

# Identifying the Temporal Dynamics and Macroeconomic Interactions of the US Economy

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## Abstract

This paper employs the VAR model to analyze interrelations among key macroeconomic variables, emphasizing unemployment, inflation, and the Fed Funds rate. The model reveals asymmetry in the unemployment-Fed Funds rate relationship, emphasizing the unique influence of unemployment. Lagged values contribute to understanding temporal dependencies, highlighting positive associations between lagged and current inflation. Impulse response analysis and the covariance matrix validate the IS-LM model and Stock and Watson's (2001) findings. Forecasts anticipate increased unemployment and a slight Fed Funds rate decrease, though accuracy tests reveal reliability issues, especially for the Fed Funds rate. ADF tests support stationarity for inflation and unemployment showing a weak indication against the unit root hypothesis for the Fed Funds rate. Lastly, SARIMA, ARIMA, and DM tests suggest performance differences, pointing to avenues for future research to enhance precision, address reliability issues, and explore variations between SARIMA and VAR models, potentially in a cross-country comparative context.

**Keywords:** Vector Autoregressive (VAR), macroeconometrics, Augmented Dickey-Fuller (ADF), SARIMA, Diebold-Mariano

## 1. Introduction

Sims (1982) introduced the Vector Autoregressive (VAR) model, a pivotal instrument in macroeconomic forecasting. Scholars such as Fomby (2013) and Sims (1986) recognize the VAR model for its capacity to incorporate uncertainty in assumption identification, rendering it particularly advantageous for policy analysis. However, Stock and Watson (2001) and Gruber (2022) propose that the VAR model has evolved further, enhancing its utility in forecasting and macroeconomic shock analysis through Bayesian methodologies, including the R2-induced Dirichlet-decomposition prior. These advancements have significantly improved the predictive performance of the VAR model, especially in high-dimensional contexts, sustaining its relevance in macroeconomic forecasting.

Stock and Watson (2001) highlight how macroeconomic variables respond to a one-unit increase in the present value of a designated VAR error. The experimental scenario assumes the specified error returns to zero in subsequent periods while other errors remain constant, particularly meaningful when correlations among errors across equations are absent. The computation of impulse responses is commonplace in recursive and structural VARs. Additionally, iterative forecasting, as suggested by Stock and Watson (2001), involves refining predicted values over successive time steps, with forecast horizons extending over 24 quarters. The approach includes calculating forecasts using the Vector Autoregressive (VAR) model, incorporating historical values, and comparing them with random walk forecasts.

Despite technical advancements, Poskitt (2017) underscores potential errors in macroeconomic forecasts employing VAR models. The author identifies two primary sources of error: estimation error and approximation error. Robertson (1999) emphasizes the need for consistent forecast records to assess the accuracy of macroeconomic forecasts. These studies collectively suggest that while VAR models may be susceptible to accuracy errors, methodologies are available to enhance their performance and evaluate their reliability.

In this context, Diebold-Mariano (1995) discusses two categories of tests for forecast accuracy: model-based and model-free. Model-based tests assume a parametric econometric model, utilizing both the model and data for accuracy evaluation. Model-free tests operate with an information set consisting only of forecasts and actual values, making them relevant when the underlying model is unknown or unavailable. The discussion primarily focuses on model-free tests, related to encompassing tests in econometrics. Previous literature emphasizes that some of these tests include non-Gaussian, non-zero-mean, serially correlated, and contemporaneously correlated forecast errors (Hendry et al., 2004; Pagan-Harding, 2011; Mariano & Brown, 1991).

Thus, this paper employs the VAR (4) model to investigate the interrelationships among key macroeconomic variables, focusing on unemployment, inflation, and the Fed Funds rate. The model highlights asymmetry in the unemployment and Fed Funds rate relationship, emphasizing the distinctive impact of unemployment. The incorporation of lagged values contributes to comprehending temporal dependencies, revealing positive associations between lagged and current inflation. Validation of the IS-LM model and Stock and Watson's (2001) findings is achieved through impulse response analysis and the covariance matrix.

Forecasting anticipates an increase in unemployment and a slight decrease in the Fed Funds rate, yet accuracy tests uncover reliability issues, particularly for the Fed Funds rate. ADF tests affirm stationarity for inflation and unemployment, indicating a weak inclination against the unit root hypothesis for the Fed Funds rate. Additionally, forecast tests like SARIMA, ARIMA, and DM tests indicate performance disparities, suggesting avenues for future research to enhance precision, address reliability concerns, and explore variations between SARIMA and VAR models.

The paper is structured as follows: Section 2 provides a comprehensive review of the methodology underlying the VAR and the employed forecast tests. In Section 3, we present the empirical results derived from the forecasts, supplemented by significance tests and graphical representations of the forecasted periods. Section 4 succinctly summarizes the outcomes obtained from the conducted exercises.

## 2. Methodology

### 2.1 Data

The dataset employed in this study was sourced from the Federal Reserve Bank of Saint Louis (FRED) through the Application Programming Interface (API) integrated with the statistical programming language R. The data encompasses quarterly data related to inflation, the federal funds rate, and the unemployment rate, spanning the period from January 1st, 1960, to June 30th, 2023. The rationale for selecting this database lies in its practicality and reliability. Additionally, the FRED database offers an accessible and democratic platform, freely available to researchers. Consequently, this study also endeavors to assess the reliability of this easily accessible database, contributing to the evaluation of its suitability for research purposes.

### 2.2 Vector Autoregressive (VAR)

First, we employ the estimation of a system of equations is conducted to delineate the dynamics of a collection of time-series data, utilizing Vector Autoregressive (VAR) models. A VAR model constitutes a system of  $n$  equations, each corresponding to a variable, within a linear framework. In this model, every variable is elucidated by its own lagged values, in addition to the contemporaneous and past values of the remaining  $n-1$  variables. This modeling approach was originally introduced by Sims (1982). To illustrate, a lag-2 VAR model involving two variables may be represented as follows:

$$X_t = \beta_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 Y_{t-1} + \beta_4 Y_{t-2} + \zeta_t \quad (1)$$

$$Y_t = \alpha_0 + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \alpha_3 Y_{t-1} + \alpha_4 Y_{t-2} + \eta_t \quad (2)$$

The following equations are estimated by Ordinary Least Squares (OLS), in which, they are characterized as the interdependencies between the two variables over time, capturing how each variable is influenced by its own past values and the past values of the other variable. Where in equation 1,  $X_t$  and  $Y_t$  represent the current values of each variable (e.g., inflation, unemployment, or fed funds rate). Next,  $X_{t-1}$  and  $X_{t-2}$  are the lagged values of  $X_t$ , this same rationale can be used for  $Y_t$ . Lastly,  $\zeta_t$  and  $\eta_t$  are the residuals, that capture the unobserved factors that influence the variable  $t$ .

Next, to supplement our analysis we used Stock and Watson (2001) VAR toolkit. The authors delve into the dynamics involving inflation, operationalized by the chained-GDP deflator, the unemployment rate, and the federal funds rate. The study involves the estimation of equations to elucidate the intricate relationships among these variables. Following the estimation, the authors employ the derived equations to simulate the typical responses of each variable to exogenous shocks. The estimated equations are utilized for the simulation of the

standard response of each variable to shocks. In the process of estimating impulse response functions, the establishment of a structured framework governing the transmission of shocks within the system becomes imperative. Typically, impulse response functions adhere to a recursive structure that incorporates contemporaneous shocks. Specifically, in this structure, variable 1 influences variable 2 and 3 at time 't', variable 2 solely impacts variable 3 at time 't', and variable 3 does not exert an influence on variable 1 and variable 2 at time 't'.

Stock and Watson (2001), present the VAR in three different structured forms (reduced, recursive, and structural). In a reduced form VAR, each variable is expressed as a linear function of its own past values, the past values of all other variables considered, and a serially uncorrelated error term. In the given example, the VAR comprises three equations representing current unemployment, inflation, and the Fed funds rate. Each equation is estimated through OLS regression, with the number of lagged values determined by various methods, using four lags in the provided examples. The error terms in the model capture the "surprise" movements in the variables after considering their past values. The passage emphasizes that if the variables are correlated, the error terms in the reduced form model will also be correlated across equations, which is a common scenario in macroeconomic applications.

Next, the recursive Vector Autoregression (VAR), which structures the error terms in each regression equation to be uncorrelated with the errors in preceding equations. This is achieved by strategically including some contemporaneous values as regressors. In the case of a three-variable VAR ordered as inflation, the unemployment rate, and the interest rate, the recursive VAR organizes equations such that inflation, the unemployment rate, and the interest rate take turns being the dependent variable. The regressors in each equation include lagged values of all three variables, and for subsequent equations, they also incorporate current values of the preceding variables. Estimation through ordinary least squares ensures uncorrelated residuals across equations. The passage highlights that the results are contingent on the order of the variables, and changing the order affects the VAR equations, coefficients, and residuals, leading to  $n!$  ( $n$  factorial) recursive VARs representing all possible orderings.

Finally, the structural VAR employs economic theory to establish contemporaneous connections among variables with causal interpretations. These VARs require "identifying assumptions" to pinpoint causal correlations, either across the entire VAR or within a specific equation. These assumptions generate instrumental variables for estimating contemporaneous links through instrumental variables regression. The researcher's creativity constrains the number of structural VARs. In a three-variable scenario, we explore two related structural VARs, each adopting a distinct assumption to identify the causal impact of monetary policy on unemployment, inflation, and interest rates. The initial VAR incorporates a version of the "Taylor rule," wherein the Federal Reserve adjusts the interest rate based on past inflation and unemployment rates, as represented by equation 3 in the structural VAR.

$$R_t = r^* + 1.5(\bar{\pi}_t - \pi^*) - 1.25(\bar{u}_t - u^*) + \text{lagged values of } R, \pi, u + \varepsilon \quad (3)$$

In this context, Stock and Watson (2001) characterize the interest rate equation in the structural VAR is represented by a Taylor rule. The dependent variable  $r^*$  signifies the desired real interest rate,  $\bar{\pi}_t$  and  $\bar{u}_t$  denote the average values of inflation and unemployment rate over the past four quarters,  $\pi^*$  and  $u^*$  are the target values of inflation and unemployment, with  $\varepsilon_t$  representing the error in the equation. This equation characterizes the interest rate response based on the Taylor rule, and the error term is interpreted as a monetary policy "shock" reflecting the deviation of actual interest rates from the Taylor rule. The shock can be estimated through a regression with the dependent variable  $R_t - 1.5(\bar{\pi}_t - \pi^*) + 1.25(\bar{u}_t - u^*)$ , and the right-hand side including a constant and lags of interest rates, unemployment, and inflation. Lastly, the authors emphasize that the Taylor rule is considered "backward-looking," prompting exploration of an alternative model variant where the Fed reacts to forecasts of inflation and unemployment four quarters ahead, replacing  $\bar{\pi}_t$  and  $\bar{u}_t$  with four-quarter ahead forecasts computed from the reduced form VAR.

### 2.3 Impulse Response Function (IRF)

According to Stock and Watson (2001), the impulse responses demonstrate the way current and subsequent values of individual variables respond to a one-unit increase in the present value of a designated VAR error. To the authors, the analytical framework presupposes that the specified error reverts to zero in subsequent periods, with all other errors maintaining a constant value of zero. Furthermore, the validity of this experimental scenario, where one error undergoes alteration while the others are maintained constant, is heightened in situations where correlations among errors across equations are absent. Consequently, the computation of impulse responses is a prevalent practice in the context of recursive and structural VARs.

## 2.4 Iterative Forecasting

Iterative forecasting is a method involving the continual refinement of predicted values over successive iterations or time steps. Following Stock and Watson (2001), we investigate forecast horizons extending over 24 quarters. The procedure comprises the calculation of forecasts  $h$  steps ahead by estimating the Vector Autoregressive (VAR) model within a specific quarter, initiating the initial forecast  $h$  steps ahead, subsequently re-estimating the VAR model through the subsequent quarter, and iteratively proceeding throughout the entire forecast period. Forecasted values are computed for each variable based on its own historical values, and for comparison purposes, a forecast utilizing a random walk (or “no change”) methodology is also included.

In this context, the forecasts used for inflation rates encompass the average value across the entire forecast period, while predictions for the unemployment rate and interest rate focus specifically on the concluding quarter of the forecast horizon. The results are shown in Table 3, elucidating the root mean square forecast error for each approach. The root mean square forecast error is derived as the square root of the average squared forecast errors over the out-of-sample period from 1960 to 2023, offering a comprehensive evaluation of forecasting precision.

## 2.5 Forecasting Tests

In this study, we employed four distinct forecasting evaluation tests commonly utilized in economic forecasting assessments, as outlined by Leitch and Tanner (1991). The tests encompassed are the Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE).

According to the authors, the Mean Absolute Error (MAE) signifies the absolute average of errors between predicted and actual values, offering an assessment of the average magnitude of errors without consideration of their direction. Lower MAE values are indicative of higher accuracy. The Mean Squared Error (MSE) represents the average of squared errors between predicted and actual values, assigning greater penalty to larger errors than MAE. This metric provides an average of squared differences, emphasizing the impact of outliers. The Root Mean Squared Error (RMSE), derived as the square root of MSE, furnishes an interpretable scale in the same units as predicted and actual values, penalizing larger errors more than MAE. Finally, the Mean Absolute Percentage Error (MAPE) quantifies the average percentage difference between predicted and actual values, calculated as the absolute percentage difference. This measure offers a percentage-based evaluation of prediction accuracy, particularly valuable in scenarios where the forecasted variable exhibits substantial variability in scale.

The calculations for each test are displayed in the following manner:

$$MAE = \frac{1}{n} \sum_{i=1}^n |Y_i - \hat{Y}_i| \quad (4)$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (5)$$

$$RMSE = \sqrt{MSE} \quad (6)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{Y_i - \hat{Y}_i}{Y_i} \right| * 100 \quad (7)$$

Where,  $n$  is the number of observations;  $Y_i$  is the actual value and  $\hat{Y}_i$  is the predicted value.

## 2.6 Augmented Dickey-Fuller Test (ADF)

Initially proposed by Dickey and Fuller (1979) and Dickey and Fuller (1981), and also Extensively covered by Hamilton (1994), and Patterson (2011). The ADF analyzes the presence of a unit root is a well-explored topic in econometrics. According to the authors, when dealing with a sequence of time series observations  $X_1, X_2, \dots, X_n$ , a commonly utilized test for evaluating the null hypothesis of a unit root is the ADF test. This test assesses the existence of a unit root in the data generating mechanism by employing the ordinary least squares (OLS) estimator  $\hat{\rho}_n$  of  $\rho_n$ , obtained through fitting the regression equation (Dickey and Fuller, (1979); Dickey and Fuller, 1981; Hamilton (1994); and, Patterson (2011)).

$$X_t = \rho X_{t-1} + \sum_{j=1}^p a_{j,p} \Delta X_{t-j} + e_{t,p} \quad (8)$$

The authors define the given notation, as  $\Delta X_t$  denotes the difference between  $X_t$  and  $X_{t-1}$ , and the order  $p$ , represented as  $p = p(n)$ , is permitted to change with  $n$ . The value of  $p$  is linked to the assumptions imposed on the underlying process. When examining the null hypothesis  $H_0: \rho = 1$ , it is commonly posited that  $X_t$  results from the integration of a linear autoregressive process with an infinite order (AR( $\infty$ )), articulated as:

$$X_t = X_{t-1} + U_t, t = \{1, 2, \dots, n\} \quad (9)$$

where,  $X_0 = 0$

$$U_t = \sum_{j=1}^{\infty} a_j U_{t-j} + e_t$$

In this context,  $\{e_t\}$  is constituted by a sequence of independent, identically distributed (i.i.d.) random variables with a mean of zero and a variance within the range  $0 < \sigma_e^2 < \infty$ . The stationarity and causality of  $\{U_t\}$  are assured by positing that  $\sum_{j=1}^{\infty} |j|^s |a_j| < \infty$  for some  $s \geq 1$  and  $\sum_{j=1}^{\infty} |j|^s |a_j| < \infty$ . For testing  $H_0$ , Dickey and Fuller (1979) introduced the studentized statistic:

$$t_n = \frac{\hat{\rho}_n - 1}{\text{std}(\hat{\rho}_n)} \quad (10)$$

The asymptotic distribution of  $t_t$  under  $H_0$  is non-standard and is well-documented in the literature. Dickey and Fuller (1979) and Dickey and Fuller (1981) derived this distribution assuming the order of the underlying autoregressive process is finite and known. In this context, Said and Dickey (1984) extended this result to cases where the innovation process  $\{U_t\}$  driving the random walk (Equation 9) is an invertible autoregressive moving-average (ARMA) process, i.e., an AR( $\infty$ ) process with exponentially decaying coefficients.

### 2.7 Seasonal Autoregressive Integrated Moving Average (SARIMA)

The SARIMA model is constructed by incorporating seasonal components into the ARIMA models:

$$\text{SARIMA}(p, d, q)(P, D, Q)[S] \quad (11)$$

Pepple and Harrison (2017) define  $p$  as the non-seasonal autoregressive order, where  $P$  represents the seasonal autoregressive order,  $q$  is the non-seasonal moving average order,  $Q$  denotes the seasonal autoregressive order, and  $d$  and  $D$  indicate the order of the common difference and seasonal difference. Model (11) can be enhanced through the following approach:

$$\begin{aligned} & (1 - \phi B^\omega - \phi_2 B^{2\omega} \dots - \phi_p B^{p\omega}) \times (1 - \phi B - \phi_2 B^2 \dots - \phi_p B^p) \times (1 - B^\omega)^D (1 - B)^d Q_n(t) \\ & = (1 - \theta_1 B^\omega - \theta_2 B^{2\omega} - \dots - \theta_{1Q} B^{Q\omega}) \times (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_p B^p) e(t) \end{aligned} \quad (12)$$

The authors define the variables accordingly, where,  $\phi$  represents the non-seasonal autoregressive parameter, and  $\theta$  denotes the non-seasonal moving average parameter. Additionally,  $\theta$  signifies the seasonal autoregressive parameter, while  $\theta$  represents the seasonal moving average parameter. The variables  $\omega$  and  $B$  stand for frequency and the differential operator, respectively.

### 2.8 Diebold and Mariano (DM) Test

Diebold (2012) asserts that the Diebold-Mariano (DM) tests focus on evaluating forecasts rather than models. This testing methodology deliberately treats forecast errors as fundamental elements. The flexibility inherent in this approach is emphasized by the authors as a strength, broadening the applicability of the DM test. This adaptability proves valuable in scenarios where models are potentially unknown or no explicit models exist.

DM makes assumptions precisely on the error loss differential (or forecast errors). Diebold-Mariano (1995) and Diebold (2012) denotes that the loss associated with the forecast error  $e_t$  as  $L(e_t)$ . Furthermore, the time- $t$  loss differential between forecasts 1 and 2 is then  $d_{12t} = L(e_{1t}) - L(e_{2t})$ . In this case, the DM requires only that the loss differential be covariance stationary. The DM model assumes that:

$$\begin{cases} E(d_{12t}) = \mu, \forall t \\ \text{cov}(d_{12t}, d_{12t}(t - \tau)) = \gamma(\tau), \forall \tau \\ 0 < \text{var}(d_{12t}) = \sigma^2 < \infty \end{cases} \quad (13)$$

$$DM_{12} = \frac{\bar{d}_{12}}{\hat{\sigma} \bar{d}_{12}} \rightarrow N(0,1)$$

$$* E(d_{12t}) \leftrightarrow E(d_{12t}) = 0$$

Lastly, the expression  $\bar{d}_{12}$  represents  $\frac{1}{T} \sum_{t=1}^T d_{12t}$  which is the sample mean loss differential, and  $\hat{\sigma} \bar{d}_{12}$  serves as a consistent estimate for the standard deviation of  $\bar{d}_{12}$ . In essence, the procedure is straightforward: If Assumption DM is satisfied, then the test statistic DM follows a limiting distribution of  $N(0,1)$ .

## 3. Results

### 3.1 ADF Test

The ADF test outcomes are presented in Table 1 and 2, with each panel focusing on a distinct economic variable. Panel A examines inflation, Panel B addresses unemployment, and Panel C assesses the Fed Funds rate. In Panel A, the Test Statistic of -2.969 is more negative than the critical value of 1%. This implies, that the null

hypothesis of a unit root is rejected, as indicates that the inflation time series variable is likely stationary after differencing. The statistical significance of the intercept and lagged first difference coefficient reinforces the rejection of the unit root hypothesis. Similarly, in Panel B, the Test Statistic of -3.5887 is more negative than the critical value of 1%.

Like inflation, the test rejects the null hypothesis for unemployment, suggesting that the time series variable is likely stationary post-differencing. The statistical significance of the intercept and lagged first difference coefficient contributes to the rejection of the unit root hypothesis. Lastly, in Panel C, the test statistic (-2.4488) is less negative than the critical value at the 1% level, but it is more negative than the critical value at the 5% level. The test provides a weak indication against the null hypothesis of a unit root for the Fed Funds rate. While the statistic is less negative than the 1% critical value, it is more negative than the 5% critical value. This suggests some evidence against a unit root, but with less certainty. Therefore, we fail to reject the null hypothesis for the Fed Funds ADF test.

In summary, the ADF test results demonstrate the successful rejection of the null hypothesis for inflation and unemployment, indicating stationarity after differencing. However, for the Fed Funds rate, there is a weak indication against the unit root hypothesis. These findings align with the conclusions drawn from accuracy tests in Section 3.5.

Table 1. ADF test results

Panel A.				
	Inflation (Intercept)	Estimate 0.29084* (0.11703)	t-value 2.485	Pr(> t ) 0.01356
	z.lag.1	-0.09047** (0.03047)	-2.970	0.00326
	z. diff. lag1	-0.26982*** (0.06367)	-4.238	3.12e-05
	z. diff. lag2	-0.07906 (0.06590)	-1.200	0.23133
	z. diff. lag3	0.02263 (0.06515)	0.347	0.72855
	z. diff. lag4	0.09922 (0.06138)	1.617	0.10716
Panel B.				
	Unemployment (Intercept)	0.70021*** (0.20258)	3.456	0.000638
	z.lag.1	-0.11922*** (0.03322)	-3.589	0.000396
	z. diff. lag1	-0.02147 (0.06319)	-0.340	0.734315
	z. diff. lag2	-0.05461 (0.06297)	-0.867	0.386564
	z. diff. lag3	0.04677 (0.06209)	0.753	0.451906
	z. diff. lag4	0.02451 (0.06145)	0.399	0.690326
Panel C.				
	Fed Funds (Intercept)	0.224550* (0.109219)	2.056	0.0408
	z.lag.1	-0.045977* (0.018776)	-2.449	0.0150
	z. diff. lag1	-0.04262 (0.061101)	-0.698	0.4861
	z. diff. lag2	-0.00419 (0.060858)	-0.069	0.9451

z. diff. lag3	0.146407* (0.060968)	2.401	0.0170
z. diff. lag4	-0.07154 (0.061540)	-1.163	0.2460

Note. Significance codes (P-values): 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1.

Table and calculations made by the authors.

Table 2. Unified ADF test results

	RSE	Degrees of Freedom	Multiple R-Squared	Adjusted R-Squared	F-Statistic	P-Value	Test-Statistic
Inflation	1.063	265	0.1336	0.1173	8.172	3.42E-04	-29.695
Unemployment	0.8288	265	0.06676	0.04915	3.791	0.002472	-35.887
Fed Funds	1.084	265	0.05485	0.03702	3.076	0.01022	-24.488

Table and calculations made by the authors.

### 3.2 VAR (4)

The response function covering the temporal interval from January 1st, 1960, to June 30th, 2023, manifests consequential effects on variables 2 and 3 at time 't'. The model VAR (4), includes lagged values up to order 4 for each endogenous variable (results can be seen in table 3). Specifically, variable 2 singularly influences variable 3 at time 't', whereas variable 3 does not exert an impact on variables 1 and 2 at the same temporal juncture. Additionally, in the formulation of the inflation equation involving the endogenous variables GDPCTPI (Inflation), UNRATE (Unemployment Rate), and FEDFUNDS (Fed Funds Rate), lagged values of these variables are employed to prognosticate the unemployment equation and the Fed Funds equation.

The outcomes evince affirmative and statistically significant coefficients for GDPCTPI.11 (Inflation at lag 1) and GDPCTPI.13 (inflation at lag 3), indicative of a positive association with the extant GDPCTPI (inflation). The model adeptly captures a lag-induced effect, signifying the enduring nature of the inflation rate. Transitioning to the prognostication of UNRATE through the utilization of lagged values of GDPCTPI, UNRATE, and FEDFUNDS, the scrutiny discerns that UNRATE.11 (Unemployment lag 1) manifests a robust positive correlation with the ongoing UNRATE. Furthermore, lagged values of GDPCTPI and FEDFUNDS substantially contribute to the anticipation of UNRATE.

In the context of forecasting FEDFUNDS (Panel C.), FEDFUNDS.11 evinces a pronounced positive association with the extant FEDFUNDS, exhibiting statistical significance. Lagged values of UNRATE and GDPCTPI also wield a noteworthy influence in predicting FEDFUNDS. Collectively, the VAR model showcases an estimable conformity to the dataset, adeptly encapsulating the intricate interrelations among the variables.

Table 3. VAR (4) results

	Estimate	t-value	Pr(> t )
Panel A.			
Inflation			
GDPCTPI.11	0.68143*** (0.06900)	9.875	<2e-16
UNRATE.11	0.16362. (0.08914)	1.836	0.0677
FEDFUNDS.11	0.08685 (0.06588)	1.318	0.1887
GDPCTPI.12	0.09163 0.08213	1.116	0.2657
UNRATE.12	-0.16267 (0.11174)	-1.456	0.1468
FEDFUNDS.12	0.01731 (0.08276)	0.209	0.8345
GDPCTPI.13	0.16086* (0.08135)	1.977	0.0492
UNRATE.13	0.20741. (0.11225)	1.848	0.0659

FEDFUNDS.I3	-0.02555 (0.08253)	-0.310	0.7571
GDPCTPL.I4	-0.02771 (0.06926)	-0.400	0.6894
UNRATE.I4	-0.16262. (0.08744)	-1.860	0.0641
FEDFUNDS.I4	-0.08343 (0.06577)	-1.269	0.2058
const	0.06385 (0.28603)	0.223	0.8235
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Panel B.			
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Unemployment			
<hr/>			
GDPCTPL.I1	-0.018572 (0.053512)	-0.347	0.72885
UNRATE.I1	0.758161*** (0.069126)	10.968	2,00E-16
FEDFUNDS.I1	-0.097099. (0.051094)	-1.900	0.05859
GDPCTPL.I2	-0.004627 (0.063692)	-0.073	0.94215
UNRATE.I2	-0.038860 (0.086654)	-0.448	0.65423
FEDFUNDS.I2	0.015425 (0.064179)	0.240	0.81027
GDPCTPL.I3	-0.004399 (0.063088)	-0.070	0.94447
UNRATE.I3	0.144744. (0.087050)	1.663	0.09768
FEDFUNDS.I3	0.085338 (0.064005)	1.333	0.18371
GDPCTPL.I4	0.046239 (0.053710)	0.861	0.39016
UNRATE.I4	-0.003897 (0.067808)	-0.057	0.95422
FEDFUNDS.I4	0.029731 (0.051001)	0.583	0.56049
const	0.601798** (0.221816)	2.713	0.00716
<hr/>			
Panel C.			
<hr/>			
Fed. Funds			
<hr/>			
GDPCTPL.I1	0.023535 (0.070921)	0.332	0.7403
UNRATE.I1	-0.186088* (0.091616)	-2.031	0.0434
FEDFUNDS.I1	0.812774*** (0.067717)	12.003	<2e-16
GDPCTPL.I2	0.036113 (0.084414)	0.428	0.6692
UNRATE.I2	0.097792 (0.114846)	0.852	0.3953
FEDFUNDS.I2	0.059088 (0.085059)	0.695	0.4879
GDPCTPL.I3	0.094562 (0.083614)	1.131	0.2592
UNRATE.I3	-0.002301 (0.115371)	-0.020	0.9841



FEDFUNDS.I3	0.150008. (0.084829)	1.768	0.0783
GDPCTPI.I4	-0.038663 (0.071184)	-0.543	0.5875
UNRATE.I4	0.062852 (0.089868)	0.699	0.4850
FEDFUNDS.I4	-0.109176 (0.067594)	-1.615	0.1076
const	0.214407 (0.293982)	0.729	0.4665

Note. Significance codes (P-values): 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘.’ 1

Table and Calculations made by the authors.

### 3.3 Residuals

The covariance matrix (table 4) delineates the extent to which the residuals associated with inflation, unemployment, and the federal funds rate exhibit collective variability. According to the findings, the correlation coefficient of -0.2929 observed between inflation and unemployment implies a moderately negative linear association. The table also suggests a negative linear association between unemployment and the Fed Funds rate. Within this framework, a negative covariance or correlation in the residuals pertaining to inflation and unemployment as well to unemployment and the Fed Funds Rate. This signifies a proclivity for these variables to exhibit opposing movements after the accommodation of the modeled relationships. To illustrate, in the event of an unforeseen rise in inflation, there exists a likelihood of an unexpected decrease in unemployment, and vice versa (in this context, the same likelihood exists between unemployment and interest rates).

Furthermore, the coefficient of 0.2402 between inflation and the residuals of the federal funds rate denotes a moderately positive linear association. The positive covariance or correlation evident in the residuals linking inflation and the federal funds rate intimates a propensity for these variables to move congruently after the incorporation of the modeled relationships. For instance, in the scenario of an unforeseen increase in inflation, a concomitant unexpected rise in interest rates may transpire, and vice versa. The comprehension of such relationships serves to enhance the precision of the model and provides insights into the collective behavior of the variables.

Table 4. Covariance matrix of residuals

	Inflation	Unemployment	Fed Funds
Inflation	1	-0.2929	0.2402
Unemployment	-0.2929	1	-0.2550
Fed Funds	0.2402	-0.2550	1

Note. Table and Calculations made by the Authors.

### 3.4 Impulse Response Function

The impulse responses for the recursive VAR, ordered in Inflation, unemployment, and Fed Funds Rate are plotted in Figure 1. Also plotted are  $\pm 1$  standard error band, which yield an approximate 66 percent confidence intervals for each of the impulse responses. In the first row, one unit shock to inflation slowly decreases over 24-quarters. In this case, inflation proceeds to fall while unemployment increases, the decrease in inflation corresponds in the increase of funds rate. One unit shock to unemployment slowly decreases over 24-quarters. As Unemployment proceeds to fall and inflation spikes (second row). The Gradual decline in unemployment corresponds in the gradual increase to the Fed Funds rate. Lastly, In the Fed Funds shocks to the variables, the gradual increase of inflation corresponds to a slump in unemployment (third row). However, both variables follow an inverse relationship throughout the 24-quarters. Conversely, the Fed Funds rate tend to decrease in this inverse relationship. Despite the changes made in the temporal interval used in this study, the results of the following tests are in accordance with the simple IS-LM framework and Stock and Watson (2001) findings.

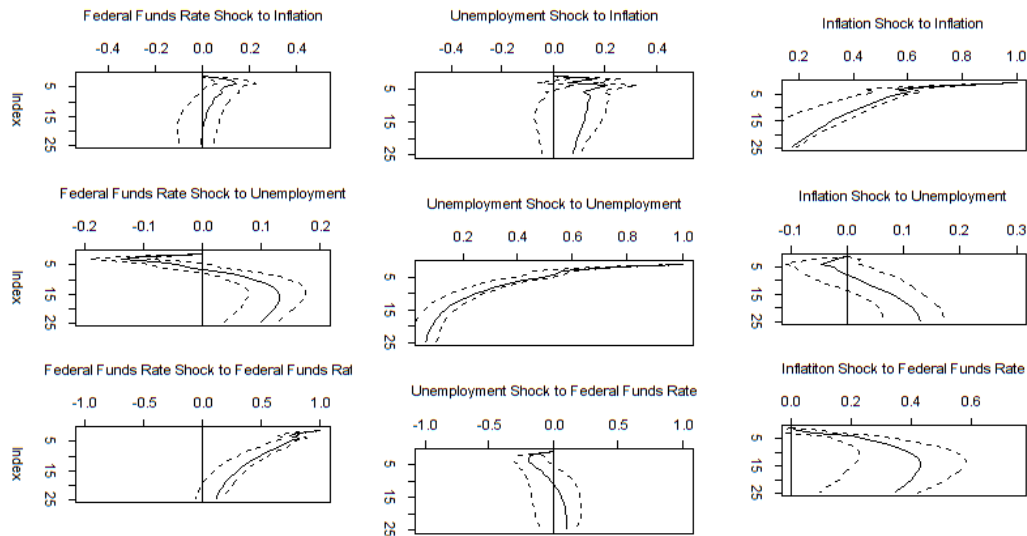


Figure 1. Inflation shocks to unemployment and federal funds rate

Note. Figure and Calculations made by the Authors.

### 3.5 VAR Iterative Forecasting Results

Employing an iterative process of incorporating previously forecasted values into our system of equations, our simple model projects change over the next three years in inflation (Panel A), unemployment (Panel B), and the federal funds rate (Panel C). Notably, our forecasts indicate an expected increase in unemployment, juxtaposed with a slight decrease in the federal funds rate. A visual representation of these projections can be gleaned from Figure 2, and a comprehensive overview is available in Table 5.

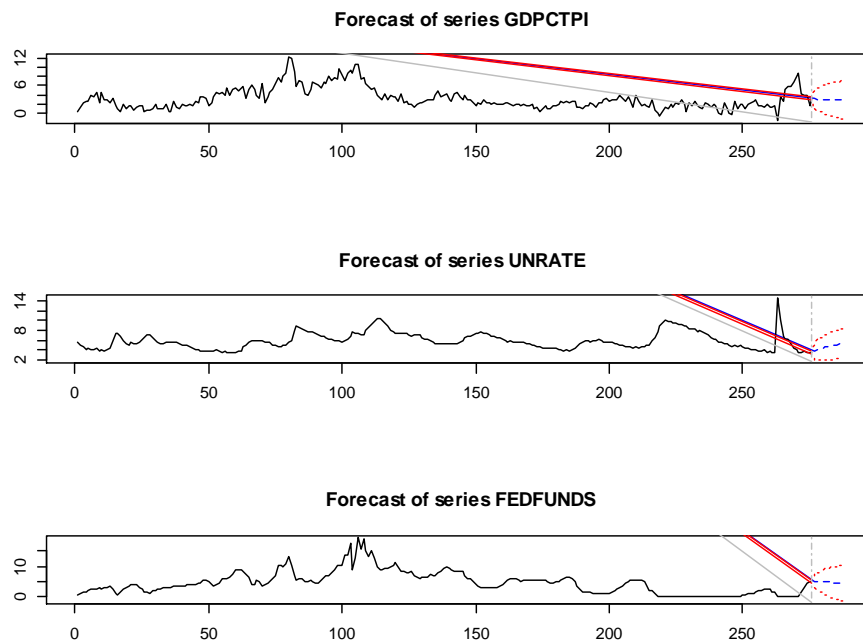


Figure 2. Iterative forecast over the next three years

Note. Figure and Calculations made by the Authors.

Table 5. VAR iterative forecasting

	Period	forecast	lower	upper	CI
<b>Panel A.</b>					
Inflation	1	3.160	107.734.568	5.243	2.083
	2	3.009	0.52734344	5.490	2.481
	3	3.035	0.26129803	5.809	2.773
	4	3.011	-0.0298488	6.051	3.040
	5	2.995	-0.2531214	6.244	3.248
	6	2.979	-0.4471899	6.405	3.426
	7	2.969	-0.6083370	6.548	3.578
	8	2.962	-0.7444929	6.669	3.706
	9	2.959	-0.8580004	6.776	3.817
	10	2.958	-0.9545616	6.871	3.913
	11	2.959	-103.609.07	6.955	3.995
	12	2.963	-110.421.72	7.030	4.067
<b>Panel B.</b>					
Unemployment	1	3.750	2.157.677	5.343	1.592
	2	4.039	1.973.435	6.105	2.065
	3	4.212	1.890.484	6.534	2.321
	4	4.442	1.927.222	6.958	2.515
	5	4.636	1.989.624	7.283	2.647
	6	4.801	2.065.800	7.536	2.735
	7	4.948	2.155.312	7.741	2.793
	8	5.072	2.239.028	7.905	2.833
	9	5.179	2.317.283	8.042	2.862
	10	5.271	2.386.516	8.156	2.884
	11	5.348	2.444.922	8.252	2.903
	12	5.415	2.493.397	8.336	2.921
<b>Panel C.</b>					
Fed Funds	1	5.208	31.166.361	7.301	2.092
	2	4.998	22.089.105	7.787	2.789
	3	5.051	17.717.622	8.331	3.280
	4	4.986	11.565.696	8.816	3.830
	5	4.876	0.6324877	9.119	4.243
	6	4.826	0.2453124	9.407	4.580
	7	4.761	-0.1175404	9.641	4.879
	8	4.700	-0.4275241	9.829	5.128
	9	4.654	-0.6873660	9.997	5.342
	10	4.613	-0.9172877	10.143	5.530
	11	4.577	-11.175.667	10.273	5.695
	12	4.549	-12.919.226	10.391	5.841

Note. Table and calculations made by the authors.

### 3.6 Accuracy Tests

This section presents the results of accuracy tests conducted on the VAR iterative forecasts. The Mean Absolute Error (MAE) test indicates a degree of reliability in the inflation forecasts, as it measures the average magnitude of errors between predicted and actual values. However, the elevated values of Mean Squared Error (MSE) and Root Mean Squared Error (RMSE) suggest suboptimal model performance. This trend persists in the notably high Mean Absolute Percentage Error (MAPE) values, indicating an unreliable forecast for inflation. Similar patterns emerge in the case of unemployment forecasts, with elevated and unreliable MAPE values.

Conversely, the Federal Funds rate tests exhibit shortcomings across various metrics, with only the MAE showing marginal levels of reliability in the forecasts. Nevertheless, the MAE for the Federal Funds rate is nearly 5%, revealing concerning patterns in the forecast accuracy. These findings collectively underscore the need for further model refinement to enhance forecast precision.

Table 6. Forecast accuracy tests

	Mean Absolute Error	Mean Squared Error	Root Mean Squared	MAPE
Panel A.	2.498	10.755	3.279	201.6
Inflation				
Panel B.				
Unemployment	2.864	14.037	3.746	157.04
Panel C.				
Fed Funds	4.967	31.188	5.584	637.53

Table and Calculations made by the Authors.

### 3.7 Interval Coverage

The examination of interval coverage reveals incongruent findings. Notably, the value attributed to the Fed Funds indicates a heightened degree of optimization in the iterative forecasts (table 7). Conversely, inflation ranks second, while unemployment occupies the last position in this regard. Such disparities stand in contrast to the outcomes derived from the accuracy tests presented in Table 6. This incongruity prompts the necessity for a more in-depth exploration into the reliability of these forecasted outcomes.

Table 7. Optimization of iterative forecasts

Index	Values
Inflation	0.7339
Unemployment	0.6086
Fed Funds	0.8393

Note. Table and calculations made by the authors.

### 3.8 Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF)

In Figure 3, discernible patterns emerge in the autocorrelation structure of the examined time series variables. Notably, inflation exhibits a robust correlation with its lagged values, showcasing a gradual decrease over successive lags. Despite this diminishing trend, the statistical significance of the correlation between inflation and its lagged values remains evident. In the case of unemployment, the correlation diminishes as the number of lags increases, and lags beyond 10 exhibit a notable negative autocorrelation. Similarly, the Fed Funds rate demonstrates a pattern akin to inflation, characterized by an initial positive correlation that gradually attenuates with increasing lags.

Furthermore, the Partial Autocorrelation Function (PACF) reveals distinctive correlations at specific lags. Inflation manifests strong correlations at lag 1, lag 2, lag 9, and lag 10, emphasizing persistent relationships. Conversely, unemployment displays a predominant correlation solely at lag 1 in its PACF. The Fed Funds rate, akin to inflation, exhibits noteworthy correlations at lag 1, lag 4, and lag 6 in its PACF.

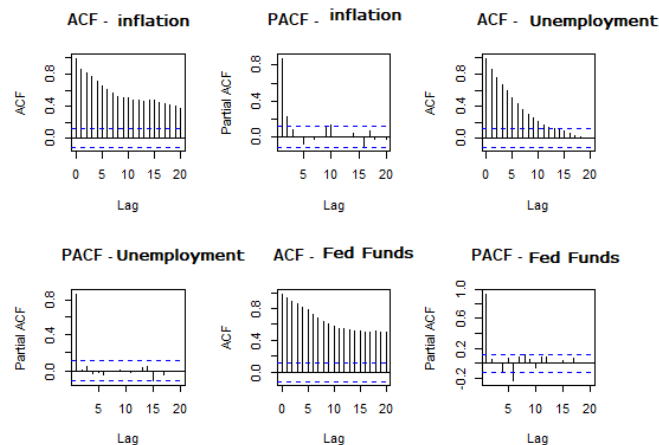


Figure 3. ACF and PACF plots

Note. Figure and Calculations made by the authors.

### 3.9 SARIMA and Diebold and Mariano Tests

The outcomes derived from the SARIMA forecasting endeavor reveal the utilization of an ARIMA (0,1,1) model for the predicted time series, as delineated in Table 8. This model incorporates a first-order differencing process aimed at achieving stationarity. The presence of a negative coefficient for  $ma1$  within the moving average component denotes an inverse correlation with lagged forecast errors. Additionally, the results present forecasted values for multiple forthcoming quarters, accompanied by their respective prediction intervals.

Table 8. SARIMA forecasts results

Panel A							
Coefficients							
	ma1					-0.3145	
	s.e.					0.0614	
	σ2 estimated					1.168	
	log likelihood					-378.22	
	AIC					760.44	
	AICc					760.49	
	BIC					767.51	
Panel B							
	Point	Forecast	Lo 80	Hi 80	Lo 95	Hi 95	
	Q3	243.665	105.143.434	3.821.866	0.3181454	4.555.155	
	Q4	243.665	0.75723733	4.116.063	-0.131790	5.005.090	
	Q1	243.665	0.50739329	4.365.907	-0.513893	5.387.194	
	Q2	243.665	0.28638584	4.586.914	-0.851895	5.725.195	
	Q3	243.665	0.08606700	4.787.233	-11.582.56	6.031.557	
	Q4	243.665	-0.0984722	4.971.772	-14.404.8	6.313.785	
	Q1	243.665	-0.2704609	5.143.761	-17.035.1	6.576.819	
	Q2	243.665	-0.4321570	5.305.457	-19.508.11	6.824.112	
	Q3	243.665	-0.5852134	5.458.514	-21.848.9	7.058.192	
	Q4	243.665	-0.7308826	5.604.183	-24.076.7	7.280.973	
	Q1	243.665	-0.87014113	5.743.441	-26.206.5	7.493.951	
	Q2	243.665	-100.376.74	5.877.068	-28.250.1	7.698.315	
Panel C							
	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
	0.009420159	1.076.626	0.7768108	1493229	5574143	0.6461126	0.0092276

Note. Figure and Calculations made by the authors.

The Diebold-Mariano (DM) tests were conducted for inflation, unemployment, and the Fed Funds rate, as outlined in Table 9. Commencing with the evaluation of inflation, the findings indicate inferior performance of the first model (SARIMA) compared to the second model (VAR). This conclusion is substantiated by the p-value of 0.08581, surpassing the customary significance level of 0.05. Consequently, within this framework, the null hypothesis asserting no disparity in forecast accuracy between the two models cannot be rejected. Furthermore, there exists insufficient evidence to assert a statistically significant difference in forecast accuracy between the two models at the 1-period forecast horizon.

Turning to the DM test for unemployment, the proximity of the p-value (0.05809) to, albeit marginally above, the conventional significance level of 0.05 implies a lack of adequate evidence to reject the null hypothesis. Therefore, the inference drawn is that there is no substantial distinction in predictive accuracy between the SARIMA and VAR models.

Conversely, in the case of the Fed Funds, the low p-value (0.02968) signifies the presence of compelling evidence to reject the null hypothesis, thereby establishing a difference in predictive accuracy between the SARIMA and VAR models. In essence, the DM test imparts statistical significance, suggesting that one of the models is likely to exhibit superior predictive accuracy relative to the other.

In summary, for inflation and unemployment, no noteworthy difference in predictive accuracy between the SARIMA and VAR models is discerned. In contrast, for the Fed Funds, a statistically significant difference is evident, with the evidence supporting the rejection of the null hypothesis, indicating a disparity in predictive accuracy that favors one of the models.

Table 9. DB test results

	DM	Forecast Horizon	Loss function	p-value	Ha
Inflation	-17.766	1	2	0.1033	two. Sided
Unemployment	21.218	1	2	0.05739	two. Sided
Fed Funds	24.928	1	2	0.02989	two. Sided

Note. Figure and Calculations made by the authors. \* Ha = Alternative Hypothesis.

#### 4. Conclusion

The VAR (4) model effectively captures intricate interrelations among variables, shedding light on the temporal effects of unemployment and the Fed Funds rate at time 't'. The model structure suggests that variable Unemployment exerts a singular influence on variable the Fed Funds rate, emphasizing the asymmetry in their relationship. However, the Fed Funds rate does not contemporaneously impact variables Inflation and Unemployment, providing nuanced insights into the complex dynamics at play. Moreover, the formulation of equations involving lagged values for predicting unemployment and the Fed Funds rate provides a comprehensive understanding of the temporal dependencies. Positive and statistically significant coefficients for lagged inflation values indicate a lasting positive association with inflation, contributing to the understanding of inflation persistence.

Next, the covariance matrix and impulse response analysis offer insights into collective variability and relationships among residuals. The findings align with established economic frameworks, such as the IS-LM model and previous research by Stock and Watson (2001), providing theoretical validation. The iterative incorporation of previously forecasted values highlights an expected increase in unemployment and a slight decrease in the federal funds rate over the next three years. However, reliability issues, particularly for the Fed Funds rate, are identified through accuracy tests.

In this context, The ADF tests successfully reject the null hypothesis for inflation and unemployment, indicating stationarity after differencing. However, a weak indication against the unit root hypothesis is observed for the Fed Funds rate. In addition, Autocorrelation and PACF analyses reveal distinctive correlations at specific lags, providing insights into persistent relationships within the variables. Lastly, the SARIMA model with an ARIMA (0,1,1) structure is employed, and DM tests suggest inferior performance for inflation, no significant difference for unemployment, and superiority for the Fed Funds rate in the VAR model.

Finally, this study identifies several avenues for further research to address existing gaps in the macroeconomic literature. First, there is a need for enhanced understanding of inflation dynamics, specifically by conducting technical investigations into the lasting positive association between lagged and current inflation. Additionally, the examination of covariance patterns and their implications on economic policy represents a pertinent area for further exploration. Furthermore, addressing reliability issues inherent in Fed Funds rate forecasts requires dedicated investigation to enhance the precision of forecasting models, while an examination of factors influencing the weak indication against the unit root hypothesis for the Fed Funds rate is warranted.

Moreover, it would be beneficial a detailed exploration of the observed performance differences in SARIMA and VAR models, particularly in the context of inflation. Lastly, further studies could broaden the scope of analysis to encompass a cross-country comparative study holds promise for providing comprehensive insights into economic variations across diverse nations.

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