\rho''(\lambda) > 0$ for all $\lambda \in [0,1] \in \mathbb{R}$, (Note 1) it then follows that $\tau/\rho(\lambda) > 1+\xi(\lambda)$ for any $\lambda \in [0,\bar{\lambda}) \in \mathbb{R}$, so that the derivative in (6) is strictly positive in this interval, as depicted in the top panel in Figure 1. For all $\lambda \in (\bar{\lambda},1] \in \mathbb{R}$, in turn, it follows that $\tau/\rho(\lambda) < 1+\xi(\lambda)$, as a result of which the derivative in (6) is strictly negative for all $\lambda \in (\bar{\lambda},1] \in \mathbb{R}$.

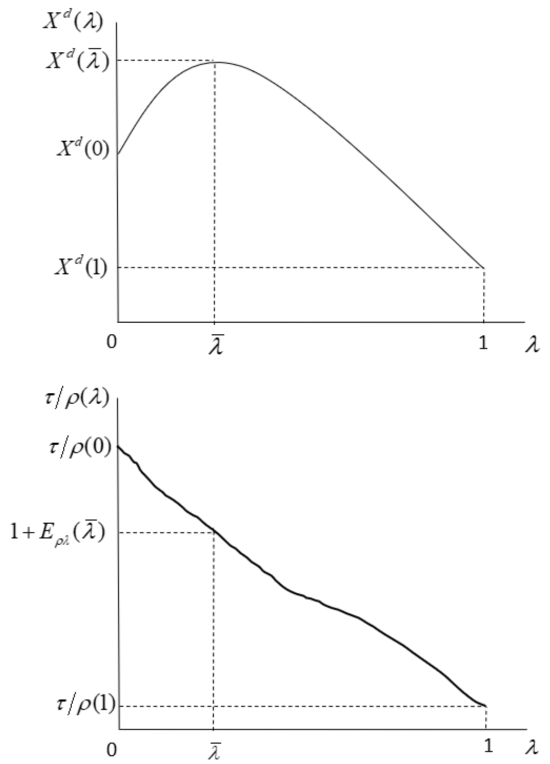

Figure 1. Total disposable income of the private sector as a function of the proportion of tax evading individuals

Of course, the disposable income of a tax evading individual who is not detected in the auditing process carried out by the government is strictly greater than the disposable income of a tax complying individual. Meanwhile, the disposable income of a tax evading individual who is caught is strictly lower than the disposable income of a tax complying individual. A tax evading individual who is not detected ends up with a disposable income which is equal to her gross income, with her successful tax evasion occurring at the expense of the actual (or disposable) government revenues. However, a tax evading individual who is detected and hence penalized with a fine, ends up with a disposable income which is strictly lower than her gross income, in benefit of the actual (or disposable) government revenues (Note 2).

As regards consumption behavior, individuals in the private sector have a common marginal propensity to consume given by $b \in (0,1) \subset \mathbb{R}$, which is therefore independent from the tax compliance strategy adopted by them. Yet the total consumption of a given subpopulation of individuals in the private sector of the economy (compliant, detected evaders and undetected evaders) depends on both the total disposable income of the private sector and the frequency of such subpopulation in the whole population of individuals. Therefore, using (5), total consumption by individuals in the private sector is given by:

$$C = bX^d(\lambda) = b\left[1-\tau+\left(\tau-\rho(\lambda)\lambda\right)\right]X \tag{7}$$

Firms make decisions to accumulate capital independently from available savings, so that firms' desired growth rate of the capital stock, assuming no depreciation, is given by:

$$g^d = \frac{I^d}{K} = \alpha + \beta u, \tag{8}$$

where α and β are strictly positive parameters. Reasonably, the desired capital accumulation in (8) features an autonomous component represented by α and an accelerator effect, the intensity of which is measured by β.

Recall the temporary supply-demand equilibrium in the product market specified in (2). As the government runs a balanced budget, which means that $X_G^d = G$, where X_G^d is given by (3), the temporary supply-demand equilibrium in the product market can be re-written as:

$$X - X_G^d - C = I^d, \tag{9}$$

which can be interpreted as the equality between private saving (note that $X - X_G^d$ represents the available income of the private sector) and desired private investment.

Normalizing (9) by the capital stock, and using (3) (with $\rho = \rho(\lambda)$) and (7), both also normalized by the capital stock, we can then solve for the rate of capacity utilization in the temporary equilibrium:

$$u^* = \frac{\alpha}{(1-b)\left[1-\tau+\left(\tau-\rho(\lambda)\right)\lambda\right]-\beta} \equiv u(\lambda) \cdot \tag{10}$$

Aggregate supply is elastic enough to allow aggregate output (and income) to be determined by aggregate demand. Hence the rate of capacity utilization adjusts to ensure that the supply-demand equilibrium condition in (9) is satisfied. To ensure that the temporary equilibrium rate of capacity utilization in (10) is stable and strictly positive, we assume that its denominator is strictly positive. The substance of this demand-led output-adjustment stability condition is that, all else constant, and given that the government runs a balanced budget (that is, public saving is equal to zero), private saving should be less responsive to changes in capacity utilization than private investment. This guarantees that any excess demand or supply in the product market is eliminated rather than exacerbated by changes in capacity utilization.

Note that the paradox of thrift holds: an increase in the propensity to save in the private sector, $1-b$, by representing an aggregate demand leakage, reduces the rate of capacity utilization in the temporary equilibrium. Meanwhile, an increase in any of the parameters of the desired investment function in (8), α and β , by increasing desired investment and hence contributing to aggregate demand formation, raises capacity utilization in the temporary equilibrium. It can be checked that an increase in the tax rate, by raising the net government revenues for any $\lambda \in [0,1) \subset \mathbb{R}$, increases the rate of capacity utilization in the temporary equilibrium when not all the individuals in the private sector adopt the tax avoiding strategy (see (2)). In fact, as the permanent running of a balanced budget by the government is equivalent to the government's propensity to spend out of its actual revenues being equal to one (that is, public saving is equal to zero), a rise in the tax rate represents a net aggregate demand injection when not all the individuals adopt the tax evading strategy. In this balanced-budget context, therefore, a rise in the tax rate is equivalent to a mechanism of "forced dissaving" or "forced spending" at the level of the aggregate economy, given that the propensity to save of the government, which is zero, is lower than the propensity to save in the private sector, which is strictly positive. However, when $\lambda = 1$, a change in the tax rate does not affect aggregate formation and hence capacity utilization in the temporary equilibrium.

Meanwhile, the effect of an increase in the proportion of tax evading individuals on capacity utilization in the temporary equilibrium is given by:

$$\frac{\partial u^*}{\partial \lambda} = -\frac{\alpha(1-b)}{D^2}\left[\frac{\tau}{\rho(\lambda)} - \left(1+\xi(\lambda)\right)\right]\rho(\lambda) = -\frac{\alpha(1-b)}{D^2}\frac{1}{X^*(\lambda)}\frac{\partial X^d(\lambda)}{\partial \lambda}, \tag{11}$$

where D is the denominator in (10), $X^*(\lambda)$ is the temporary equilibrium level of aggregate income corresponding to the rate of capacity utilization in the temporary equilibrium, and $\xi(\lambda)$ is the elasticity of ρ , the expected penalty for evading taxation, with respect to the proportion of tax evading individuals in the private sector. Therefore, there is a precise relationship between the response of the temporary equilibrium capacity utilization to the proportion of tax evading individuals and the response of the total disposable income of the private sector, X^d , to the same proportion (see (6): $sign(\partial u^*(\lambda)/\partial \lambda) = -sign(\partial X^d(\lambda)/\partial \lambda)$. The intuition is straightforward. Both the disposable income of the private sector in (5) and the actual government revenues (or disposable income of the government) in (3) contribute to aggregate formation and hence to the temporary equilibrium level of aggregate income (and thus, given the capital stock, to the rate of capacity utilization in the temporary equilibrium). As seen in (Note 2), however, the disposable income of the private sector and the disposable income of the government move in opposite directions, so that a change in the proportion of tax evading individuals that increases the former, reduces the latter, and vice versa. Suppose, for example, an increase in the proportion of tax evading individuals, which results in an increase (decrease) in the disposable income of the private sector (government) when $\lambda \in [0, \bar{\lambda}) \in \mathbb{R}$ in Figure 1. Given that the propensity to spend out of the disposable income of the balanced-budget government is greater than the propensity to consume of the private sector (see (7)), this transfer of disposable income to the private sector reduces aggregate demand and hence capacity utilization. Moreover, this fall in capacity utilization reduces desired investment through the accelerator effect in (8), which causes a further decrease in aggregate demand and hence capacity utilization due to the multiplier effect of investment demand. Now suppose the same increase in the proportion of tax evading individuals, which results in a fall (rise) in the disposable income of the private sector (government) when

$\lambda \in (\bar{\lambda},1] \in \mathbb{R}$ in Figure 1. Considering that the propensity to spend out of the disposable income of the balanced-budget government is greater than the propensity to consume of the private sector, this transfer of disposable income now in favor of the government raises aggregate demand and hence capacity utilization. Besides, rise fall in capacity utilization raises desired investment through the accelerator effect, which causes a further increase in aggregate demand and hence capacity utilization due to the multiplier effect of investment demand.

Let us now compute the output growth rate in the temporary equilibrium by substituting u^* defined in (10) into g^d given in (8). We get:

$$g^* = \alpha + \frac{\alpha\beta}{(1-b)[1-\tau+\lambda(\tau-\rho(\lambda))]-\beta}. \tag{12}$$

Therefore, the temporary equilibrium output growth is also parameterized by the proportion of tax evading individuals in the private sector, which is predetermined at each point in time. Given that $\partial g^* / \partial u^* = \beta > 0$, all the qualitative comparative statics results obtained above for u^* hold for g^* as well.

However, it should be recalled that the frequency distribution of tax compliance strategies is predetermined at each point in time, varying over time according to the evolutionary dynamic to be specified and explored in the next section. We will then be able to investigate whether the impact of a given change in the proportion of tax evading individuals on capacity utilization and output growth in the evolutionary equilibrium is qualitatively the same as it is in the temporary equilibrium. In other words, we will interestingly be able to explore an issue bearing important implications, which is whether a change in the proportion of tax evading individuals that has a temporary or transitory effect on capacity utilization and output growth has a persistent or permanent effect as well.

3. Evolutionary Dynamics of Tax Compliance and Equilibrium Selection

Let us explore the possibility that the economy achieves an *evolutionary equilibrium*, i.e. a microeconomic state featuring the constancy of the frequency distribution of tax compliance strategies, (λ, 1-λ), across individuals in the private sector. In the transition dynamics to the evolutionary equilibrium, our earlier assumptions about the stable adjustment in the product market ensure that the temporary equilibrium values of the rates of capacity utilization and output growth are always attained. The economy then evolves over time driven by the interaction between the macrostate (capacity utilization and output growth) and the microstate (frequency distribution of tax compliance strategies). A key role is played in this dynamic interaction by the disposable income of individuals in the private sector and their consumption behavior. As seen in the preceding section, the gross income accruing to a given individual is part of the aggregate income, which is determined by aggregate demand. The latter, in turn, is formed by three components: investment of private firms, consumption of the private individuals, and government spending. As specified earlier, investment demand features an accelerator effect, while consumption demand and government spending depend on the choice of tax compliance strategy made by private individuals and the outcome of the tax auditing system implemented by the balanced-budget government. Ultimately the net or disposable private income and the net government revenues (which include collected tax and fines) as source of aggregate demand formation (and hence aggregate output determination) evolve over time driven by the evolutionary dynamics of the frequency distribution of tax compliance strategies across individuals in the private sector.

While the frequency distribution of tax compliance strategies is a predetermined variable in each point in time, it varies between consecutive temporary equilibria according to an evolutionary dynamic based on the payoffs of the two tax compliance strategies. Reasonably, we specify the payoff of each tax compliance strategy as the expected net or disposable income associated with that strategy, as formally defined in (3). As our endogenous macroeconomic variables of interest are the rates of capacity utilization and output growth, we conveniently normalize the payoffs associated with the two tax compliance strategies by the capital stock: (Note 3)

$$\pi_e = [1-\rho(\lambda)]u(\lambda) \tag{13}$$

and

$$\pi_c = (1-\tau)u(\lambda), \tag{14}$$

where π_e denotes the payoff of the tax evading strategy and π_c the payoff of the tax complying strategy, while $u(\lambda)$ is given by the temporary equilibrium capacity utilization in (10). Recall that as the full-capacity output to capital ratio is normalized to one, we measure the rate of capacity utilization by the output to capital ratio. Hence the microdynamics of the frequency distribution of tax compliance strategies across private individuals both affect and are in turn affected by the macrodynamics of the level of economic activity as measured by the rate of capacity utilization.

Drawing on these payoffs, we will assume that the selection process of tax compliance strategies across agents in the economy is described as a *replicator dynamic*: (Note 4)

$$\frac{\dot{\lambda}}{\lambda} = \pi_e - \bar{\pi}, \tag{15}$$

where $\dot{\lambda} \equiv d\lambda/dt$ is the instantaneous rate of change of the proportion of tax evading individuals in the private sector in a given point in time, implying that $\dot{\lambda}/\lambda$ is the instantaneous growth rate of this proportion, whereas $\bar{\pi} = \lambda\pi_e + (1-\lambda)\pi_c$ is the average payoff across individual taxpayers. According to the replicator dynamic in (15), the frequency of the tax evading strategy across individual taxpayers increases (decreases) when it has above-average (below-average) payoff, which represents a frequency-dependent selection mechanism. (Note 5)

Substituting the payoffs in (13)-(14) into the replicator dynamic in (13), simple algebraic manipulation yields:

$$\dot{\lambda} = \lambda(1-\lambda)(\pi_e - \pi_c) = \lambda(1-\lambda)[\tau - \rho(\lambda)]u(\lambda). \tag{16}$$

Therefore, for a given proportion of tax evading individuals, λ, and, consequently, for a given amount of evaded taxes, the effort of the government to recover some of the loss in tax revenues is determined, which implies the expected penalty $\rho(\lambda)$. Therefore, if the tax rate is strictly greater (smaller) than the expected penalty, the payoff yielded by the tax evading strategy is then strictly greater (smaller) than the payoff yielded by the tax complying strategy, so that the proportion of tax evading individuals increases (decreases).

As it turns out, the state transition of the economy is driven by the ordinary differential equation in (16), the state space of which is represented by $\Theta = \{\lambda \in \mathbb{R} : 0 \le \lambda \le 1\}$. Let us show that the replicator dynamic in (16) has three equilibria. These are two monomorphic equilibria featuring survival of only one tax compliance strategy in each, and one polymorphic equilibrium, now featuring the survival of both tax compliance strategies.

Considering that $\tau - \gamma < \tau - \rho(1) < 0 < \tau - \rho(0) < \tau$ and $0 < u(\lambda) \le 1$, it is straightforward to verify that $\dot{\lambda} = 0$ in (16) both at λ=0 and λ=1. Therefore, all firms evading their legally due tax obligations (λ=0) and all firms complying with such tax obligations (λ=1) are both monomorphic evolutionary equilibria.

Meanwhile, the existence of a polymorphic evolutionary equilibrium for the replicator dynamic formally specified in (16) materializes if, and only if, there is a $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\dot{\lambda} = 0$ or, equivalently, if the following evolutionary equilibrium condition holds:

$$\tau - \rho(\lambda^*) = 0, \tag{17}$$

considering again that $\tau - \gamma < \tau - \rho(1) < 0 < \tau - \rho(0) < \tau$ and $0 < u(\lambda) \le 1$. As $\tau - \rho(0) > 0$, $\tau - \rho(1) < 0$ and the left-hand side in (17) is a continuous function over the state space $\Theta = \{\lambda \in \mathbb{R} : 0 \le \lambda \le 1\}$, we can then apply the intermediate value theorem to readily conclude that there is some $\lambda^* \in (0,1) \subset \mathbb{R}$ where (17) holds. Moreover, as $\rho'(\lambda)$>0 for all $\lambda \in \Theta$, it follows that the differential given by $\tau - \rho(\lambda)$ is a strictly decreasing function of the proportion of tax evading individuals, λ. As a result, since the function $\tau - \rho(\lambda)$ is continuous over the state space Θ, there is only one polymorphic evolutionary equilibrium represented by $\lambda^* \in \Theta$ implicitly defined by (17). It is also worth noting that $\lambda^* > \bar{\lambda}$, given that substituting (17) into (6) implies that $\partial X^d(\lambda^*)/\partial\lambda = -\xi(\lambda^*)\rho(\lambda^*)X^*(\lambda^*) < 0$, where $X^*(\lambda^*)$ represents the level of aggregate income achieved in the evolutionary equilibrium.

The stability properties of the three evolutionary equilibria can be readily inferred from the sign of $\tau - \rho(\lambda)$. Since $\lambda(1-\lambda) > 0$ for all $\lambda \in (0,1) \subset \mathbb{R}$, we know that $sign(\dot{\lambda}) = sign(\tau - \rho(\lambda))$, where $sign(\cdot)$ stands for the sign function. As $\tau - \rho(0) > 0$, $\tau - \rho(1) < 0$ and $\tau - \rho(\lambda)$ is a strictly decreasing function of λ, we can then conclude that $\tau - \rho(\lambda) > 0$ for all $\lambda \in [0, \lambda^*) \subset \mathbb{R}$ and $\tau - \rho(\lambda) < 0$ for all $\lambda \in (\lambda^*, 1] \subset \mathbb{R}$. Consequently, $\dot{\lambda} > 0$ for all $\lambda \in [0, \lambda^*) \subset \mathbb{R}$ and $\dot{\lambda} < 0$ for all $\lambda \in (\lambda^*, 1] \subset \mathbb{R}$. As a result, both monomorphic evolutionary equilibria are unstable (repulsors) while the polymorphic evolutionary equilibrium is asymptotically stable (an attractor). In keeping with the empirical evidence, in our analytical framework heterogeneity in tax compliance behavior emerges as a persistent outcome (Slemrod, 2007; Buehn & Schneider, 2016; Alm, 2019). In effect, the analytical framework developed in this paper provides a logically consistent and behaviorally sound evolutionary explanation for the persistent coexistence of both tax complying and tax evading behavior in the population of taxpayers, in addition to exploring how this persistent microdiversity co-evolves with the macrodynamics of the economy. The transition dynamics to the evolutionary equilibrium configuration is depicted in Figure 2.

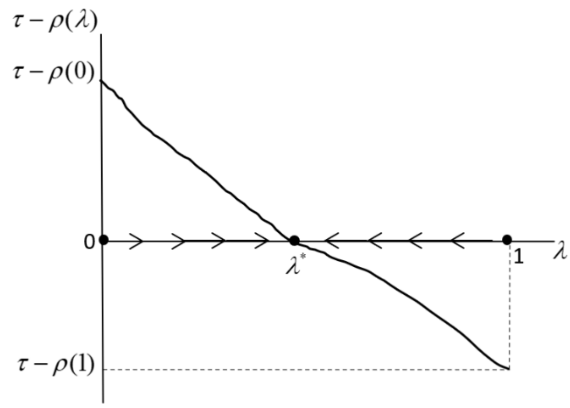

Figure 2. Convergence to the unique polymorphic evolutionary equilibrium

The substance of the convergence to the unique polymorphic evolutionary equilibrium is best understood by noting that, in our analytical framework, one key feature of the evolutionary microdynamics of the frequency distribution of tax compliance strategies across private individuals is that there is strategic substitutability in the choice of tax compliance behavior: an individual's decision to evade taxation has a negative payoff externality on the other tax evading individuals. The reason is that this decision, by raising the expected penalty represented by $\rho(\lambda)$ in (16) due to the increase in the penalty rate, γ, by the government, reduces the respective payoff differential. In Figure 1, when convergence to the unique polymorphic evolutionary equilibrium occurs from the left (right), an individual's choice of the tax evading (complying) strategy reduces (raises) the respective payoff differential given by $\tau-\rho(\lambda)>0$ ($\tau-\rho(\lambda)<0$).

The convergence to the unique polymorphic evolutionary equilibrium described in Figure 2 can be related to the response of the rate of capacity utilization to changes in the proportion of tax evading individuals derived in (11), noting that $\lambda^*\in(\bar{\lambda},1)\in\mathbb{R}$. Recalling Figure 1, when convergence to the unique polymorphic evolutionary equilibrium in Figure 2 occurs from the left, so that the proportion of tax evading individuals is rising, this is accompanied by an increase (decrease) in the disposable income of the private sector (government) while $\lambda\in[0,\bar{\lambda})\in\mathbb{R}$. As the propensity to spend out of the disposable income of the balanced-budget government is greater than the propensity to consume of the private sector, this transfer of disposable income to the private sector ends up reducing aggregate demand and hence capacity utilization and (per (12)) output growth. The convergence to the polymorphic equibrium continues when $\lambda\in(\underline{\lambda},\lambda^*)\in\mathbb{R}$, but now the increase in the proportion of tax evading individuals leads to a fall (rise) in the disposable income of the private sector (government). Considering that the propensity to consume out of the disposable income of the balanced-budget government is greater than the propensity to consume of the private sector, this transfer of disposable income now in favor of the government ends up raising aggregate demand and hence capacity utilization and output growth. Therefore, the convergence to the polymorphic equilibrium is not accompanied by falling rates of capacity utilization and output growth while $\lambda\in(\bar{\lambda},\lambda^*)\in\mathbb{R}$.

Having identified all the three existing evolutionary equilibria and their respective stability properties, we can then establish how exogenous changes in the tax rate impact on the proportion of tax evading individuals in the evolutionary equilibrium. Applying the implicit function theorem to the polymorphic evolutionary equilibrium condition in (17), we have:

$$\frac{\partial\lambda^*}{\partial\tau}=\frac{1}{\rho'(\lambda^*)}>0\,. \tag{18}$$

Therefore, the higher the tax rate, the higher the frequency of tax evading individuals in the private sector and, consequently, the volume tax evasion in the economy, which is in keeping with the empirical evidence (Slemrod, 2007; Alm, 2019). It should be recalled, however, that the proportion of tax evading individuals is a direct estimate of the gross instead of the net tax gap, as the probability with which a tax evading individual is caught and hence inevitably penalized with a fine proportional to her gross income is given by $\varepsilon\in(0,1)\subset\mathbb{R}$.

Based on (11) and (18) and considering that $\partial X^d(\lambda^*)/\partial\lambda=-\xi(\lambda^*)\rho(\lambda^*)X^*<0$, it follows that:

$$\frac{\partial u(\lambda^*)}{\partial\tau}=\frac{\partial u(\lambda^*)}{\partial\lambda}\frac{\partial\lambda^*}{\partial\tau}=-\frac{\alpha(1-b)}{D^2}\frac{1}{X^*(\lambda^*)}\frac{\partial X^d(\lambda^*)}{\partial\lambda}\frac{\partial\lambda^*}{\partial\tau}>0, \tag{19}$$

recalling that D is the denominator in (10). Given that $\partial g^* / \partial u^* = \beta > 0$, all the qualitative comparative statics results obtained above for u^* hold for g^* as well.

4. Conclusions

In the analytical framework developed here the economy evolves over time driven by the interaction between the macrostate (capacity utilization and output growth) and the microstate (frequency distribution of tax compliance strategies across individuals). An important role is played in this dynamic interaction by the disposable income of individuals in the private sector and their consumption behavior. Another important role is played by the actual amount of revenues (income taxes voluntarily paid by complying individuals plus fines paid by tax evading individuals who are detected) collected by the government, which runs a balanced budget and hence does not save. These two sources of aggregate demand formation depend on the pervasiveness of tax evasion (proxied by the proportion of tax evading individuals). As a result, the macrodynamic of the rates of capacity utilization and output growth is coevolutionarily coupled to the microdynamic of tax compliance behavior across individuals.

The evolutionary microdynamic of tax compliance across individuals is driven by a replicator dynamic. In this evolutionary dynamic, the frequency of the tax evading strategy across individuals increases (decreases) when it has above-average (below-average) payoff, which represents a frequency-dependent selection mechanism. This evolutionary dynamic has three equilibria. These are two monomorphic equilibria featuring survival of only one tax compliance strategy in each, and one polymorphic equilibrium, now featuring the survival of both tax compliance strategies. Both monomorphic evolutionary equilibria are unstable (repulsors) while the polymorphic evolutionary equilibrium is asymptotically stable (an attractor). Therefore, in keeping with the empirical evidence, heterogeneity in tax compliance behavior emerges evolutionarily as a persistent outcome. The transition dynamic towards the evolutionary equilibrium features a non-linear response of the rates of capacity utilization and output growth to changes in the proportion of tax evaders in the economy.

The analytical framework set forth here replicates several pieces of empirical evidence on tax evasion. First, the proportion of non-complying taxpayers (and hence the volume of tax evasion) depends on the tax rate and the expected cost of tax evasion. Second, heterogeneity in tax compliance behavior across taxpayers is persistent instead of temporary. Third, under such heterogeneity, the immediate (temporary equilibrium) impact of a change in the proportion of tax evading individuals on the rates of capacity utilization and output growth is non-linear. Fourth, the proportion of non-complying taxpayers and the rates of capacity utilization and output growth vary positively with the tax rate in the evolutionary equilibrium.

Acknowledgments

The authors gratefully acknowledge research funding provided by CNPq (National Council of Scientific and Technological Development – Brazil) [grants 313628/2021-1 (JJS) and 316740/2023-3 (GTL)]. This study was also financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001 and the São Paulo Research Foundation – Brazil (FAPESP) [grant 2019/03148–5]. The authors contributed equally to this study.

References

Alm, J. (2019). What motivates tax compliance? *Journal of Economic Surveys, 33*(2), 353-388. https://doi.org/10.1111/joes.12272

Bethencourt, C., & Kunze, L. (2019). Tax evasion, social norms, and economic growth, *Journal of Public Economic Theory, 21*, 332-346. https://doi.org/10.1111/jpet.12346

Buehn, A., & Schneider, F. (2016). Size and Development of Tax Evasion in 38 OECD Countries: What do we (not) know? *Journal of Economics and Political Economy, 3*(1), 1-11. https://doi.org/10.1453/jepe.v3i1.634

Caballé, J., & Panadés, J. (1997). Tax evasion and economic growth. *Public Finance = Finances Publiques*, *52*(3-4), 318-340. Retrieved from https://ideas.repec.org/a/pfi/pubfin/v52y1997i3-4p318-40.html

Célimène, F., Dufrénot, G., Mophou, G., & N'Guérékata, G. (2016). Tax evasion, tax corruption and stochastic growth. *Economic Modelling, 52*, 251-258. https://doi.org/10.1016/j.econmod.2014.10.055

Cerqueti, R., & Coppier, R. (2011). Economic growth, corruption and tax evasion. *Economic Modelling, 28*(1), 489-500. https://doi.org/10.1016/j.econmod.2010.07.006

Chen, B. L. (2003). Tax evasion in a model of endogenous growth. *Review of Economic Dynamics, 6*(2), 381-403. https://doi.org/10.1016/S1094-2025(03)00002-4

Hofbauer, J., & Sigmund, K. (1998). *Evolutionary games and population dynamics*. Cambridge: Cambridge

University Press. https://doi.org/10.1017/CBO9781139173179

Internal Revenue Service (IRS). (2022). *Federal Tax Compliance Research: Tax Gap Estimates for Tax Years 2014-2016*. Washington, DC: IRS Office of Research, Analysis, and Statistics. Retrieved from https://www.irs.gov/pub/irs-pdf/p5869.pdf

Ivanyna, M., Moumouras, A., & Rangazas, P. (2016). The culture of corruption, tax evasion, and economic growth. *Economic Inquiry*, 54, 520-542. https://doi.org/10.1111/ecin.12228

Slemrod, J. (2007). Cheating ourselves: The economics of tax evasion. *Journal of Economic Perspectives, 21*, 25-48. https://doi.org/10.1257/jep.21.1.25

Varvarigos, D. (2017). Cultural norms, the persistence of tax evasion, and economic growth. *Economic Theory, 63*(4), 961-995. https://doi.org/10.1007/s00199-016-0976-1

Vasilopoulou, Y., & Thomakos, D. (2017). Tax evasion, tax administration, and the impact of growth: Tax enforcement as regulatory failure in a high tax rates, high tax evasion, and low-growth economic environment. In Y. Vasilopoulou, & D. Thomakos (Eds.), *Taxation in Crisis. Tax Policy and the Quest for Economic Growth* (pp. 175-203). Springer. https://doi.org/10.1007/978-3-319-65310-5_8

Vega-Redondo, F. (1996). *Evolution, Games and Economic Behaviour*. Oxford: Oxford University Press. https://doi.org/10.1093/0198774729.001.0001

Weibull, J. W. (1995). *Evolutionary game theory*. Cambridge, MA: The MIT Press.

Notes

Note 1. Based on the derivative in (6) and recalling that $\rho'(\lambda) > 0$ and $\rho''(\lambda) > 0$ for all $\lambda \in [0,1] \in \mathbb{R}$, we know that:

$$\frac{\partial^2 X^d(\lambda)}{\partial \lambda^2} = -\rho''(\lambda)\lambda - 2\rho'(\lambda) < 0, \text{ for all } \lambda \in [0,1] \in \mathbb{R}.$$

Therefore, considering that the first order derivative in (6) is null at $\bar{\lambda}$, we can infer that such a derivative is strictly positive for any $\lambda \in [0,\bar{\lambda}) \in \mathbb{R}$ and is strictly negative for all $\lambda \in (\bar{\lambda},1] \in \mathbb{R}$. The behavior of the expression $\tau/\rho(\lambda) - (1+\xi(\lambda))$ in (6) has the same sign as the first order derivative in (6) and, consequently, that expression has the same behavior with respect to λ as this derivative.

Note 2. In effect, if we add together the total disposable income of the subpopulation of tax evading individuals who are not detected, $(1-\varepsilon)\lambda X$, the total disposable income of the same subpopulation which is detected and hence penalized with a fine, $\varepsilon(1-\gamma)\lambda X$, the total disposable income of the subpopulation of tax complying individuals, $(1-\tau)(1-\lambda)X$, and the actual (or disposable) government revenues, $[\tau(1-\lambda)+\rho\lambda]X$, the total sum is equal to the aggregate income: $[(1-\varepsilon)\lambda + \varepsilon(1-\gamma)\lambda + (1-\tau)(1-\lambda) + \tau(1-\lambda) + \rho\lambda]X = X$.

Note 3. Let H be the (constant) measure of individuals in the economy and let H_e be the measure of tax evading individuals. Therefore, the measure of tax complying individuals is H-H_e. By construction, we have $\lambda = H_e/H$. We can define the payoff of the tax evading strategy as follows:

$$\pi_e \equiv \frac{1}{K}\frac{X_e^d}{H_e} = \frac{X}{K}\frac{X_e^d}{X}\frac{1}{H_e} = \frac{X}{K}\frac{X_e^d}{X}\frac{1}{\lambda H}.$$

For simplicity and without loss of generality, we normalize H=1. Inserting the total disposable income of tax evaders in (4) and the temporary equilibrium capacity utilization in (10) into the expression above, we obtain the payoff in (13). Analogously, the payoff of a tax complying strategy can be written as:

$$\pi_c \equiv \frac{1}{K}\frac{X_c^d}{(H-H_e)} = \frac{X}{K}\frac{X_c^d}{X}\frac{1}{(H-H_e)} = \frac{X}{K}\frac{X_c^d}{X}\frac{1}{(1-\lambda)H}.$$

Normalizing H=1 and using the total disposable income of the subpopulation of tax complying individuals in (4) and the temporary equilibrium capacity utilization in (10), the payoff in (14) follows.

Note 4. On evolutionary dynamics, see Hofbauer and Sigmund (1998), Vega-Redondo (1996) and Weibull (1995).

Note 5. This replicator dynamic can be formally derived from a model of (social or individual) learning as in Weibull (1995, sec. 4.4).