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On the Interest Rate Derivatives Pricing with Discrete Probability Distribution and Calibration with Genetic Algorithm

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Abstract

Bond prices and fixed-income derivatives intricately depend on the ever-evolving landscape of interest rates. This study introduces an exceptionally efficient semi-analytical pricing methodology designed for discretely updated path-dependent interest rate options. Our approach involves the derivation of an analytical solution for the characteristic function of the logarithm of the discretely discounted strike price, enabling the computation of coefficients for the Fourier-cosine series governing the path-dependent option pricing. The distinctive feature of our pricing method lies in the utilization of a modified Skellam probability distribution to model interest rate increments, resulting in a remarkably swift and precise calculation process. Unlike existing solutions for similar pricing challenges, our proposed formula considerably enhances computational efficiency. Moreover, we delve into the influence of central bank monetary decision probabilities on option prices via a comprehensive series of numerical experiments. In an effort to further fine-tune the calibration parameter values for the yield curve, we employ a genetic algorithm, thus contributing to the heightened precision of our pricing model. This research, with its innovative approach, not only refines pricing procedures but also offers valuable insights into the dynamic interplay between monetary policies and fixed-income derivatives.

Keywords: discrete interest rates, interest rate derivatives, path-dependent option pricing, bond pricing, genetic algorithm, cos method

1. Introduction

Finite state physical systems are a class of systems that can be described by a finite number of states and transitions between those states. These systems have a discrete and well-defined set of possible states, and their behavior can be modeled using finitestate machines or automata. Finite state physical systems have applications in various fields, including cyber-physical systems (Kazemi, Safavi, Arefi, & Naseri, 2022), computer science (Pretorius & Bosch, 2003), natural language processing and spellchecking (Pirinen & Lindén, 2014), reinforcement learning, and economics.

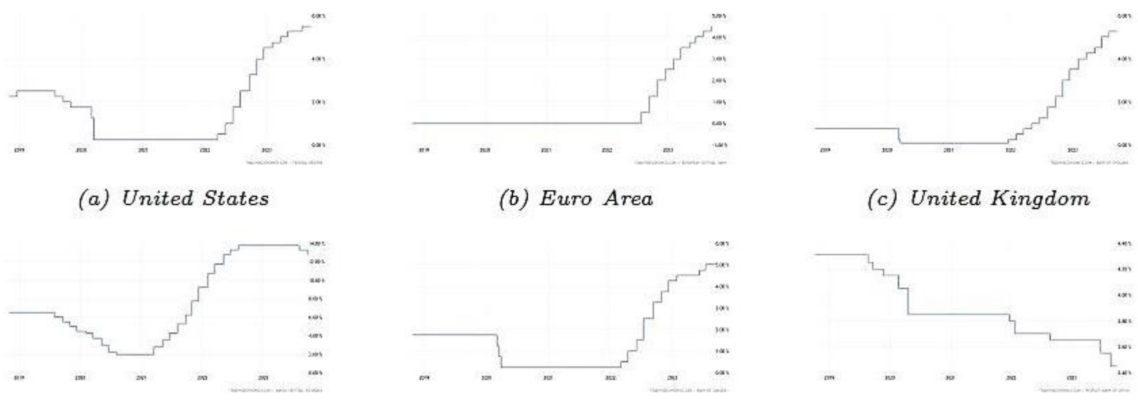
Interest rates, which jump in a discrete set of states (Heidari & Wu, 2009), play a crucial role in the functioning of financial markets and have a significant impact on economic activity. It can be found in Di Matteo, Airoldi, and Scalas (2004), for example, an examination that the daily increment of the interest rate series is non-Gaussian, non-Brownian, and follows a leptokurtic distribution. In this sense, addressing the incidence of jumps is a key feature of interest rate modeling to consider (Yu & Ning, 2019). The occurrence of interest rate jumps may lead to a significant impact on financial prices and are often associated with unexpected events or news releases (Das, 2002).

Econophysics, an interdisciplinary field that applies concepts and methods from physics to economics and finance, has made significant contributions to the modeling and analysis of interest rates. A vast number of scholars have investigated the influence of interest rates on various financial and economic factors (Broga, Viegas, & Jensen, 2016; Li, Lu, Jiang, & Petrova, 2021; Gomes, Mendes, & Mendes, 2008). Interest rates are closely linked to various economic variables, such as inflation, economic growth, and exchange rates. Econophysics has explored the relationship between interest rates and these economic variables using statistical and econometric techniques. For example, studies have examined the relationship between interest rates and inflation uncertainty and prediction (see e.g. Fama, 1975 and Söderlind, 1998). In Shaukat, Zhu, and Khan (2019), is studied the mechanism through which the real interest rate establishes a negative impact on economic growth. These models provide insights into the transmission mechanisms between interest rates and economic variables and can help in understanding the impact of interest rate changes on the broader economy (Backus & Zin, 1993).

In this paper, we deal with discrete probability distribution to model the interest rate jump dynamics. Discrete random events play a significant role in both physical systems and econophysics. In physical systems, discrete random events are often modeled using stochastic approaches such as discrete-event dynamic systems (DEDS) (Wang & Chen, 2005). DEDS is a stochastic approach that models discrete-time random sources in physical systems. It considers the noise part as a Gaussian white noise or a Brownian motion. This approach is used to study contaminant transport models under random sources in groundwater systems (Wang & Chen, 2005), for example. On the other hand, discrete compounding interest rates are a fundamental concept in economics that plays a crucial role in various financial calculations and transactions. Unlike continuous compounding, which assumes that interest is continuously compounded over time, discrete compounding assumes that interest is compounded at specific intervals, such as annually, semi-annually, quarterly, or monthly (Fabozzi, 2000).

Utilizing a discrete probability model, this study undertakes an examination of the monetary influence exerted by decisions made during central bank meetings on the pricing of interest rate derivatives. Central bank meetings hold a pivotal role in the determination of market prices, particularly concerning interest rates. In light of the gradual evolution of market-driven interest rate processes, central banks have established policy operational frameworks aimed at guiding and regulating market interest rates effectively (Liping & Wu, 2016). Through the provision of lucid and transparent communication, central banks possess the capacity to mold the behavior of market participants and, consequently, shape market prices (Kuncoro, 2020). The discernible impact of central bank meetings on market prices is further observable through the yield curve, which delineates the correlation between interest rates pertaining to bonds with varying maturities. It has been empirically established that central bank monetary policy actions and communication influence the shape and dynamics of the yield curve (Brand, Bunčić, & Turunen, 2010). Furthermore, it is imperative to recognize that the reach of central bank meetings extends beyond the confines of domestic markets. The decisions and communication strategies employed by major central banks, such as the Federal Reserve and the European Central Bank, possess the potential for global ramifications (Brusa, Savor, & Wilson, 2019). Changes in interest rates and market prices within one nation can transmit ripples to other countries through diverse channels, including shifts in capital flows and fluctuations in exchange rates. Consequently, central bank meetings and their associated outcomes are subjects of vigilant scrutiny by market participants on a global scale.

The period between central bank meetings, often several weeks, is characterized by a stable interest rate environment, where rates remain unchanged. This phenomenon can be observed in major economies, exhibited in Figure 1.



(a) United States *(b) Euro Area* *(c) United Kingdom*

(d) Brazil *(e) Canada* *(f) China*

Figure 1. Interest rates (2018-2023)

Source: Trading Economics.

The stability in interest rates during these periods can offer economic predictability and stability. Still, it also poses challenges related to market expectations, investor behavior, and the need for effective central bank communication and intervention when necessary. Constant interest rates between meetings are an integral part of the broader monetary policy landscape, requiring a delicate balance to ensure economic growth, price stability, and financial market resilience.

A comprehensive examination of how the discrete set of states of the interest rate scheduled jumps affect the bond and option prices is still missing. Unlike other widely used continuous time and state financial models such as the Vasicek model Vasicek (1977) applied in Xiao, Zhang, Zhang, and Chen (2014), Cox, Ingersoll, and Ross (1986), J. Hull and White (1990), and others found in Bouziane (2008) and da Silva, Baczynski, and Bragança (2019), this paper deals with the problem of modeling discrete path-dependent interest rate products, such as bond and options. It can be found in Heidari and Wu (2009) a term structure model that incorporates an anticipated jump component with known arrival times but random continuous jump size. We propose to characterize interest rate changes triggered by scheduled central bank meetings adopting discrete jump sizes through the modified version of the Skellam distribution introduced in da Silva, Baczynski, and Vicente (2023).

This paper focuses on modeling discrete path-dependent interest rate products using the modified Skellam distribution due to its inherent suitability for capturing the discrete set of states associated with scheduled jumps in interest rates. Unlike the prevalent continuous time and state financial models, such as the Vasicek model and others referenced in literature, our emphasis lies in addressing the intricacies of discrete path-dependent financial products like bonds and options.

The choice to utilize the modified Skellam distribution stems from its ability to effectively model discrete events, aligning precisely with the nature of jumps observed in interest rates. These discrete jumps, delineated within the domain of {…,-0.50%,-0.25%,0,0.25%,0.50%,…}, demand a model that can adequately encapsulate their discrete nature, a requirement not adequately met by continuous-time models.

Moreover, the modified Skellam distribution, introduced in da Silva et al. (2023), was originally tailored to calculate discrete probabilities of jumps, a crucial aspect when dealing with products traded in markets exhibiting discrete behaviors, such as those in the Brazilian stock exchange. By focusing on the modified Skellam distribution for modeling discrete path-dependent interest rate products, this paper addresses the nuanced dynamics of these financial instruments, acknowledging the discrete nature of interest rate changes and emphasizing the necessity for a modeling framework aligned with such discrete phenomena.

The Skellam distribution is also applied in other areas of science, such as in the evaluation of the intensity difference of pixels in the spatial and temporal domains in camera research (Hwang, Kim, & Kweon, 2007) and in the study of football results with the number of goals scored by each side (Karlis & Ntzoufras, 2009).

We deal with the accumulated, discretely compounded, interest rate index y_{t_n} given by

$$y_{t_n} = y_{t_0} \prod_{t_j=t_0}^{t_n} \left(1 + d_{t_j}\right)^{\frac{1}{252}} \tag{1}$$

where t_j denotes the end of the day j and d_{t_j} assigns the corresponding discrete interest rate expressed as an annually effective rate. y_{t_0} is the value of the index at time t_0. Considering discrete compounding as in (1), zero-coupon bond which pays 1 at maturity t_n can be obtained via y_{t_0}/y_{t_n}. We also deal with the problem of calculate the price of an interest rate option contingent on the index shown in Equation (1), in which the payoff is given by

$$C_{t_n} = Nd\max\left(y_{t_n} - K, 0\right) \tag{2}$$

where d is a constant that defines if it is a call option d=1 or if it is a put option d=-1 and N is the number of contracts. Correspondingly, the discretely monitored price for the call option with strike K and maturity in t_n is given by

$$C_{t_0} = N\mathbb{E}\left[\frac{1}{\prod_{t_j=t_0}^{t_n}\left(1+d_{t_j}\right)^{\frac{1}{252}}} d\max\left(y_{t_n} - K, 0\right) \mid \mathcal{F}_{t_0}\right] \tag{3}$$

The path-dependent option problem was addressed by da Silva et al. (2016), where the price of the call option (2) was calculated via the discretization of the partial differential equations, due to the Feynman-Kac theorem (Oksendal, 2007), governing the option pricing problem and solving it through the introduced modified full-implicit finite difference method. da Silva et al. (2016) compared the prices under the discrete and continuous updating of the interest rate index and showed significant discrepancies. However, solving this two-dimension PDE is computationally intensive and time-consuming. Authors also assume that the short-term interest rate follows a continuous mean-reverting stochastic process governed by the Vasicek model (Vasicek, 1977) which is not supported by observing the interest rate historical data exhibited in Figure 1.

In Baczynski, Otazú, and Vicente (2017), the same discrete updating problem was addressed by solving a time and value discretization of the interest rate process in conjunction with prices of Arrow-Debreu securities with unitary lifetime obtained via the Feymann-Kac formula. The method is also computationally intensive and hugely time-consuming.

For the first time in the econophysics and financial literature, this study explores analytically the discrete updating of interest rates in space and time in conjunction with the pricing of bonds and path-dependent interest rate options. To capture the behavior of interest rate jumps during scheduled central bank meetings, we use the modified Skellam distribution introduced by da Silva et al. (2023). We adapt the COS method (Fang & Oosterlee, 2008) to calculate the option prices.

Unlike other works in calibration that use the method of maximum entropy to minimize the error between market and model prices (Gomes-Gonçalves, Gzyl, & Mayoral, 2018; and Malhotra, Srivastava, and Taneja, 2019), we employ a genetic algorithm (GA) in order to develop optimal or nearly optimal calibrated parameters.

The paper is organized as follows. Section 2 provides the analytical solution for the characteristic function of the discounted strike price required to calculate bond and option pricing. In Section 3 we provide a series of numerical results: we calculate the interest rate option prices and show yield curves implied by the proposed model. We also propose a genetic algorithm method to calibrate bond prices. Section 4 concludes the paper.

2. Main Result

Options are financial derivatives that give investors the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset at a predetermined price within a specified period. Call and put options play a crucial role in financial markets, providing investors with opportunities to hedge risk, speculate on price movements, and generate income (J.C. Hull, 2009).

Options on equity securities have existed for more than a century. In Kairys and Valerio (1997), authors explored the market for equity options in the 1870s, highlighting the existence of trading in an order-driven over-the-counter market before the listing of option contracts. This historical perspective emphasizes the long-standing presence of options in financial markets and their evolution over time.

Interest rate derivatives are a cornerstone of modern financial markets, playing a pivotal role in managing and hedging interest rate risk, speculating on future interest rate movements, and optimizing investment portfolios. These sophisticated financial instruments derive their value from underlying interest rates, making them integral to the world of fixed-income securities.

We devised the option pricing problem (3) through the COS method introduced by Fang and Oosterlee (2008). The COS method, which is based on the Fouriercosine series, is an intriguing, rapid, and accurate derivatives pricing method, that was first applied to the path-dependent interest rate option by da Silva et al. (2019).

The modular pricing engine of the COS method is supplied by two terms, as shown in Appendix A: the coefficients of the cosine series connected with the payoff function and the characteristic function related to the random variable's density function.

The second part is to find the characteristic function associated with the logarithm of the discretely discounted strike price given by

$$\ln\left[K\prod_{t_j=t_0}^{t_n}\left(1+d_{t_j}\right)^{-\frac{1}{252}}\right]\triangleq\ln\left(k_{t_n}\right) \tag{4}$$

which is also used to calculate the prices of bonds and plot the term structure of the interest rates.

Theorem 2.1. Suppose that the the logarithm of the strike price of a derivative product is discretely (daily) discounted. Then, the characteristic function associated with the logarithm of the discretely discounted strike price is given by

$$\phi\left(\ln\left(k_{t_n}\right),u\right)=\exp\left(iu\ln\left(k_{t_0}\right)\right)\exp\left(-r_{t_0}\frac{iu(n+1)}{252}\right)\prod_{j=0}^{n-1}\phi\left(r_{t_{j+1}}-r_{t_j},\frac{u(j-n)}{252}\right), \tag{5}$$

where where r_t is the continuously compounding interest rate at time t and k_t is the discretely discounted strike price from t, n times (days).

Proof. We note that

$$\ln\left(k_{t_n}\right)=\ln\left(k_{t_0}\right)-\frac{1}{252}\sum_{i=t_0}^{t_n}\ln\left(1+d_{t_i}\right)=\ln\left(k_{t_0}\right)-\frac{1}{252}\sum_{i=t_0}^{t_n}\ln\left(X_i\right) \tag{6}$$

then,

$$\begin{aligned}\phi(\ln(k_{t_n}),u) &= \mathbb{E}\left[\exp\left(iu\ln(k_{t_n})\right) \mid \mathcal{F}_{t_0}\right] \\ &= \mathbb{E}\left[\exp\left(iu\ln(k_{t_0})\right) \mid \mathcal{F}_{t_0}\right]\mathbb{E}\left[\exp\left(iu\left(\ln(k_{t_n})-\ln(k_{t_0})\right)\right) \mid \mathcal{F}_{t_0}\right]\end{aligned} \tag{7}$$

We have that the discounted strike price at time t_0 is known. Then

$$\mathbb{E}\left[\exp\left(iu\ln(k_{t_0})\right) \mid \mathcal{F}_{t_0}\right] = \exp\left(iu\ln(k_{t_0})\right) \tag{8}$$

Manipulating the term inside the remaining expectation in (7), we have

$$\begin{aligned}\ln(k_{t_n})-\ln(k_{t_0}) &= -\tfrac{1}{252}\{n[\ln(X_{t_1})-\ln(X_{t_0})]+(n-1)[\ln(X_{t_2})-\ln(X_{t_1})]+\cdots \\ &\quad \ldots.+2[\ln(X_{t_{n-1}})-\ln(X_{t_{n-2}})]+[\ln(X_{t_n})-\ln(X_{t_{n-1}})]\}-\tfrac{(n+1)}{252}\ln(X_{t_0}) \\ &= -\textstyle\sum_{j=0}^{n-1}\left\{\tfrac{(n-t_j)}{252}\left[\ln\left(X_{t_{j+1}}\right)-\ln\left(X_{t_j}\right)\right]\right\}-\tfrac{(n+1)}{252}\ln(X_{t_0}),\end{aligned} \tag{9}$$

where $X_i = 1 + d_{t_i}$. Substituting in (8) and (9) in Equation (7), we have

$$\begin{aligned}\phi(\ln(k_{t_n}),u) &= \exp\left(iu\ln(k_{t_0})\right)\times \\ &\mathbb{E}\left[\exp\left\{iu\left(-\textstyle\sum_{j=0}^{n-1}\left\{\tfrac{(n-j)}{252}\left[\ln\left(X_{t_{j+1}}\right)-\ln\left(X_{t_j}\right)\right]\right\}-\tfrac{(n+1)}{252}\ln(X_{t_0})\right)\right\} \mid \mathcal{F}_{t_0}\right].\end{aligned} \tag{10}$$

Assuming that the process $\ln(X_{t_i})$ have independent increments with respect to $\mathcal{F}_{t_0}$, Equation (10) becomes

$$\phi(\ln(k_{t_n}),u) = \exp\left(iu\ln(k_{t_0})\right)\mathbb{E}\left[\exp\left(-\tfrac{iu(n+1)}{252}\ln(X_{t_0})\right) \mid \mathcal{F}_{t_0}\right]\times \tag{11}$$

$$\textstyle\prod_{j=0}^{n-1}\mathbb{E}\left[\exp\left\{-\tfrac{iu(n-j)}{252}\left[\ln\left(X_{t_{j+1}}\right)-\ln\left(X_{t_j}\right)\right]\right\} \mid \mathcal{F}_{t_0}\right]$$

The index evolves according to the rate $d_t = e^{r_t} - 1$. Then,

$$r_t = \ln(1 + d_t) \tag{12}$$

So, the characteristic function (11) becomes

$$\begin{aligned}\phi(\ln(k_{t_n}),u) &= \exp\left(iu\ln(k_{t_0})\right)\mathbb{E}\left[\exp\left(-\tfrac{iu(n+1)}{252}r_{t_0}\right) \mid \mathcal{F}_{t_0}\right]\times \\ &\textstyle\prod_{j=0}^{n-1}\mathbb{E}\left[\exp\left\{-\tfrac{iu(n-j)}{252}\left[r_{t_{j+1}}-r_{t_j}\right]\right\} \mid \mathcal{F}_{t_0}\right] \\ &= \exp\left(iu\ln(k_{t_0})\right)\phi\left(r_{t_0},-\tfrac{u(n+1)}{252}\right)\textstyle\prod_{j=0}^{n-1}\phi\left(r_{t_{j+1}}-r_{t_j},-\tfrac{u(n-j)}{252}\right) \\ &= \exp\left(iu\ln(k_{t_0})\right)\exp\left(-r_{t_0}\tfrac{u(n+1)}{252}\right)\textstyle\prod_{j=0}^{n-1}\phi\left(r_{t_{j+1}}-r_{t_j},-\tfrac{u(n-j)}{252}\right).\end{aligned} \tag{13}$$

The characteristic function (5) of the logarithm of the discretely discounted strike price is a function of the characteristic functions of the continuously compounded interest rate increments.

Theorem 2.2. Suppose that the payoff of a derivative product is given by the path-dependent interest rate call options payoff shown in Eq (3). The B_j coefficients of the COS method (see Appendix A) associated to the path-dependent interest rate call option are given by

$$B_0 = \int_a^{\ln y}(y - e^x)dx = y\ln(y) + (-a-1)y + \mathrm{e}^a, \tag{14}$$

and

$$\begin{aligned}B_j &= \int_a^{\ln y}(y-e^x)\cos\left(\tfrac{\pi j(x-a)}{b-a}\right)dx \\ &= \frac{(b-a)^2\left((b-a)y\sin\left(\frac{\pi j\cdot(\ln(y)-a)}{b-a}\right)-\pi jy\cos\left(\frac{\pi j\cdot(\ln(y)-a)}{b-a}\right)+\pi\mathrm{e}^a j\right)}{\pi j\cdot(\pi^2 j^2+(b-a)^2)}.\end{aligned} \tag{15}$$

Note that the B_j coefficients (14) and (15) are given by the Equation (13) using $g(x) = y - e^x$, where

$$x = \ln\left[K\textstyle\prod_{t_j=t_0}^{t_n}\left(1+d_{t_j}\right)^{-\frac{1}{252}}\right] \triangleq \ln(k_{t_n}) \tag{16}$$

stands for the random variable of the model, i.e. the logarithm of the discretely t_n- discounted strike price k. Details about the integration limits *a* and *b* are shown in Appendix A.

3. Results

We apply the theorems developed above to (i) obtain the price of zero-coupon bonds and calculate the yield curve; (ii) calculate the path-dependent option prices via the COS method; (iii) calibrate model parameters through a genetic algorithm.

3.1 Bond Pricing

We apply a modified version of a discrete probability distribution to define the discretely monitored updating of the interest rates.

In our research, we employ a specialized adaptation of a discrete probability distribution to facilitate the discretely monitored updating of interest rates. To elaborate on this distribution, the Skellam distribution characterizes the discrete probability of the difference between two statistically independent random variables, denoted as N_1 and N_2. These variables follow a Poisson distribution, with respective expected values μ_1 and μ_2, and their difference is supported on the set {…,-2,-1,0,1,2,…}. For comprehensive details, Johnson, Kemp, and Kotz's work in 2006 provides an insightful exploration of this distribution.

Our approach extends beyond the standard Skellam distribution. In this study, we leverage the modified Skellam distribution introduced by da Silva et al. (2023) in Theorem 2.1. This adaptation is fundamental to our methodology, particularly in the pricing of bonds and path-dependent options. Notably, da Silva et al. (2023) originally applied the modified Skellam distribution to calculate discrete probabilities of jumps, primarily focusing on simple derivative products traded within the Brazilian stock exchange.

In our context, the significance of employing the modified Skellam distribution lies in its utilization within Equation (17).

$$\phi(u) = e^{-(\mu_1+\mu_2)+\mu_1 e^{\frac{iu}{400}}+\mu_2 e^{-\frac{iu}{400}}}. \tag{17}$$

This equation outlines the characteristic function that underpins our model, specifically designed to accommodate interest rate increments falling within the domain {…,-0.50%,-0.25%,0,0.25%,0.50%,…}, as articulated in Equation (13).

By integrating the modified Skellam distribution as introduced in da Silva et al. (2023), our model aligns with the discrete monitoring requirements necessary for our interest rate updates, ensuring their relevance within the specified domain of {…,-0.50%,-0.25%,0,0.25%,0.50%,…}. This tailored application facilitates our exploration of interest rate dynamics in a manner congruent with our research objectives.

Zero-coupon bond prices which pays $k_{t_n} = 1_{t_n}$ at maturity t_n, are obtained putting u=-i into the characteristic function (5) as follows:

$$\begin{aligned}\phi\left(\ln\left(1_{t_n}\right), u\right) &= \phi(\ln(1), -i) \\ &= \exp\left(-r_{t_0}\frac{(n+1)}{252}\right)\prod_{j=0}^{n-1} \phi\left(r_{t_{j+1}} - r_{t_j}, \frac{-i(j-n)}{252}\right) \\ &= B(t,T)\end{aligned} \tag{18}$$

which are only computed on the days that central banks meet.

In Figure (2) we show a possible term structure of the interest rates. Suppose that μ_1=4 and μ_2=0.5 for the meetings which will occur in 150 and 300 business days, and μ_i=0, for i=1,2, otherwise. Using (18), the bond price for 2 years (504 business days) is given by:

$$\phi\left(\ln\left(1_{\{504\}}\right), u\right) = e^{\left(-\frac{(505)r_t}{252}\right)} e^{-(4+0.5)+4e^{\frac{1}{400}}+0.5e^{-\frac{1}{400}}} e^{-(4+0.5)+4e^{\frac{1}{400}}+0.5e^{-\frac{1}{400}}}. \tag{19}$$

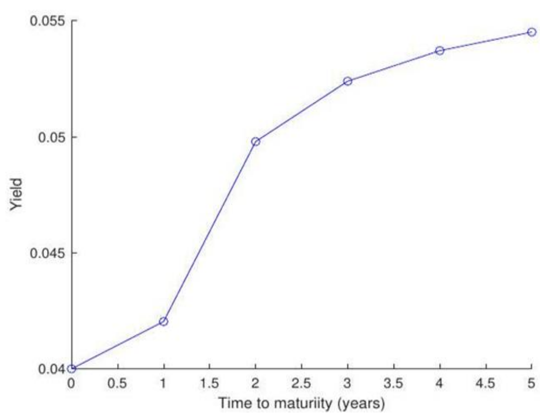


Figure 2. Increasing yield curve

In Figure (3) we suppose that μ_1=4 and μ_2=0.5 for the meeting which will occur in 150 business days and μ_1=0.5 and μ_2=4 for the meeting which will occur 300 business days. Note that, in this case, we have a humped shape as a consequence of the probability of the rates dropping in the long run. Using (18), the bond price for 2 years (504 business days) is given by:

$$\phi\left(\ln\left(1_{\{504\}}\right), u\right) = e^{\left(-\frac{(505)r_t}{252}\right)} e^{-(4+0.5)+4e^{\frac{1}{400}}+0.5e^{-\frac{1}{400}}} e^{-(0.5+4)+0.5e^{\frac{1}{400}}+4e^{-\frac{1}{400}}} \tag{20}$$

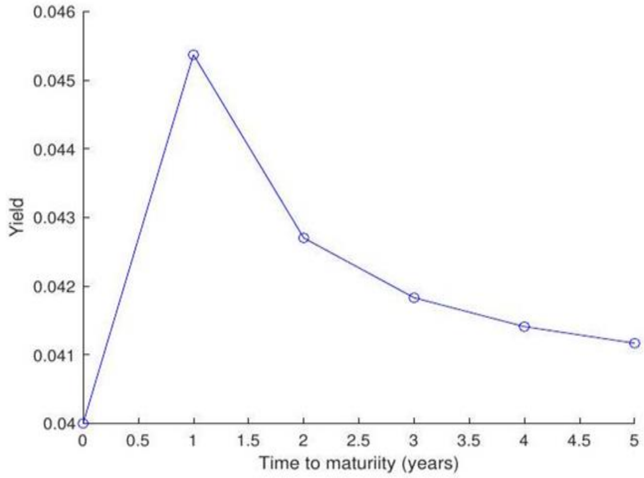


Figure 3. Humped yield curve

3.2 Option Pricing

Assume that $t_n = 54$ days and that there is a central bank meeting on the 45th day. The path-dependent interest rate call option is given by the formula (12), in which the B_j coefficients are given by (14) and (15), and the A_j coefficients (5) depend on the characteristic function (5) given by the Theorem 2.1. Since the interest rate remains constant between two consecutive central bank meetings, and consequently, the characteristic functions of the interest rate increments are equal to 1 between them, the characteristic function (5) depends only on the computation of

$$\phi\left(r_{t_{46}} - r_{t_{45}}, u\right).$$

To summarize, characteristic functions are computed just the day after the scheduled central bank meeting days.

In Figure 4 we depicted the call option prices with $r_{t_0} = 0.04, y_{t_0} = 260000$ and $K = 262000$. We fix the parameters $\mu_2 = 0.5$ and vary μ_1 from 3 to 4 , thus increasing the probability of a large interest rate movement in the next meeting. We note that, as expected, the higher the probability of a large interest rate jump, the higher the call option price. For μ_1=3, the highest probability is concentrated in an interest rate jump of +0.5% in the next meeting, while for μ_1=4 the concentration is in +0.75%.

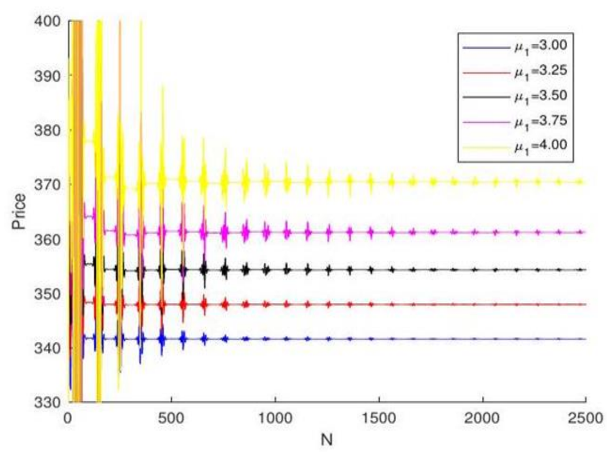


Figure 4. Option prices with increasing μ_1

In contrast to other methods for discretely updating path-dependent option pricing, this is an extremely efficient method. The computer used for all experiments has an Intel Core i5 CPU, 2.53GHz. MATLAB R2022a takes 4.5 seconds to calculate 2500 prices, varying n from 1 to 2500 in the cosine series (12) for each of the five levels of μ_1.

3.3 Implied Volatilities

Implied volatility is a measure of market expectations for future price fluctuations. In the context of interest rate derivatives, implied volatility is particularly crucial for pricing options, such as interest rate caps, floors, and swaptions. Implied volatility reflects the anticipated future volatility of interest rates. It is an essential component of pricing models like the Black-76 formula (Black, 1976), which is used to determine the fair value of options on interest-rate futures and forwards.

The Black-76 formula is a mathematical model that is used to determine the theoretical pricing of European-style options on interest rate futures or forwards. The Black-76 formula, developed as an extension of the original Black-Scholes model for stock options, provides a framework for pricing interest rate options by including implied volatility as a critical input.

The model assumes that the present value of the forward y_{t_n} follows the geometric Brownian motion as below

$$d\bar{y}(t) = \sigma\bar{y}(t)dW(t) \tag{21}$$

where σ is the volatility and W(t) is the standard Wiener processes. The call price formula is given by

$$\begin{gathered} C(t,T) = D(t,T)[y(t)N(d_1) - KN(d_2)] \\ d_1 = \frac{\ln\frac{y(t)}{K} + \frac{\sigma_y^2 T}{2}}{\sigma_y\sqrt{T}}, \\ d_2 = \frac{\ln\frac{y(t)}{K} - \frac{\sigma_y^2 T}{2}}{\sigma_y\sqrt{T}}, \end{gathered} \tag{22}$$

where D(t,T) is the discount factor, y(t) is the spot index and N(·) denotes the cumulative standard normal distribution function.

In practice, the volatility σ is not readily observable but is calculated using the Black-76 method from option prices. To compute the implied volatility that would make the theoretical option price match the observed market price, market players utilize a reverse engineering method, such as the Newton-Raphson method, to determine the implied volatility that minimizes the difference between the model-derived option price and the actual market price.

In Figure 5 we show that the implied volatilities of the path-dependent option with the Skellam distribution model and that calculated via the duly parametrized Vasicek model with κ=1.5, θ=0.05 and σ=3.5% (see Vieira & Pereira, 2000) are similar.

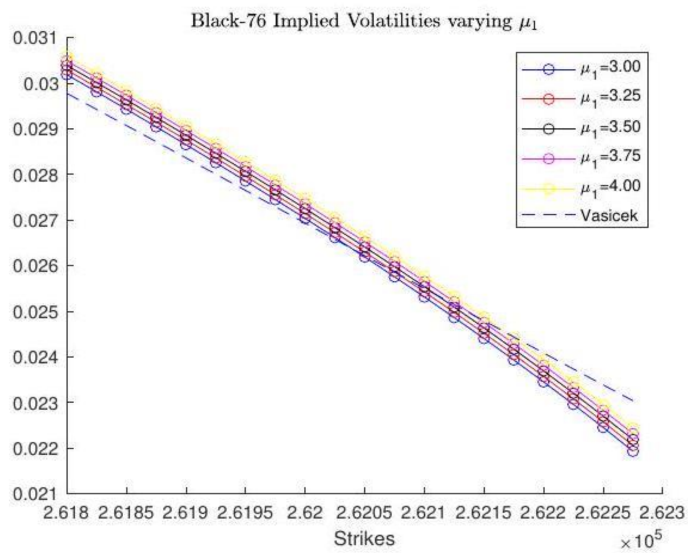


Figure 5. Black-76 implied volatilities

3.4 Calibration

The potent class of optimization and search algorithms known as genetic algorithms (GAs) used in the current work were inspired by the natural selection process. In order to develop optimal or nearly optimal solutions for a variety of complicated problems, GAs work by computationally simulating biological concepts including inheritance, mutation, and selection. The technique has three key principles selection, recombination (crossover), and mutation, which guide the algorithm at its heart. Recombination combines properties of solutions to produce offspring, whereas selection allows potential solutions to be picked based on their fit to a particular environment. Small random variations introduced by mutation allow for the investigation of new search space areas. Numerous industries use genetic algorithms, illustrating their adaptability and efficiency in tackling challenging issues. Among the important fields are artificial intelligence and machine learning, which are used to generate neural network designs, optimize model hyperparameters, and choose features, with Goldberg (1989) being a classic reference on the subject. The efficiency of genetic algorithms in dealing with complicated and expansive search areas, where other techniques may falter due to their exponential nature, is one of their notable properties. They can adjust to shifting environmental conditions, making it possible to keep looking for the best answers even as the situation changes frequently. As shown in Cavuoti et al. (2013), they can be successfully parallelized, enabling the efficient use of computational resources.

This work proposes to use Genetic Algorithms (GA) to determine the parameters μ_1 and μ_2 of the bond price formula (18). The procedure is designed to investigate possible solutions in the space inspired by the natural evolution process. Each member of the GA's population represents a set of factors that influence how effectively the minimization works and the GA operates throughout generations created by these individuals.

By treating the pair of parameters μ_1 and μ_2 as the genes of these individuals, it is possible to create an initial population and use genetic operators to enhance the outcomes. To create new individuals, the recombination of genetic operator crossover and mutation are imposed under a certain probability distribution. We define crossover only over the values of μ_1 for individuals selected to be changed by this function. The value of μ_2 experiences a genetic alteration due to the mutation function. These options seem plausible given that there are only two parameters that represent an individual gene.

Additionally, a selection procedure is required to pick fitter individuals for the following generation, ensuring that the top ones are kept and have a higher chance of reproducing. In this sense, the tournament technique with a size of 5 is employed, forcing individuals to compete against each other based on their m_1 e m_2 evaluated.

The optimization includes a constraint on the parameters, namely $\mu_1 > 0, \mu_2 > 0$.

We consider a zero-coupon bond with a maturity equal to 1 year and a yield of 8%. The central bank meeting is set to take place in 150 business days. In order to determine the potential values of μ_1 and μ_2 that minimize the error between the market price and the bond price formula (18), we ran the experiments with a set of initial conditions given by r_{t_0} ={7.50%,7.75%,8.00%,8.25%,8.50%} and select the best individuals over generations loop. This will allow us to check the variance, namely $\mu_1 + \mu_2$, of the modified discrete Skellam distribution for each of these results and select the pair with the best (minimum) individual variance. The GA settings utilized were as follows: the number of generations n_r=50; population size n_p=3000; probability of crossover of 50%; and 20% of mutation rate. When the error is less than 10^{-6}, the algorithm terminates and the convergence is considered complete.

The results for each initial condition r_{t_0}={7.50%,7.75%,8.00%,8.25%,8.50%} are depicted in Figures 6, 7, 8, 9 and 10, respectively. In the left panels are shown the Genetic Algorithms convergence. The right panels show the resulting probability distribution implied by the calibrated model prices. As expected, for actual interest rates below the bond yield, we have a resulting right-tailed distribution. Conversely, for actual interest rates above the bond yield, we have a resulting left-tailed distribution. For the case where the bond yield is equal to the actual interest rate, we have the highest leptokurtic distribution.

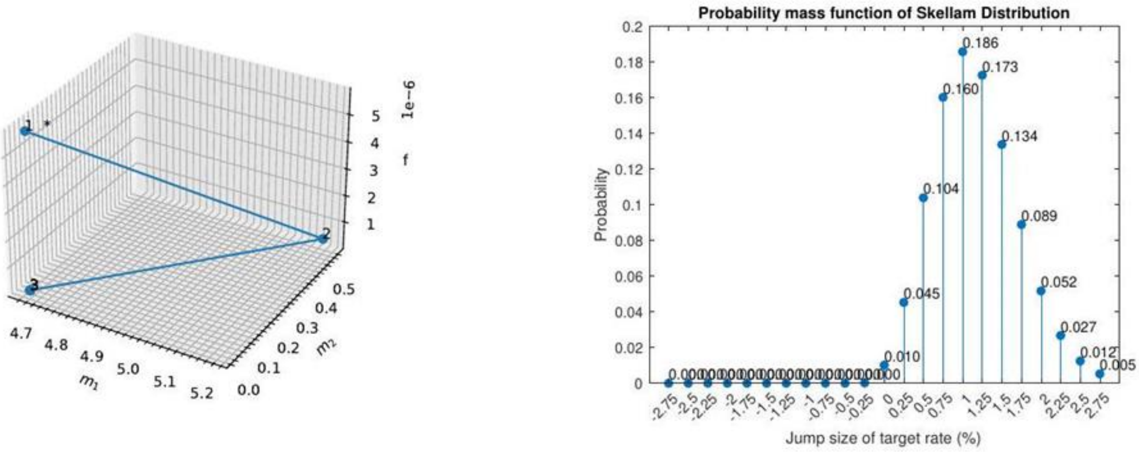


Figure 6. $r_{t_0} = 7.50\%$. (Left) GA convergence (Right) Modified Skellam distribution with $\mu_1 = 4.66$ and $\mu_2 = 0.02$

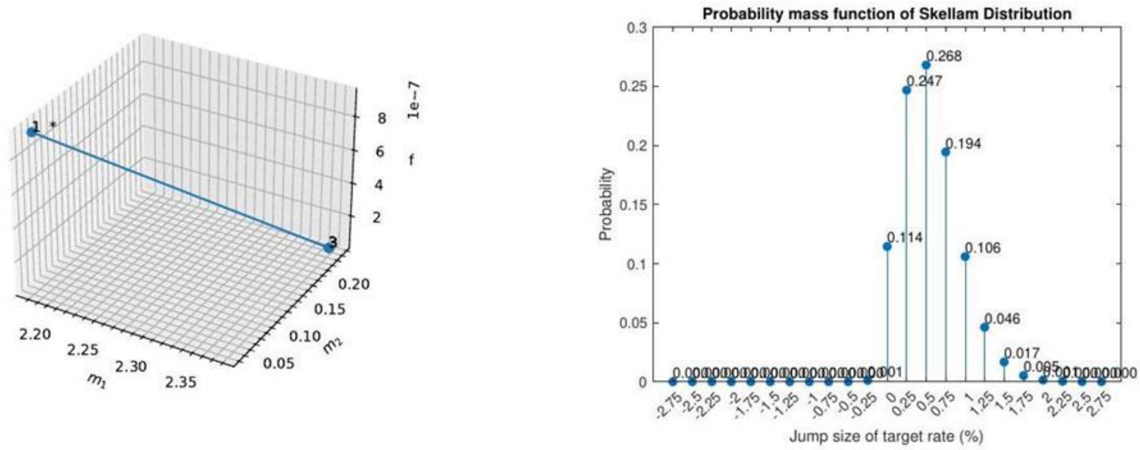


Figure 7. $r_{t_0} = 7.75\%$. (Left) GA convergence (Right) Modified Skellam distribution with $\mu_1 = 2.18$ and $\mu_2 = 0.01$

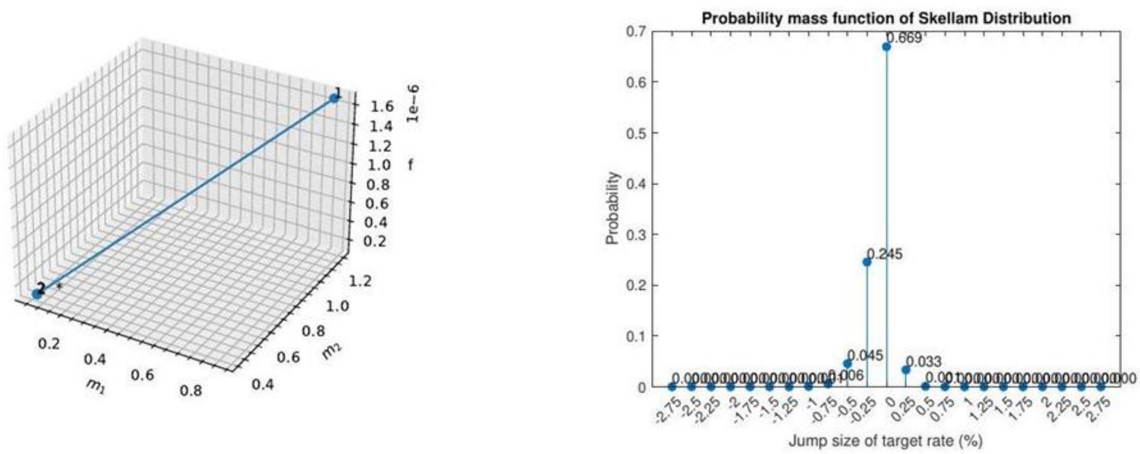


Figure 8. $r_{t_0} = 8.00\%$. (Left) GA convergence (Right) Modified Skellam distribution with $\mu_1 = 0.05$ and $\mu_2 = 0.37$

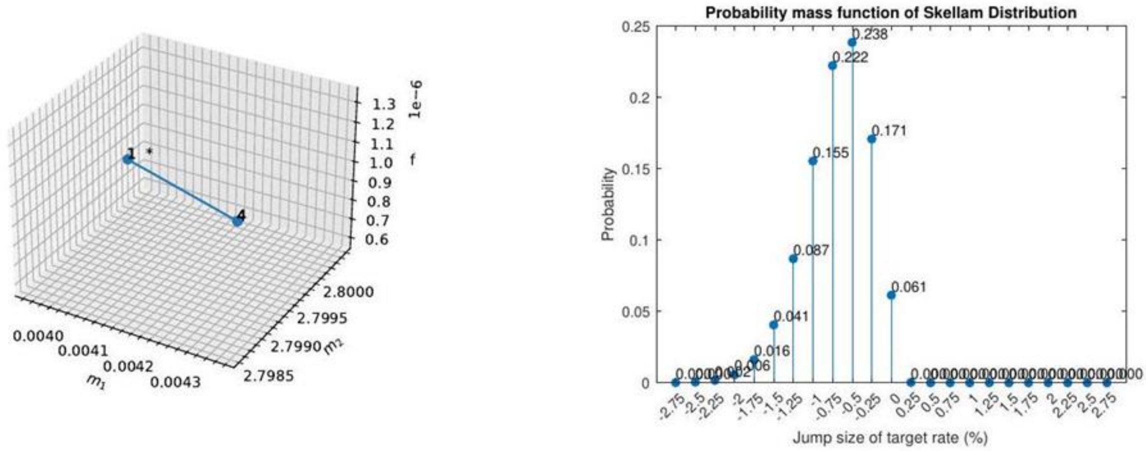


Figure 9. $r_{t_0} = 8.25\%$. (Left) GA convergence (Right) Modified Skellam distribution with $\mu_1 = 0.004$ and $\mu_2 = 2.798$

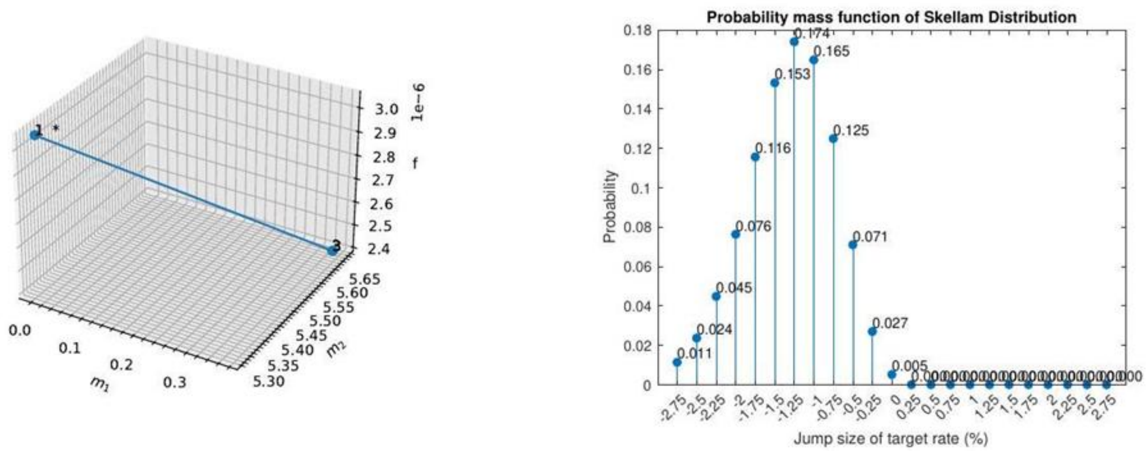


Figure 10. $r_{t_0} = 8.50\%$. (Left) GA convergence (Right) Modified Skellam distribution with $\mu_1 = 0.003$ and $\mu_2 = 5.283$

We highlight that the procedure can be bootstrapped to calibrate the next vertices of the term structure of the interest rates by using the previously calibrated parameters. Numerical challenges appear when there is more than one central bank meeting between vertices. In this case, we have a higher dimension calibration procedure which is multiple of 2 times the number of meetings.

The genetic algorithm calibration technique necessitates a large number of error assessments between the function that generates the bond price and the market price. This guarantees that our model converges in a reasonable amount of time, usually within a few seconds. However, the same cannot be said for examples documented in the literature (da Silva et al., 2016; and Baczynski et al., 2017) in which a considerable number of Partial Differential Equations must be solved to get at each price, resulting in calibration time frames that are practically unacceptably long.

4. Discussion

The implications of the research findings in this study are multifaceted and have significant relevance within the realm of financial mathematics and econophysics. Here, we delve into the implications, comparisons with prior work, limitations, theoretical and practical significance, and highlight avenues for future research.

The outcomes of this research hold great promise for practical applications in the financial industry. The introduced efficient pricing methodology, specifically tailored for discretely updated path-dependent interest rate options, opens doors to more precise and timely pricing of complex financial/fixed-income instruments. These instruments are pivotal in managing interest rate risk, optimizing investment portfolios and enhancing financial decision-making. The ability to accurately compute implied volatilities is particularly significant, as it empowers market participants to make informed decisions in a dynamic financial landscape.

This research distinguishes itself from earlier methods by shifting its focus to discrete path-dependent interest rate products. Unlike previous models that predominantly relied on continuous-time models and partial differential equations, the approach presented here aligns more closely with the realities of financial markets. By modeling interest rate jumps during scheduled central bank meetings discretely, it offers a more realistic representation of market dynamics, thereby contributing to the refinement of pricing methodologies.

While this study is a leap forward in interest rate option pricing, it is vital to acknowledge its limitations. The modified Skellam distribution employed in this research assumes non-time-dependent parameters. While the approach used here is highly effective, it's important to consider the inherent characteristics of real-world interest rate movements. In reality, the probabilities associated with interest rate changes may exhibit a decreasing variance as time progresses, eventually converging to a limited set of possibilities as the time to a jump approaches zero. This distinction emphasizes the need for further exploration and modeling to better align with the nuanced behavior of interest rates in the temporal dimension. Future research might delve into time-dependent parameters, providing a more accurate representation of the dynamic nature of interest rate movements and their associated probabilities.

This research carries both theoretical and practical significance. The introduction of a novel approach to pricing path-dependent interest rate options enriches the domain of financial mathematics and econophysics. The practical applications of the efficient pricing methodology and the precise calculation of implied volatilities can contribute to better risk management, investment strategies, and financial decision-making in the real world.

The study has identified a notable gap in the existing literature, pertaining to the comprehensive examination of how discrete interest rate jumps affect bond and option prices. Future research endeavors can build upon this foundation, potentially incorporating more complex models that account for additional factors influencing interest rate movements. This avenue of research has the potential to refine our understanding of interest rate derivatives and their pricing methodologies.

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Appendix A

The COS method

Let $f: [0,\pi] \longrightarrow \mathbb{R}$ be an integrable function. Then, the Fourier-cosine series of f is given by

$$f(\xi) = \frac{a_0}{2} + \sum_{j=1}^{\infty} a_j \cos(j\xi),\ \xi \in [0,\pi] \tag{A1}$$

where

$$a_j = \frac{2}{\pi}\int_0^{\pi} f(\xi) cos(j\xi) d\xi,\ j = 0,1,2,\ldots \tag{A2}$$

For functions supported in any arbitrary interval [a,b], a change of variable ξ=π (x-a)/(b-a) is considered.

Fang and Oosterlee (2008) suggested the following alternative for the integration limits [a,b] in order for the approximation to be considered good:

$$a = c_1 - L\sqrt{c_2 + \sqrt{c_4}}\ b = c_1 + L\sqrt{c_2 + \sqrt{c_4}} \tag{A3}$$

with L=10. The coefficients c_k are the k-th cumulant of x given by

$$c_k = \frac{1}{i^k}\frac{d^k}{du^k}h(u)\Big|_{u=0} \tag{A4}$$

where the cumulant generating function is given by

$$h(u) = \ln \mathbb{E}[e^{iuX}]. \tag{A5}$$

Remind that the domain of f, typically, is not [a,b]. This interval is chosen in order to capture as much probability as possible from f.

Then, the Fourier-cosine series expansion of f in the interval [a,b] is

$$f(x) = \frac{a_0}{2} + \sum_{j=1}^{\infty} a_j \cos\left(j\pi\frac{x-a}{b-a}\right) \tag{A6}$$

where

$$a_j = \frac{2}{b-a}\int_a^b f(x)\cos\left(j\pi\frac{x-a}{b-a}\right)dx,\ j = 0,1,2,\ldots \tag{A7}$$

Let us assume that $f \in L^1(\mathbb{R})$. By the Euler's identity, the coefficients of the Fourier-cosine expansion of f are

$$\begin{aligned} a_j &= \frac{2}{b-a}\int_a^b f(x)\Re\left[\exp\left(ij\pi\frac{x-a}{b-a}\right)\right]dx \\ &= \frac{2}{b-a}\Re\left(\exp\left(-ij\pi\frac{a}{b-a}\right)\int_a^b f(x)\exp\left(ij\pi\frac{x}{b-a}\right)dx\right). \end{aligned} \quad \text{(A8)}$$

Let X be a continuous random variable. If f, with domain in $\mathbb{R}$, is a probability density function of X, then

$$a_j \approx \frac{2}{b-a}\Re\left(\exp\left(-ij\pi\frac{a}{b-a}\right)\hat{f}\left(\frac{j\pi}{b-a}\right)\right) \triangleq A_j \quad \text{(A9)}$$

where $\hat{f}$ is the characteristic function of X, that is

$$\hat{f}(u) = \int_{\mathbb{R}} \exp(ixu)f(x)dx \quad \text{(A10)}$$

which approximates

$$\int_a^b \exp(ixu)f(x)dx \quad \text{(A11)}$$

Therefore, the approximation of f is given by the following Fourier-cosine series

$$f(x) \approx \frac{A_0}{2} + \sum_{j=1}^n A_j\cos\left(j\pi\frac{x-a}{b-a}\right),\ x \in [a,b] \quad \text{(A12)}$$

for a given n.

Let the continuous real-valued random variable Z(t,T) be a function of the underlying source of risk - the interest rate process $\{r(s), s \in [t,T]\}$ - experienced by a European call option maturing at time T. So, we may write Z(t,T) ≡ Z(t, $\{r(s), s \in [t,T]\}$). Let $f(\cdot \mid r(t))$ be the risk-neutral probability density function of Z(t,T) conditional to r(t) and g(Z(t,T)) be the discounted payoff function of the option. Then the price of this option at time t is

$$\begin{aligned} C(t,T) &= \mathbb{E}[g(Z(t,T)) \mid r(t)] \\ &= \int_{\mathbb{R}} g(w)f(w \mid r(t))dw \end{aligned} \quad \text{(A13)}$$

where E is the risk-neutral expected value. Truncating f in the interval [a,b] we have:

$$C(t,T) \approx \int_a^b g(w)f(w \mid r(t))dw \quad \text{(A14)}$$

Using f(x) as in (A12) we have

$$C(t,T) \approx \frac{A_0}{2}\int_a^b g(x)dx + \sum_{j=1}^n A_j\int_a^b g(x)\cos\left(j\pi\frac{x-a}{b-a}\right)dx \quad \text{(A15)}$$

Hence, the series approximation of the option price is given by

$$C(t,T) \approx \frac{A_0 B_0}{2} + \sum_{j=1}^n A_j B_j \quad \text{(A16)}$$

where the A_k coefficients are given by (A9) and

$$B_j = \int_a^b g(x)\cos\left(j\pi\frac{x-a}{b-a}\right)dx, \text{ for } j = 0,1,\dots,n. \quad \text{(A17)}$$

In order to employ the COS method to calculate the price of the path-dependent interest rate call option, it is necessary to find the B_j coefficients (A17) associated with the payoff function of the contract.

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