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Systematic Risk and Corporate Business Performance

Marco A. Paganini[1]

[1] Managing Director, TRU Consulting, Milan, Italy

Correspondence: Marco A. Paganini, TRU Consulting, Via Ippolito Rosellini, 8, 20124, Milan, Italy. Tel: 39-02-36518-169. E-mail: mp@truconsulting.it

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Abstract

The paper investigates the link between systematic risk and corporate business performance, represented mainly by the degree of operative and financial leverage. Although theoretical contributions link the value of the common stock to corporate performance, CAPM does not identify a satisfactory relation between the latter and β, setting aside the relation to the corporate capital structure. A detailed analysis of CAPM highlights two relevant anomalies: short sales and R^2 low values explaining the fundamental relation between stock and stock market excess return. Using an alternative approach, we highlight how CAPM, on one side, can be an incomplete theory to explain the stock returns and, on the other side, that the portfolio risk could be equivalent to the underlying corporate businesses portfolio, filtered by the feedback effect of the stock market. The empirical evidence descending from the analysis of several portfolios with an increasing number of stocks belonging to the S&P 500 Index reveals that the optimisation process leads to progressively higher β paired with a simultaneous R^2 deterioration; furthermore, β appears subject to sudden oscillations. Overall, β does not adequately represent the relation between stock risk and return. The integration of the joint performance of the stock market and corporate business in an MLR relation leads to a clear improvement in R^2 thanks to the surfacing of the correlation between these two explanatory variables, a condition entirely ignored by CAPM.

Keywords: CAPM, corporate business, DOL, DFL, efficient frontier, performance, short sales, systematic risk

1. Introduction

The present paper investigates the relation between systematic risk and its underlying determinants. Systematic or non-diversifiable risk derives from CAPM theory, developed mainly by Sharpe (1964) and Lintner (1965a). CAPM identifies a primary relation between the stock excess return, compared to the risk-free asset return, and the stock market excess return through a variable named β, specific to the common stock, which measures its risk compared to the market portfolio, even if at the outset such a risk was related only to the portfolio to be optimised.

From the ensuing CAPM developments, numerous studies have examined the empirical evidence, the theoretical implications, and multiple practical applications related to the cost of equity and the impact of the capital structure on systematic risk and capital budgeting, to list a few.

CAPM is undoubtedly the most famous and used corporate financial theory, but it has always generated heated debate between its proponents and detractors. It gave rise to doubts about its validity as the empirical evidence did not fully support its theoretical conclusions (Jensen, 1972). Other doubts arise from the absence or tenuousness of the link between β and its underlying determinants, primarily related to corporate business since the relation with the capital structure is due to Hamada (1972) and Rubinstein (1973). The essay by Mandelker et al. (1984) represents the only significant exception. The present paper builds on this essay and attempts to verify the relation between the degree of operative and financial leverage on one side and the systematic risk and stock return on the other.

The paper presents three main Sections, each divided into several subsections. Section 2 defines the degree of operative and financial leverage and recalls their importance as measures of corporate performance. The contribution by Mandelker et al. (1984) is analysed, together with the approach by Miller et al. (1961), in determining the relation between corporate performance and stock values and returns.

In Section 3, referring to the essays by Lintner (1965a) and Merton (1972), we recap the concepts of efficient frontier and portfolio optimisation. We will subsequently analyse two of CAPM's questionable issues: the role of short sales and the inability to explain a significant fraction of the stock return variability. Such topics offer the opportunity to propose an alternative approach to explain stock returns determined by the stock market's and corporate business's joint performance. The portfolio risk originates from the mutual combination of the underlying corporate businesses, filtered by the stock market feedback effect, highlighting the correlation between these two explanatory variables, a condition entirely ignored by CAPM. From this perspective, CAPM could prove to be an incomplete theory of stock returns, considering that returns are assumed to be exogenous data.

Section 4 analyses CAPM's asset allocation and security market line using 100 stocks of the S&P 500 Index in the 1991-2020 timespan without using homogeneous stock portfolios. Subsequently, we shall conduct an integration test of the joint performance of the stock market and the corporate business to evaluate the goodness-in-fit of an MLR relation in explaining the stock return variability.

Conclusions follow in Section 5.

2. Mandelker–Ghon Rhee and Modigliani-Miller Contributions

2.1 The Relation between the DOL-DFL Nexus and Corporate Performance

What role do DOL and DFL play in explaining corporate performance? If we define the degree of operating and financial leverage, namely DOL and DFL, in the following way:

$$DOL_t = \frac{\Delta\%EBIT_t}{\Delta\%S_t} \tag{1}$$

$$DFL_t = \frac{\Delta\%\pi_t}{\Delta\%EBIT_t} \tag{2}$$

$$\Delta\%\pi_t = \frac{\pi_t}{\pi_{t\text{-}1}} - 1 = DOL_t * DFL_t * \Delta\%S_t = DTL_t * \Delta\%S_t \tag{3}$$

where π states the corporate net profit, $\Delta\%S_t$ and $\Delta\%EBIT_t$ represent the percentage change of Revenue and EBIT between two consecutive periods.

For more details on the definitions, determinants, and impact of DOL and DFL, please refer in full to the essays by Paganini (2019, 2021).

2.2 The Mandelker-Ghon Rhee Equation

The essay by Mandelker et al. (1984) represents a starting point for deriving a link between the risky asset β and the underlying corporate business. Starting from the β classic definition as the ratio over a given timespan between the covariance $Cov(R_i, R_P)$ of common stock and portfolio returns on one side and the portfolio variance σ_P^2 on the other, through a series of algebraic steps, the authors determine its equivalence with two measures of corporate performance, DOL and DFL and an intrinsic β; the equation is the following:

$$\beta_i = \frac{Cov(R_i, R_P)}{\sigma_P^2} = DOL * DFL * \beta_i^o \tag{4}$$

where β_i^o definition is the following:

$$\beta_i^o = \frac{Cov\left(\frac{\pi_{it\text{-}1}}{S_{it\text{-}1}} * \frac{S_{it}}{E_{it\text{-}1}}, R_P\right)}{\sigma_P^2} \tag{5}$$

where:

$\frac{\pi_{it\text{-}1}}{S_{it\text{-}1}}$ = net profit margin of the period *t-1;*

$\frac{S_{it}}{E_{it\text{-}1}}$ = equity turnover for the period from *t-1* to *t.*

β_i^o measures the intrinsic business risk of common stocks, magnified by the appropriate DOL and DFL based on management decisions to impact systematic risk β_i. Equation (4) is an alternative risk decomposition to the Hamada (1972) and Rubinstein (1973) relations.

The appeal of their paper lies in the emphasis placed on the joint role played by DOL and DFL on the systematic risk measured by β. The income statement and the capital structure impact systematic risk, and equation (4) provides the transmission mechanism. By studying the empirical evidence, their paper suggests that DOL and DFL influence a relevant share of β change. It also has two distinctive features:

1) They do not consider DOL and DFL as random variables.

2) Consequently, they conceive β in a limited period.

Suppose we drop hypothesis 2 and calculate β over a long timespan: hypothesis 1, that DOL and DFL are not random variables, would fail, with the unfortunate consequence that equation (4) would cease to have cogent validity. Mandelker et al. consider β a measure of systematic risk limited to a narrow, if not instantaneous, period, as Black (1972).

Undoubtedly, the systematic risk formalized by β is relative to a limited period of three to five years, though not necessarily short. Thinking that β is constant outside the timespan from *t-n* to *t* is equivalent to maintaining that the market trend is perfectly cyclical; in such a hypothesis, the *t-n* return would be replaced by an identical *t+1* return, leaving the ratio between covariance and variance unchanged over time in this and each subsequent period of the rolling timespan. Since the financial market is not cyclical, β is bound to change over time: even when we divide the timespan into two or more periods, it changes. Consequently, remaining within the timespan from *t-n* to *t*, we can cast doubt on whether to consider β a constant measure of systematic risk and their determinants DOL, DFL, and β_i^o as well.

Let us consider the return R_i of the i^{th} common stock; based on the classic definition of its return, we can compute it as the asset appreciation increased by the dividends paid on the common stock compared to its value at the beginning of the timespan.

The equation is the following:

$$R_i = \frac{V_t + D_t + V_{t-1}}{V_{t-1}} = \left(\frac{V_t}{V_{t-1}} - 1\right) + \frac{D_t}{V_{t-1}} = cg_t + d_t \tag{6}$$

from which we obtain a breakdown of the common stock return into a capital gain rate and a dividend rate. We report this distinction in the systematic risk definition:

$$\beta_i = \frac{Cov(R_i, R_P)}{\sigma_P^2} = \frac{Cov(cg_t + d_t, R_P)}{\sigma_P^2} = \frac{Cov(cg_t, R_P)}{\sigma_P^2} + \frac{Cov(d_t, R_P)}{\sigma_P^2} = \beta_{cg+}\beta_d \tag{7}$$

From the classic definition of the common stock β, we have obtained its decomposition into a β ascribable to the capital gain rate and the dividend rate of the asset. If the covariance of the dividend rate to the portfolio return were relatively low or zero, the common stock β would be mainly, if not entirely, ascribable to the capital gain rate. In this way, the linkage between corporate performance and systematic risk disappears. What initially looked promising, on a closer investigation, turns out to be disappointing. Assuming the payout ratio δ_t, the link between corporate performance and systematic risk could take the following form:

$$R_i = cg_t + d_t = cg_t + \frac{\delta_t * \pi_{t-1} * (1 + DOL_t * DOL_t * \Delta\%S_t)}{V_{t-1}} \tag{6 bis}$$

by using equation (3) in the following form:

$$\pi_t = \pi_{t-1} * (1 + DOL_t * DOL_t * \Delta\%S_t) \tag{3 bis}$$

If the correlation and covariance between the dividend rate and the portfolio return are very low or negligible, the explicit link between corporate performance and systematic risk is severed. Consequently, the linkage between DOL, DFL and β, reflected in the capital gain rate, becomes invisible in the hypothesis that such a link exists.

2.3 The Modigliani-Miller Equation

Fortunately, the essay by Miller et al. (1961), even if dated, can help us to untangle what has arisen around the risky asset return, apparently independent of corporate performance. Equation [5] of such a paper allows us to determine the stock value at time *t* based on the net profit π_t and the investment needs I_t to increase the corporate physical capital. From this relation, we obtain the following equation:

$$R_i = \left(\frac{V_t + \pi_t - I_t}{V_{t-1}} - 1\right) = \left(\frac{V_t}{V_{t-1}} - 1\right) + \left(\frac{\pi_t - I_t}{V_{t-1}}\right) = cg_t + cf_t \tag{6 ter}$$

since the difference between the net profit and the asset increase can be considered an approximate measure of the corporate cash flow cf_t, before any intervention in the capital structure.

From this relation, Miller et al. (1961) derive the well-known theorem that the stock value is a function of its current net profit π_t, its growth rate *k*, the rate of return *r* and the market rate *R*:

$$V_t = \frac{\pi_t * (1-k)}{R - k * r} \tag{8}$$

This conclusion is adopted in a different form by Fama et al. (2015) in their paper "Five-factor asset pricing model".

The other conclusion Miller et al. (1961) reached is the role of dividends: the dividend is considered a financial illusion. Therefore, the value of the firm and its common stock are determined exclusively by business considerations and not by the method of packaging and distributing the fruits deriving from the income capacity. What matters is the income capacity of corporate assets and its investment policy. Therefore, these factors are reflected in the stock return, even if not easily visible.

At this point, it is evident that the corporate value is determined, in summary, by extracting net profit from the current and future assets, the latter determined by the investment policy. We can identify these factors in equation (3 bis), which describes the corporate economic and financial dynamics. Such dynamics depend on the initial situation, represented by the net profit $\pi_{t\text{-}1}$ in the previous period *t-1*, and on how the business evolves in period *t*, based on the sales growth, DOL and DFL. The Modigliani-Miller equation (8) provides the fundamental insight that stock return and net profit are interdependent.

It would be very intriguing and powerful to link corporate performance to systematic risk analytically, but the characteristics of the equations examined up to now do not allow for an explicit, simple, and linear relation among stock return, systematic risk, and corporate performance.

2.4 The Dilution

Another element that plays a crucial role in determining the profitability of a common stock, which influences EPS, is the number of shares outstanding or dilution. This element operates exclusively at the price and return level of the common stock. It is possible that when the corporate daily management fails to produce an EPS level considered satisfactory, the Board of Directors could use the dilution to achieve the EPS target, provided the financial resources are available without any legal obstacle to implementing such a program. After all, a temporary reduction in the outstanding shares could also be a good business for the firm.

Moving from net profit to EPS, equation (3 bis) changes to the following form to consider the dilution:

$$EPS_t = \frac{EPS_{t\text{-}1} * (1 + DOL_t * DOL_t * \Delta\%S_t)}{1 + \Delta\%N_t} \tag{3 ter}$$

where $\Delta\%N_t$ is the percentage change in the shares outstanding between periods *t-1* and *t*.

A decrease in dilution leads to an increase in EPS, other conditions being equal; the reduction of the outstanding shares certainly impacts DFL, partially offsetting the dilution effect and making the equity contraction perhaps less favourable.

3. Critical Review of CAPM

3.1 CAPM Based on Lintner and Merton Contributions

The two seminal papers by Sharpe (1964) and Lintner (1965a) laid the foundation for the CAPM. In particular, the latter allows us to systematize simultaneously and analytically the following topics:

1) the role of short sales;
2) the role of risk-free assets;
3) the optimal mix of investments in risky assets, with and without short sales;
4) the portfolio risk;
5) the risky asset contribution to the portfolio risk.

The essay is much richer in additional information than those mentioned above, such as the market price implications of portfolio optimisation, corporate capital budgeting, and corporate project portfolio optimisation.

The optimal portfolio investment mix in risky assets is determined, in the mean-standard deviation plane, through a tangent line to the efficient frontier of the portfolio of risky assets and intercept on the ordinates equal to the return of the risk-free asset $\overline{R_F}$. The procedure followed by Lintner maximizes the slope θ of the market opportunity line by setting its partial derivative to the weight assumed by the i^{th} risky asset equal to zero. We do not know whether Lintner realized that the efficient frontier of the portfolio of risky assets in the mean-standard deviation plane was a shifted hyperbola: Lintner (1965b) stated such a frontier as an envelope. The analytical determination of this frontier was developed by Merton (1972) and published about seven years after Lintner's paper. The critical difference between Lintner's and Merton's approach concerns the valuation of short sales: for Lintner, going short on a risky asset is an investment like going long, while for Merton, a short position needs offsetting with more long positions. In fact, for Lintner, the sum of the percentage weights of investments in individual risky assets is equal to 1 only if assumed in absolute value, as in equation (9), while it is always equal to 1 for Merton, as in equation (10):

$$\sum_{i=1}^{m} |{}_{L}w_i| = 1 \tag{9}$$

$$\sum_{i=1}^{m} {}_{M}w_i = 1 \tag{10}$$

The immediate consequence of such a difference could involve a translation of Lintner's conic section compared to Merton's towards the southwest along the market opportunity line with slope θ and intercept $\overline{R_F}$. The translation along the abscissa is equal to:

$$\Delta x = {}_{M}x_{PT} * \left(\sum_{i=1}^{m} {}_{L}w_i - 1 \right) \tag{11}$$

where:

${}_{M}x_{PT}$ = point of tangency of the market opportunity line with Merton's conic section;

${}_{L}w_i$ = Lintner's weight in the i^{th} risky asset.

With short sales, the sum of Lintner's weights ${}_{L}w_i$ is always less than one, and consequently, the translation Δx on the abscissa of Lintner's point of tangency is negative. The market opportunity line, with intercept $\overline{R_F}$ and slope θ, will have the following equation:

$$R_P = \overline{R_F} + \theta * \sigma_P \tag{12}$$

where the slope θ is:

$$\theta = \sqrt{B + \overline{R_F} * (C * \overline{R_F} - 2A)} \tag{13}$$

while A, B and C are the parameters of the Merton conic:

$$A = \mathbf{1}^{\mathrm{T}} \boldsymbol{\Omega}^{-1} \boldsymbol{R} \tag{14}$$

$$B = \boldsymbol{R}^{\mathrm{T}} \boldsymbol{\Omega}^{-1} \boldsymbol{R} \tag{15}$$

$$C = \mathbf{1}^{\mathrm{T}} \boldsymbol{\Omega}^{-1} \mathbf{1} \tag{16}$$

where:

$\boldsymbol{R}$ = column vector of m risky asset returns

$\boldsymbol{\Omega}^{-1}$ = inverse of the covariance matrix of m risky asset returns

$\mathbf{1}$ = unit column vector of size m.

Matrices and vectors are in boldface, while the superscript T indicates the transposed matrix/vector.

Merton's conic section linking the return R_P to the standard deviation σ_P of the portfolio of m risky assets in the mean-standard deviation plane is the following:

$$R_P = \overline{R_P} \pm \sqrt{\frac{D}{C}\left(\sigma_P^2 - \overline{V_P}\right)} \tag{17}$$

where:

$\overline{R_P}$ = minimum portfolio return, equal to the *A/C* ratio (minimum of the efficient frontier)

$\overline{V_P}$ = minimum portfolio variance, equal to *1/C*

$D = BC - A^2$

The equation (17), with only the positive radical, represents the portfolio efficient frontier.

We can locate the point of tangency of Merton's conic section with the market opportunity line as a simple geometric problem of a line passing through $\overline{R_F}$ tangent to a given conic. In contrast, the conic descends from an optimisation procedure to minimize the portfolio variance as a measure of risk. The point of tangency ${}_{M}x_{PT}$ of Merton's conic with the market opportunity line is equal to the standard deviation resulting from the following equation:

$${}_{M}x_{PT} = \frac{\theta * (C * \overline{R_F} - A)}{D - C * \theta^2} \tag{18}$$

The points of tangency of Merton's and Lintner's conics overlap if there are no short sales; they differ in the opposite case, but both are tangent to the same straight line. Such conics are hyperbolas with different geometric centres but the same shape. The matrix representation of conic sections is the reference for more details.

The geometric translation of Lintner's hyperbola is due to the peculiar assessment adopted with equation (9) for short positions. This difference has no significant impact if we can borrow or lend at the same interest rate equal to $\overline{R_F}$. The investor can choose the final mix of his portfolio between risky and risk-free assets, determining his position on the market opportunity line based on personal preferences or utility curves. Since Lintner's optimal portfolio positions are southwest of Merton's, the former portfolio may generate more debt than the latter, notwithstanding its overinvestment required by the presence of short positions. Lintner's assessment of short sales seems orthodox from an economic perspective, and it should be more advisable than the Merton solution, which is algebraically simpler due to relation (10).

If we need to get Merton's weights directly (Nocedal et al., 1999), we must set the following system of equations:

$$\begin{bmatrix} \boldsymbol{\Omega} & -\boldsymbol{A}^T \\ \boldsymbol{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} {}_M\boldsymbol{W} \\ \boldsymbol{\lambda_m} \end{bmatrix} = \boldsymbol{K} \begin{bmatrix} {}_M\boldsymbol{W} \\ \boldsymbol{\lambda_v} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z} \\ R_P \\ 1 \end{bmatrix} \tag{19}$$

where:

$\boldsymbol{\Omega}$ = *m x m* covariance matrix of risky asset returns

$\boldsymbol{A}$ = *2 x m* Jacobian matrix of constraints (risky asset returns and unit weights)

${}_M\boldsymbol{W}$ = column vector of *m* Merton's weights

$\boldsymbol{\lambda_v}$ = column vector of the 2 Lagrangians

$\boldsymbol{Z}$ = zero column vector of *m* first-order necessary condition

R_P = scalar of the target portfolio return

1 = scalar of the sum of the stock weights in the portfolio

From this system of equations, we can obtain Merton's weights ${}_M\boldsymbol{W}$, and the Lagrangians $\boldsymbol{\lambda_v}$ by solving:

$$\boldsymbol{K}^{-1} \begin{bmatrix} \boldsymbol{Z} \\ R_P \\ 1 \end{bmatrix} = \begin{bmatrix} {}_M\boldsymbol{W} \\ \boldsymbol{\lambda_v} \end{bmatrix} \tag{19 bis}$$

Using equations (12), (13), and (18), we obtain the optimal portfolio return R_M, given the risk-free rate $\overline{R_F}$. By entering the R_M value as the R_P target, we get Merton's weights of the optimal portfolio, while by inserting any other figure, we obtain the corresponding mix of the portfolio frontier, efficient or not.

Interestingly, Lintner (1965a) does not use the β definition in his paper, resorting to an alternative measure λ, defined below:

$$\lambda = \frac{\sum_{i=1}^{m} w_i (R_i - \overline{R_F})}{\sum_{i=1}^{m} \sum_{j=1}^{m} w_i w_j Cov(R_i, R_j)} = \frac{R_M - \overline{R_F}}{\sigma_M^2} \tag{20}$$

The subscript M to the portfolio return and variance indicates that it is optimised. We get used to the following formulation:

$$R_i - \overline{R_F} = \beta_i * (R_M - \overline{R_F}) \tag{21}$$

Lintner limits himself to writing the first three members of the following equation that correspond, after a series of transformations, to the fifth member, undoubtedly equivalent to the second member of (21):

$$R_i - \overline{R_F} = z_i \sigma_i^2 + \sum_j z_j \sigma_{ij} = \lambda w_i \sigma_i^2 + \sum_j \lambda w_j \sigma_{ij} = \lambda \left(w_i \sigma_i^2 + \sum_j w_j \sigma_{ij} \right) = \lambda * Cov(R_i, R_P) \tag{21 bis}$$

The relation (21 bis) is the necessary and sufficient condition to obtain the weights w_i that guarantee a single solution at the maximum of θ to w_i. Lintner interprets the riskiness of an asset within a portfolio based on its variance and covariance with all the other stocks in the portfolio, not based on the standard deviation of its returns. Therefore, λ represents the return/risk required by the investor to maintain a position on the i^{th} asset within the portfolio for any stock, given its risk represented by $Cov(R_i, R_M)$; such a risk, therefore, changes according to which stocks are held in the portfolio since it is not an absolute measure of asset riskiness. The required return/risk λ to keep an asset is the same for all stocks in the portfolio but is conceptually different from θ that determines the investment size in the optimal portfolio and risk-free assets.

For reasons we will explain in subsection 4.6, relation (21) is exclusively valid ex-post; therefore, we will never use the expression expected returns.

The CAPM standard formulation is extremely assertive in believing that the return of a risky asset is due to its risk profile measured by β. In fact, from (21), we see that the stock excess return is commensurate with the optimal portfolio excess return, considered equivalent to the market portfolio (Fama et al., 1973), through a specific β. This armoured relation does not leave much room for the fundamental determinants of the common stock risk. It is worth reiterating that (21) is a portfolio equilibrium condition of a non-deterministic nature.

3.2 Flaws in CAPM Theoretical Framework

CAPM standard configuration presents some theoretical inconsistencies. The most significant is the role of short positions within the optimal portfolio. Lintner notices this inconsistency and identifies an alternative solution, constraining the portfolio positions to be exclusively equal to or greater than zero through the KKT condition (Kuhn et al., 2013; Nocedal et al., 1999; Ghojogh et al., 2021). However, such a solution may be suboptimal compared to the optimised portfolio. How do we reconcile the optimised portfolio that needs short sales with the market portfolio that only has long positions? Now, as the number *m* of risky assets in the portfolio increases, two crucial phenomena occur:

1) the short positions progressively increase towards 50%;

2) some stock weights become extreme, both in long and short positions.

Levy et al. (2001) treat such an issue theoretically, concluding that the characteristic that makes an asset *good* in a sizeable portfolio, even with only 100 risky assets, is not quickly evident. The negative weights that generate short sales depend on the values assumed by z_i, that is, the sum of all the stock excess returns multiplied by the corresponding element v_{ij} deriving from the inverse of the covariance matrix:

$$w_i^o = \frac{z_i^o}{\lambda^o} = \frac{\sum_{j=1}^{m}(R_j - \overline{R_F})v_{ij}}{\lambda^o} \tag{22}$$

Given that λ^o is positive and common to all assets, even assuming that all excess returns are positive, a negative value of w_i^o in the optimal portfolio depends on the v_{ij} values in the inverse matrix in correspondence with the asset column considered. If the sum of these values based on the excess returns of all *m* stocks in the portfolio is negative, then we have a short sale. Thus, the specific characteristics of the risky asset do not necessarily determine its positive or negative weight, mainly depending on the property of the inverse matrix. The particular asset combination determines the covariance matrix, and its inverse establishes the weight sign.

Furthermore, we must consider that each row of the covariance matrix is orthonormal to each column of its inverse matrix and vice versa. Given that its product is 1 when *i=j* and 0 when *i≠j*, even starting from a matrix with positive covariances, it is inevitable that many v_{ij} elements outside the main diagonal are negative, determining the almost automatic presence of negative weights originating short sales. As the number of common stocks in the portfolio increases, the appearance of negative v_{ij} is physiological as the number of elements below the main diagonal (while those above the diagonal are the transposition of those below, being the matrix symmetric) is preponderantly compared to those on the diagonal. Such a property seems unrelated to any economic explanation of what stocks are short-sold.

If we wish to go deeper into the topic, it would be necessary to examine the essay by Stevens (1995) that takes its cue from Anderson et al. (1981). All the elements of each specific row of the inverse matrix are the ratio between the same denominator $\sigma_i^2(1 - D_i)$ and a numerator based on $-\beta_{ij}$ when *i≠j* and 1 when *i=j*; through a few algebraic steps, (22) becomes the following:

$$w_i^o = \frac{1}{\lambda^o}\frac{[(R_i - \overline{R_F}) - \sum_{j=2}^{n}(R_j - \overline{R_F})\beta_{ij}]}{\sigma_i^2(1 - D_i)} \tag{22 bis}$$

where:

$D_i = \frac{\sigma_{.i}^{\mathrm{T}}\,\Omega_{n-1}^{-1}\,\sigma_{.1}}{\sigma_i^2} = R^2$ of the multiple regression between the i^{th} asset and the other *n-1* assets

$\boldsymbol{\beta}^{\mathrm{T}} = \boldsymbol{\sigma}_{.i}^{\mathrm{T}}\,\boldsymbol{\Omega}_{n-1}^{-1}$ = vector of multiple regression coefficients β_{ij} of the i^{th} asset to the other *n-1* assets

$\boldsymbol{\Omega}_{n-1}^{-1}$= inverse matrix of the *n-1* assets obtained by discarding the row and column containing the i^{th} asset

$\boldsymbol{\sigma}_{.1}$= column vector of the *n-1* covariances obtained by discarding the variance σ_i^2 of the i^{th} asset

From (22 bis), we see that the optimal weight w_i^o will be positive only when the excess return of the i^{th} asset exceeds the mean of the excess returns of the other *n-1* assets weighted on the specific multiple regression coefficients β_{ij} between the i^{th} asset and the other *n-1* assets:

$$(R_i - \overline{R_F}) > \sum_{j=2}^{n} (R_j - \overline{R_F}) \beta_{ij}$$

The result obtained with (22 bis) allows us to understand the sign of the weight in the optimal portfolio: if the stock excess return is not high enough and is correlated positively with the other *n-1* stocks with high excess returns through a high β_{ij} coefficient, then its weight could be negative. Conversely, a low-excess return asset negatively correlated to the other *n-1* stocks could have a positive weight. Lintner (1965a) also reaches the same conclusion logically. In short, the positive correlation between two common stocks, high excess return and a high β_{ij} coefficient to the i^{th} asset leads to a decrease in the optimal weight w_i^o of the i^{th} stock. If the number of combinations of this kind is high enough, the weight will become negative, and such a change of state occurs faster the lower the excess return of the i^{th} asset is.

Eventually, Levy et al. (2001) observe that the Sharpe ratio tends to halve by banning short sales, implying a high implicit cost for the investor.

The price adjustment process of risky assets is not understandable before a significant discrepancy between the weight assumed in an optimal and the market portfolio, if not a generic down or upward pressure for excess/deficient assets held compared to the optimal portfolio. We will see the implications of asset pricing with empirical evidence.

This perspective leads to a further consideration: Lintner's analysis of the optimal portfolio concerns *m* assets with *m,* which need not necessarily tend to infinity. The number of common stocks does not necessarily have to equal the market portfolio. If we limit ourselves to an analysis of *m* risky assets of which we know the returns, variances and covariances deriving from their time series, we obtain some critical information:

1) the risk-free return $\overline{R_F}$ is a datum of the moment in which we carry out the ex-post analysis of the times series; it is an element not entirely extraneous to the computation of β as long as R_M is the return on the optimised portfolio of *m* risky assets.
2) The β weighted mean of the *m* risky assets always equals 1.
3) The α weighted mean of the *m* risky assets always equals 0.
4) The regression R^2 of each stock against the optimal portfolio build with the same *m* assets is relatively low.
5) The t-stat measurements confirm the null hypothesis for α and the alternative for β.
6) By increasing the number and frequency of the observations, there is no significant improvement in R^2.

We must assess F and t-stat with caution for the reasons stated in subsection 4.3.

Comparing *m* risky asset returns to a market index return, such as the S&P 500 Index, we get the same conclusions as in points 4, 5 and 6 above. Conclusions 1, 2, and 3 are not necessarily valid when the benchmark index is not coming from the optimised portfolio for the reasons we will see in subsection 3.3; for the moment, it is enough to observe that these are pure algebraic consequences of having chosen a regression where the explanatory variable, the return on the optimised portfolio, descends from the variable we would explain. Conclusions from 4 to 6 above rely on the hypothesis that the stock return distribution is normal despite showing "fatter tails".

The present paper fully shares the observations by Roll and Ross, separately and jointly, expressed in their multiple essays about CAPM at the level of individual risky assets, which we can summarize:

1) The linearity relation between return and β holds regardless of the chosen market portfolio or a set of *m* stocks, whether efficient or not (Roll, 1977); the efficiency of the market portfolio and CAPM are equivalent (Ross, 1977).
2) CAPM is not testable without knowing the proper market portfolio mix (Roll, 1977; Roll et al., 1994). Shifting to a market index, we cannot improve its testability;
3) Given the previous points above, the theory is not testable (Roll, 1977; Gibbons et al., 1989) at the risk of turning out to be a tautology;
4) CAPM's ability to explain stock price changes is modest (Roll, 1988).

Roll (1988) argues that the R^2 regression of the monthly returns of single assets, not a homogeneous asset portfolio in terms of risk, to a market index does not deviate much from 0.30. Adding a sector factor, we reach 0.35, thus leaving 65% of the variance of this return completely unexplained.

We recall that R^2 equals:

$$R^2 = \rho^2(R_i R_M) = \frac{[Cov(R_i, R_M)]^2}{\sigma_i^2 \sigma_M^2} = \beta_i^2 \frac{\sigma_M^2}{\sigma_i^2} \tag{23}$$

Consequently, the share of the R_i variance unexplained equals:

$$1 - R^2 = 1 - \beta_i^2 \frac{\sigma_M^2}{\sigma_i^2} = \frac{\sigma_i^2 - \beta_i^2 \sigma_M^2}{\sigma_i^2} \tag{24}$$

while the R_i variance unexplained equals:

$$\sigma_\varepsilon^2 = \sigma_i^2 - \beta_i^2 \sigma_M^2 \tag{25}$$

For a demonstration, see Appendix B.2 by Ciech (2016). The i^{th} stock variance σ_i^2 is due to a component linked to systematic risk and a residual component ε unrelated to the market return. Suppose now that there exists a fictitious variable *X*, uncorrelated to the return of the market portfolio R_M, such that it can explain the residual variance σ_ε^2; we can then write the following relation:

$$\sigma_i^2 = (\beta_i^M \sigma_M)^2 + (\beta_i^X \sigma_X)^2 \tag{25 bis}$$

Since the share of systematic risk is lower than the unexplained one, we can write the following relation:

$$(\beta_i^X \sigma_X)^2 > (\beta_i^M \sigma_M)^2 \tag{26}$$

from which we get the following condition:

$$\rho^2(XR_i) > \rho^2(R_i R_M) \tag{27}$$

It follows that CAPM is unable to explain most of the common stock risk, essentially the correlation between R_i and R_M is not adequate to explain the variability of the former; it can identify the non-diversifiable part of the risk but leaves the diversifiable part unexplained without explaining to what the first risk component is ascribable. This issue has already been addressed by Lintner (1965b) when he deals with the advantage deriving from diversification: in the case in which all the covariances of the assets are zero, all the risk would be non-systematic, and the benefit of diversification would be substantial; in the opposite case, all the residual variances would be zero; consequently, all the asset returns would be perfectly correlated with each other, and diversification would cease to have effects. Portfolio diversification takes advantage of assets correlated negatively with other common stocks and, above all, from residual variances greater than zero with consequent imperfect correlations between assets.

A polynomial may explain a larger share of common stock return variability. The Mandelker-Ghon Rhee and Modigliani-Miller equations, already mentioned, provide clues that a multiple regression equation like this perhaps is needed:

$$R_i = \alpha_i + \beta_i^M * R_M + \beta_i^\psi * \psi_i \tag{28}$$

where ψ_i is a corporate performance measure, β_i^M and β_i^ψ are the stock market and corporate performance coefficients or regressors linking the stock market and corporate performance variables to the common stock return.

CAPM's worth lies in the ability to select an optimal portfolio starting from *m* risky assets and maximizing the utility to risk-averse investors, given their indifference curves and the return of the risk-free asset. In essence, CAPM states the best risky asset portfolio to invest in and how intensively to use it by combining it with risk-free assets.

Again, Lintner (1965 b) is illuminating: "The goal of diversification is not to avoid or minimize risk *per se* but to select the best portfolio, i.e., the best combination of risk and expected return from the portfolio mix".

If we want to have an explanation of the behaviour of the risk/return ratio of single assets, we must look elsewhere.

3.3 An Alternative Approach: A Model Under Certainty Conditions

Let us imagine for simplicity that we have a common stock whose variance is intelligible at 100% using the following equation:

$$R_i = a_i + b_i R_P + c_i \psi_i \tag{29}$$

where the coefficients a_i, b_i, and c_i are not regressors, at least for the time being. The stock market performance R_P as well as the corporate performance ψ_i explain the stock return. The variance of the common stock with these characteristics will be the following:

$$\sigma_i^2 = b_i^2 \sigma_P^2 + c_i^2 \sigma_\psi^2 + 2 b_i c_i Cov(\psi_i, R_P) \tag{30}$$

where:

R_i = i^{th} common stock return

R_P = stock market return

ψ_i = corporate performance related to the i^{th} common stock

σ_x^2 = X variance operator

$Cov(X,Y)$ = X and Y covariance operator

Suppose we have a portfolio made up of *m* risky assets with the same algebraic characteristics as the previous common stock; the mean and the variance of the portfolio return will vary according to the weight assumed by the investment w_i in every single asset of the portfolio:

$$R_P = \sum_{i=1}^{m} w_i R_i = \sum_{i=1}^{m} w_i a_i + \sum_{i=1}^{m} w_i b_i R_P + \sum_{i=1}^{m} w_i c_i \psi_i \tag{31}$$

$$\sigma_P^2 = \sum_{i=1}^{m} w_i b_i \sigma_P^2 + \sum_{i=1}^{m} w_i c_i Cov(\psi_i, R_P) \tag{32}$$

From the previous two equations, we obtain the following:

$$R_P = \frac{\sum_{i=1}^{m} w_i a_i + \sum_{i=1}^{m} w_i c_i \psi_i}{1 - \sum_{i=1}^{m} w_i b_i} \tag{33}$$

$$\sigma_P^2 = \frac{\sum_{i=1}^{m} w_i c_i Cov(\psi_i, R_P)}{1 - \sum_{i=1}^{m} w_i b_i} \tag{34}$$

Suppose, by reducing to absurd, that both ψ_i and $Cov(\psi_i, R_P)$ are null, what would happen to such a system? Equation (32) would become:

$$\sigma_P^2 = \sum_{i=1}^{m} w_i b_i \sigma_P^2 \tag{32 bis}$$

from which we can obtain, dividing the portfolio variance by itself, that:

$$\frac{\sigma_P^2}{\sigma_P^2} = \frac{\sum_{i=1}^{m} w_i b_i \sigma_P^2}{\sigma_P^2} = \sum_{i=1}^{m} w_i b_i = 1 \tag{35}$$

What happens in CAPM doing the same operation? Let us examine the portfolio variance σ_P^2:

$$\sigma_P^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} w_i w_j Cov(R_i, R_j) = \sum_{i=1}^{m} w_i Cov(R_i, R_P) \tag{36}$$

Dividing both members of (36) by the portfolio variance σ_P^2 we obtain:

$$\frac{1}{\sigma_P^2} \sum_{i=1}^{m} w_i Cov(R_i, R_P) = \sum_{i=1}^{m} w_i \frac{Cov(R_i, R_P)}{\sigma_P^2} = \sum_{i=1}^{m} w_i \beta_i = 1 \tag{37}$$

The above condition occurs when the weight of each asset is constant across the timespan analysis, i.e., when R_P is endogenous to the model.

If this occurs, the following condition occurs:

$$\sum_{i=1}^{m} w_i \frac{Cov(R_i, R_P)}{\sigma_P^2} = \frac{Cov(\sum_{i=1}^{m} w_i R_i, R_P)}{\sigma_P^2} = \frac{Cov(R_P, R_P)}{\sigma_P^2} = \frac{\sigma_P^2}{\sigma_P^2} = 1 \tag{37 bis}$$

from which we also get the following:

$$\sum_{i=1}^{m} w_i \alpha_i = \sum_{i=1}^{m} w_i (1-\beta_i)\overline{R_F} = \overline{R_F} \sum_{i=1}^{m} w_i (1-\beta_i) = \overline{R_F}\left(\sum_{i=1}^{m} w_i - \sum_{i=1}^{m} w_i \beta_i\right) = 0 \tag{38}$$

Therefore, in CAPM asset allocation, the weighted means of α and β must necessarily converge towards 0 and 1, respectively, and this occurs only when the search for both the regressors refers to the portfolio of *m* risky assets because the mean portfolio return R_P is endogenous to the model. If, on the other hand, we compute the regression against a market index, there is no guarantee that the weight of each asset is constant over time; on the contrary, precisely the opposite occurs without paying attention to the weight of each stock within the market portfolio. In this context, an average weight for the whole period is meaningless. Consequently, both α and β weighted means may diverge from their theoretical values of 0 and 1. Empirical tests should take it into account. Such a result does not depend on the certainty conditions of the hypothesized system; it is a general conclusion that is also valid for CAPM.

We must go back to equation (35): in the hypothesis that all the weights of each asset of (35) coincide with those of (37), we can conclude that:

$$b_i = \beta_i$$

It follows that assuming that both ψ_i and $Cov(\psi_i, R_P)$ are null, equations (31) and (32) could be compatible with CAPM. However, in such a system, the portfolio return would be infinite as the denominator of (33) would collapse to zero. Indeed, the denominator of (34) would also collapse to zero, like its numerator, leaving the portfolio variance undetermined. In this system, both ψ_i and $Cov(\psi_i, R_P)$ cannot, by definition, be null.

In short, if we ignore both ψ_i and $Cov(\psi_i, R_P)$, we create an incomplete system. Hence, CAPM, interpreted through (21), implicitly assumes no relation between the portfolio return and the corporate performance of the risky assets held in the portfolio, so the covariance between these variables is null. The asset returns, variances and covariances are exogenous market variables. Indeed, the approach that CAPM has been adopting over time would appear rudimentary.

3.4 Further Development Under Certainty Conditions

At this point, we can take a further step forward. If we perfect the equation (34) by inserting the equation (33) in place of R_P in $Cov(\psi_i, R_P)$, we get the following result:

$$\sigma_P^2 = \frac{\sum_{i=1}^{m} w_i c_i Cov(\psi_i, R_P)}{1-\sum_{i=1}^{m} w_i b_i} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} w_i c_i w_j c_j Cov(\psi_i, \psi_j)}{(1-\sum_{i=1}^{m} w_i b_i)^2} \quad (39)$$

In the system we are illustrating, the portfolio variance σ_P^2 is determined by the ratio between:

a. the double summation of the corporate performance covariances weighted both on the weight w_i assumed by each asset in the portfolio and the coefficient c_i which measures the transferability of the corporate performance on the asset return;
b. the square of the difference between 1 and the weighted mean transferability coefficient b_i of the stock market performance on the asset return based on the weight assumed by each asset in the portfolio.

To fully understand the meaning of equation (39), one last algebraic transformation is needed; exchanging the denominator of the second member with the first member, we obtain:

$$(1-\sum_{i=1}^{m} w_i b_i)^2 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} w_i c_i w_j c_j Cov(\psi_i, \psi_j)}{\sigma_P^2} \quad (40)$$

from which we get:

$$\sum_{i=1}^{m} w_i b_i + \left[\frac{\sum_{i=1}^{m}\sum_{j=1}^{m} w_i c_i w_j c_j Cov(\psi_i, \psi_j)}{\sigma_P^2}\right]^{\frac{1}{2}} = \sum_{i=1}^{m} w_i b_i + \frac{\left[\sum_{i=1}^{m}\sum_{j=1}^{m} w_i c_i w_j c_j Cov(\psi_i, \psi_j)\right]^{1/2}}{\sigma_P} = MR + FR = 1 \quad (41)$$

where:

MR = share of portfolio risk arising from the stock market

FR = share of the portfolio risk coming from the joint risk of the corporate businesses

Equation (41) represents the breakdown of portfolio risk in the mean-standard deviation plane between the share ascribable to stock market risk and the residual share from the joint corporate business performance. Therefore, for each portfolio of *m* risky assets, we can decompose its risk into a share relating to the stock market and the residual share deriving from corporate performance, the one not explained by CAPM.

CAPM assumes that risky asset returns are exogenous data, showing which is the optimal way of building the portfolio, but is unable to explain in depth the asset returns due to its essential incompleteness: the absence of a formalized link between corporate performance ψ_i and R_i makes it disputable. Instead, CAPM introduces a feedback effect of the stock market on the common stock returns, which we must carefully evaluate to understand the portfolio risk measured by its standard deviation σ_P, easily derivable from (39):

$$\sigma_P = \frac{\left[\sum_{i=1}^{m}\sum_{j=1}^{m} w_i c_i w_j c_j Cov(\psi_i, \psi_j)\right]^{1/2}}{1-\sum_{i=1}^{m} w_i b_i} = \frac{\left[\sum_{i=1}^{m}\sum_{j=1}^{m} w_i c_i w_j c_j Cov(\psi_i, \psi_j)\right]^{1/2}}{1-MR} \quad (39\ bis)$$

The feedback effect comes into play with the denominator of (39 bis), exactly as in a closed-loop system whose operation we will mention in subsection 3.5. We will see later what kind of operation we can obtain with such a denominator; for the time being, we observe that the closer the risk share of the stock market $MR = \sum_{i=1}^{m} w_i b_i$ approaches 1, the greater the system instability will be. The meaning of the numerator of (39 bis) is simply the square root of the risk deriving from the covariance matrix of corporate performance multiplied by the column vector obtained with the product of the weight w_i assumed by the i^{th} asset in the portfolio by the transferability coefficient c_i of the corporate performance, multiplied again by the transposition of the same column vector.

The stock portfolio risk derives essentially from the joint corporate portfolio businesses, suitably filtered by the feedback effect of the stock market. The effort, which is not entirely trivial, will be to search for the transfer function of corporate performance on the common stock returns, while CAPM provides a bright explanation of the feedback transfer function. Another issue is that the equation system (29), (30), (33) and (39) allows multiple solutions.

It is necessary to quantify the corporate performance ψ_i and the parameter c_i that allows the corporate performance transferability on the stock return to understand the transfer function. All this for every single asset in the portfolio, a gigantic task since it is not predictable *a priori* which process conveys the corporate performance ψ_i nor the transferability parameter c_i.

3.5 Closed-Loop System

In subsection 3.4, we realized a topic underestimated in CAPM, if not completely ignored: the impact each stock in the portfolio has on the portfolio itself and the feedback effect of the latter on any asset in the portfolio. In CAPM R_p and R_i are independent, thanks to the fact that their values are exogenous variables. In the real stock market R_i influences R_p and CAPM theorizes the feedback of R_p on R_i through equation (21).

In closed-loop system theory, the output signal y_t, for control purposes, is the input, via the β stage, into the mixer, which adds or differs from the input signal x_t. In the case under analysis, the signal is added to the input signal, originating positive feedback, as in Figure 1:

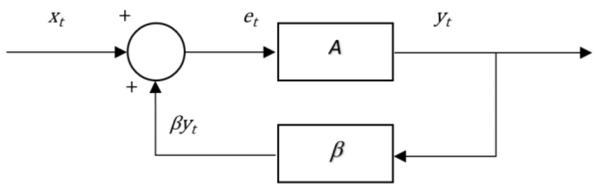

Figure 1. Closed-loop system

The algebraic relations in such a system are the following:

$$e_t = x_t + \beta y_t \tag{42}$$

$$y_t = Ae_t = A(x_t + \beta y_t) \tag{43}$$

$$y_t(1 - A\beta) = Ax_t \tag{44}$$

From equation (44) arises the following relation, which typifies the ratio between the output and the input signal:

$$\frac{y_t}{x_t} = \frac{A}{1-A\beta} \tag{45}$$

A and β are the transfer functions (Millman et al., 1972). We can point out that the system shows a strong discontinuity if $A\beta$ is equal or close to 1. Typically, a system of this kind is an oscillator characterized by intrinsic instability, just the opposite of systems in which the output signal subtracts from the input signal. An oscillating system is not necessarily unstable: if it simply oscillated between two predetermined states, it would be considered stable. The stock market fluctuates for several reasons: the stream of news relating to the firms, the industries in which firms operate, macroeconomic and political information, and, in general, all the information relevant to the firm participating in the stock market. There is no guarantee that the stock market is stable, and it is difficult to determine the conditions for stabilizing it, provided it is functional.

It follows that, as in closed-loop systems, it is perfectly useless to continuously examine the progress of a signal, i.e., instant by instant, being able to obtain the same result with an appropriate sampling of the input signal and predict the behaviour of the output signal based on the knowledge of the transfer functions. From this point of view, corporate finance is still an immature theory as the transfer function β, which does not coincide with systematic risk, appears sufficiently clear and studied by CAPM, while the transfer function *A* has not been well turned inside out or does not have a universally accepted and shared solution.

The second members of equation (45) and (39 bis) are very similar; first, the denominator represents the feedback effect while the numerator represents the transfer function *A*, which, in the case of a risky asset, links the corporate performance x_t to the return y_t of the specific asset. The feedback effect acts on the corporate performance, adding βy_t as portfolio return, and giving rise to the signal e_t which, suitably transformed, allows us to obtain the return y_t. *Mutatis mutandis*, no logical difference can be deduced by replacing an electrical signal with economic-financial information relating to risky assets, the stock market and corporate performance.

3.6 Incompleteness Consequences

We have already examined how the R^2 achievable with CAPM is low enough, leaving much of the risky asset return variability unexplained. Furthermore, CAPM does not allow us to decompose systematic risk into a share ascribable to the stock market and corporate performance. CAPM's supposed incompleteness causes both such problems and has some operational implications. In a context where equations (29) and (30) represent the asset return, try to estimate the parameters a_i and b_i, completely ignoring the existence of a second variable and its parameter c_i, leads to serious estimation errors, downloading the value of the latter parameter on the former two.

Let us examine the error that occurs in the estimation of β_i defined in a classical way as the ratio between the covariance $Cov(R_i, R_P)$ of the risky asset and portfolio returns and the portfolio variance σ_P^2. We identify this estimator with $\widehat{\beta}_\iota$, assuming we know the true ${}_2\beta_M$ obtainable with precision through a multiple regression or MLR in which R^2 is equal to 1. We can obtain this value of ${}_2\beta_M$ analytically through the MLR regressor and subtract $\widehat{\beta}_\iota$ from it, we get the following error:

$$ {}_2\beta_M - \widehat{\beta}_\iota = \frac{\rho^2(R_P\psi_i)\left[\widehat{\beta}_\iota - \frac{Cov(\psi_i,R_i)}{Cov(\psi_i,R_P)}\right]}{1-\rho^2(R_P\psi_i)} \tag{46} $$

where:

$$ {}_2\beta_M = \frac{\sigma_{\psi_i}^2 Cov(R_i,R_P) - Cov(\psi_i,R_P)\,Cov(\psi_i,R_i)}{\sigma_P^2\sigma_{\psi_i}^2 - [Cov(\psi_i,R_P)]^2} = \text{stock market performance regressor} $$

Now, this error tends to zero in two conditions, assuming that MLR allows us to reach an R^2 equal to 1:

1) when the correlation between the stock market and corporate performance is zero or

2) when the estimator $\widehat{\beta}_\iota$ is equal to the ratio of the covariances of corporate performance to stock return and stock market performance.

Both these two conditions seem unfeasible; therefore, ignoring the existence of $Cov(\psi_i, R_i)$ implies the presence of an error in $\widehat{\beta}_\iota$ estimation: the higher the correlation between the stock market and corporate performance, the higher the error.

Such an error reverberates in the intercept estimation $\widehat{\alpha}_\iota$, resulting in the following error:

$$ \alpha_i - \widehat{\alpha}_\iota = -\left({}_2\beta_M - \widehat{\beta}_\iota\right)\mu(R_P) - {}_2\beta_\psi\,\mu(\psi_i) \tag{47} $$

where:

$$ {}_2\beta_\psi = \frac{\sigma_P^2 Cov(\psi_i,R_i) - Cov(\psi_i,R_P)\,Cov(R_i,R_P)}{\sigma_P^2\sigma_{\psi_i}^2 - [Cov(\psi_i,R_P)]^2} = \text{corporate performance regressor} $$

Also, for the intercept, we can point out that the error would tend to zero only if the correlation between the stock market and corporate performance is null, a condition that is not impossible but not readily achievable.

The errors represented by equations (46) and (47) appear large enough to justify a poor result of R^2. Furthermore, this result should direct research towards a better understanding of the stock return pattern.

The fourth point concerns the R^2 partitioning of the MLR regression or commonality analysis (Nathans et al., 2012). We should ask ourselves whether and how R^2 can be broken down into shares of the explanatory variables R_P and ψ_i of the asset return. If we now compare the R^2 of the simple regressions of R_P and ψ_i against R_i, R_M^2 and R_ψ^2 respectively, with the MLR $R_{M+\psi}^2$, we realize that their difference will hardly be zero and will give rise to an overlap or bridge effect, depending on whether the sign is positive or negative:

$$ R_M^2 + R_\psi^2 - R_{M+\psi}^2 = OL_{M+\psi} \tag{48} $$

From relation (48), we can obtain the net contribution of the variables R_P e ψ_i on R^2, respectively ΔR_M^2 and ΔR_ψ^2, with the overlap/bridge effect $OL_{M+\psi}$:

$$ R_{M+\psi}^2 = (R_M^2 - OL_{M+\psi}) + (R_\psi^2 - OL_{M+\psi}) + OL_{M+\psi} = \Delta R_M^2 + \Delta R_\psi^2 + OL_{M+\psi} = R_M^2 + R_\psi^2 - OL_{M+\psi} \tag{49} $$

Here, it is not as essential to establish how relevant the net contribution or the overlap of the explanatory variables is as to note that R^2 could be the result of the effect of a ghost variable that does not appear in the OLS regression. Determining the net and overlapping effects of the explanatory variables on R^2 is complex as many variables impact the risky asset return R_i, many of which are entirely unknown: even the MLR with three or four regressors fails to reach an R^2 equal to 100%. First, it is necessary to resort to $\overline{R^2}$, an adjusted measure of R^2, every time the number of regressors increases. Consequently, already with a single regressor, it is convenient to use immediately $\overline{R^2}$ to put the first two explanatory variables on the same level of importance, regardless of which of the two we use first; otherwise, the arbitrary choice of the first regressor can pollute the result.

The overlap effect can be determined, as we have already seen, in a simple way as the difference between the sum of the $\overline{R^2}$ of the two OLS regressions and that coming from the MLR; analytically, the following equation represents the overlap:

$$OL_{M+\psi} = \frac{\sigma_M^2(\beta_M^2 - {}_2\beta_M^2) + \sigma_\psi^2(\beta_\psi^2 - {}_2\beta_\psi^2) - 2\,{}_2\beta_M\,{}_2\beta_\psi\,Cov(R_P, \psi_i)}{\sigma_i^2} \tag{50}$$

The overlap depends on the correlation between the two explanatory variables R_P e ψ_i. Indeed, the coefficients of the multiple regressions ${}_2\beta_M$ and ${}_2\beta_\psi$ in the case of covariance $Cov(R_P, \psi_i) = 0$, collapse on the simple regressors β_M and β_ψ, making the overlap null. Consequently, the greater the correlation $\rho(R_P\psi_i)$, the greater the overlap is.

We do not know the overlap effect of a second explanatory variable from the CAPM empirical evidence. CAPM ignores such a problem. Consequently, Roll's estimate that CAPM can explain only 30% of a stock return variability could depend on an omitted variable with a substantial overlap effect.

The results are even more complicated to understand because, in CAPM empirical evidence, common stocks are combined in portfolios to avoid EIV problems. We will deal with this issue in subsection 4.5.

The fifth problem concerns the transition from analysing a portfolio of *m* risky assets to a market portfolio in which the mix changes over time. This assessment will result in the following:

1) A market portfolio return R_P^* different from the optimised portfolio return R_M.

2) A variable market portfolio mix w_i^* different from the constant and optimised portfolio mix w_i.

Using the OLS regression of R_P^* against R_i, we reach α and β estimates different from those achievable with an optimised portfolio, where the weights w_i are constant all along the timespan.

3.7 Summary of CAPM Incompleteness

We try to summarize what has been highlighted so far by a simple comparison between a two-variable model to CAPM, essentially based on the optimised or market portfolio return as the only explanatory variable of the risky asset return:

1) The R^2 achievable with CAPM is too modest; clearly, there is an external explanatory variable to CAPM that justifies the remaining risky asset return variability, but for now, we do not know what it is.

2) The systematic risk may be ascribable to corporate performance risk rather than stock market risk: CAPM can provide limited clues about such a decomposition, mainly due to the corporate capital structure.

3) Each asset influences the portfolio or the market portfolio return, and the latter affects the former through a feedback effect, with undeniable oscillating consequences.

4) The α and β descending from risky assets and optimal portfolio return regression are not free to assume a correct value since their weighted mean must be constrained to 0 and 1, respectively. The market portfolio does not make this fluctuation unrestrained; on the contrary, it soils it.

5) MLR highlights overlap effects that we can conveniently anatomise to determine the impacts of two or more explanatory variables on $\overline{R^2}$. This effect depends on the existence of a correlation between the explanatory variables. CAPM could reach a modest R^2 also thanks to this overlap effect. So, there are one or more ghost variables that limit the CAPM explanatory power on one side and the other the R^2 obtained may not be ascribable to CAPM due to such omitted variables.

6) The market portfolio is very different from the Lintner or Merton optimised portfolio. Several essays by Roll (1977, 1988), Ross (1977), and Roll et al. (1994) are enlightening. Even replacing the return of the optimised portfolio of *m* risky assets with the market portfolio return, albeit represented by a primary market index, does not add more sharpness to CAPM's significance.

CAPM is an essential corporate finance theory, but we should take for what it is worth:

a. To determine the efficient frontier of the portfolio of *m* risky assets with the return vector $\boldsymbol{R}$ and the covariance matrix $\boldsymbol{\Omega}$.

b. To determine the portfolio's optimal mix, given the current level of risk-free asset return $\overline{R_F}$.

c. To evaluate the distance between the current portfolio and the optimal one.

d. To evaluate the distance between the optimal portfolio and the market one.

e. To define a different investment allocation by borrowing or lending sums at the $\overline{R_F}$ rate.

In the absence of better or equally simple alternatives, CAPM can provide captivating explanations of the stock market operations, even if not always accurate or validated by empirical evidence. Therefore, CAPM is a theory that links the portfolio of *m* risky assets and allows us to determine the optimal portfolio's correct risk/return profile, given a set of ex-post information, which means the vector of risky assets returns $\boldsymbol{R}$ and their covariances matrix $\boldsymbol{\Omega}$.

Given the risk-free asset return $\overline{R_F}$ at time *t*, the previous set of information allows us to determine the optimal portfolio with the investment share for every single risky asset that allows maximizing the investor utility, who will be able to adjust the risk/return profile of the overall portfolio by borrowing or lending at the rate $\overline{R_F}$, even if this is not strictly necessary due to the presence of the orthogonal portfolio. In this regard, see Black (1972). How close or far the stock market is from an optimal condition can be assessed by comparing the mix of the optimised portfolio with the market portfolio. Some common stocks in the current portfolio will be in excess compared to the optimal portfolio mix and will be sold to invest in stocks that will appear in shortage. These movements will generate price and return changes, which will again modify the optimal portfolio, perhaps accompanying this movement with sensitive changes in the $\overline{R_F}$ rate.

4. Empirical Evidence

4.1 Objectives of the Analysis

Having concluded the CAPM theoretical examination, the time has come to analyse empirically some essential topics highlighted in Section 3.

First, the strategy is to verify the asset allocation of 100 common stocks included in the S&P 500 Index in May 2022. We started with creating a 10-stock portfolio with no short sales, gradually expanding the portfolio to 25, 50, 75 and 100 stocks, both with and without short sales, getting nine optimised portfolios, correlating them with the performance of the S&P 500 Index. We shall track and explain the trends of some CAPM parameters of the optimised portfolios and their single common stocks.

Later, the analysis focused on the security market line or β, using the 30-year time series of monthly returns over 5, 10 and 30-year timespan and relating them to the return of the S&P 500 Index as a proxy of the market portfolio. For each common stock compared to the S&P 500, we present the 5-year rolling β for each month from January 1992 to December 2020.

Lastly, to explain stock return, we shall integrate into an MLR the S&P 500 Index return and the corporate performance measurable by a sufficiently large set of business variables, notably some DOL and DFL variables.

We shall present the outputs from these test batteries and draw some preliminary considerations.

4.2 Empirical Evidence Data

To empirically verify the previous three topics, i.e., asset allocation, security market line, and integration of stock market return with corporate performance, we use multiple sources of information, easily accessible to even non-professional investors, as CAPM prescribes.

First, we select the 100 nonfinancial risky assets from those in the S&P Index in May 2022, representing over 50% of the index mix.

The monthly market prices of the 100 common stocks and the value of the S&P 500 Index come from Yahoo! Finance. We limit the analysis to 30 years, from January 1991 to December 2020. The S&P 500 Index and 68 stocks are present in each of the 360 months of such a timespan, while the remaining 32 stocks progressively join as they land on the stock exchange. The prices need adjustments for dividend payouts, splits, and other equity operations. Based on these quotations, we computed the monthly returns of 100 risky assets and the market index; we compared such data with those coming from Portfolio Visualizer, not detecting significant differences in terms of mean returns but only modest differences between the monthly returns caused primarily by the presence of dividends.

For the optimised portfolios computation, we used both the *tout-court* and the lognormal returns, i.e., $ln(1 + R_i)$, without highlighting appreciable or significant differences in the final outputs. Consequently, we use the lognormal return for the asset allocation, while for the subsequent analyses, we use the return *tout-court*.

Concerning the financial statements and outstanding shares, we used Bloomberg data and the annual reports (form type 10-K) available on the Security and Exchange Commission's EDGAR website. Bloomberg data are not ideally suited to the purposes of this study as the time series available often include ex-post adjustments. Regarding comparability, Bloomberg's work is impeccable, but we prefer to use the original financial statements without any ex-post adjustments for the current analysis. We use Yahoo! Finance, Bloomberg and EDGAR for dividends, splits, and other equity adjustments, correcting them when and where necessary. When deemed necessary, we resort to the dividend and stock split histories published directly by the companies.

4.3 Distribution of Stock Returns

For the distribution of stock returns, we refer to the essays by Mandelbrot (1963), Fama (1963), Fama (1965) and Officer (1972). Briefly, stock prices follow a random-walk behaviour based on two assumptions:

1) successive price changes are independent and
2) they conform to some probability distribution.

While there are no doubts regarding the first point relating to the independence of successive price changes, the distribution does not appear to be perfectly described by a Gaussian; Mandelbrot's hypothesis seems more fitting. In particular, the study by Officer (1972) believes that a symmetric stable class of distributions better describes the distribution of returns due to tails that are fatter than the Gaussian but with properties inconsistent with the stable hypothesis, such as the behaviour of the sample standard deviation.

Fame et al. (1973), based on the results of Fama (1965) and Blume (1970), believe that the interpretation of t-stat, valid for normal distributions, applied to the distribution of stock returns leads to overestimate probabilities and significance levels. The values of F, t-stat and P-Value presented in the following subsections must take this topic into account even though Fama et al. (1973) write that "as one is not concerned with precise estimates of probability levels, interpreting *t-statistics* in the usual way does not lead to serious errors".

4.4 Asset Allocation

The first objective was to verify the behaviour of the optimised portfolios' main parameters as the number of stocks increases, correlating them with single common stocks and the S&P 500 Index return. The analysis starts with a set of ten common stocks to avoid short sales without constraints, and we progressively increase the number of stocks to 25, 50, 75 and 100, first with and then without short sales. We show the data collected from the nine portfolios in Table 1.

Table 1 allows us to observe the following topics as the number of risky assets in the portfolio increases:

1) The return and the standard deviation of the optimised portfolio decrease while in the optimised portfolio *sine*, without short sales, decreases mainly only the standard deviation.
2) Both the slope θ of the market opportunity line and the return/risk parameter λ of the portfolio increase; it means that the standard deviation is declining faster than the portfolio return, at least;
3) With short sales, the number of active stocks is always equal to the number of selected stocks, while the number of stocks sold short progressively increases up to 46%.
4) Without short sales, not all the selected stocks are active; indeed, the percentage of inactive stocks proliferates.
5) Lintner's conic sections differ from Merton's when short sales are involved.
6) The market opportunity line is always tangent to Merton's and Lintner's conic sections, with and without short sales, but the portfolio optimal mix with the KKT condition active does not lie on the market opportunity line.
7) The *A*, *B*, *C* and *D* parameters of the conic sections progressively increase in value; they indicate non-degenerate conics and are shifted hyperbolas. We can refer to the matrix representation of conic sections for more details.
8) The centres of Merton's conic sections always have standard deviations equal to zero, but returns seem roughly constant.
9) The centres of Lintner's conic sections with short sales show a progressive reduction in the abscissa and ordinate; the abscissa is always negative, while the ordinate becomes negative as the number of common stocks increases. Since the centre on the ordinates of each conic corresponds to its minimum point $\overline{R_P}$, Lintner's conic section overflows more into the second Cartesian quadrant, generating a portfolio with zero

standard deviation and return lower than risk-free assets. Hence, this portfolio lies below the market opportunity line.

10) The orthogonal portfolios always lie on the inefficient part of Merton's conic sections.

Table 1. Comparison of the main portfolio parameters: Lintner's weights with short sales

Item	Parameters	Stocks in Portfolio								
		Base	**With Short Sales**				**Without Short Sales**			
		10	**25**	**50**	**75**	**100**	**25**	**50**	**75**	**100**
Portfolio Data	$\Sigma_L w_i$	100.000%	69.956%	31.665%	25.456%	17.442%	100.000%	100.000%	100.000%	100.000%
	σ_M	3.952%	2.402%	1.108%	0.855%	0.557%	3.411%	3.078%	2.850%	2.738%
	R_M	1.278%	0.849%	0.531%	0.464%	0.367%	1.181%	1.246%	1.184%	1.201%
	R_F	0.0075%	0.0075%	0.0075%	0.0075%	0.0075%	0.0075%	0.0075%	0.0075%	0.0075%
	θ	32.145%	35.026%	47.196%	53.344%	64.569%	34.390%	40.237%	41.294%	43.612%
	λ	8.134	14.585	42.583	62.381	115.921	10.081	13.071	14.491	15.931
	Matrix Precision	0.000E+00	0.000E+00	-2.887E-15	5.329E-15	8.771E-15	0.000E+00	-2.887E-15	5.329E-15	8.771E-15
	Stocks #	10.00	25.00	50.00	75.00	100.00	25.00	50.00	75.00	100.00
	Active Stocks	10.00	25.00	50.00	75.00	100.00	16.00	17.00	22.00	29.00
	Short Sales	0.00	5.00	21.00	32.00	46.00	0.00	0.00	0.00	0.00
	Data	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly
	Start	Jan-91	Jan-91	Jan-91	Jan-91	Jan-91	Jan-91	Jan-91	Jan-91	Jan-91
	End	Dec-20	Dec-20	Dec-20	Dec-20	Dec-20	Dec-20	Dec-20	Dec-20	Dec-20
	Months	360	360	360	360	360	360	360	360	360
Conic Section	A	8.202	10.320	13.635	16.086	20.449	10.320	13.635	16.086	20.449
	B	0.105	0.124	0.225	0.287	0.420	0.124	0.225	0.287	0.420
	C	902.796	1,557.086	2,008.177	2,751.865	3,065.779	1,557.086	2,008.177	2,751.865	3,065.779
	D	27.127	86.924	265.489	530.914	869.352	86.924	265.489	530.914	869.352
Merton Version	minimum R_p	0.908%	0.663%	0.679%	0.585%	0.667%	0.663%	0.679%	0.585%	0.667%
	minimum σ^2_p	0.111%	0.064%	0.050%	0.036%	0.033%	0.064%	0.050%	0.036%	0.033%
	minimum σ_p	3.328%	2.534%	2.232%	1.906%	1.806%	2.534%	2.232%	1.906%	1.806%
	θ=m	32.145%	35.026%	47.196%	53.344%	64.569%	35.026%	47.196%	53.344%	64.569%
	$_M x_{PT}$	3.952%	3.433%	3.500%	3.359%	3.193%	3.433%	3.500%	3.359%	3.193%
	$_M y_{PT}$	1.278%	1.210%	1.659%	1.799%	2.069%	1.210%	1.659%	1.799%	2.069%
	$_M x_C$	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
	$_M y_C$	0.908%	0.663%	0.679%	0.585%	0.667%	0.663%	0.679%	0.585%	0.667%
Lintner Version	minimum R_p	0.908%	0.302%	-0.450%	-0.751%	-1.035%	0.663%	0.679%	0.585%	0.667%
	minimum σ^2_p	0.111%	0.023%	0.000%	0.004%	0.007%	0.064%	0.050%	0.036%	0.033%
	minimum σ_p	3.328%	1.503%	-0.160%	-0.598%	-0.830%	2.534%	2.232%	1.906%	1.806%
	θ=m	32.145%	35.026%	47.196%	53.344%	64.569%	35.026%	47.196%	53.344%	64.569%
	$_L x_{PT}$	3.952%	2.402%	1.108%	0.855%	0.557%	3.433%	3.500%	3.359%	3.193%
	$_L y_{PT}$	1.278%	0.849%	0.531%	0.464%	0.367%	1.210%	1.659%	1.799%	2.069%
	$_L x_C$	0.000%	-1.031%	-2.392%	-2.504%	-2.636%	0.000%	0.000%	0.000%	0.000%
	$_L y_C$	0.908%	0.302%	-0.450%	-0.751%	-1.035%	0.663%	0.679%	0.585%	0.667%
Orthogonal Portfolio	R_z	0.00750%	0.00750%	0.00750%	0.00750%	0.00750%	0.00750%	0.00750%	0.00750%	0.00750%
	σ^2_z	0.38092%	0.14113%	0.08390%	0.05360%	0.04796%	0.14113%	0.08390%	0.05360%	0.04796%
	σ_z	6.17187%	3.75679%	2.89654%	2.31514%	2.18990%	3.75679%	2.89654%	2.31514%	2.18990%

According to the Lintner and Merton methodologies to assess weights, we have traced in Figure 2 the three primary conic sections, one with the 10-stock portfolio and the others with two 100-stock portfolios. Apart from the southwest shift of Lintner's conic section compared to Merton's, already mentioned, we can note that the conic section tends to move west as the number of stocks in the portfolio increases, with a marked enlargement of the shape, which allows it to have a higher return for the same standard deviation. Consequently, the market opportunity line must have a steeper slope, allowing for intercepting higher indifference curves.

We can see the trajectories of the following points:

1) Merton's point of tangency (squared indicator): it moves west and then heads north; this movement

indicates a return increase with the same portfolio risk.

2) Lintner's point of tangency (round indicator): moves in a westerly and slightly southerly direction; such a movement indicates a progressive portfolio risk reduction joint to a less than proportional reduction in the portfolio return.

3) The optimal point with short sales constraints due to the KKT condition (triangle-shaped): it moves westward and is always suboptimal compared to Merton's and Lintner's point of tangency, given that they coincide in the absence of short sales; this shift indicates a progressive reduction of the risk with an almost constant portfolio return.

4) The minimum point of Merton's conic section (diamond shape): it moves westward and slightly southward, indicating a reduction in portfolio risk with roughly the same return.

The main conclusion is that by expunging short sales through the KKT condition, the optimal portfolio does not lie on the market opportunity line and, therefore, must be considered suboptimal. Secondly, whenever the number of common stocks in the portfolio increases, it is necessary to have a growing share of stocks sold short; the market portfolio, to be efficient, should have a negative quotation for many stocks listed on the stock exchange. Since this cannot happen, the market portfolio is not as efficient as an optimised portfolio: better, the market portfolio is neither efficient nor optimal, and the alleged syllogism that the market portfolio is efficient is not confirmed. Thirdly, even having expunged short sales, the optimised portfolio *sine* has a limited set of active common stocks compared to those selected from the stock market index: more than 70% of the stocks do not enter the 100-stock optimised portfolio *sine*. See Levy (1983) for such an issue. Finally, we can point out that the increase in the number of stocks entails an appreciable risk reduction and a modest return increase. So, the 100-stock optimised portfolio *sine* indicates diversification's true potential: risk reduction with substantial return stability.

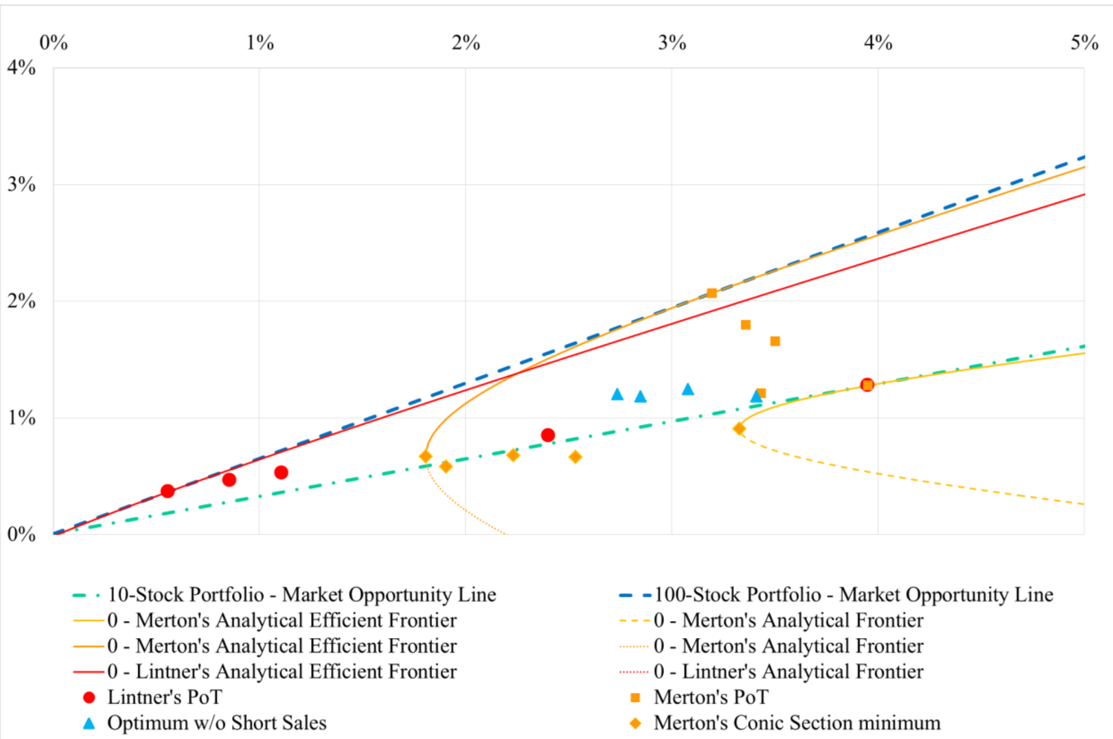

Figure 2. Conic sections, market opportunity lines and trajectories

4.4.1 Optimised Portfolios and S&P 500 Index

A legitimate question arises: how can we consider the market portfolio efficient if it includes risky assets that investors should not purchase to maximize θ?

Table 2. Regression between optimised portfolio against S&P 500 Index returns

Portofolio against S&P 500 Returns	Stocks in Portfolio								
	Base	With Short Sales				Without Short Sales			
	10	25	50	75	100	25	50	75	100
#Records	360	360	360	360	360	360	360	360	360
#Empty Records	0	0	0	0	0	0	0	0	0
Mean	0.675%	0.675%	0.675%	0.675%	0.675%	0.675%	0.675%	0.675%	0.675%
Variance	0.179%	0.179%	0.179%	0.179%	0.179%	0.179%	0.179%	0.179%	0.179%
STD Deviation	4.227%	4.227%	4.227%	4.227%	4.227%	4.227%	4.227%	4.227%	4.227%
Covariance	0.126%	0.075%	0.026%	0.017%	0.009%	0.115%	0.101%	0.095%	0.091%
OLS Intercept α	-0.004	-0.004	-0.004	-0.004	-0.004	-0.005	-0.007	-0.007	-0.008
OLS Slope β	0.805	1.306	2.113	2.292	2.844	0.985	1.066	1.173	1.210
Correlation vs. R_M	75.291%	74.215%	55.414%	46.371%	37.478%	79.485%	77.649%	79.046%	78.348%
R^2 vs. R_M	56.687%	55.079%	30.707%	21.502%	14.046%	63.179%	60.294%	62.483%	61.385%
R^2 Adj. vs. R_M	56.566%	54.953%	30.514%	21.283%	13.806%	63.076%	60.183%	62.378%	61.277%
Standard Error	0.028	0.028	0.035	0.038	0.039	0.026	0.027	0.026	0.026
F	468.547	438.949	158.649	98.065	58.501	614.262	543.634	596.223	569.092
P-Value	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
SIG?	True	True	True	True	True	True	True	True	True
α Standard Error	0.002	0.002	0.002	0.002	0.002	0.001	0.002	0.001	0.002
t-stat	-2.289	-2.728	-2.163	-1.721	-1.487	-3.398	-4.302	-4.813	-5.132
P-Value	1.133%	0.334%	1.560%	4.309%	6.897%	0.038%	0.001%	0.000%	0.000%
SIG?	True	True	True	True	False	True	True	True	True
β Standard Error	0.037	0.062	0.168	0.231	0.372	0.040	0.046	0.048	0.051
t-stat	21.646	20.951	12.596	9.903	7.649	24.784	23.316	24.418	23.856
P-Value	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
SIG?	True	True	True	True	True	True	True	True	True

We try to answer this question by correlating the nine optimised portfolio returns against the S&P 500 Index return. We show the results in Table 2. In this analysis, the independent variable is the optimised portfolio return. Since the index mix has changed over the 30 years while the optimised portfolio has a fixed structure, the correlation will be less than 100%. We also recall the 68 stocks over thirty years in the optimised portfolios, and the remaining 32 stocks included progressively over the next 23 years. We could have performed the asset allocation only over the last seven years, but with only 84 monthly observations, we would have had a covariance matrix with rank 83, which would have allowed us to create an optimised portfolio of only 83 stocks; alternatively, we should have shifted to weekly observations, but this would not have allowed us to compare these analyses with the subsequent ones.

If we focus on the correlation index and R^2, it appears that short sales constraint allows the return of the optimised portfolio *sine* to be more correlated to the S&P 500 Index return, while the R^2 allows explaining more than 60% of the latter variance with the former. This element collapses rapidly as the number of stocks and short sales increase. Over 41% of the investment in the 100-stock portfolio is due to short sales for 46 stocks, while long positions are less than 59% of the investment for 54 stocks. Its R^2 is only 14%.

Market portfolio and optimal portfolios are strongly correlated only without short sales. Nevertheless, even without short sales, we can see that the market portfolio return is lower with a higher standard deviation than the 100-stock optimised portfolio *sine*. Comparing the θ of the latter portfolio, we can see from Table 1 that it is worth 43.6% against 15.8% of the former. This comparison also demonstrates that the market portfolio is suboptimal to the optimised portfolio *sine*. However, there is more: as the number of stocks in the portfolio increases, θ increases steadily. If we had included all the S&P 500 stocks in the portfolio, we might have gotten an even higher θ. We have already seen that over 70% of stocks are inactive; this percentage could rise further with a 500-stock portfolio, just as it has steadily risen from 25 stocks (with 36% of inactive stocks) to 100 stocks (71 % inactive). Indeed, the portfolio mix weighs heavily on this result.

4.4.2 β Behaviour with Short Sales

A fundamental point is missing to complete the asset allocation analysis: how do the common stock βs behave within the optimised portfolios?

We will first analyse the β behaviour of the optimised portfolio with short sales. The fundamental point is that 100% of the stocks are active; consequently, when the stocks increase from *m* to *n*, we witness homogeneous behaviour of all the *m* stocks already in the portfolio: all the βs increase homogeneously (see Figure 3 on the left), and such growth is related to the trend of the portfolio excess return, $R_M - \overline{R_F} = X_M$. We analyse the β percentage change of a generic stock, exploiting CAPM equilibrium relations, through a few algebraic passages we obtain:

$$\Delta\%\beta_i = \frac{\beta_i^n}{\beta_i^m} - 1 = \frac{\Delta\% Cov(R_i, R_M) - \Delta\%\sigma_M^2}{1+\Delta\%\sigma_M^2} = \frac{-\Delta\% X_M}{1+\Delta\% X_M} \tag{51}$$

where:

$$1 + \Delta\% X_M = \frac{R_M^n - \overline{R_F}}{R_M^m - \overline{R_F}} = \frac{X_M^n}{X_M^m} \tag{52}$$

The facts are as follows:

1) The β of all common stocks increases as the number of stocks in the portfolio rises; 79 of the 100 stocks in the portfolio have a β greater than two, while only four stocks have a β less than one.

2) The portfolio excess return X_M trend involves such an increase, which, as we have seen, decreases as the number of stocks in the portfolio grows, and this does not depend on the absolute risk level of the single stock.

3) Consequently, the increase is homogeneous for all common stocks.

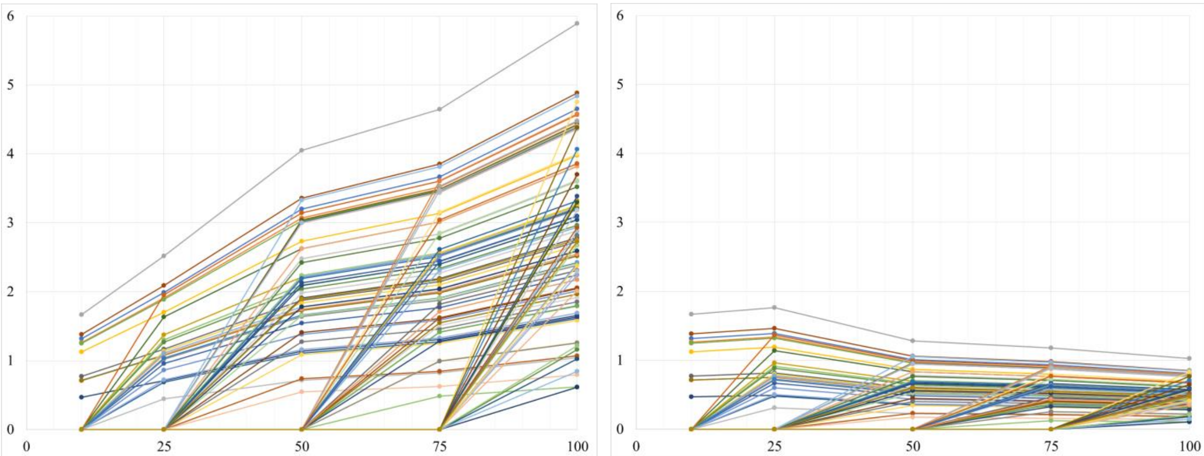

Figure 3. β trends inside five optimised portfolios: Lintner's weights on the left and Merton's on the right

The β continuous increase can only come from reducing the optimised portfolio variance faster than the covariance of the stock return compared to the portfolio return. As the number of stocks grows, both θ and λ increase; if they do not increase, the incremental stock should have zero weight. To keep constant the stock excess return to $\overline{R_F}$, the more λ increases, the more the covariance $Cov(R_i, R_M)$ must decrease. But the optimised portfolio variance shrinks even more, and this leads to the β increase, an increase measurable more simply employing the percentage change of the portfolio excess return $\Delta\% X_M$. Although β is a measure of the risk of the common stock compared to the optimised portfolio, its dynamic depends on the $\Delta\% X_M$ trend via λ.

What happens if we base the portfolio weights on Merton's method instead of Lintner's? Relations (51) and (52) are still valid, but using Merton's conic section, there is a tendency to increase the portfolio return with equal standard deviation, implying that β must decrease. With a 100-stock portfolio, the contraction is so powerful that only one of the β exceeds 1.0: see Figure 3 on the right. Such a trend, apparently illogical from an economic perspective, leads us again to favour Lintner's assessment of the portfolio weights. Certainly, Merton's evaluation is more valuable in the optimisation stage but less economically understandable.

At this point, it is unclear why we observe β, a specific indicator of common stock, instead of watching λ, common to all stocks in the portfolio. By precisely defining the market portfolio mix and size, it would be easy to verify its optimisation to discover that the market is perhaps not as optimised as we usually consider it, even if it remains an excellent tool for sharing and containing risks (Lintner, 1970).

Furthermore, the β weighted mean of the optimised portfolio equals one due to many negative weights, just as the α weighted mean is null. These topics are sufficiently substantiated and highlight the characteristics the stock β should have if the market portfolio were optimised: a very high β, even though their weighted mean is 1.

4.4.3 β Behaviour without Short Sales

What characteristics must stocks possess to have a positive or negative weight? Observing the covariance matrix, its inverse, and the stock excess return X_i, it does not appear at first sight identifiable what characteristic determines its sign and value. As already extensively treated in subsection 3.2, everything depends on the v_{ij} elements of the inverse matrix.

Table 3. Common stock layers in optimised portfolio *sine*

Layer	Stocks in Portfolio	10	25	50	75	100
1	10	10	9	7	6	6
2	+15	-	7	4	3	3 (+1-1)
3	+25	-	-	6	6	7 (+1-1)
4	+25	-	-	-	7	7 (+1-1)
5	+25	-	-	-	-	6
	Total	**10**	**16**	**17**	**22**	**29**

The last issue introduces the β behaviour in the optimised portfolio *sine*. Even for the latter, it is not easy to understand the characteristics that the common stocks excluded must have since it is not sufficient to have a negative weight in the optimised portfolio with short sales to be a candidate for taking on a zero weight in the optimised portfolio *sine*. The zero-weight choice mainly depends on a nonlinear system where even the v_{ij} elements of the inverse matrix do not play an explanatory role.

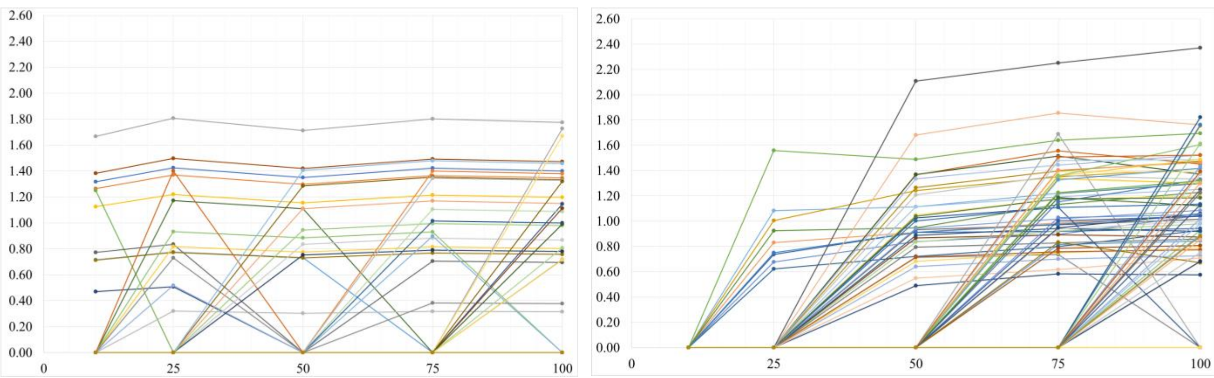

Figure 4. β trends inside five optimised portfolios *sine*: on the left, stocks included, and on the right, the excluded ones

Having said all this, let us see the behaviour of the stocks in the portfolio *sine*. First, we focus on the dynamic of inclusions and exclusions. From Table 3, it is possible to examine the stratification of the stocks in the five portfolios *sine*; from 10 to 75 stocks, the dynamic presents exclusively stopping stocks, while moving from 75 to 100 stocks appear assets not previously selected.

In Figure 4 on the left, we can examine the overall β dynamics, assuming that their value nullifies when they leave the portfolio so as not to pollute the overall picture. We will exclude or include stocks from the analysis as they leave or enter the portfolios. Also, in the optimised portfolios *sine*, we have homogeneity in the β dynamics, which remains confined between 0.3 and 1.8, with some oscillations, without a decisive and constant increase as in the optimised portfolios with short sales. Determining the β dynamic appears problematic as we initially witness a stopping and rising dynamic altogether, subsequently mixed up by repechages of excluded stocks.

The βs follow the same dynamic already highlighted by equation (51). All this is for the stocks included in the portfolio. Therefore, the relative β constancy follows the same dynamics of the portfolio excess returns: the portfolio variance shrinks while its return is relatively constant.

Considering that the portfolios *sine* are sub-optimised compared to those with short sales, it follows that low βs are typical of a sub-optimised stock market.

What happens to the βs of the excluded stocks? From Figure 4 on the right, we can examine such trends. They present the most disparate dynamics; they have no connection with the portfolio return trend or each other. Their values are lower than those of the optimised portfolio with short sales. Apart from one stock, the other 70 stocks show values between 0.50 and 1.90. We must ask ourselves the meaning of common stock βs in a non-optimised portfolio or for stocks excluded *stricto sensu* from the portfolio, even the market portfolio.

4.4.4 Correlation and Determination Indices

The last issue concerns the trend of the correlation and determination indices of single stock returns compared to the optimised portfolio return. Again, we examine the relations between these indices in a portfolio of *n* stocks starting from a portfolio of *m* stocks with $n > m$. The relation for the correlation index is as follows:

$$\Delta\%\rho = \frac{\rho_{iM}^{n}}{\rho_{iM}^{m}} - 1 = \frac{\Delta\% Cov(R_i, R_M) - \Delta\%\sigma_M}{1 + \Delta\%\sigma_M} \tag{53}$$

From this, we can conclude that in an optimised portfolio of any kind, the correlation between stock and portfolio returns tends to decrease as the covariance reduction exceeds the reduction of the portfolio standard deviation. Although the covariance and standard deviation dynamics depend on the weight assessment by Lintner or Merton, this cannot influence the correlation between the common stock and optimised portfolio returns. As we have already seen above, the equilibrium relation requires that when the number of common stocks in the portfolio increases, there is an inverse relation between the covariance $Cov(R_i, R_M)$ and λ: the former decreases as the latter increases, linked to the θ rise. Furthermore, as the number of stocks increases, the portfolio variance decreases, which leads to the standard deviation shrinkage. The covariance reduction is faster than the standard deviation shrinkage. Consequently, the correlation index decreases when we move from *m* to *n* stocks, leading to an automatic R^2 contraction:

$$\Delta\%R^2 = (1 + \Delta\%\rho)^2 - 1 \tag{54}$$

Table 4. Correlation e determination index dynamics referred to the number of stocks in the portfolio

Stocks in Portfolio	With Short Sales		Without Short Sales	
	$\Delta\%\rho$	$\Delta\%R^2$	$\Delta\%\rho$	$\Delta\%R^2$
25	-8.2%	-15.8%	-6.5%	-12.6%
50	-25.8%	-44.9%	-14.5%	-26.9%
75	-11.5%	-21.7%	-2.5%	-5.0%
100	-17.4%	-31.7%	-5.3%	-10.3%

In optimised portfolios with short sales, the dynamics of the correlation and determination indices of the stock returns compared to portfolio return are always completely homogeneous for every stock, except for those just included since all the stocks are always active. In optimised portfolios *sine*, we must exclude the stopping stocks to appreciate the trend homogeneity of correlation and determination indices. We show such dynamics in Table 4.

Within an optimised portfolio with short sales, as the number of stocks increases, we can observe the following events:

1) β tends to increase with Lintner's weights (the opposite with Merton's weights);
2) ρ and R^2 tend to decrease.

It follows that trying to explain the common stock return against the optimised portfolio return through OLS is misleading, while the β trend is due to the increase in the risk perception of common stock compared to the optimised portfolio variance that decreases continuously. We can also apply the same considerations to an optimised portfolio *sine*, albeit in a much softer way. The emphasized tendency towards an R^2 reduction is due to portfolio optimisation; consequently, Roll (1988) absurdly should rejoice in having an R^2 around 30%; such a parameter should be much lower if the market portfolio were optimised. With a 100-stock optimised portfolio with short sales, the mean R^2 is around 4.5%. It is a further clue that the market portfolio is far from optimised; this is a market failure, and CAPM should be cleared, not blamed.

4.5 Security Market Lines

After analysing the asset allocation, we must look at the monthly return of the 100 common stocks compared to the market portfolio return using the S&P 500 Index as its proxy. We analysed three periods ending in 2020: 30 years from 1991, 10 years from 2011 and 5 years from 2016. We present the data in Tables 5, 6a and 6b. We replaced the company's names with aliases. Table 5 summarizes the data from the three periods.

Briefly, we can see that using the monthly returns, we get the following results:

1) The intercepts and slopes of the regressions are considered admissible based on the F test, but as the time window analysis shrinks, its goodness-in-fit decreases.

2) While the slopes align with the goodness-in-fit of the regression, measured by t-stat and P-Value, the intercepts fail to pass the null hypothesis easily; the more the time window shrinks, the less they are significant: in the 5-year timespan, only 16 intercepts are significant.

3) The determination index R^2 increases on average as the time window decreases and remains at relatively low levels, around 30%, but not insignificant, as expected in an optimised portfolio with short sales.

4) Moving from monthly to weekly returns R^2 worse, but this result is not due to better portfolio optimisation but to more erratic price movements.

5) The intercepts have an arithmetic mean value between 0.7% and 1.0%, statistically insignificant, while the slopes have values that fluctuate around 1.

On the one hand, the analysis confirms that if we think of explaining stock excess return through the market portfolio excess return by using β, we risk incurring excessive risks since the relation does not have an outstanding predictive value if measured by R^2, on the other hand, the βs oscillating around one and the R^2s suggest that the market portfolio is not optimised. All this confirms that the security market line of each stock has the meaning of a portfolio optimisation condition purely, without any predictive worth.

Table 5. Summary of β analyses

Timespan	Item	Obs.	Standard Error	α	β	R^2	α t-stat	β t-stat	α P-Value	β P-Value	F	F P-Value
30Y	μ	318.83	0.0844	1.008%	1.0083	22.562%	2.028	9.470	5.918%	0.017%	100.17	0.034%
	σ	75.44	0.0406	0.685%	0.394	9.673%	0.8477	3.2381	9.120%	0.155%	63.08	0.310%
	SIG?					**5**			**56**	**100**		**100**
10Y	μ	119.11	0.0593	0.709%	0.9451	30.330%	1.289	7.216	11.233%	0.177%	59.99	0.355%
	σ	4.42	0.0260	0.807%	0.389	14.799%	1.2390	2.8143	12.595%	1.012%	44.42	2.023%
	SIG?					**26**			**31**	**98**		**98**
5Y	μ	60	0.0603	0.694%	0.9464	33.637%	0.8827	5.5167	16.420%	0.520%	34.64	1.039%
	σ	0.0000	0.0244	1.088%	0.418	15.125%	1.1698	2.0512	14.107%	2.298%	24.41	4.596%
	SIG?					**37**			**16**	**94**		**94**
5Y monthly rolling β	μ	290.55	0.0770	0.995%	1.0338	25.968%	0.9980	4.5089	18.396%	1.547%	25.14	3.094%
	σ	93.3196	0.0332	0.637%	0.375	9.427%	0.4265	1.3102	6.058%	2.550%	13.67	5.099%
	SIG?					**6**			**1**	**79**		**79**

Table 5 shows the mean rolling β of all common stocks recalculated monthly from 1992 to 2020. The analysis is captivating, but the summary does not do justice to the data detail shown in Table 7. It is necessary to examine the diagrams to understand their meaning. We report in Figure 5 the diagrams of three stocks issued by historical corporations, which can represent what a rolling β means.

In Figure 5, we show in blue with the scale on the left the 5-year rolling β and the 30-year stationary β, while all the other variables have the scale on the right. With a solid blue line, we have the 5-year rolling β while the trend of the relative intercept α, R^2, and the P-Value of the rolling regression have dashed lines. The 30-year stationary β, its intercept α and the R^2 of the regression have a dotted line with a double point, shown purely for comparison purposes with the 5-year rolling analysis.

Such analysis highlights:

1) a substantial variability of the rolling β, which contrasts with the presumed constancy of the stationary β,

2) an equally significant variability of the R^2 of the rolling β regression,

3) a modest oscillation of the rolling α and

4) a localized P-Value movement of the rolling β.

In the first diagram on top of Figure 5, the first stock highlights a marked oscillation in rolling β corresponding with the 2008 crisis, also felt in the third diagram in the bottom but not in the second in the middle, which is instead affected by a period of the corporate downturn that began in 1992 and ended in 1996. The overall graphical analysis of the 100 common stocks shows abrupt changes in the 5-year rolling β in a period of one or few months, paired with other sharp α, R^2 and P-Value movements, not always synchronous with their respective rolling β.

Such trends are not unrelated to stock market trends and corporate performance. The security market line does not appear insensitive to such changes. It is unclear what the transmission mechanism of the corporate or industry performance on the stock return is: CAPM cannot specify it, being persuasive in allocating assets but less effective in explaining the individual stock price and return trends.

Table 6a. 30-Year, 10-Year, and 5-Year Stationary β Data

30Y period (1991-2020)

#	Conventional Name	Obs.	Standard Error	Intercept	Beta	R^2	Intercep t t-stat	Beta t-stat	Intercept P-Value	Beta P-Value	F	F P-Value
1	Achernar	276	0.0652	0.616%	0.9456	29.456%	1.557	10.696	6.032%	0.000%	114.41	0.000%
2	Adhil	341	0.0895	1.438%	1.0266	19.000%	2.922	8.917	0.186%	0.000%	79.52	0.000%
3	Albaldah	102	0.0959	1.472%	1.1108	16.602%	1.493	4.462	6.927%	0.001%	19.91	0.002%
4	Albali	360	0.0554	0.646%	0.3317	5.954%	2.174	4.761	1.516%	0.000%	22.67	0.000%
5	Alchiba	360	0.0669	0.374%	0.7874	19.671%	1.043	9.363	14.886%	0.000%	87.67	0.000%
6	Alcor	248	0.0828	1.850%	0.5596	7.996%	3.497	4.624	0.028%	0.000%	21.38	0.001%
7	Alcyone	360	0.0484	0.658%	0.6332	23.216%	2.535	10.404	0.584%	0.000%	108.24	0.000%
8	Aldebaran	360	0.0683	0.815%	0.7697	18.325%	2.225	8.962	1.334%	0.000%	80.32	0.000%
9	Alderamin	360	0.0736	0.990%	0.9981	24.528%	2.511	10.786	0.625%	0.000%	116.35	0.000%
10	Aldhanab	360	0.0854	0.878%	1.3112	29.404%	1.918	12.211	2.794%	0.000%	149.11	0.000%
11	Algol	360	0.0715	0.981%	0.7199	15.189%	2.558	8.007	0.547%	0.000%	64.12	0.000%
12	Algorab	355	0.2656	2.488%	1.0436	2.671%	1.739	3.112	4.143%	0.100%	9.69	0.201%
13	Alhena	360	0.0595	0.767%	0.8163	24.985%	2.407	10.920	0.829%	0.000%	119.24	0.000%
14	Alioth	252	0.0513	0.383%	0.7880	31.268%	1.179	10.665	11.976%	0.000%	113.73	0.000%
15	Alkarab	360	0.0576	0.686%	0.5876	15.528%	2.222	8.112	1.346%	0.000%	65.81	0.000%
16	Almaaz	360	0.0680	0.755%	0.9511	25.719%	2.073	11.133	1.945%	0.000%	123.95	0.000%
17	Alnair	341	0.0564	0.740%	0.9296	32.624%	2.387	12.812	0.877%	0.000%	164.14	0.000%
18	Alpheratz	360	0.0513	0.736%	0.4341	11.231%	2.677	6.730	0.388%	0.000%	45.29	0.000%
19	Alrakis	360	0.0791	1.156%	1.1054	25.687%	2.728	11.124	0.334%	0.000%	123.74	0.000%
20	Altair	360	0.0681	0.709%	1.0503	29.604%	1.943	12.270	2.637%	0.000%	150.55	0.000%
21	Alzirr	360	0.0614	-0.168%	1.2460	42.180%	-0.511	16.160	30.500%	0.000%	261.16	0.000%
22	Aniara	135	0.0710	1.893%	1.0176	25.273%	3.001	6.707	0.161%	0.000%	44.98	0.000%
23	Antares	360	0.0456	0.206%	0.6904	28.808%	0.843	12.036	19.987%	0.000%	144.87	0.000%
24	Arcturus	360	0.0482	0.400%	0.7598	30.530%	1.549	12.543	6.118%	0.000%	157.33	0.000%
25	Aspidiske	260	0.1753	1.202%	1.9302	18.823%	1.098	7.735	13.655%	0.000%	59.82	0.000%
26	Atlas	360	0.0992	0.899%	1.6783	33.602%	1.691	13.460	4.590%	0.000%	181.17	0.000%
27	Bellatrix	360	0.0624	0.397%	0.6751	17.130%	1.187	8.603	11.793%	0.000%	74.00	0.000%
28	Betelgeuse	360	0.0498	0.995%	0.1353	1.291%	3.733	2.164	0.011%	1.556%	4.68	3.112%
29	Bharani	360	0.0511	1.077%	0.3208	6.513%	3.933	4.994	0.005%	0.000%	24.94	0.000%
30	Canopus	360	0.1297	1.186%	0.7250	5.234%	1.706	4.447	4.448%	0.001%	19.77	0.001%
31	Capella	360	0.0563	0.320%	0.6337	18.294%	1.059	8.953	14.505%	0.000%	80.16	0.000%
32	Castor	360	0.0511	0.747%	0.6980	24.836%	2.730	10.876	0.333%	0.000%	118.29	0.000%
33	Copernicus	360	0.0648	0.485%	0.6657	15.746%	1.397	8.180	8.161%	0.000%	66.91	0.000%
34	Deneb	243	0.2260	1.705%	1.9498	12.440%	1.169	5.852	12.187%	0.000%	34.24	0.000%
35	Diadem	360	0.0757	1.088%	0.8574	18.482%	2.681	9.009	0.383%	0.000%	81.17	0.000%
36	Dubhe	304	0.1297	0.347%	1.6545	23.958%	0.459	9.755	32.313%	0.000%	95.15	0.000%
37	Electra	273	0.1480	1.261%	1.1117	10.070%	1.397	5.509	8.179%	0.000%	30.34	0.000%
38	Furud	360	0.0854	1.174%	0.8151	13.881%	2.566	7.596	0.535%	0.000%	57.70	0.000%
39	Gienah	195	0.0762	1.321%	1.0447	25.337%	2.387	8.093	0.898%	0.000%	65.49	0.000%
40	Hamal	360	0.0565	0.718%	0.8845	30.244%	2.371	12.459	0.914%	0.000%	155.22	0.000%
41	Intercrus	360	0.0570	0.358%	1.1174	40.429%	1.172	15.587	12.094%	0.000%	242.96	0.000%
42	Izar	346	0.1159	1.482%	0.8319	8.341%	2.343	5.595	0.986%	0.000%	31.30	0.000%
43	Jabbah	360	0.0564	1.082%	0.9200	32.010%	3.582	12.983	0.019%	0.000%	168.55	0.000%
44	Kang	360	0.0608	0.781%	0.6284	15.895%	2.397	8.225	0.853%	0.000%	67.66	0.000%
45	Keid	262	0.1584	2.706%	1.9903	23.147%	2.746	8.849	0.323%	0.000%	78.31	0.000%
46	Kitalpha	360	0.0481	0.526%	0.8109	33.480%	2.042	13.423	2.093%	0.000%	180.19	0.000%
47	Kochab	360	0.1132	1.273%	1.4360	22.145%	2.099	10.091	1.827%	0.000%	101.83	0.000%
48	Kraz	360	0.0741	0.363%	1.0646	26.743%	0.914	11.432	18.074%	0.000%	130.69	0.000%
49	Libertas	197	0.0950	1.761%	1.4106	28.263%	2.564	8.765	0.555%	0.000%	76.83	0.000%
50	Markab	360	0.0492	0.369%	1.1397	48.669%	1.400	18.424	8.125%	0.000%	339.43	0.000%

10Y period (2011-2020)

#	Conventional Name	Obs.	Standard Error	Intercept	Beta	R^2	Intercep t t-stat	Beta t-stat	Intercept P-Value	Beta P-Value	F	F P-Value
1	Achernar	120	0.0485	0.382%	1.0543	42.126%	0.836	9.268	20.255%	0.000%	85.89	0.000%
2	Adhil	120	0.0521	1.127%	0.7919	26.218%	2.296	6.475	1.173%	0.000%	41.93	0.000%
3	Albaldah	102	0.0959	1.472%	1.1108	16.602%	1.493	4.462	6.927%	0.001%	19.91	0.002%
4	Albali	120	0.0448	0.786%	0.1382	1.447%	1.863	1.316	3.245%	9.534%	1.73	19.068%
5	Alchiba	120	0.0567	0.148%	0.8051	23.677%	0.277	6.050	39.116%	0.000%	36.61	0.000%
6	Alcor	120	0.0809	1.139%	0.8528	14.624%	1.495	4.496	6.882%	0.001%	20.21	0.002%
7	Alcyone	120	0.0351	0.459%	0.6686	35.783%	1.385	8.109	8.426%	0.000%	65.75	0.000%
8	Aldebaran	120	0.0399	1.091%	0.6161	26.885%	2.904	6.587	0.220%	0.000%	43.39	0.000%
9	Alderamin	120	0.0416	0.630%	0.9128	42.538%	1.607	9.346	5.540%	0.000%	87.35	0.000%
10	Aldhanab	120	0.0597	0.727%	0.7891	21.192%	1.293	5.633	9.932%	0.000%	31.73	0.000%
11	Algol	120	0.0466	0.942%	0.7849	30.396%	2.145	7.179	1.699%	0.000%	51.53	0.000%
12	Algorab	120	0.1174	2.388%	0.5167	2.895%	2.159	1.876	1.644%	3.159%	3.52	6.317%
13	Alhena	120	0.0406	0.448%	0.8363	39.548%	1.172	8.786	12.176%	0.000%	77.20	0.000%
14	Alioth	120	0.0513	0.220%	0.9383	34.009%	0.456	7.798	32.468%	0.000%	60.81	0.000%
15	Alkarab	120	0.0476	0.777%	0.3758	8.746%	1.731	3.363	4.305%	0.052%	11.31	0.104%
16	Almaaz	120	0.0468	1.102%	0.8951	35.992%	2.497	8.146	0.696%	0.000%	66.35	0.000%
17	Alnair	120	0.0342	0.298%	0.8485	48.730%	0.926	10.590	17.825%	0.000%	112.15	0.000%
18	Alpheratz	120	0.0368	0.563%	0.4250	17.017%	1.623	4.919	5.364%	0.000%	24.20	0.000%
19	Alrakis	120	0.0569	0.755%	1.2435	42.403%	1.409	9.321	8.079%	0.000%	86.87	0.000%
20	Altair	120	0.0567	0.346%	1.0819	35.957%	0.648	8.140	25.917%	0.000%	66.25	0.000%
21	Alzirr	120	0.0766	-0.912%	1.1278	25.006%	-1.264	6.273	10.443%	0.000%	39.35	0.000%
22	Aniara	120	0.0725	1.816%	1.0287	23.658%	2.658	6.047	0.448%	0.000%	36.57	0.000%
23	Antares	120	0.0440	-1.131%	1.1788	52.543%	-2.730	11.430	0.365%	0.000%	130.65	0.000%
24	Arcturus	120	0.0351	-0.033%	0.9957	55.391%	-0.101	12.105	45.972%	0.000%	146.52	0.000%
25	Aspidiske	120	0.0672	0.445%	1.3682	38.947%	0.703	8.676	24.175%	0.000%	75.28	0.000%
26	Atlas	120	0.0497	0.355%	1.2523	49.393%	0.757	10.732	22.513%	0.000%	115.17	0.000%
27	Bellatrix	120	0.0627	0.636%	0.5660	11.135%	1.076	3.845	14.203%	0.010%	14.79	0.020%
28	Betelgeuse	120	0.0445	0.647%	0.2399	4.274%	1.542	2.295	6.288%	1.174%	5.27	2.348%
29	Bharani	120	0.0447	1.604%	0.2474	4.508%	3.811	2.360	0.011%	0.995%	5.57	1.990%
30	Canopus	120	0.0534	0.438%	0.6012	16.342%	0.871	4.801	19.281%	0.000%	23.05	0.000%
31	Capella	120	0.0437	0.552%	0.3513	9.041%	1.341	3.425	9.121%	0.042%	11.73	0.085%
32	Castor	120	0.0378	0.655%	0.5470	24.338%	1.838	6.161	3.427%	0.000%	37.96	0.000%
33	Copernicus	120	0.0442	0.556%	0.5377	18.542%	1.334	5.183	9.240%	0.000%	26.86	0.000%
34	Deneb	120	0.0571	1.680%	0.4987	10.523%	3.126	3.725	0.112%	0.015%	13.88	0.030%
35	Diadem	120	0.0558	1.140%	0.7552	22.008%	2.169	5.770	1.606%	0.000%	33.30	0.000%
36	Dubhe	120	0.1399	-1.474%	2.3383	30.071%	-1.118	7.123	13.287%	0.000%	50.74	0.000%
37	Electra	120	0.0466	1.141%	0.3427	7.701%	2.602	3.138	0.522%	0.108%	9.85	0.215%
38	Furud	120	0.0567	1.075%	0.7418	20.866%	2.012	5.578	2.324%	0.000%	31.11	0.000%
39	Gienah	120	0.0535	0.746%	0.9557	32.944%	1.479	7.614	7.090%	0.000%	57.97	0.000%
40	Hamal	120	0.0437	0.570%	1.0570	47.375%	1.384	10.307	8.451%	0.000%	106.23	0.000%
41	Intercrus	120	0.0471	0.397%	1.2781	53.139%	0.894	11.568	18.653%	0.000%	133.81	0.000%
42	Izar	120	0.0694	0.714%	0.6704	12.569%	1.093	4.119	13.839%	0.004%	16.96	0.007%
43	Jabbah	120	0.0496	1.402%	0.9969	38.359%	3.002	8.569	0.164%	0.000%	73.43	0.000%
44	Kang	120	0.0431	0.340%	0.8690	38.522%	0.838	8.599	20.175%	0.000%	73.94	0.000%
45	Keid	120	0.1062	2.320%	1.4051	21.245%	2.319	5.642	1.105%	0.000%	31.83	0.000%
46	Kitalpha	120	0.0385	0.578%	0.7758	38.483%	1.594	8.592	5.682%	0.000%	73.82	0.000%
47	Kochab	120	0.0479	1.454%	1.1058	45.041%	3.219	9.834	0.083%	0.000%	96.71	0.000%
48	Kraz	120	0.0773	0.247%	1.4096	33.866%	0.340	7.773	36.735%	0.000%	60.43	0.000%
49	Libertas	120	0.0697	0.676%	1.2851	34.384%	1.030	7.863	15.259%	0.000%	61.83	0.000%
50	Markab	120	0.0423	-0.570%	1.3052	59.506%	-1.433	13.168	7.724%	0.000%	173.40	0.000%

5Y period (2016-2020)

#	Conventional Name	Obs.	Standard Error	Intercept	Beta	R^2	Intercept t-stat	Beta t-stat	Intercept P-Value	Beta P-Value	F	F P-Value
1	Achernar	60	0.0426	0.897%	0.8258	42.485%	1.5782	6.5455	5.998%	0.000%	42.84	0.000%
2	Adhil	60	0.0526	0.420%	0.8091	31.799%	0.6000	5.2003	27.543%	0.000%	27.04	0.000%
3	Albaldah	60	0.0623	0.599%	1.1830	41.544%	0.7224	6.4203	23.647%	0.000%	41.22	0.000%
4	Albali	60	0.0471	0.658%	0.2058	3.621%	1.0476	1.4761	14.958%	7.266%	2.18	14.533%
5	Alchiba	60	0.0692	-0.984%	0.8144	21.415%	-1.0670	3.9756	14.520%	0.010%	15.81	0.020%
6	Alcor	60	0.0827	1.423%	0.9734	21.422%	1.2906	3.9765	10.099%	0.010%	15.81	0.020%
7	Alcyone	60	0.0384	0.290%	0.6871	38.633%	0.5664	6.0426	28.664%	0.000%	36.51	0.000%
8	Aldebaran	60	0.0425	0.970%	0.6578	32.076%	1.7140	5.2335	4.594%	0.000%	27.39	0.000%
9	Alderamin	60	0.0429	0.908%	0.8859	45.611%	1.5879	6.9741	5.887%	0.000%	48.64	0.000%
10	Aldhanab	60	0.0655	0.539%	0.7322	19.736%	0.6169	3.7765	26.986%	0.019%	14.26	0.038%
11	Algol	60	0.0507	0.278%	0.7506	30.163%	0.4110	5.0050	34.131%	0.000%	25.05	0.001%
12	Algorab	60	0.1044	-0.003%	0.2987	1.587%	-0.0024	0.9671	49.907%	16.875%	0.94	33.750%
13	Alhena	60	0.0450	0.226%	0.7178	33.410%	0.3771	5.3944	35.374%	0.000%	29.10	0.000%
14	Alioth	60	0.0663	0.423%	0.9665	29.502%	0.4793	4.9266	31.677%	0.000%	24.27	0.001%
15	Alkarab	60	0.0473	1.283%	0.4303	14.002%	2.0338	3.0731	2.328%	0.161%	9.44	0.323%
16	Almaaz	60	0.0465	1.240%	0.8585	40.173%	2.0013	6.2407	2.502%	0.000%	38.95	0.000%
17	Alnair	60	0.0402	1.012%	0.8051	44.097%	1.8872	6.7639	3.207%	0.000%	45.75	0.000%
18	Alpheratz	60	0.0376	0.850%	0.3797	16.740%	1.6984	3.4149	4.740%	0.059%	11.66	0.117%
19	Alrakis	60	0.0607	0.242%	1.3626	49.790%	0.2992	7.5838	38.293%	0.000%	57.51	0.000%
20	Altair	60	0.0612	1.484%	0.9750	33.351%	1.8205	5.3873	3.692%	0.000%	29.02	0.000%
21	Alzirr	60	0.1012	-2.121%	1.0799	18.311%	-1.5721	3.6057	6.068%	0.032%	13.00	0.065%
22	Aniara	60	0.0571	1.177%	1.0954	41.983%	1.5455	6.4784	6.383%	0.000%	41.97	0.000%
23	Antares	60	0.0512	-1.846%	1.3312	57.074%	-2.7033	8.7816	0.450%	0.000%	77.12	0.000%
24	Arcturus	60	0.0422	-0.351%	0.9055	47.504%	-0.6228	7.2446	26.794%	0.000%	52.48	0.000%
25	Aspidiske	60	0.0618	-0.113%	1.2338	43.988%	-0.1378	6.7491	44.545%	0.000%	45.55	0.000%
26	Atlas	60	0.0568	0.707%	1.2955	50.592%	0.9332	7.7064	17.728%	0.000%	59.39	0.000%
27	Bellatrix	60	0.0711	-0.370%	0.6549	14.313%	-0.3906	3.1126	34.877%	0.144%	9.69	0.288%
28	Betelgeuse	60	0.0500	0.563%	0.3724	9.847%	0.8440	2.5169	20.106%	0.731%	6.33	1.463%
29	Bharani	60	0.0467	1.978%	0.1572	2.187%	3.1821	1.1387	0.117%	12.976%	1.30	25.953%
30	Canopus	60	0.0591	-0.606%	0.6070	17.202%	-0.7694	3.4713	22.239%	0.049%	12.05	0.098%
31	Capella	60	0.0460	0.438%	0.4071	13.347%	0.7132	2.9890	23.928%	0.205%	8.93	0.410%
32	Castor	60	0.0422	0.692%	0.5777	27.006%	1.2314	4.6323	11.156%	0.001%	21.46	0.002%
33	Copernicus	60	0.0447	0.596%	0.4548	16.948%	1.0013	3.4404	16.043%	0.054%	11.84	0.108%
34	Deneb	60	0.0544	1.446%	0.2884	5.240%	1.9937	1.7908	2.545%	3.927%	3.21	7.854%
35	Diadem	60	0.0539	0.740%	0.8342	32.052%	1.0310	5.2307	15.341%	0.000%	27.36	0.000%
36	Dubhe	60	0.1609	1.413%	2.2635	28.040%	0.6588	4.7540	25.631%	0.001%	22.60	0.001%
37	Electra	60	0.0506	1.423%	0.2513	4.626%	2.1084	1.6773	1.966%	4.944%	2.81	9.887%
38	Furud	60	0.0605	0.466%	0.7537	23.430%	0.5779	4.2128	28.280%	0.004%	17.75	0.009%
39	Gienah	60	0.0446	0.442%	0.9949	49.484%	0.7437	7.5376	23.004%	0.000%	56.82	0.000%
40	Hamal	60	0.0461	0.831%	1.0935	52.600%	1.3541	8.0226	9.048%	0.000%	64.36	0.000%
41	Intercrus	60	0.0561	-0.043%	1.2028	47.533%	-0.0573	7.2488	47.727%	0.000%	52.55	0.000%
42	Izar	60	0.0639	-1.044%	0.5430	12.446%	-1.2257	2.8714	11.263%	0.285%	8.25	0.570%
43	Jabbah	60	0.0484	1.311%	0.9910	45.213%	2.0324	6.9185	2.335%	0.000%	47.87	0.000%
44	Kang	60	0.0517	0.226%	0.8021	32.121%	0.3279	5.2389	37.208%	0.000%	27.45	0.000%
45	Keid	60	0.1120	3.919%	1.4478	24.765%	2.6260	4.3694	0.552%	0.003%	19.09	0.005%
46	Kitalpha	60	0.0483	0.594%	0.7123	29.975%	0.9229	4.9827	17.994%	0.000%	24.83	0.001%
47	Kochab	60	0.0485	1.941%	0.9716	44.195%	3.0060	6.7774	0.195%	0.000%	45.93	0.000%
48	Kraz	60	0.0993	-0.234%	1.6543	35.367%	-0.1768	5.6336	43.014%	0.000%	31.74	0.000%
49	Libertas	60	0.0684	0.782%	1.1719	36.647%	0.8581	5.7923	19.717%	0.000%	33.55	0.000%
50	Markab	60	0.0490	-0.780%	1.4111	62.021%	-1.1945	9.7322	11.857%	0.000%	94.72	0.000%

Table 6b. 30-Year, 10-Year, and 5-Year Stationary β Data

30Y period (1991-2020)

#	Conventional Name	Obs.	Standard Error	Intercept	Beta	R^2	Intercep t t-stat	Beta t-stat	Intercept P-Value	Beta P-Value	F	F P-Value
51	Matar	360	0.0863	0.757%	1.4257	32.562%	1.637	13.148	5.124%	0.000%	172.86	0.000%
52	Merak	347	0.1344	1.588%	1.4069	16.179%	2.168	8.160	1.542%	0.000%	66.59	0.000%
53	Merga	360	0.0672	0.717%	0.9751	27.150%	1.994	11.551	2.348%	0.000%	133.42	0.000%
54	Merope	130	0.0632	1.739%	0.9063	25.553%	3.043	6.628	0.142%	0.000%	43.93	0.000%
55	Mesarthim	360	0.0634	0.671%	1.0176	31.316%	1.976	12.776	2.449%	0.000%	163.23	0.000%
56	Mimosa	360	0.0476	0.504%	0.6966	27.505%	1.980	11.655	2.426%	0.000%	135.83	0.000%
57	Mira	360	0.0470	0.716%	0.3133	7.298%	2.847	5.309	0.233%	0.000%	28.18	0.000%
58	Mizar	152	0.0531	0.254%	0.8700	36.521%	0.583	9.290	28.050%	0.000%	86.30	0.000%
59	Musica	360	0.0685	0.683%	0.5817	11.305%	1.861	6.755	3.176%	0.000%	45.63	0.000%
60	Nashira	232	0.0539	0.939%	1.1103	44.008%	2.627	13.445	0.460%	0.000%	180.78	0.000%
61	Nekkar	360	0.0511	0.288%	0.8261	31.595%	1.051	12.859	14.707%	0.000%	165.35	0.000%
62	Nembus	229	0.0696	1.023%	0.7440	17.217%	2.200	6.871	1.442%	0.000%	47.21	0.000%
63	Okab	360	0.0600	0.877%	1.0198	33.802%	2.729	13.520	0.334%	0.000%	182.80	0.000%
64	Polaris	233	0.0614	0.528%	0.6040	15.281%	1.300	6.455	9.751%	0.000%	41.67	0.000%
65	Pollux	163	0.1105	0.710%	0.7097	7.902%	0.812	3.717	20.906%	0.014%	13.81	0.028%
66	Procyon	101	0.0854	2.263%	1.1382	21.034%	2.567	5.135	0.588%	0.000%	26.37	0.000%
67	Proxima Centauri	245	0.1477	2.204%	1.2097	11.406%	2.321	5.593	1.056%	0.000%	31.28	0.000%
68	Ran	360	0.0510	0.507%	0.6200	20.730%	1.854	9.676	3.229%	0.000%	93.62	0.000%
69	Regulus	360	0.0722	1.079%	1.2087	33.119%	2.787	13.315	0.280%	0.000%	177.28	0.000%
70	Rigel	360	0.1042	1.385%	1.2052	19.121%	2.480	9.200	0.681%	0.000%	84.64	0.000%
71	Rukbat	360	0.0568	0.263%	0.6740	19.947%	0.864	9.445	19.405%	0.000%	89.20	0.000%
72	Sabik	360	0.1377	1.393%	1.9370	25.914%	1.888	11.190	2.994%	0.000%	125.22	0.000%
73	Salm	360	0.0971	0.986%	1.7860	37.421%	1.894	14.631	2.950%	0.000%	214.08	0.000%
74	Sceptrum	360	0.1069	-0.020%	1.5111	26.101%	-0.035	11.245	48.595%	0.000%	126.45	0.000%
75	Segin	282	0.1539	2.933%	1.7958	21.358%	3.170	8.720	0.085%	0.000%	76.05	0.000%
76	Sheratan	94	0.0693	0.741%	0.9954	24.581%	1.001	5.476	15.975%	0.000%	29.99	0.000%
77	Sirius	360	0.0727	0.664%	0.9925	24.789%	1.703	10.863	4.468%	0.000%	118.00	0.000%
78	Spica	360	0.0768	1.295%	0.8508	17.817%	3.144	8.810	0.090%	0.000%	77.61	0.000%
79	Subra	94	0.0485	1.173%	0.7204	25.828%	2.263	5.660	1.299%	0.000%	32.04	0.000%
80	Syrma	360	0.1126	1.521%	1.2724	18.417%	2.520	8.990	0.609%	0.000%	80.82	0.000%
81	Tabit	360	0.0653	1.024%	0.5645	11.664%	2.924	6.875	0.184%	0.000%	47.27	0.000%
82	Talitha	360	0.0566	0.534%	0.7879	25.554%	1.762	11.085	3.945%	0.000%	122.89	0.000%
83	Tarazed	360	0.1644	1.033%	2.2116	24.241%	1.172	10.703	12.094%	0.000%	114.55	0.000%
84	Tarf	360	0.0986	0.714%	1.0518	16.760%	1.352	8.490	8.857%	0.000%	72.08	0.000%
85	Tejat	360	0.0777	1.497%	0.8663	18.007%	3.593	8.867	0.019%	0.000%	78.62	0.000%
86	Thuban	360	0.0715	0.912%	1.2317	34.419%	2.380	13.707	0.891%	0.000%	187.89	0.000%
87	Tiaki	360	0.0540	0.675%	0.6476	20.255%	2.331	9.536	1.015%	0.000%	90.93	0.000%
88	Timir	300	0.0673	0.864%	0.9694	28.858%	2.195	10.995	1.448%	0.000%	120.88	0.000%
89	Titawin	360	0.0641	0.091%	1.0479	32.115%	0.265	13.014	39.569%	0.000%	169.36	0.000%
90	Toliman	125	0.1757	4.035%	1.5353	10.788%	2.478	3.857	0.728%	0.009%	14.87	0.018%
91	Tonatiuh	360	0.0865	1.090%	1.4830	34.195%	2.351	13.639	0.964%	0.000%	186.03	0.000%
92	Tureis	332	0.1132	1.418%	1.1613	16.109%	2.248	7.960	1.261%	0.000%	63.37	0.000%
93	Uruk	360	0.0615	0.549%	1.2232	41.154%	1.664	15.823	4.846%	0.000%	250.36	0.000%
94	Vega	222	0.1610	3.493%	1.0057	6.628%	3.190	3.952	0.082%	0.005%	15.62	0.010%
95	Veritate	360	0.0663	0.563%	0.9456	26.481%	1.587	11.356	5.669%	0.000%	128.95	0.000%
96	Wezen	360	0.0515	0.780%	0.5603	17.288%	2.825	8.650	0.250%	0.000%	74.83	0.000%
97	Xihe	360	0.0677	0.182%	1.0389	29.420%	0.501	12.216	30.845%	0.000%	149.22	0.000%
98	Yildun	352	0.1541	1.552%	1.0335	7.408%	1.861	5.292	3.178%	0.000%	28.00	0.000%
99	Zaurak	360	0.0556	1.136%	0.9066	31.975%	3.813	12.972	0.008%	0.000%	168.28	0.000%
100	Zibal	360	0.1438	1.201%	1.6904	19.648%	1.559	9.356	5.999%	0.000%	87.54	0.000%

10Y period (2011-2020)

#	Conventional Name	Obs.	Standard Error	Intercept	Beta	R^2	Intercep t t-stat	Beta t-stat	Intercept P-Value	Beta P-Value	F	F P-Value
51	Matar	120	0.0407	0.614%	1.1515	55.153%	1.599	12.047	5.621%	0.000%	145.12	0.000%
52	Merak	120	0.0802	0.266%	1.3388	30.025%	0.353	7.116	36.253%	0.000%	50.63	0.000%
53	Merga	120	0.0652	0.585%	0.8147	19.364%	0.952	5.323	17.145%	0.000%	28.34	0.000%
54	Merope	120	0.0628	1.721%	0.9272	25.144%	2.910	6.296	0.216%	0.000%	39.64	0.000%
55	Mesarthim	120	0.0531	0.398%	1.2312	45.256%	0.795	9.877	21.410%	0.000%	97.55	0.000%
56	Mimosa	120	0.0335	0.459%	0.5567	29.808%	1.454	7.079	7.426%	0.000%	50.11	0.000%
57	Mira	120	0.0385	0.650%	0.2327	5.316%	1.792	2.574	3.785%	0.565%	6.63	1.129%
58	Mizar	120	0.0551	-0.009%	0.9031	29.290%	-0.018	6.991	49.287%	0.000%	48.88	0.000%
59	Musica	120	0.0515	1.445%	0.2759	4.237%	2.980	2.285	0.175%	1.205%	5.22	2.411%
60	Nashira	120	0.0373	0.653%	1.0973	57.110%	1.856	12.535	3.295%	0.000%	157.12	0.000%
61	Nekkar	120	0.0478	-0.744%	1.2405	50.877%	-1.651	11.055	5.069%	0.000%	122.21	0.000%
62	Nembus	120	0.0621	0.986%	0.8451	22.187%	1.685	5.801	4.735%	0.000%	33.65	0.000%
63	Okab	120	0.0382	1.069%	0.9722	49.981%	2.974	10.859	0.178%	0.000%	117.91	0.000%
64	Polaris	120	0.0665	0.554%	0.7515	16.438%	0.884	4.818	18.919%	0.000%	23.21	0.000%
65	Pollux	120	0.0962	1.323%	0.7198	7.937%	1.460	3.189	7.350%	0.091%	10.17	0.183%
66	Procyon	101	0.0854	2.263%	1.1382	21.034%	2.567	5.135	0.588%	0.000%	26.37	0.000%
67	Proxima Centauri	120	0.0664	1.289%	0.8889	21.649%	2.062	5.710	2.070%	0.000%	32.60	0.000%
68	Ran	120	0.0360	0.220%	0.5614	27.296%	0.649	6.656	25.865%	0.000%	44.30	0.000%
69	Regulus	120	0.0471	1.240%	0.8686	34.339%	2.793	7.856	0.305%	0.000%	61.71	0.000%
70	Rigel	120	0.0439	-0.022%	0.9077	39.653%	-0.052	8.805	47.914%	0.000%	77.53	0.000%
71	Rukbat	120	0.0433	0.019%	0.5451	19.612%	0.047	5.365	48.136%	0.000%	28.79	0.000%
72	Sabik	120	0.0734	1.020%	1.3288	33.532%	1.475	7.715	7.139%	0.000%	59.53	0.000%
73	Salm	120	0.0687	0.500%	1.6191	46.097%	0.772	10.046	22.071%	0.000%	100.91	0.000%
74	Sceptrum	120	0.0626	-1.148%	1.2654	38.610%	-1.946	8.615	2.703%	0.000%	74.21	0.000%
75	Segin	120	0.0690	1.666%	1.1007	28.177%	2.564	6.804	0.580%	0.000%	46.29	0.000%
76	Sheratan	94	0.0693	0.741%	0.9954	24.581%	1.001	5.476	15.975%	0.000%	29.99	0.000%
77	Sirius	120	0.0439	0.586%	1.0731	47.947%	1.417	10.426	7.955%	0.000%	108.69	0.000%
78	Spica	120	0.0447	0.985%	0.7983	32.907%	2.337	7.608	1.055%	0.000%	57.88	0.000%
79	Subra	94	0.0485	1.173%	0.7204	25.828%	2.263	5.660	1.299%	0.000%	32.04	0.000%
80	Syrma	120	0.0654	1.353%	1.1455	32.107%	2.197	7.470	1.497%	0.000%	55.80	0.000%
81	Tabit	120	0.0427	0.989%	0.7906	34.572%	2.461	7.896	0.766%	0.000%	62.35	0.000%
82	Talitha	120	0.0440	0.364%	0.7512	30.969%	0.879	7.276	19.073%	0.000%	52.94	0.000%
83	Tarazed	120	0.1378	1.112%	2.2281	28.706%	0.857	6.893	19.659%	0.000%	47.51	0.000%
84	Tarf	120	0.0780	-1.017%	1.8507	46.420%	-1.384	10.111	8.447%	0.000%	102.23	0.000%
85	Tejat	120	0.0479	1.499%	0.6982	24.616%	3.318	6.207	0.060%	0.000%	38.53	0.000%
86	Thuban	120	0.0607	0.143%	1.2282	38.703%	0.251	8.632	40.114%	0.000%	74.50	0.000%
87	Tiaki	120	0.0358	0.376%	0.5418	26.065%	1.114	6.450	13.380%	0.000%	41.60	0.000%
88	Timir	120	0.0525	0.967%	0.9157	31.937%	1.957	7.441	2.634%	0.000%	55.37	0.000%
89	Titawin	120	0.0464	-0.651%	0.9858	41.057%	-1.491	9.066	6.935%	0.000%	82.19	0.000%
90	Toliman	120	0.1669	3.777%	1.7534	14.530%	2.403	4.479	0.892%	0.001%	20.06	0.002%
91	Tonatiuh	120	0.0584	0.102%	1.0572	33.508%	0.185	7.711	42.686%	0.000%	59.46	0.000%
92	Tureis	120	0.0422	0.952%	1.0176	47.214%	2.393	10.274	0.915%	0.000%	105.55	0.000%
93	Uruk	120	0.0288	0.395%	1.1379	70.552%	1.454	16.814	7.424%	0.000%	282.71	0.000%
94	Vega	120	0.1610	2.725%	1.1554	7.352%	1.797	3.060	3.746%	0.137%	9.36	0.274%
95	Veritate	120	0.0512	0.288%	1.2933	49.603%	0.598	10.777	27.545%	0.000%	116.14	0.000%
96	Wezen	120	0.0405	0.797%	0.8449	40.076%	2.087	8.883	1.952%	0.000%	78.91	0.000%
97	Xihe	120	0.0720	-1.015%	1.5170	40.587%	-1.495	8.978	6.873%	0.000%	80.61	0.000%
98	Yildun	120	0.1132	1.568%	0.6708	5.127%	1.470	2.525	7.216%	0.644%	6.38	1.289%
99	Zaurak	120	0.0377	0.807%	0.8993	46.707%	2.271	10.169	1.247%	0.000%	103.42	0.000%
100	Zibal	120	0.1063	1.021%	1.6364	26.722%	1.019	6.560	15.509%	0.000%	43.03	0.000%

5Y period (2016-2020)

#	Conventional Name	Obs.	Standard Error	Intercept	Beta	R^2	Intercept t-stat	Beta t-stat	Intercept P-Value	Beta P-Value	F	F P-Value
51	Matar	60	0.0415	1.071%	1.0691	56.653%	1.9371	8.7065	2.880%	0.000%	75.80	0.000%
52	Merak	60	0.1009	1.240%	1.3796	26.913%	0.9226	4.6214	18.003%	0.001%	21.36	0.002%
53	Merga	60	0.0755	1.040%	0.9605	24.166%	1.0339	4.2992	15.274%	0.003%	18.48	0.007%
54	Merope	60	0.0588	1.306%	1.0029	36.457%	1.6674	5.7686	5.042%	0.000%	33.28	0.000%
55	Mesarthim	60	0.0599	1.240%	1.1698	42.916%	1.5540	6.6034	6.282%	0.000%	43.60	0.000%
56	Mimosa	60	0.0361	0.349%	0.5863	34.202%	0.7261	5.4908	23.535%	0.000%	30.15	0.000%
57	Mira	60	0.0413	0.283%	0.2919	8.965%	0.5144	2.3899	30.446%	1.006%	5.71	2.012%
58	Mizar	60	0.0635	-0.323%	0.8158	24.559%	-0.3816	4.3453	35.206%	0.003%	18.88	0.006%
59	Musica	60	0.0629	1.161%	0.3649	6.220%	1.3851	1.9614	8.566%	2.732%	3.85	5.464%
60	Nashira	60	0.0359	0.646%	1.0870	64.417%	1.3529	10.2468	9.067%	0.000%	105.00	0.000%
61	Nekkar	60	0.0553	-0.901%	1.3068	52.372%	-1.2229	7.9861	11.316%	0.000%	63.78	0.000%
62	Nembus	60	0.0613	0.662%	1.0146	35.035%	0.8108	5.5927	21.039%	0.000%	31.28	0.000%
63	Okab	60	0.0396	0.393%	1.0284	57.015%	0.7452	8.7711	22.958%	0.000%	76.93	0.000%
64	Polaris	60	0.0421	0.027%	0.6360	30.988%	0.0473	5.1033	48.121%	0.000%	26.04	0.000%
65	Pollux	60	0.0537	1.636%	0.5474	16.965%	2.2847	3.4424	1.300%	0.054%	11.85	0.108%
66	Procyon	60	0.0775	2.266%	1.1488	30.211%	2.1947	5.0108	1.610%	0.000%	25.11	0.001%
67	Proxima Centauri	60	0.0529	1.667%	0.9768	40.211%	2.3671	6.2457	1.064%	0.000%	39.01	0.000%
68	Ran	60	0.0369	0.130%	0.5869	33.247%	0.2642	5.3747	39.628%	0.000%	28.89	0.000%
69	Regulus	60	0.0386	1.714%	0.8227	47.196%	3.3310	7.2000	0.075%	0.000%	51.84	0.000%
70	Rigel	60	0.0422	0.392%	0.7529	38.579%	0.6984	6.0357	24.387%	0.000%	36.43	0.000%
71	Rukbat	60	0.0477	-0.441%	0.6937	29.387%	-0.6933	4.9131	24.544%	0.000%	24.14	0.001%
72	Sabik	60	0.0816	2.143%	1.3511	35.044%	1.9695	5.5938	2.684%	0.000%	31.29	0.000%
73	Salm	60	0.0774	1.540%	1.4767	41.742%	1.4928	6.4464	7.046%	0.000%	41.56	0.000%
74	Sceptrum	60	0.0667	-1.341%	1.2510	40.894%	-1.5074	6.3348	6.857%	0.000%	40.13	0.000%
75	Segin	60	0.0643	1.647%	1.1986	40.634%	1.9219	6.3007	2.977%	0.000%	39.70	0.000%
76	Sheratan	60	0.0781	0.795%	0.8289	18.141%	0.7633	3.5852	22.419%	0.035%	12.85	0.069%
77	Sirius	60	0.0485	0.279%	1.0170	46.439%	0.4326	7.0914	33.345%	0.000%	50.29	0.000%
78	Spica	60	0.0485	0.364%	0.9332	42.204%	0.5644	6.5080	28.733%	0.000%	42.35	0.000%
79	Subra	60	0.0446	1.529%	0.6762	31.147%	2.5712	5.1222	0.636%	0.000%	26.24	0.000%
80	Syrma	60	0.0666	1.803%	1.2814	42.165%	2.0313	6.5027	2.341%	0.000%	42.28	0.000%
81	Tabit	60	0.0452	0.211%	0.9043	44.054%	0.3493	6.7581	36.406%	0.000%	45.67	0.000%
82	Talitha	60	0.0525	0.027%	0.6884	25.329%	0.0385	4.4356	48.471%	0.002%	19.67	0.004%
83	Tarazed	60	0.1415	4.806%	2.2839	33.897%	2.5480	5.4536	0.675%	0.000%	29.74	0.000%
84	Tarf	60	0.0814	-1.930%	2.1077	56.906%	-1.7793	8.7515	4.022%	0.000%	76.59	0.000%
85	Tejat	60	0.0501	1.312%	0.7445	30.295%	1.9646	5.0208	2.713%	0.000%	25.21	0.001%
86	Thuban	60	0.0599	1.374%	0.9764	34.389%	1.7231	5.5137	4.510%	0.000%	30.40	0.000%
87	Tiaki	60	0.0404	0.073%	0.5836	29.172%	0.1361	4.8876	44.611%	0.000%	23.89	0.001%
88	Timir	60	0.0487	1.178%	0.8610	38.149%	1.8175	5.9811	3.716%	0.000%	35.77	0.000%
89	Titawin	60	0.0478	-0.896%	1.2283	56.517%	-1.4059	8.6825	8.255%	0.000%	75.39	0.000%
90	Toliman	60	0.1692	3.713%	2.1904	24.825%	1.6471	4.3765	5.248%	0.003%	19.15	0.005%
91	Tonatiuh	60	0.0577	0.290%	0.9518	34.876%	0.3769	5.5732	35.383%	0.000%	31.06	0.000%
92	Tureis	60	0.0428	1.468%	0.9868	51.125%	2.5732	7.7892	0.633%	0.000%	60.67	0.000%
93	Uruk	60	0.0317	0.384%	1.1173	70.966%	0.9096	11.9065	18.340%	0.000%	141.76	0.000%
94	Vega	60	0.1001	2.128%	0.9243	14.389%	1.5961	3.1222	5.795%	0.140%	9.75	0.280%
95	Veritate	60	0.0463	0.678%	1.3695	63.275%	1.0991	9.9966	13.813%	0.000%	99.93	0.000%
96	Wezen	60	0.0458	0.905%	0.8275	39.093%	1.4819	6.1014	7.189%	0.000%	37.23	0.000%
97	Xihe	60	0.0834	-1.483%	1.8848	50.126%	-1.3340	7.6350	9.371%	0.000%	58.29	0.000%
98	Yildun	60	0.0895	0.644%	0.7812	13.059%	0.5401	2.9515	29.558%	0.228%	8.71	0.456%
99	Zaurak	60	0.0433	1.309%	0.7254	35.564%	2.2673	5.6578	1.356%	0.000%	32.01	0.000%
100	Zibal	60	0.1036	1.991%	1.3632	25.430%	1.4424	4.4474	7.728%	0.002%	19.78	0.004%

Table 7. 5-Year Monthly Rolling β in 1992-2020 Timespan

Conventional Name	Obs.	Standard Error	Intercept	Beta	R^2	Intercept t-stat	Beta t-stat	Intercept P-Value	Beta P-Value	F	F P-Value
Achernar	217	0.0559	0.656%	1.1252	35.844%	1.0487	5.8712	21.243%	5.500%	47.79	11.000%
Adhil	282	0.0845	1.322%	0.9804	20.936%	1.2616	3.8742	16.406%	0.720%	17.22	1.441%
Albaldah	43	0.0716	1.729%	0.8975	19.535%	1.7051	3.6332	6.910%	2.324%	16.94	4.647%
Albali	348	0.0519	0.760%	0.3593	10.379%	1.2418	2.2487	14.332%	11.032%	7.60	22.065%
Alchiba	348	0.0636	0.368%	0.8659	25.307%	0.4798	4.4338	22.703%	0.298%	22.09	0.597%
Alcor	189	0.0720	1.589%	0.5644	9.193%	1.6992	2.3604	7.177%	2.682%	6.03	5.364%
Alcyone	348	0.0452	0.596%	0.6945	28.880%	1.0010	4.8376	18.137%	1.549%	27.63	3.098%
Aldebaran	348	0.0661	0.674%	0.8358	24.682%	0.9654	4.3037	17.959%	1.658%	20.79	3.316%
Alderamin	348	0.0705	0.883%	1.0871	28.397%	1.0044	4.7843	20.018%	0.749%	25.35	1.498%
Aldhanab	348	0.0809	1.049%	1.3768	32.628%	0.9513	5.3719	18.322%	0.029%	31.45	0.058%
Algol	348	0.0675	0.877%	0.7249	23.080%	1.2083	3.8510	17.995%	5.959%	23.80	11.918%
Algorab	296	0.2394	2.750%	1.2972	8.224%	0.9126	2.1089	22.184%	7.369%	5.42	14.737%
Alhena	348	0.0586	0.780%	0.8312	25.519%	0.9003	4.4365	21.891%	1.351%	22.71	2.703%
Alioth	193	0.0441	0.189%	0.8326	36.167%	0.3353	5.8357	31.915%	0.041%	37.23	0.082%
Alkarab	348	0.0549	0.611%	0.6419	18.693%	0.8372	3.6052	20.896%	3.197%	18.30	6.394%
Almaaz	348	0.0660	0.675%	1.0417	31.717%	1.0283	5.2074	13.990%	0.436%	29.43	0.872%
Alnair	282	0.0526	0.609%	0.9455	39.629%	1.0047	6.2948	17.336%	0.001%	42.31	0.002%
Alpheratz	348	0.0483	0.619%	0.5492	20.665%	0.9980	3.6303	18.559%	4.622%	17.58	9.243%
Alrakis	348	0.0798	1.118%	1.1530	27.167%	1.0025	4.6357	19.686%	0.672%	23.72	1.344%
Altair	348	0.0670	0.635%	1.0624	29.423%	0.7265	5.0624	21.919%	0.124%	31.07	0.247%
Alzirr	348	0.0491	0.006%	1.2113	50.157%	0.1803	7.9884	22.122%	0.023%	69.78	0.046%
Aniara	76	0.0752	2.227%	0.9788	16.842%	2.1884	3.4078	2.395%	0.209%	12.34	0.418%
Antares	348	0.0409	0.403%	0.6360	27.359%	0.7121	4.7111	13.737%	0.031%	24.17	0.062%
Arcturus	348	0.0458	0.461%	0.8011	35.403%	0.7637	5.9038	22.807%	0.083%	42.14	0.166%
Aspidiske	201	0.1305	2.391%	1.7473	23.427%	1.4047	4.1785	16.887%	0.585%	18.87	1.171%
Atlas	348	0.0888	0.836%	1.6605	38.505%	0.5981	6.1398	22.720%	0.009%	40.44	0.017%
Bellatrix	348	0.0592	0.369%	0.7662	22.933%	0.5372	4.1405	23.064%	0.768%	20.82	1.536%
Betelgeuse	348	0.0467	0.964%	0.1670	7.142%	1.5490	1.2979	9.225%	12.461%	4.81	24.922%
Bharani	348	0.0491	0.979%	0.3556	11.902%	1.5518	2.4595	9.892%	10.410%	9.16	20.820%
Canopus	348	0.1046	1.400%	0.8228	14.048%	1.0983	2.9237	17.717%	3.791%	11.37	7.581%
Capella	348	0.0553	0.426%	0.6835	19.956%	0.6302	3.7639	26.661%	0.552%	15.36	1.104%
Castor	348	0.0488	0.683%	0.7873	28.602%	1.2553	4.8475	18.433%	0.022%	24.98	0.045%
Copernicus	348	0.0626	0.412%	0.7723	21.485%	0.5715	3.9354	26.389%	1.004%	18.10	2.007%
Deneb	184	0.1211	2.147%	1.5888	24.726%	1.5743	4.3511	8.781%	0.741%	22.65	1.483%
Diadem	348	0.0751	1.133%	0.9223	21.251%	1.2846	3.9344	13.752%	0.297%	17.44	0.595%
Dubhe	245	0.1288	0.420%	1.7845	25.742%	0.3616	4.5136	14.848%	0.102%	23.17	0.203%
Electra	214	0.1067	1.664%	1.1868	18.713%	1.5827	3.6052	9.927%	0.708%	14.25	1.415%
Furud	348	0.0839	1.250%	0.8985	15.975%	1.0332	3.2696	19.443%	0.662%	11.83	1.324%
Gienah	136	0.0650	0.905%	1.0461	30.917%	1.0509	5.1219	16.702%	0.004%	27.23	0.007%
Hamal	348	0.0544	0.661%	0.9047	32.327%	0.9914	5.3825	19.478%	0.389%	34.45	0.778%
Intercrus	348	0.0541	0.362%	1.1267	42.582%	0.6330	6.9122	26.631%	0.002%	54.61	0.004%
Izar	287	0.1044	1.616%	0.9963	13.727%	1.2062	3.0046	14.521%	0.611%	9.56	1.222%
Jabbah	348	0.0554	0.980%	0.9464	31.565%	1.4013	5.2034	13.146%	0.017%	28.50	0.033%
Kang	348	0.0562	0.726%	0.7116	25.307%	1.0260	4.3726	18.983%	3.319%	24.35	6.638%
Keid	203	0.1338	2.092%	2.1207	27.444%	1.2291	4.6918	18.166%	0.021%	22.77	0.043%
Kitalpha	348	0.0458	0.544%	0.8337	35.885%	0.9386	5.8122	17.842%	0.063%	36.98	0.126%
Kochab	348	0.1087	1.350%	1.4703	29.915%	1.0725	5.0929	21.253%	0.591%	31.96	1.181%
Kraz	348	0.0658	0.579%	0.9986	28.841%	0.7232	4.9065	24.264%	0.253%	27.89	0.506%
Libertas	138	0.0820	1.689%	1.3602	32.193%	1.4553	5.2710	11.687%	0.007%	28.87	0.015%
Markab	348	0.0457	0.453%	1.1095	49.540%	0.7078	7.9857	20.412%	0.000%	71.14	0.000%
Matar	348	0.0826	0.660%	1.5256	38.831%	0.6544	6.1762	25.623%	0.009%	40.72	0.017%
Merak	288	0.1153	1.376%	1.4138	25.751%	0.6077	4.4822	20.718%	0.372%	22.27	0.743%
Merga	348	0.0651	0.643%	0.9656	27.460%	0.6954	4.6965	26.223%	0.441%	25.29	0.881%
Merope	71	0.0624	1.530%	1.0403	24.122%	1.8037	4.2905	6.498%	0.072%	19.63	0.144%
Mesarthim	348	0.0610	0.558%	1.0784	34.768%	0.7569	5.7345	18.405%	0.168%	38.53	0.336%
Mimosa	348	0.0454	0.443%	0.7636	29.799%	0.8342	4.9906	18.789%	0.212%	27.11	0.425%
Mira	348	0.0460	0.813%	0.3135	9.170%	1.3476	2.2303	12.705%	6.407%	6.59	12.815%
Mizar	93	0.0504	0.106%	0.9385	31.585%	0.2024	5.2100	33.048%	0.022%	28.53	0.043%
Musica	348	0.0635	0.586%	0.6649	18.645%	0.6971	3.5104	22.174%	4.157%	17.64	8.314%
Nashira	173	0.0452	0.770%	1.0305	43.580%	1.2963	6.7820	12.710%	0.000%	47.68	0.001%
Nekkar	348	0.0489	0.480%	0.7714	28.543%	0.7177	4.8867	17.671%	0.051%	27.01	0.103%
Nembus	170	0.0678	0.887%	0.7900	18.689%	1.0921	3.5333	19.322%	2.714%	14.32	5.428%
Okab	348	0.0600	1.066%	1.0580	34.568%	1.3955	5.6067	15.103%	0.071%	34.18	0.141%
Polaris	174	0.0618	0.428%	0.7198	18.508%	0.4624	3.5964	29.389%	0.297%	13.77	0.594%
Pollux	104	0.1068	1.176%	0.6687	6.022%	1.0695	1.8266	19.261%	7.142%	3.80	14.283%
Procyon	42	0.0829	1.874%	1.2751	22.223%	1.7171	4.0499	5.735%	0.152%	17.58	0.303%
Proxima Centauri	186	0.1159	2.631%	1.1803	18.936%	1.6249	3.5616	11.138%	2.291%	15.78	4.581%
Ran	348	0.0488	0.510%	0.6746	25.406%	0.9231	4.4260	22.642%	0.725%	22.05	1.450%
Regulus	348	0.0714	1.172%	1.1932	32.484%	1.2186	5.3244	19.004%	0.030%	30.12	0.060%
Rigel	348	0.0977	1.390%	1.3384	30.696%	0.8857	5.1357	20.430%	0.589%	30.79	1.179%
Rukbat	348	0.0548	0.435%	0.6874	21.009%	0.7026	3.9024	24.414%	0.383%	16.99	0.767%
Sabik	348	0.1257	1.219%	2.0085	32.960%	0.7814	5.4084	21.310%	0.312%	32.47	0.624%
Salm	348	0.0933	0.760%	1.8876	40.039%	0.4761	6.3242	24.788%	0.005%	42.23	0.011%
Sceptrum	348	0.0946	0.223%	1.4318	28.321%	-0.0195	4.7897	24.398%	0.099%	23.94	0.199%
Segin	224	0.1155	2.337%	1.6492	25.949%	1.6047	4.5147	8.667%	0.036%	21.35	0.072%
Sheratan	35	0.0676	0.499%	1.1247	25.104%	0.5553	4.4260	29.485%	0.052%	21.56	0.103%
Sirius	348	0.0715	0.564%	1.0794	28.868%	0.6422	4.8979	27.712%	0.115%	26.67	0.230%
Spica	342	0.0747	1.129%	0.8854	19.423%	1.2774	3.7084	16.143%	0.469%	14.65	0.937%
Subra	35	0.0478	1.389%	0.8645	27.967%	2.2018	4.7485	2.718%	0.002%	22.76	0.003%
Syrma	348	0.1082	1.527%	1.2958	22.235%	1.2214	4.0003	16.396%	1.645%	18.16	3.290%
Tabit	348	0.0612	0.934%	0.5764	21.204%	1.3672	3.4472	16.069%	5.121%	19.89	10.242%
Talitha	348	0.0542	0.506%	0.8438	28.079%	0.6333	4.7728	21.314%	0.135%	24.90	0.270%
Tarazed	348	0.1615	0.733%	2.2164	25.017%	0.2925	4.3359	26.392%	1.869%	21.50	3.738%
Tarf	314	0.0948	1.036%	0.8772	15.001%	0.7790	3.0533	19.794%	3.683%	12.21	7.366%
Tejat	348	0.0752	1.497%	0.9798	20.656%	1.6876	3.7916	11.575%	2.289%	16.62	4.578%
Thuban	348	0.0698	0.913%	1.2729	34.971%	0.9838	5.8047	17.756%	0.030%	39.43	0.060%
Tiaki	348	0.0507	0.607%	0.7080	24.754%	0.9515	4.3244	19.993%	1.528%	21.72	3.056%
Timir	241	0.0654	0.863%	0.9191	27.145%	1.0490	4.6910	20.263%	0.327%	25.59	0.654%
Titawin	348	0.0606	0.241%	1.0127	32.439%	0.2526	5.3247	18.810%	0.447%	31.81	0.894%
Toliman	66	0.1498	2.791%	0.9999	5.312%	1.3050	1.6924	13.109%	7.867%	3.45	15.735%
Tonatiuh	310	0.0783	1.046%	1.4958	38.535%	0.8000	6.1286	22.509%	0.027%	40.29	0.053%
Tureis	273	0.0956	1.216%	1.1545	25.360%	1.1369	4.4485	17.057%	0.160%	21.91	0.319%
Uruk	348	0.0555	0.557%	1.2027	48.451%	0.9589	8.1030	20.235%	0.003%	78.12	0.006%
Vega	163	0.1612	3.432%	1.1300	6.927%	1.6601	2.0026	7.695%	4.541%	4.45	9.082%
Veritate	348	0.0624	0.512%	0.9834	32.184%	0.6464	5.3338	21.121%	0.473%	33.33	0.946%
Wezen	348	0.0495	0.589%	0.6557	23.353%	0.9018	4.1457	18.761%	2.677%	22.93	5.354%
Xihe	348	0.0633	0.469%	0.9158	25.809%	0.5932	4.5317	23.123%	0.184%	23.48	0.369%
Yildun	293	0.1525	1.623%	1.1293	9.438%	0.7834	2.2965	23.375%	5.735%	6.52	11.470%
Zaurak	348	0.0546	1.010%	0.9792	39.452%	1.4492	6.5284	11.029%	0.035%	50.95	0.070%
Zibal	348	0.1445	1.077%	1.7517	20.601%	0.4965	3.8477	25.557%	0.502%	16.47	1.004%

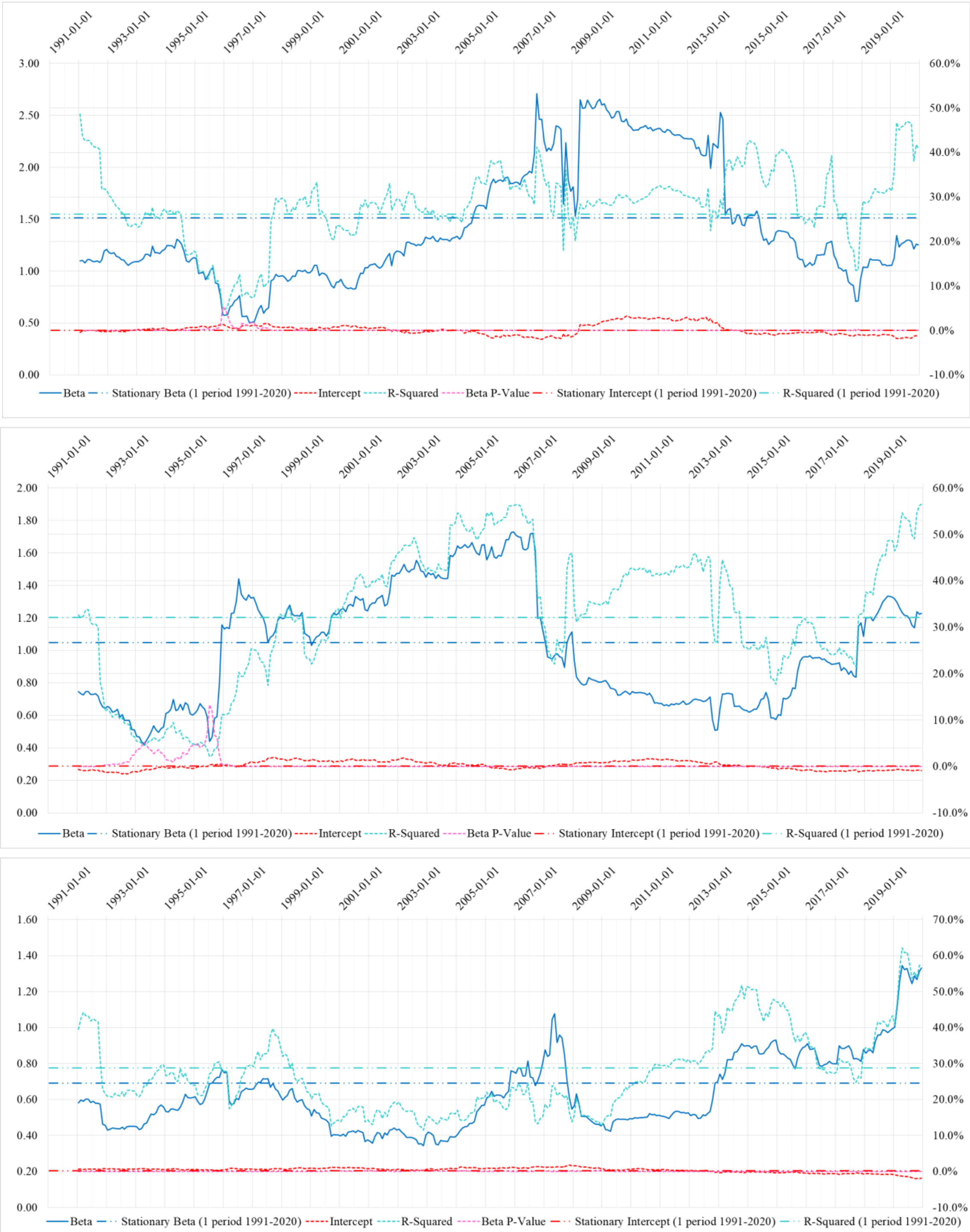

Figure 5. 5-Year Monthly Rolling β of Three Historical Corporations of the Automotive, IT and Energy Industries

Furthermore, we must ask ourselves how it is possible to create homogeneous β portfolios if this variable could undergo sudden fluctuations due to internal or external causes to the firm: over a 5-year timespan, we can get the false feeling that β is or has been constant when it can change suddenly and abruptly in one month. The first stock on top of Figure 5, fell from 2.50 to 1.50 between 2007 and 2009 to suddenly rise again towards 2.50 with an R^2 in between 20% and 40%; a similar trend would not have emerged by combining this stock in a portfolio of 50/100 other stocks in a regime where β computation takes place annually or biannually. Portfolios can mitigate EIV bias but have other drawbacks that mask factors that characterize individual stocks (Jegadeesh et al., 2015).

4.6 First Conclusions on the Empirical Evidence

From the previous subsections 4.4 and 4.5, we have verified that CAPM can perform an essential function in contouring the portfolio efficient frontier of *m* risky assets and determining the optimal combination that maximizes the investor utility through the slope of the market opportunity line. Once this function is completed, the prospect of using the equation (21) reported below falls sharply.

$$R_i - \overline{R_F} = \beta_i * (R_M - \overline{R_F}) = \lambda * Cov(R_i, R_M) \tag{21}$$

We reiterate that equation (21) is an equilibrium condition to minimise portfolio risk. Furthermore, it is a stochastic relation whose only constant element is $\overline{R_F}$. Thinking that (21) is verified every instant is a pious illusion, like thinking that β_i remains constant in time. Equation (21) is a valid relation for a specific portfolio aimed at minimizing its risk. Assigning such a relation to a different task involves fatal errors for the reasons we are about to present.

We have seen that β takes on different meanings depending on its use: within an optimised portfolio or as a generic measure of a stock risk compared to a market index. Let us examine the results of the first kind using Figure 6, which presents the 360 observations of stock monthly returns in the 1991-2020 timespan.

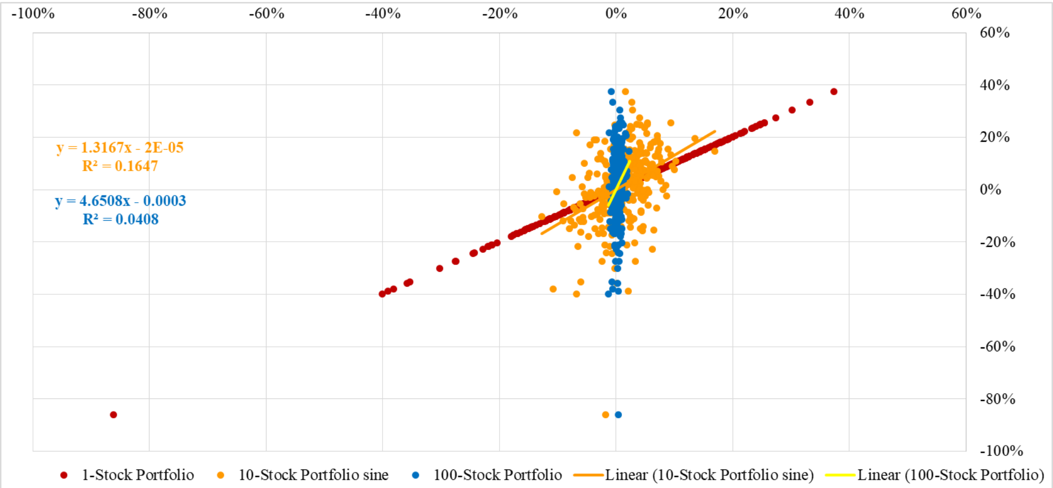

Figure 6. Portfolio Optimisation Effect: Portfolio Return on the Abscissa and Stock Return on the Ordinate

If we assumed that such a risky asset was the only one available on the market, the portfolio returns would be equal to the stock returns and the observations would be distributed in Figure 6 as the amaranth points along a bisector of the Cartesian axes: intercept equal to 0, slope equal to 1 and such a distribution would have an R^2 equal to 1. Combining the common stock into the 10-stock optimised portfolio analysed already, we know there would be no short sales, and the orange dots represent the distribution of the observations. The optimisation process causes a squeeze of the portfolio returns along the abscissa while there is no change along the ordinate. The security market line increases from 1 to approximately 1.317; there is no significant change in the intercept but R^2 plummets to approximately 16.5%.

Moving to the 100-stock optimised portfolio with short sales and Lintner's weights, portfolio returns squeeze further with the same stock return distribution as before: the observations in Figure 6 are in blue. The optimisation is so effective that the portfolio variance collapses, originating a sharp rise in the yellow regression line, with a slope of 4.65, intercept practically zero, and R^2 at 4.1%. We have already presented the dynamics of these parameters with equations (51) and (54).

In summary, portfolio optimisation is so efficient in containing return variance that it makes the common stock appear riskier: the squeeze effect is more prominent as the number of common stocks increases. At the same time, the relation between portfolio and stock returns becomes less and less significant.

The inclusion of risk-free assets does not change β because $\overline{R_F}$ is constant, so it does not affect the variance, covariance, correlation, and determination indices.

In Figure 7 on the left, we replaced the 100-stock portfolio with the S&P 500 Index (turquoise dots). The index composed of 500 stocks has squeezed returns like the 10-stock optimised portfolio, while on the right, we have replaced the S&P 500 Index with the 100-stock optimised portfolio *sine*, of which only 29 stocks are active (red dots). Now, it appears evident that the effectiveness of the optimisation with short sales constraints exceeds the

S&P 500 Index. It follows that the slope of the regression line of the former is steeper than the latter, and, as we have also seen in the previous case, there are algebraic and stochastic reasons which should make us reflect on the optimisation effectiveness of the market portfolio represented by a proxy. If the market portfolio represented by the S&P 500 proxy were as effective as the 100-stock optimised portfolio *sine*, there would be two numerical consequences: it would have a steeper slope and a lower R^2 but unfortunately, this does not happen. Table 8 compares the essential parameters of the five possible portfolios to the sample common stock to assess their internal dynamics: the variable θ allows us to establish their optimisation ranking.

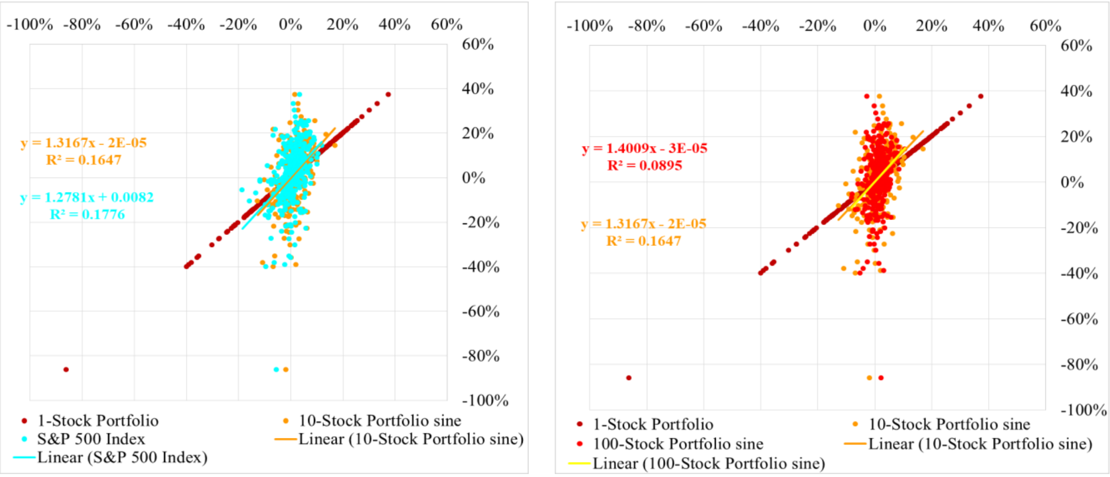

Figure 7. Portfolio Optimisation Effect: Portfolio Returns on the Abscissa and Stock Returns on the Ordinate

Table 8. Regression of five portfolios against a sample stock (benchmark)

Portfolios	**1-stock (benchmark)**	**10-stock without short sales**	**100-stock with short sales (Lintner's weights)**	**100-stock with short sales (Merton's weights)**	**100-stock without short sales**	**S&P 500 Index**
Return	1.680%	1.278%	0.367%	2.069%	1.201%	0.675%
Variance	1.644%	0.156%	0.003%	0.102%	0.075%	0.179%
$Cov(R_i, R_M)$	1.644%	0.206%	0.014%	0.083%	0.105%	0.228%
β	1.000	1.317	4.651	0.811	1.401	1.278
α	0.000	0.000	0.000	0.000	0.000	0.008
R^2	100.0%	16.472%	4.082%	4.082%	8.948%	17.756%
θ	13.046%	32.145%	64.569%	64.569%	43.612%	15.800%

Even the 10-stock portfolio *sine* beats the S&P 500 Index, which is slightly better than the single stock itself, despite having the latter a nine times higher variance. The only element ignored by this analysis is that the five portfolios are based on ex-post decisions while the single stock and S&P 500 Index are potentially resulting from ex-ante choices: if an investor decided to invest in 1991, he could have bought only the single stock and the S&P 500 Index while no one could have invested in the 100-stock portfolios and only with much good luck could it have been possible to opt for the 10-stock portfolio that would initially have been composed by five stocks with five subsequent additions between 1997 and 2012, but probably with different weights from those coming from equation (19 bis). Ex-post, we know the covariance matrix for 1991-2020; ex-ante, we should have had the crystal ball or a better forecast tool than that provided by CAPM. The approach used by Black et al. (1992) is encouraging. It would be interesting to use the data for the first n months to optimise the portfolio for the subsequent month through all the 30 years and compare that result with the 30-year static optimisation and the S&P 500 Index.

The outcome is that equation (21), of pure stochastic nature, is instrumental only for portfolio optimisation and performs a magnificent job. It loses worth when the portfolio is not optimised, such as the market portfolio or its proxy, and even when the risky asset, excluded from the market portfolio, is compared to this latter. In any case, its predictive reliability can only be modest, being a stochastic relation with a percentage of the explained variance lower the broader the stock set held in the portfolio is. Of course, it plays an essential role in defining the risk/return ratio of the stock, but alas, it cannot quantify it correctly outside a specifically optimised portfolio.

CAPM empirical evidence studies assume that the market is efficient without any checking about its optimisation, and such a fault has led to nothing. Extremely refined statistical techniques employed to validate CAPMs clash with the empirical evidence that:

1) The market portfolio might be neither efficient nor optimised;
2) The better the portfolio optimisation, with many uncorrelated stocks and short sales, the worse the explained variance of the relation (21);
3) β changes over time, even within a few weeks.

CAPM takes the risky asset prices, returns, variances and covariances as exogenous data. It cannot explain the performance of the capital market by itself owing to its incompleteness. If we want a theory explaining the stock market returns, we must look outside CAPM.

4.7 Integration of Stock Market with Corporate Performance

After analysing the asset allocation and security market lines of 100 stocks embedded in the S&P 500 Index, we investigate the possibility of explaining stock returns based on the stock market's and corporate business's joint performance.

4.7.1 Stock Market and Corporate Business Performance Data Sampling

As far as the stock returns are concerned, we said everything already; for the stock market performance, we will use the S&P 500 Index return as a proxy for the market portfolio, while for the corporate business performance, we will use some variables linked to the DOL and DFL definition, already presented in subsection 2.1. Table 9 summarizes the fourteen variables employed according to their kind.

Table 9. Corporate performance variables and ratios

Statement of Income Variables	Ratio between Variables	Ratio between Ratios
Total Revenue	Total Revenue Growth	DOL
EBIT	EBIT Growth	DFL
Net Income	Net profit Growth	DTL
Adjusted Basic EPS	Adjusted Diluted EPS Growth	
Adjusted Diluted EPS	Risk Rate	
Dividends		

Most variables are selected from the income statement except for dividends, partly associated with the net income trend. The ratios between variables are simply ratios between a specific quantity at time t to time $t-1$ to determine its growth trend. The Risk Rate at time t is the reciprocal of the ratio between EBIT and Total Revenue at time $t-1$. It indicates the risk of the corporate business, albeit in a rough form: the higher this ratio, the greater the chance that unfavourable changes, even modest, of unit prices, unit variable costs, volumes and sales or manufacturing mixes cause adverse effects on corporate profitability. Instead, DTL is simply the DOL by DFL product.

The first problem in managing a heterogeneous mass of similar financial and market data is their sampling; the stock and stock market index quotations are even at an intra-daily level while listed firms release quarterly financial statements. Even choosing a monthly frequency for market data, as has been done already, would cause a significant mismatch with quarterly financial statement data. Furthermore, the comparison between quarterly corporate data should involve a seasonal adjustment to be meaningful. Even so, there would still be the problem of defining the trend at the quarter level versus the previous quarter or quarter of the last year, avoiding seasonal data adjustment. We discarded the use of quarterly data owing to the consideration that the stock market operates skilful professional investors who can evaluate the overall corporate performance by inferring it from the quarterly data and converting it into an annual projection; an excellent quarterly performance combined with a good corporate knowledge would allow the expert analyst to understand the real corporate trend or even to predict it in advance, without having information other than that available on the market. For this reason, we decided to use stock annual returns by relating them to annual financial statement data.

4.7.2 Regression Procedure

We regress the stock market and corporate performance of the year t on the stock return of the same year: for the S&P 500 Index return, there are no problems of synchronicity with the stock return, but with the corporate performance it is necessary to make a logical leap and link it to the stock return in advance of the moment in which the financial statements for the same year are made available, perhaps several months in advance. We have hypothesized that the market does not have to wait for the end of the financial period to find out how the firm is doing: it knows beforehand its performance. Therefore, we paired the annual corporate performance at time t, relative to the previous 12 months, with the annual stock market and the specific stock returns in one of the 12 months of the same financial period.

But which month in particular? The one which maximizes R^2 within the 12 months of the financial period.

Of course, it was necessary to convert the monthly time series from month/year into month/financial period as the start and end date of the financial period do not always coincide with the solar one and are specific for each firm. Fortunately, there are very few cases of firms that have changed the closing date of the financial period by one or more months while scheduling the end of the financial period on a specific variable day within a limited period of a few days around the end of a month is relatively frequent. In this case, we assumed the month-end date as the end of the financial period.

Once we acquired the financial statements, we based the analysis for each stock on the following process:

1) *Stage 1*: simple regression at the annual level of the S&P 500 Index return as the explanatory variable of the stock return, identifying the month of the financial period in which R^2 is maximized on a thirty-year or the available timespan.

2) *Stage 2*: once defined the month in which there is the maximum variability explained of the stock return against the index return and vice versa, we carried out an analysis of the correlations with all the fourteen variables of Table 9 on the available timespan for the stock to identify the possible candidates for the subsequent multiple linear regression or MLR.

3) *Stage 3*: MLR to identify the best second regressor for the same month identified in Stage 1 based on the criterion to maximize R^2 in the absence of multicollinearity.

4) *Stage 4*: search for a better month than the one already identified in Stage 1. If we observe a better month, the analysis restarts from Stage 2.

5) *Stage 5*: identification of the third and fourth regressor that maximize R^2, in the absence of multicollinearity. In some cases, Stage 5 changed the reference month identified with Stages 1 and 4, restarting the analysis from Stage 2.

The described process makes it possible to identify the best MLR for each stock, using three explanatory variables of corporate performance and the stock market performance over a sufficiently long time.

The method of choosing the month of the financial period in which to match the stock and stock market returns with the financial statement data might seem arbitrary and opportunistic, but if it allows explaining a significant share of the stock return variability over 30 years for 68 stocks out of 100, could prove decisive. Considering that the month of the financial period that shows tremendous significance is between 5 and 6, it seems to indicate that even before the availability of the half-year report, the trends of stock return R_i and corporate performance ψ_i are already correlated, in part mitigated by the presence of R_M. This undeniable fact should not necessarily lead to insider trading actions or other criminally relevant conduct but only the possibility of skilfully analysing corporate performance in advance based on incomplete information.

For each stock, we will present the details of the simple regressions against the S&P 500 Index return and MLR against the same return and the corporate performance's best variable. For each stock, we will also show the R^2 of the simple regressions against each of the corporate performance's fourteen variables, determining an overall ranking based on the highest R^2. We will also analyse the best MLR with two regressors focusing on the overlap or the bridge discovered with the commonality analysis, verifying the absence of multicollinearity.

4.7.3 OLS Results

We can start with the summary of the OLS regression of R_M against R_i with the annual return data, shown in Table 10; we present the details in Table 11: on average, we analysed around 26 periods for every stock, of which 68 stocks had 30 periods, and only four stocks had less than ten periods. Based on the F test, 83 regressions are significant. Based on t-stats, 70 intercepts did not pass the null hypothesis test, while the βs are significant in 83 regressions. We can see that shifting from monthly to annual 30-year returns, R^2 increases by more than 13%, to 36.3% in Table 10 from 22.5% in Table 5, with 41 stocks exceeding 40% versus only five stocks with monthly returns.

Table 10. Summary of OLS with annual returns

OLS	Obs.	Month	Standard Error	α	β	R^2	α t-stat	β t-stat	α P-Value	β P-Value	F	F P-Value
μ	26.38	5.68	0.344	9.636%	1.460	36.320%	1.542	3.719	12.219%	2.283%	17.15	4.565%
σ	6.504	3.616	0.338	9.537%	1.447	19.738%	1.0671	1.8214	13.360%	6.352%	15.58	12.704%
Count						**41**			**30**	**83**		**83**

Table 11. OLS with annual returns for 100 stocks

#	Conventional Name	Obs.	Month	Standard Error	Intercept	Beta	R^2	Intercept t-stat	Beta t-stat	Intercept P-Value	Beta P-Value	F	F P-Value
1	Achernar	23	2	0.225	7.753%	1.300	57.031%	1.567	5.279	6.605%	0.002%	27.87	0.003%
2	Adhil	28	6	0.257	13.133%	1.334	50.022%	2.478	5.101	1.002%	0.001%	26.02	0.003%
3	Albaldah	8	12	0.220	5.084%	2.486	70.993%	0.434	3.832	33.981%	0.432%	14.68	0.864%
4	Albali	30	1	0.140	5.838%	0.610	36.362%	1.998	4.000	2.773%	0.021%	16.00	0.042%
5	Alchiba	30	10	0.235	2.475%	1.041	30.806%	0.484	3.531	31.592%	0.073%	12.47	0.146%
6	Alcor	20	1	0.301	22.262%	0.398	5.787%	3.133	1.052	0.287%	15.347%	1.11	30.694%
7	Alcyone	30	11	0.143	6.931%	0.699	35.045%	2.220	3.887	1.736%	0.028%	15.11	0.057%
8	Aldebaran	30	3	0.192	5.308%	1.161	52.606%	1.334	5.575	9.647%	0.000%	31.08	0.001%
9	Alderamin	30	9	0.257	12.684%	0.931	23.721%	2.281	2.951	1.519%	0.317%	8.71	0.634%
10	Aldhanab	30	8	0.335	1.411%	2.537	55.900%	0.194	5.958	42.395%	0.000%	35.49	0.000%
11	Algol	30	9	0.243	14.133%	0.562	11.269%	2.692	1.886	0.593%	3.487%	3.56	6.974%
12	Algorab	29	3	0.845	20.149%	1.686	12.453%	1.169	1.960	12.634%	3.021%	3.84	6.042%
13	Alhena	30	3	0.225	8.604%	1.248	38.552%	1.794	4.191	4.179%	0.013%	17.57	0.025%
14	Alioth	21	2	0.095	2.185%	0.707	70.228%	1.007	6.695	16.320%	0.000%	44.82	0.000%
15	Alkarab	30	2	0.246	9.593%	0.607	16.886%	1.887	2.385	3.481%	1.204%	5.69	2.408%
16	Almaaz	30	3	0.263	10.252%	0.838	25.792%	1.923	3.120	3.233%	0.208%	9.73	0.417%
17	Alnair	28	3	0.186	11.092%	0.455	18.120%	2.885	2.399	0.389%	1.196%	5.75	2.392%
18	Alpheratz	30	6	0.119	7.716%	0.581	41.671%	3.110	4.473	0.214%	0.006%	20.00	0.012%
19	Alrakis	30	2	0.284	15.993%	0.741	18.490%	2.724	2.520	0.550%	0.885%	6.35	1.771%
20	Altair	30	3	0.311	8.431%	0.932	21.158%	1.294	2.741	10.307%	0.527%	7.51	1.054%
21	Alzirr	30	1	0.188	-1.502%	1.291	58.595%	-0.382	6.295	35.259%	0.000%	39.62	0.000%
22	Aniara	11	6	0.318	3.828%	3.453	40.536%	0.247	2.477	40.517%	1.758%	6.14	3.517%
23	Antares	30	9	0.178	0.906%	0.874	36.365%	0.235	4.000	40.782%	0.021%	16.00	0.042%
24	Arcturus	30	2	0.182	4.952%	0.786	39.340%	1.328	4.261	9.743%	0.010%	18.16	0.021%
25	Aspidiske	21	2	0.542	20.718%	2.454	46.847%	1.682	4.092	5.444%	0.031%	16.75	0.062%
26	Atlas	30	2	0.394	10.533%	1.470	29.591%	1.286	3.430	10.445%	0.094%	11.77	0.189%
27	Bellatrix	30	10	0.182	-1.263%	1.446	59.005%	-0.320	6.348	37.567%	0.000%	40.30	0.000%
28	Betelgeuse	30	7	0.155	15.916%	-0.308	7.401%	4.767	-1.496	0.003%	7.293%	2.24	14.585%
29	Bharani	30	10	0.143	10.463%	0.637	31.129%	3.373	3.557	0.110%	0.068%	12.66	0.136%
30	Canopus	30	11	0.248	11.927%	0.539	9.629%	2.200	1.727	1.810%	4.757%	2.98	9.514%
31	Capella	30	1	0.122	3.673%	0.578	40.031%	1.433	4.323	8.149%	0.009%	18.69	0.018%
32	Castor	30	12	0.205	9.770%	0.682	24.669%	2.245	3.028	1.641%	0.262%	9.17	0.524%
33	Copernicus	30	4	0.209	3.426%	1.118	41.594%	0.780	4.465	22.100%	0.006%	19.94	0.012%
34	Deneb	20	9	0.321	11.164%	2.151	53.057%	1.461	4.510	8.063%	0.014%	20.34	0.027%
35	Diadem	30	8	0.324	15.651%	0.818	15.996%	2.307	2.309	1.434%	1.427%	5.33	2.853%
36	Dubhe	25	2	0.489	0.649%	1.531	28.186%	0.061	3.005	47.595%	0.316%	9.03	0.632%
37	Electra	22	1	0.297	12.345%	1.443	43.021%	1.845	3.886	3.996%	0.046%	15.10	0.092%
38	Furud	30	10	0.528	10.798%	1.579	16.859%	0.941	2.383	17.746%	1.210%	5.68	2.421%
39	Gienah	16	8	0.093	9.176%	1.111	68.777%	3.314	5.553	0.256%	0.004%	30.84	0.007%
40	Hamal	30	5	0.220	11.390%	0.642	15.160%	2.432	2.237	1.082%	1.672%	5.00	3.344%
41	Intercrus	30	5	0.133	1.238%	1.303	76.960%	0.455	9.671	32.647%	0.000%	93.53	0.000%
42	Izar	28	9	0.545	12.363%	2.157	28.807%	1.039	3.244	15.429%	0.162%	10.52	0.323%
43	Jabbah	30	1	0.152	7.589%	1.216	65.842%	2.392	7.347	1.184%	0.000%	53.97	0.000%
44	Kang	30	5	0.130	9.132%	0.672	48.092%	3.423	5.093	0.096%	0.001%	25.94	0.002%
45	Keid	20	2	0.824	37.132%	2.118	20.642%	1.923	2.164	3.519%	2.209%	4.68	4.417%
46	Kitalpha	30	10	0.112	7.245%	0.939	63.452%	3.066	6.972	0.238%	0.000%	48.61	0.000%
47	Kochab	30	9	0.530	7.899%	2.834	38.785%	0.686	4.212	24.923%	0.012%	17.74	0.024%
48	Kraz	30	8	0.278	4.322%	0.985	21.811%	0.716	2.795	23.986%	0.464%	7.81	0.927%
49	Libertas	15	1	0.340	22.310%	1.827	56.201%	2.357	4.084	1.738%	0.065%	16.68	0.129%
50	Markab	30	1	0.154	5.800%	1.104	60.685%	1.802	6.574	4.112%	0.000%	43.22	0.000%
51	Matar	30	1	0.268	6.169%	1.792	57.305%	1.101	6.130	14.007%	0.000%	37.58	0.000%
52	Merak	28	11	1.531	12.570%	4.812	18.562%	0.370	2.434	35.709%	1.104%	5.93	2.209%
53	Merga	30	2	0.223	4.768%	1.258	51.325%	1.031	5.434	15.560%	0.000%	29.52	0.001%
54	Merope	10	12	0.208	14.434%	1.747	53.564%	1.503	3.038	8.567%	0.806%	9.23	1.612%
55	Mesarthim	30	7	0.301	14.434%	0.399	3.425%	2.227	0.996	1.708%	16.377%	0.99	32.754%
56	Mimosa	30	5	0.128	7.370%	0.532	26.501%	2.699	3.177	0.583%	0.180%	10.10	0.361%
57	Mira	30	12	0.187	8.641%	0.341	8.986%	2.181	1.663	1.888%	5.377%	2.76	10.753%
58	Mizar	12	2	0.182	4.604%	0.704	27.151%	0.647	1.931	26.623%	4.118%	3.73	8.236%
59	Musica	30	7	0.213	0.456%	1.511	50.346%	0.099	5.328	46.082%	0.001%	28.39	0.001%
60	Nashira	19	7	0.130	7.510%	1.123	73.543%	2.382	6.874	1.460%	0.000%	47.26	0.000%
61	Nekkar	30	5	0.137	4.067%	0.831	43.712%	1.400	4.663	8.626%	0.003%	21.74	0.007%
62	Nembus	19	2	0.203	8.388%	1.238	62.567%	1.693	5.331	5.439%	0.003%	28.41	0.006%
63	Okab	30	2	0.290	13.498%	1.282	39.332%	2.246	4.261	1.638%	0.010%	18.15	0.021%
64	Polaris	19	2	0.087	3.141%	0.567	65.452%	1.473	5.675	7.956%	0.001%	32.21	0.003%
65	Pollux	13	9	0.184	-0.097%	1.643	61.844%	-0.017	4.222	49.346%	0.072%	17.83	0.143%
66	Procyon	8	7	0.203	18.181%	2.206	32.327%	1.091	1.693	15.852%	7.070%	2.87	14.141%
67	Proxima Centauri	20	8	0.516	23.794%	1.807	20.678%	1.928	2.166	3.487%	2.198%	4.69	4.397%
68	Ran	30	11	0.142	4.151%	0.765	39.499%	1.335	4.276	9.628%	0.010%	18.28	0.020%
69	Regulus	30	1	0.328	14.548%	1.664	34.396%	2.079	3.831	2.344%	0.033%	14.68	0.066%
70	Rigel	30	5	0.563	13.502%	1.958	22.545%	1.133	2.855	13.348%	0.401%	8.15	0.802%
71	Rukbat	30	1	0.163	2.693%	0.823	43.167%	0.788	4.612	21.878%	0.004%	21.27	0.008%
72	Sabik	30	2	0.795	9.253%	3.916	35.595%	0.553	3.934	29.231%	0.025%	15.48	0.050%
73	Salm	30	2	0.452	11.218%	2.054	38.479%	1.196	4.185	12.085%	0.013%	17.51	0.026%
74	Sceptrum	30	4	0.280	-7.347%	1.773	49.950%	-1.249	5.286	11.105%	0.001%	27.94	0.001%
75	Segin	23	6	1.916	40.959%	5.912	17.989%	0.931	2.146	18.116%	2.185%	4.61	4.370%
76	Sheratan	7	11	0.175	-16.158%	3.304	68.713%	-1.292	3.314	12.646%	1.058%	10.98	2.115%
77	Sirius	30	4	0.263	6.517%	1.159	32.538%	1.178	3.675	12.427%	0.050%	13.51	0.100%
78	Spica	30	9	0.420	16.325%	0.955	11.094%	1.838	1.869	3.834%	3.604%	3.49	7.209%
79	Subra	7	1	0.124	10.895%	1.621	73.309%	1.752	3.706	7.012%	0.696%	13.73	1.392%
80	Syrma	30	2	0.643	21.946%	1.405	10.023%	1.605	1.766	5.984%	4.414%	3.12	8.828%
81	Tabit	30	12	0.277	16.089%	0.279	2.905%	2.730	0.915	0.542%	18.395%	0.84	36.789%
82	Talitha	30	4	0.266	3.725%	1.544	45.631%	0.667	4.848	25.525%	0.002%	23.50	0.004%
83	Tarazed	30	5	0.863	20.894%	1.076	3.159%	1.138	0.956	13.233%	17.370%	0.91	34.741%
84	Tarf	30	5	0.338	12.472%	0.341	2.089%	1.733	0.773	4.707%	22.302%	0.60	44.604%
85	Tejat	30	8	0.424	22.563%	0.829	7.809%	2.449	1.540	1.042%	6.739%	2.37	13.477%
86	Thuban	30	1	0.311	7.401%	1.162	29.534%	1.138	3.426	13.235%	0.096%	11.74	0.191%
87	Tiaki	30	11	0.122	6.306%	0.805	49.537%	2.364	5.243	1.263%	0.001%	27.49	0.001%
88	Timir	24	8	0.280	6.723%	1.560	56.262%	1.075	5.320	14.694%	0.001%	28.30	0.002%
89	Titawin	30	7	0.273	-0.229%	1.381	34.060%	-0.039	3.803	48.463%	0.036%	14.46	0.071%
90	Toliman	10	11	2.335	-33.897%	12.456	17.681%	-0.248	1.311	40.532%	11.315%	1.72	22.629%
91	Tonatiuh	30	6	0.461	11.424%	2.289	42.661%	1.190	4.564	12.202%	0.005%	20.83	0.009%
92	Tureis	27	9	0.319	14.325%	1.270	30.444%	2.055	3.308	2.523%	0.142%	10.94	0.285%
93	Uruk	30	11	0.136	3.864%	1.459	72.266%	1.302	8.542	10.172%	0.000%	72.96	0.000%
94	Vega	18	5	0.790	38.320%	3.182	25.528%	1.798	2.342	4.550%	1.622%	5.48	3.245%
95	Veritate	30	2	0.259	9.156%	0.740	22.033%	1.722	2.813	4.809%	0.444%	7.91	0.887%
96	Wezen	30	7	0.149	8.345%	0.722	32.166%	2.598	3.644	0.739%	0.054%	13.28	0.108%
97	Xihe	30	4	0.205	3.875%	0.869	30.863%	0.899	3.535	18.826%	0.072%	12.50	0.144%
98	Yildun	29	12	0.754	28.217%	0.005	0.000%	1.758	0.005	4.504%	49.789%	0.00	99.577%
99	Zaurak	30	3	0.136	13.815%	0.853	57.374%	5.015	6.139	0.001%	0.000%	37.69	0.000%
100	Zibal	30	7	0.759	19.171%	1.802	16.159%	1.246	2.323	11.161%	1.383%	5.40	2.766%

Table 12. R^2 matrix between corporate performance variables and S&P 500 Index return against the stock returns

#	Conventional Name	R^2 Matrix														
		Total Revenue	EBIT	Net Income	Adj Basic EPS	Adj Diluted EPS	Adj Dividends	Revenue Growth	EBIT Growth	Net Income Growth	Adj EPS Growth	DOL	DFL	DTL	Risk Rate	S&P 500 Annual Returns
1	Achernar	0.343%	1.467%	0.287%	0.966%	0.922%	0.257%	5.591%	8.725%	8.682%	7.475%	46.885%	0.267%	42.211%	27.697%	57.031%
2	Adhil	2.490%	0.034%	0.023%	0.062%	0.073%	1.153%	2.767%	0.767%	0.157%	0.154%	0.648%	0.012%	0.161%	0.003%	50.022%
3	Albaldah	11.012%	18.209%	20.915%	20.279%	20.294%	0.000%	4.613%	45.929%	49.571%	49.785%	44.142%	52.185%	49.217%	52.196%	70.993%
4	Albali	0.022%	5.443%	14.889%	43.775%	43.348%	0.000%	5.036%	0.739%	13.554%	14.766%	6.981%	0.736%	7.558%	6.608%	36.362%
5	Alchiba	4.747%	3.771%	3.643%	3.494%	3.569%	6.378%	25.518%	1.573%	6.554%	5.990%	4.252%	0.397%	4.144%	0.697%	30.806%
6	Alcor	2.363%	0.038%	0.192%	0.185%	0.172%	0.000%	0.003%	8.506%	1.018%	1.113%	0.203%	1.056%	2.353%	0.068%	5.787%
7	Alcyone	8.286%	10.486%	13.584%	12.988%	13.149%	5.246%	21.154%	7.148%	0.400%	0.472%	2.546%	0.361%	0.795%	15.755%	35.045%
8	Aldebaran	1.369%	2.113%	2.101%	2.114%	2.070%	0.011%	1.349%	27.548%	16.842%	15.666%	0.754%	0.178%	6.351%	1.608%	52.606%
9	Alderamin	5.589%	7.352%	4.457%	4.005%	4.063%	1.829%	11.665%	25.322%	9.048%	8.934%	3.321%	0.026%	12.112%	19.601%	23.721%
10	Aldhanab	5.265%	0.130%	0.031%	0.309%	0.382%	6.457%	36.224%	11.127%	11.301%	21.943%	7.217%	8.663%	6.562%	4.679%	55.900%
11	Algol	12.540%	0.134%	0.001%	0.115%	0.107%	0.463%	2.356%	6.639%	3.596%	3.914%	5.129%	15.446%	2.957%	6.344%	11.269%
12	Algorab	0.759%	0.710%	0.454%	0.216%	0.245%	0.000%	1.315%	2.350%	2.146%	1.736%	0.335%	7.542%	0.259%	5.938%	12.453%
13	Alhena	12.271%	18.914%	18.044%	18.456%	18.578%	7.612%	11.267%	0.101%	0.102%	0.752%	5.644%	0.001%	0.340%	6.307%	38.552%
14	Alioth	0.365%	3.709%	13.211%	10.278%	10.163%	0.293%	17.633%	0.287%	0.614%	0.577%	0.112%	8.084%	0.194%	0.004%	70.228%
15	Alkarab	11.650%	12.491%	13.400%	10.857%	10.915%	6.178%	24.944%	32.808%	3.730%	2.825%	0.287%	0.000%	0.033%	1.484%	16.886%
16	Almaaz	0.282%	0.165%	0.199%	0.592%	0.582%	0.798%	9.856%	2.822%	0.475%	0.473%	0.637%	0.838%	0.397%	0.127%	25.792%
17	Alnair	0.164%	0.115%	0.212%	0.381%	0.384%	2.663%	29.652%	6.243%	2.096%	1.390%	2.450%	3.777%	1.154%	1.703%	18.120%
18	Alpheratz	5.729%	2.793%	3.509%	2.507%	2.553%	3.221%	11.365%	0.426%	3.262%	3.594%	0.359%	0.676%	0.115%	0.017%	41.671%
19	Alrakis	9.183%	8.398%	8.340%	3.494%	3.406%	1.398%	24.232%	33.870%	0.676%	0.556%	7.580%	0.062%	6.726%	14.566%	18.490%
20	Altair	1.370%	5.341%	5.134%	2.386%	2.349%	0.032%	57.113%	41.997%	2.990%	2.989%	0.707%	1.825%	0.077%	0.226%	21.158%
21	Alzirr	4.518%	2.032%	7.636%	8.973%	8.850%	0.967%	22.486%	0.106%	1.667%	1.551%	12.696%	0.496%	9.850%	21.219%	58.595%
22	Aniara	3.466%	4.677%	5.035%	4.274%	4.352%	7.087%	23.661%	5.273%	4.658%	4.123%	8.873%	11.319%	0.043%	6.965%	40.536%
23	Antares	0.003%	0.295%	7.655%	5.831%	5.810%	19.607%	17.530%	2.095%	24.476%	25.217%	2.147%	24.711%	5.448%	0.105%	36.365%
24	Arcturus	0.019%	0.007%	0.000%	0.138%	0.170%	1.694%	20.545%	0.288%	4.736%	3.865%	12.736%	0.004%	9.847%	0.610%	39.340%
25	Aspidiske	2.065%	0.653%	1.377%	1.500%	1.542%	0.000%	2.323%	63.791%	6.816%	8.318%	8.584%	3.176%	0.528%	19.036%	46.847%
26	Atlas	0.022%	0.659%	0.242%	0.061%	0.021%	1.930%	55.397%	30.893%	34.486%	32.841%	0.288%	3.428%	0.066%	0.266%	29.591%
27	Bellatrix	1.691%	0.019%	0.415%	0.634%	0.677%	3.839%	0.838%	3.397%	2.075%	2.581%	2.754%	18.253%	7.222%	3.154%	59.005%
28	Betelgeuse	8.724%	0.183%	0.000%	0.394%	0.377%	6.249%	0.265%	3.215%	1.540%	1.127%	0.309%	6.430%	0.168%	0.187%	7.401%
29	Bharani	1.914%	4.352%	2.281%	1.680%	1.673%	7.311%	0.019%	2.133%	6.741%	7.769%	4.615%	3.555%	0.712%	0.388%	31.129%
30	Canopus	3.298%	1.008%	0.322%	0.129%	0.131%	2.347%	11.686%	2.383%	5.873%	5.911%	0.007%	3.252%	0.081%	0.308%	9.629%
31	Capella	0.756%	0.240%	0.199%	0.199%	0.182%	0.497%	1.167%	1.126%	0.022%	0.028%	6.030%	1.821%	0.415%	1.069%	40.031%
32	Castor	0.239%	0.046%	0.080%	0.096%	0.102%	0.046%	5.462%	13.162%	13.958%	12.432%	1.817%	0.001%	5.426%	1.631%	24.669%
33	Copernicus	0.809%	4.366%	11.981%	16.813%	17.256%	4.927%	4.456%	1.212%	1.556%	1.396%	0.989%	3.280%	0.344%	0.748%	41.594%
34	Deneb	0.959%	2.053%	2.996%	34.186%	34.262%	0.579%	36.479%	2.866%	5.922%	6.137%	0.253%	0.032%	5.362%	0.419%	53.057%
35	Diadem	1.313%	0.237%	0.426%	0.559%	0.557%	0.666%	55.479%	46.226%	23.911%	21.916%	0.006%	1.217%	0.121%	2.283%	15.996%
36	Dubhe	0.338%	1.555%	1.469%	0.248%	0.237%	3.898%	4.116%	2.553%	1.350%	1.652%	3.254%	2.803%	2.759%	2.851%	28.186%
37	Electra	0.152%	0.224%	3.808%	10.233%	10.352%	0.300%	1.935%	27.130%	9.884%	9.775%	30.741%	0.011%	0.089%	48.539%	43.021%
38	Furud	8.539%	5.511%	5.810%	3.937%	3.968%	1.354%	51.770%	69.409%	69.926%	68.648%	28.254%	0.156%	0.949%	61.281%	16.859%
39	Gienah	5.171%	4.733%	5.152%	5.256%	5.175%	0.000%	0.013%	7.107%	1.065%	0.985%	15.505%	3.592%	0.798%	0.212%	68.777%
40	Hamal	5.282%	4.021%	4.632%	3.190%	3.177%	0.243%	54.403%	0.351%	2.583%	2.559%	0.376%	1.149%	0.000%	0.137%	15.160%
41	Intercrus	0.034%	1.268%	0.873%	0.404%	0.376%	0.019%	5.038%	5.020%	10.138%	9.646%	9.401%	1.329%	6.544%	4.839%	76.960%
42	Izar	6.977%	4.621%	4.096%	4.429%	4.773%	8.970%	2.388%	0.841%	0.339%	0.474%	0.417%	3.856%	0.021%	0.134%	28.807%
43	Jabbah	1.161%	2.722%	3.299%	3.783%	3.741%	4.170%	6.778%	0.009%	0.011%	0.009%	0.002%	0.756%	0.027%	0.128%	65.842%
44	Kang	1.034%	2.668%	3.222%	1.876%	1.901%	0.390%	1.345%	0.270%	0.424%	0.458%	25.374%	1.271%	22.386%	0.000%	48.092%
45	Keid	0.425%	6.225%	6.048%	6.564%	6.006%	1.719%	47.538%	2.451%	1.672%	1.106%	0.990%	2.992%	0.095%	1.326%	20.642%
46	Kitalpha	2.566%	4.133%	4.540%	2.256%	2.252%	0.330%	6.100%	12.354%	1.103%	0.340%	7.612%	1.796%	2.944%	18.175%	63.452%
47	Kochab	0.043%	0.205%	0.705%	0.828%	0.792%	2.334%	0.568%	10.933%	2.227%	2.874%	0.001%	2.094%	0.016%	3.698%	38.785%
48	Kraz	2.553%	26.566%	33.725%	33.805%	33.726%	10.071%	1.009%	3.605%	1.960%	1.821%	1.204%	4.543%	9.463%	29.581%	21.811%
49	Libertas	1.319%	0.007%	0.083%	0.091%	0.092%	0.000%	2.221%	3.967%	2.413%	2.256%	4.573%	2.991%	2.171%	0.182%	56.201%
50	Markab	2.636%	3.026%	3.083%	3.238%	3.231%	2.873%	0.461%	0.431%	5.722%	6.354%	0.360%	0.466%	1.310%	0.026%	60.685%
51	Matar	1.779%	1.782%	4.023%	1.352%	1.261%	0.091%	17.405%	2.196%	9.843%	10.691%	5.518%	5.169%	4.779%	2.174%	57.305%
52	Merak	5.152%	5.354%	3.855%	3.342%	3.328%	4.480%	0.073%	2.056%	0.716%	0.529%	1.941%	0.002%	0.042%	2.242%	18.562%
53	Merga	4.856%	3.373%	0.006%	0.092%	0.118%	2.216%	0.818%	10.130%	0.728%	0.527%	1.937%	3.881%	2.261%	2.192%	51.325%
54	Merope	2.327%	0.204%	1.935%	1.153%	0.999%	0.000%	4.729%	3.684%	6.607%	5.206%	3.202%	13.312%	5.975%	24.894%	53.564%
55	Mesarthim	3.405%	0.936%	6.759%	5.176%	5.008%	0.011%	47.100%	2.064%	16.259%	16.235%	0.147%	15.676%	10.635%	1.097%	3.425%
56	Mimosa	0.097%	2.345%	8.261%	7.616%	7.589%	1.206%	2.444%	5.153%	11.679%	12.265%	18.832%	0.191%	9.944%	5.566%	26.501%
57	Mira	0.080%	0.518%	0.565%	0.652%	0.627%	1.624%	28.623%	1.744%	2.759%	2.211%	3.146%	0.095%	2.958%	0.000%	8.986%
58	Mizar	4.750%	20.545%	4.039%	1.303%	1.334%	33.252%	47.722%	47.766%	7.849%	13.839%	12.525%	3.826%	9.664%	14.653%	27.151%
59	Musica	0.400%	0.021%	0.084%	0.194%	0.191%	0.220%	4.958%	4.869%	0.184%	0.178%	6.794%	0.159%	0.039%	3.375%	50.346%
60	Nashira	0.813%	0.784%	0.902%	0.684%	0.624%	1.196%	11.266%	0.001%	1.118%	4.031%	1.298%	9.349%	35.369%	2.080%	73.543%
61	Nekkar	3.152%	15.149%	15.884%	16.179%	16.167%	5.431%	12.542%	5.499%	0.605%	0.598%	1.188%	11.491%	0.442%	0.410%	43.712%
62	Nembus	0.024%	2.119%	2.263%	0.000%	0.013%	0.797%	0.749%	3.983%	2.043%	20.214%	3.679%	2.919%	3.363%	5.489%	62.567%
63	Okab	13.271%	6.083%	5.415%	2.485%	2.537%	2.020%	26.605%	33.065%	34.792%	29.891%	0.081%	0.009%	0.166%	5.583%	39.332%
64	Polaris	2.368%	3.941%	0.147%	0.010%	0.017%	0.063%	0.088%	2.669%	0.411%	0.741%	0.035%	1.868%	0.367%	0.000%	65.452%
65	Pollux	39.324%	23.519%	15.928%	7.621%	7.683%	0.000%	4.930%	11.464%	0.063%	0.058%	17.919%	43.724%	6.254%	10.142%	61.844%
66	Procyon	14.558%	76.638%	45.159%	48.234%	49.879%	0.000%	0.331%	4.840%	9.387%	10.381%	2.861%	5.938%	9.298%	0.241%	32.327%
67	Proxima Centauri	1.130%	1.251%	0.789%	0.586%	0.638%	0.000%	6.064%	17.944%	20.336%	27.258%	3.671%	7.794%	5.194%	11.922%	20.678%
68	Ran	3.878%	5.350%	2.166%	2.320%	2.368%	4.398%	5.951%	0.121%	0.097%	0.120%	14.759%	0.051%	5.653%	0.137%	39.499%
69	Regulus	7.844%	9.088%	7.440%	4.605%	4.705%	3.222%	49.838%	19.678%	17.107%	12.960%	0.161%	0.347%	0.053%	0.302%	34.396%
70	Rigel	8.726%	8.772%	5.773%	6.214%	6.396%	4.371%	11.909%	43.029%	1.352%	1.443%	5.216%	0.880%	0.015%	26.348%	22.545%
71	Rukbat	0.559%	1.459%	1.842%	0.556%	0.581%	2.397%	16.366%	17.127%	0.025%	0.876%	1.417%	1.760%	0.168%	1.186%	43.167%
72	Sabik	0.506%	0.008%	0.025%	0.004%	0.011%	0.327%	28.691%	2.102%	1.240%	1.480%	0.021%	0.201%	0.819%	2.278%	35.595%
73	Salm	0.135%	7.105%	6.857%	4.394%	4.156%	0.837%	47.066%	5.834%	1.794%	1.483%	0.010%	0.283%	0.337%	1.643%	38.479%
74	Sceptrum	1.450%	16.663%	15.618%	18.479%	17.901%	5.495%	16.782%	12.707%	19.246%	17.001%	4.422%	7.261%	1.691%	0.330%	49.950%
75	Segin	2.187%	1.008%	0.719%	0.779%	0.778%	0.000%	90.181%	6.427%	5.254%	3.794%	0.003%	0.043%	0.396%	1.290%	17.989%
76	Sheratan	0.482%	3.867%	28.269%	27.556%	27.379%	5.727%	0.010%	38.124%	47.222%	48.049%	17.220%	26.038%	43.866%	40.844%	68.713%
77	Sirius	0.857%	0.512%	0.054%	0.480%	0.424%	0.460%	1.127%	2.510%	3.300%	5.762%	0.631%	21.692%	30.964%	3.049%	32.538%
78	Spica	0.711%	0.375%	0.145%	0.274%	0.265%	0.530%	57.706%	76.501%	64.380%	68.599%	11.452%	8.084%	61.412%	21.827%	11.094%
79	Subra	27.028%	14.281%	13.024%	13.728%	13.797%	28.909%	2.359%	33.248%	19.188%	18.423%	0.067%	8.065%	0.318%	40.806%	73.309%
80	Syrma	0.052%	0.035%	0.023%	0.026%	0.029%	0.416%	46.589%	0.007%	15.560%	13.508%	0.002%	0.000%	0.005%	0.011%	10.023%
81	Tabit	1.781%	2.101%	1.795%	0.906%	0.941%	0.009%	0.401%	1.785%	0.630%	2.293%	23.944%	0.301%	7.208%	1.302%	2.905%
82	Talitha	17.223%	12.395%	6.984%	4.396%	4.595%	16.815%	0.527%	0.030%	0.366%	0.269%	1.496%	0.367%	0.284%	0.267%	45.631%
83	Tarazed	2.697%	25.691%	22.988%	30.686%	29.307%	0.000%	47.282%	7.354%	22.979%	21.734%	3.233%	0.326%	5.406%	3.661%	3.159%
84	Tarf	0.100%	15.089%	12.460%	14.340%	14.177%	11.037%	48.314%	13.984%	0.569%	0.398%	7.843%	7.550%	0.910%	7.934%	2.089%
85	Tejat	3.505%	3.063%	2.099%	1.569%	1.560%	0.345%	0.098%	1.437%	6.670%	4.725%	2.123%	0.258%	1.642%	2.623%	7.809%
86	Thuban	1.427%	8.891%	6.340%	4.717%	4.541%	0.639%	69.859%	22.912%	0.685%	0.552%	1.379%	1.366%	1.297%	6.094%	29.534%
87	Tiaki	3.584%	5.990%	4.810%	4.606%	4.414%	4.935%	9.362%	10.127%	6.214%	4.938%	4.135%	0.102%	3.937%	19.390%	49.537%
88	Timir	0.828%	1.455%	0.586%	0.539%	0.525%	0.126%	14.931%	44.928%	33.331%	31.548%	17.706%	0.955%	19.241%	24.823%	56.262%
89	Titawin	1.380%	3.250%	2.930%	0.000%	0.001%	6.591%	41.377%	1.274%	0.453%	0.785%	0.668%	0.459%	0.056%	0.136%	34.060%
90	Toliman	28.249%	70.516%	47.066%	51.538%	48.915%	0.000%	1.660%	87.071%	30.666%	24.955%	85.780%	13.987%	42.486%	82.611%	17.681%
91	Tonatiuh	20.800%	11.642%	13.304%	11.655%	11.818%	3.179%	29.030%	22.821%	0.080%	0.101%	2.361%	0.238%	0.011%	0.791%	42.661%
92	Tureis	1.700%	1.785%	1.358%	0.890%	0.905%	0.089%	8.568%	37.307%	2.706%	2.801%	37.594%	1.346%	19.805%	37.239%	30.444%
93	Uruk	2.917%	0.006%	0.732%	0.768%	0.767%	0.185%	0.943%	8.269%	0.678%	0.993%	4.240%	3.881%	0.371%	7.481%	72.266%
94	Vega	1.254%	1.218%	1.312%	1.151%	1.157%	0.000%	7.774%	8.009%	46.079%	49.463%	14.506%	6.118%	39.650%	9.086%	25.528%
95	Veritate	3.156%	5.867%	15.047%	11.868%	11.791%	0.029%	8.212%	6.891%	6.841%	6.612%	4.051%	6.842%	6.531%	2.391%	22.033%
96	Wezen	0.000%	0.207%	0.311%	0.670%	0.717%	1.707%	0.067%	0.164%	4.098%	4.174%	0.283%	2.926%	0.324%	0.293%	32.166%
97	Xihe	11.396%	17.382%	11.457%	13.390%	13.262%	0.006%	10.966%	1.422%	2.892%	2.095%	6.197%	0.357%	0.280%	19.214%	30.863%
98	Yildun	0.105%	0.162%	0.102%	0.043%	0.041%	0.000%	0.055%	1.406%	0.009%	0.004%	1.633%	16.615%	0.057%	2.874%	0.000%
99	Zaurak	7.259%	6.063%	5.092%	5.441%	5.360%	1.648%	23.843%	11.473%	2.644%	1.796%	2.372%	0.376%	2.466%	2.131%	57.374%
100	Zibal	1.247%	10.462%	10.389%	17.985%	17.968%	4.473%	74.980%	0.041%	7.819%	7.202%	2.771%	7.873%	9.577%	0.103%	16.159%

4.7.4 The R^2 between Explanatory Variables and Stock Returns

Before looking at MLRs, it is worth examining which of the stock market and corporate performance variables shows the highest R^2 against each of the 100 stock returns. The details of this analysis are in Table 12 for all the R^2 of the fourteen variables plus the S&P 500 Index return, while in Table 13, we present the summary.

Table 13. Summary of R^2 data

R^2 Matrix	Annual Returns vs. main variables														
	Total Revenue	EBIT	Net Income	Adj Basic EPS	Adj Diluted EPS	Adj Dividends	Revenue Growth	EBIT Growth	Net Income Growth	Adj EPS Growth	DOL	DFL	DTL	Risk Rate	S&P 500 Annual Returns
μ	4.263%	6.264%	6.089%	6.463%	6.426%	2.999%	17.364%	12.948%	8.836%	9.051%	6.948%	4.853%	6.529%	8.577%	36.320%
σ	6.386%	11.363%	8.608%	10.364%	10.270%	5.272%	20.362%	18.028%	13.559%	13.679%	12.112%	8.273%	11.902%	14.490%	19.738%
Count	**1**	**1**	**0**	**2**	**0**	**0**	**18**	**9**	**1**	**2**	**2**	**2**	**0**	**1**	**61**

The S&P 500 Index return shows the highest mean R^2 with 36.3%, already established in Table 10, and in 61 cases, it results in having the highest value of R^2 compared to the other fourteen corporate performance variables. Not surprisingly, Revenue Growth and EBIT Growth present 18 and 9 cases, respectively, with the highest R^2 value with a rounded mean of 17.3% and 12.9%. Four variables never reached the maximum R^2: Net Income, Adjusted Diluted EPS, Adjusted Dividends and DTL. The other eight variables compete for the remaining 12 places.

It seems important to underline two topics relating to the annual stock returns:

1) Although the annual stock return is strongly correlated to the S&P 500 Index return, not unrelated to the fact that the latter derives from a weighted mean of the former, at least in the last seven years, the market also seems to appreciate other corporate performance indicators in 39 cases, and Revenue Growth and EBIT Growth play this role in 27 instances.

2) Dividends show a mean R^2 of less than 3%, with only six cases exceeding 10%: the market does not appreciate this form of equity remuneration, confirming the thesis by Modigliani-Miller that dividends are a financial illusion.

4.7.5 MLR Summary

Table 14 shows the MLR data analysis summary, while the details for each stock are in Tables 15a and 15b.

The F test appears significant in 94 cases, while only 28 intercepts, 81 stock market regressors, and 68 corporate performance regressors pass the null hypothesis test.

Table 14. Summary of MLR analysis

MLR	General Info				Value				R^2					t-stat			P-Value			F	
	Obs.	Month	Standard Error	Condition Number	Intercept	$_2\beta_M$	β_M % Change	$\rho(MR)$	R^2	Adjusted R^2	Stock Market Contribution	Bridge / (Overlap)	Corporate Performance Contribution	Intercept	$_2\beta_M$	$_2\beta_R$	Intercept	$_2\beta_M$	$_2\beta_R$	F	F P-Value
μ	26.38	5.68	0.267	27,227.71	4.492%	1.220	-15.975%	9.559%	55.861%	51.877%	33.405%	-3.573%	22.045%	0.911	3.764	1.697	14.098%	4.102%	3.514%	18.38	1.023%
σ	6.504	3.616	0.200	90,216.07	16.245%	1.116	37.279%	23.165%	17.958%	18.923%	20.283%	11.168%	23.228%	1.649	2.089	3.319	13.882%	11.102%	5.584%	14.01	4.404%
Count									**81**	**76**							**28**	**81**	**68**		**94**

Compared to the significant 100 β regressors of the 30-year OLS with monthly returns shown in subsection 4.3, it would appear like a significant step backwards, but if we look at the MLR $\overline{R^2}$ and R^2, we get a mean value of 51.8% and 55.8%, respectively, starting from 22,5% in Table 5.

4.7.6 The Commonality Analysis

The mean contribution of the stock market explanatory variable through $\overline{R^2}$ is 33.4%, while the corporate performance variable reaches 22.0% with a mean overlap of 3.5% (data in Table 14).

We should investigate further such mean results by separating the overlap from bridge cases. From Figure 8 on the left, we can examine the bridge case, where there is no apparent overlap between the $\overline{R^2}$ of the stock market and corporate performance variables, $\overline{R^2_M}$ and $\overline{R^2_\psi}$, respectively, while on the right, we have the overlap case. First, the transition from the monthly to the annual return is more noticeable in the bridge than in the overlap case by more than 5%. The adjustment due to the transition from one regressor to two regressors results in a reduction of about 3%.

With two regressors, the gross contribution of the stock market is higher than 5% in the bridge case, while the corporate performance contribution is only 6%, the same value as the mean bridge. In the overlap case, the gross contribution of the stock market is reduced by an overlap of 10%, while the corporate performance contributes almost 33%, higher than the contribution of the stock market. The $R^2_{M+\psi}$ is greater than 4% in the overlap than the bridge case.

Table 15a. MLR details

#	Conventional Name	Obs.	Month	Standard Error	Collinearity Test	Condition Number	Value					R^2					t-stat			P-Value			F		MLR Changes		
							Intercept	$_2\beta_M$	$_2\beta_R$	2nd Regressor Name	ρ(MR)	R^2	Adjusted R^2	Stock Market Contribution	Bridge / (Overlap)	Corporate Performance Contribution	Intercept	$_2\beta_M$	$_2\beta_R$	Intercept	$_2\beta_M$	$_2\beta_R$	F	F P-Value	R-squared Increase	R-squared % Change	β_M % Change
1	Achernar	23	2	0.197	False	2575.48	7.417%	0.941	0.0003195	DOL	52.402%	68.544%	65.399%	54.985%	-33.942%	44.355%	1.709	3.711	2.706	5.146%	0.069%	0.681%	21.791	0.001%	8.367%	14.671%	-27.643%
2	Adhil	28	6	0.258	False	11.09	13.365%	1.344	0.0200654	EBIT Growth	-4.141%	51.389%	47.500%	48.099%	2.451%	-3.050%	2.504	5.102	0.839	0.959%	0.001%	20.480%	13.214	0.012%	-2.521%	-5.040%	0.685%
3	Albaldah	8	12	0.156	False	160318.44	28.116%	2.463	-0.0000142	EBIT	-1.855%	87.898%	83.058%	66.158%	12.322%	4.577%	2.337	5.366	-2.643	3.331%	0.151%	2.291%	18.159	0.509%	12.065%	16.995%	-0.905%
4	Albali	30	1	0.114	False	28.62	-9.306%	0.426	0.0432832	Adj EPS	35.787%	58.828%	55.778%	34.089%	-19.636%	41.325%	-2.017	3.186	3.838	2.688%	0.181%	0.034%	19.289	0.001%	19.416%	53.397%	-30.125%
5	Alchiba	30	10	0.204	False	7.09	-2.953%	0.932	0.4359707	Revenue Growth	13.259%	49.764%	46.043%	28.335%	-5.150%	22.858%	-0.622	3.610	3.192	26.954%	0.062%	0.178%	13.373	0.009%	15.237%	49.461%	-10.494%
6	Alcor	20	1	0.302	False	6.74	26.053%	0.231	-0.0994543	EBIT Growth	-43.844%	10.079%	-0.500%	0.553%	-4.477%	3.424%	3.142	0.545	-0.901	0.297%	29.636%	19.016%	0.953	40.536%	-6.288%	-108.648%	-42.009%
7	Alcyone	30	11	0.131	False	7.78	2.670%	0.752	0.4385574	EBIT Growth	-12.955%	47.085%	43.165%	32.725%	6.608%	3.832%	0.798	4.514	2.479	21.594%	0.006%	0.987%	12.012	0.019%	8.120%	23.171%	7.658%
8	Aldebaran	30	3	0.184	False	6.25	5.964%	1.061	0.1148717	Net Profit Growth	26.738%	57.652%	54.515%	50.913%	-10.270%	13.872%	1.550	5.101	1.794	6.639%	0.001%	4.204%	18.379	0.001%	1.909%	3.629%	-8.594%
9	Alderamin	30	9	0.227	False	7.07	7.963%	0.805	0.2623843	EBIT Growth	14.975%	42.659%	38.411%	20.997%	-5.240%	22.654%	1.544	2.857	2.986	6.717%	0.406%	0.297%	10.043	0.055%	14.690%	61.930%	-13.533%
10	Aldhanab	30	8	0.309	False	9.54	-6.022%	2.024	1.0667200	Revenue Growth	46.920%	63.983%	61.315%	54.325%	-26.957%	33.946%	-0.819	4.562	2.461	21.012%	0.005%	1.026%	23.982	0.000%	5.414%	9.686%	-20.203%
11	Algol	30	9	0.216	False	151.55	16.167%	0.706	-0.0055199	DFL	18.302%	32.638%	27.648%	8.100%	7.122%	12.427%	3.432	2.625	-2.927	0.097%	0.705%	0.344%	6.541	0.483%	16.379%	145.348%	25.636%
12	Algorab	29	3	0.830	False	11.23	37.945%	1.592	-0.1435037	DFL	-7.939%	18.573%	12.309%	9.210%	-1.018%	4.117%	1.791	1.877	-1.398	4.251%	3.591%	8.698%	2.965	6.918%	-0.144%	-1.153%	-5.583%
13	Alhena	30	3	0.199	False	24225.62	24.621%	1.185	-0.0000570	EBIT	-8.052%	53.463%	50.016%	36.357%	-2.359%	16.018%	3.564	4.477	-2.941	0.069%	0.006%	0.332%	15.509	0.003%	11.464%	29.738%	-5.024%
14	Alioth	21	2	0.084	False	284941.44	18.898%	0.768	-0.0000033	Total Revenue	25.091%	78.047%	75.607%	68.661%	11.825%	-4.879%	2.750	7.981	-2.532	0.659%	0.000%	1.044%	31.996	0.000%	5.379%	7.660%	8.649%
15	Alkarab	30	2	0.209	False	10.32	-0.751%	0.462	1.3251562	EBIT Growth	19.196%	42.213%	37.932%	13.918%	-6.393%	30.408%	-0.143	2.096	3.440	44.379%	2.278%	0.095%	9.862	0.061%	21.046%	124.638%	-23.955%
16	Almaaz	30	3	0.254	False	5.75	6.319%	0.797	0.2604253	Revenue Growth	9.185%	32.998%	28.035%	23.142%	-1.744%	6.637%	1.118	3.054	1.704	13.667%	0.252%	4.993%	6.649	0.449%	2.243%	8.695%	-4.876%
17	Alnair	28	3	0.164	False	6.15	7.183%	0.334	0.4843519	Revenue Growth	24.166%	38.837%	33.944%	14.970%	-7.973%	26.946%	1.971	1.938	2.910	2.997%	3.203%	0.374%	7.937	0.214%	15.824%	87.331%	-26.629%
18	Alpheratz	30	6	0.115	False	14.50	5.983%	0.547	0.5223152	Revenue Growth	15.451%	47.443%	43.550%	39.588%	-4.237%	8.199%	2.300	4.305	1.722	1.470%	0.010%	4.825%	12.187	0.017%	1.879%	4.510%	-5.821%
19	Alrakis	30	2	0.230	False	5.85	6.196%	0.657	0.6512182	EBIT Growth	8.947%	48.268%	44.436%	15.579%	-2.651%	31.508%	1.153	2.741	3.942	12.946%	0.536%	0.026%	12.596	0.014%	25.946%	140.326%	-11.400%
20	Altair	30	3	0.213	False	8.89	0.521%	0.569	1.9040089	Revenue Growth	26.258%	64.460%	61.828%	18.342%	-12.097%	55.582%	0.112	2.363	5.736	45.596%	1.281%	0.000%	24.486	0.000%	40.669%	192.215%	-38.930%
21	Alzirr	30	1	0.121	False	49.54	-8.758%	1.332	0.0190421	Risk Rate	-4.806%	83.396%	82.166%	57.116%	6.645%	18.406%	-3.150	10.055	6.351	0.198%	0.000%	0.000%	67.806	0.000%	23.571%	40.228%	3.131%
22	Aniara	11	6	0.241	False	15.49	-20.036%	3.692	0.6858918	Revenue Growth	-8.140%	69.702%	62.127%	33.929%	13.019%	15.179%	-1.378	3.487	2.775	10.277%	0.412%	1.205%	9.202	0.843%	21.591%	53.263%	6.928%
23	Antares	30	9	0.141	False	10.83	-5.430%	0.877	0.0837910	DFL	-0.375%	61.302%	58.435%	34.092%	2.321%	22.022%	-1.590	5.053	4.171	6.174%	0.001%	0.014%	21.385	0.000%	22.070%	60.692%	0.311%
24	Arcturus	30	2	0.155	False	190.96	7.502%	0.875	-0.0030407	DTL	16.635%	57.321%	54.160%	37.173%	10.359%	6.627%	2.292	5.480	-3.373	1.497%	0.000%	0.113%	18.131	0.001%	14.820%	37.671%	11.405%
25	Aspidiske	21	2	0.353	False	6.26	11.217%	1.522	0.4994236	EBIT Growth	41.880%	78.643%	76.270%	44.050%	-29.665%	61.885%	1.363	3.538	5.177	9.480%	0.117%	0.003%	33.141	0.000%	29.423%	62.807%	-37.995%
26	Atlas	30	2	0.291	False	7.13	2.334%	0.808	1.3855525	Revenue Growth	39.038%	62.974%	60.231%	27.077%	-20.649%	53.804%	0.372	2.351	4.934	35.635%	1.315%	0.002%	22.961	0.000%	30.640%	103.543%	-45.037%
27	Bellatrix	30	10	0.174	False	49.81	1.593%	1.325	-0.0094882	DFL	-27.855%	63.935%	61.264%	57.540%	-11.610%	15.333%	0.393	5.848	-1.921	34.868%	0.000%	3.265%	23.933	0.000%	2.259%	3.829%	-8.384%
28	Betelgeuse	30	7	0.151	False	110193.58	29.541%	-0.291	-0.0000096	Total Revenue	5.532%	15.282%	9.007%	4.094%	-0.551%	5.464%	3.214	-1.446	-1.585	0.169%	7.987%	6.232%	2.435	10.658%	1.606%	21.699%	-5.717%
29	Bharani	30	10	0.139	False	90135.17	0.084%	0.687	0.0000084	Total Revenue	-17.901%	36.991%	32.323%	28.669%	5.244%	-1.590%	0.012	3.877	1.585	49.541%	0.031%	6.231%	7.925	0.196%	1.195%	3.838%	7.896%
30	Canopus	30	11	0.243	False	7.12	12.365%	0.562	0.0705693	Adj EPS Growth	-5.159%	16.362%	10.166%	6.401%	1.214%	2.551%	2.325	1.837	1.474	1.393%	3.864%	7.599%	2.641	8.963%	0.537%	5.581%	4.320%
31	Capella	30	1	0.121	False	1015.98	3.660%	0.591	-0.0001669	DFL	7.586%	43.398%	39.205%	37.890%	3.001%	-1.685%	1.443	4.453	-1.267	8.021%	0.007%	10.795%	10.351	0.046%	-0.826%	-2.064%	2.206%
32	Castor	30	12	0.187	False	6.16	5.463%	0.719	0.3444788	Adj EPS Growth	-6.845%	39.684%	35.216%	21.979%	3.933%	9.304%	1.271	3.493	2.593	10.730%	0.083%	0.760%	8.882	0.109%	10.547%	42.755%	5.353%
33	Copernicus	30	4	0.210	False	44223.84	13.394%	1.089	-0.0000156	EBIT*	-13.687%	43.078%	38.862%	39.508%	-1.597%	0.951%	1.057	4.285	-0.839	14.998%	0.010%	20.439%	10.217	0.050%	-2.732%	-6.568%	-2.610%
34	Deneb	20	9	0.306	False	9.37	21.473%	1.677	-0.1672609	Revenue Growth	-53.455%	59.505%	54.741%	50.449%	-28.658%	32.950%	2.232	3.109	-1.645	1.970%	0.319%	5.913%	12.490	0.046%	1.684%	3.175%	-22.050%
35	Diadem	30	8	0.210	False	9.14	-5.466%	0.668	2.1696813	Revenue Growth	10.277%	66.050%	63.535%	12.996%	-3.350%	53.889%	-0.990	2.899	6.309	16.553%	0.367%	0.000%	26.265	0.000%	47.539%	297.194%	-18.275%
36	Dubhe	25	2	0.486	False	573.56	2.034%	1.555	-0.0010297	DOL	3.987%	32.255%	26.097%	25.064%	1.985%	-0.952%	0.191	3.069	-1.150	42.509%	0.281%	13.133%	5.237	1.379%	-2.089%	-7.413%	1.516%
37	Electra	22	1	0.247	False	44.34	8.014%	0.931	0.0301389	Risk Rate	46.594%	62.557%	58.616%	40.172%	-27.522%	45.966%	1.398	2.667	3.149	8.677%	0.639%	0.199%	15.872	0.009%	15.595%	36.249%	-35.486%
38	Furud	30	10	0.299	False	32.48	8.901%	0.826	0.0981187	Net Profit Growth	25.017%	74.253%	72.346%	13.890%	-10.396%	68.852%	1.367	2.130	7.758	9.139%	2.121%	0.000%	38.934	0.000%	55.487%	329.122%	-47.675%
39	Gienah	16	8	0.092	False	23.30	6.588%	1.181	0.0136938	DTL	-29.938%	71.553%	67.177%	66.547%	6.918%	-6.288%	1.841	5.686	1.126	4.426%	0.004%	14.018%	16.350	0.028%	-1.600%	-2.327%	6.304%
40	Hamal	30	5	0.164	False	13.24	9.873%	-0.003	1.7450992	Revenue Growth	52.990%	54.403%	51.026%	12.130%	-13.879%	52.775%	2.813	-0.014	4.821	0.452%	49.466%	0.002%	16.107	0.002%	35.866%	236.586%	-100.532%
41	Intercrus	30	5	0.132	False	10.16	1.892%	1.266	0.0150006	Adj EPS Growth	24.947%	77.858%	76.218%	76.138%	-6.339%	6.419%	0.678	9.120	1.046	25.172%	0.000%	15.241%	47.469	0.000%	-0.743%	-0.965%	-2.782%
42	Izar	28	9	0.525	False	89698.15	26.306%	2.187	-0.0000162	Total Revenue	2.644%	36.559%	31.483%	26.069%	2.016%	3.399%	1.884	3.414	-1.748	3.563%	0.109%	4.639%	7.203	0.339%	2.677%	9.291%	1.372%
43	Jabbah	30	1	0.145	False	629.05	8.454%	1.257	-0.0005011	DFL	13.646%	69.824%	67.588%	64.622%	5.755%	-2.788%	2.753	7.861	-1.888	0.521%	0.000%	3.494%	31.237	0.000%	1.747%	2.653%	3.388%
44	Kang	30	5	0.130	False	14.07	6.422%	0.678	0.3487882	Revenue Growth	-3.985%	50.157%	46.465%	46.239%	2.406%	-2.179%	1.738	5.142	1.058	4.680%	0.001%	14.980%	13.585	0.008%	-1.627%	-3.383%	0.826%
45	Keid	20	2	0.540	False	5.46	-8.037%	2.101	2.1307132	Revenue Growth	0.529%	67.850%	64.068%	16.233%	3.212%	44.623%	-0.517	3.277	4.996	30.588%	0.222%	0.006%	17.939	0.006%	43.426%	210.376%	-0.800%
46	Kitalpha	30	10	0.106	False	42.11	-21.961%	0.867	0.0558425	Risk Rate	25.521%	68.772%	66.459%	62.146%	-10.940%	15.252%	-1.592	6.614	2.145	6.155%	0.000%	2.056%	29.731	0.000%	3.008%	4.740%	-7.643%
47	Kochab	30	9	0.531	False	7.96	5.774%	2.620	0.1376663	EBIT Growth	31.580%	40.778%	36.392%	36.598%	-7.958%	7.752%	0.491	3.689	0.953	31.354%	0.050%	17.442%	9.296	0.085%	-2.393%	-6.170%	-7.546%
48	Kraz	30	8	0.211	False	46.50	-4.963%	1.002	0.0301385	Adj EPS	-1.360%	56.285%	53.047%	19.019%	2.669%	31.359%	-0.990	3.733	4.614	16.557%	0.045%	0.004%	17.382	0.001%	31.236%	143.210%	1.710%
49	Libertas	15	1	0.339	False	36481.26	31.306%	1.874	-0.0000184	Total Revenue	9.966%	59.831%	53.136%	52.831%	6.576%	-6.271%	2.447	4.181	-1.041	1.537%	0.064%	15.912%	8.937	0.420%	-3.065%	-5.453%	2.545%
50	Markab	30	1	0.148	False	6.14	6.847%	1.085	0.0681933	Adj EPS Growth	6.662%	64.710%	62.096%	59.281%	-0.195%	3.009%	2.166	6.682	1.755	1.967%	0.000%	4.532%	24.755	0.000%	1.411%	2.325%	-1.719%

Table 15b. MLR details

#	Conventional Name	Obs.	Month	Standard Error	Collinearity Test	Condition Number	Value					R^2					t-stat			P-Value			F		MLR Changes		
							Intercept	$_2\beta_M$	$_2\beta_R$	2nd Regressor Name	ρ(MR)	R^2	Adjusted R^2	Stock Market Contribution	Bridge / (Overlap)	Corporate Performance Contribution	Intercept	$_2\beta_M$	$_2\beta_R$	Intercept	$_2\beta_M$	$_2\beta_R$	F	F P-Value	R-squared Increase	R-squared % Change	β_M % Change
51	Matar	30	1	0.246	False	7.19	4.515%	1.665	0.7541950	Revenue Growth	18.933%	65.084%	62.498%	55.781%	-7.737%	14.455%	0.868	6.072	2.453	19.654%	0.000%	1.046%	25.164	0.000%	5.193%	9.061%	-7.104%
52	Merak	28	11	1.474	False	10.93	-2.383%	6.390	-0.3752372	Net Profit Growth	42.848%	27.439%	21.634%	15.430%	9.307%	-3.103%	-0.071	3.034	-1.749	47.216%	0.278%	4.630%	4.727	1.815%	3.072%	16.547%	32.793%
53	Merga	30	2	0.215	False	6328.12	5.710%	1.301	0.0000654	DTL	-10.659%	56.525%	53.305%	49.587%	4.948%	-1.229%	1.275	5.805	1.797	10.666%	0.000%	4.176%	17.552	0.001%	1.980%	3.857%	3.412%
54	Merope	10	12	0.193	False	39.78	10.925%	1.836	-0.0214887	Net Profit Growth	10.968%	65.080%	55.103%	47.759%	12.411%	-5.067%	1.188	3.424	-1.519	13.687%	0.554%	8.623%	6.523	2.516%	1.539%	2.874%	5.117%
55	Mesarthim	30	7	0.227	False	11.81	14.325%	0.077	2.2674177	Revenue Growth	22.020%	47.221%	43.312%	-0.024%	-1.875%	45.211%	2.936	0.249	4.733	0.336%	40.266%	0.003%	12.078	0.018%	39.887%	1164.582%	-80.722%
56	Mimosa	30	5	0.118	False	7.41	8.712%	0.540	-0.0769387	Adj EPS Growth	2.214%	39.584%	35.109%	23.876%	2.101%	9.132%	3.375	3.494	-2.418	0.113%	0.083%	1.131%	8.845	0.111%	8.608%	32.482%	1.556%
57	Mira	30	12	0.156	False	6.13	5.003%	0.366	0.5502011	Revenue Growth	-3.974%	38.945%	34.423%	5.736%	2.613%	26.074%	1.449	2.137	3.640	7.942%	2.093%	0.057%	8.611	0.128%	25.437%	283.070%	7.263%
58	Mizar	12	2	0.144	False	11.95	5.697%	0.476	1.2776509	EBIT Growth	28.513%	59.193%	50.124%	19.867%	-12.284%	42.542%	1.011	1.588	2.658	16.912%	7.343%	1.306%	6.527	1.771%	22.973%	84.611%	-32.316%
59	Musica	30	7	0.215	False	48.40	2.245%	1.595	-0.0064403	Risk Rate	38.310%	51.256%	47.645%	48.573%	-0.852%	-0.076%	0.426	5.150	-0.710	33.691%	0.001%	24.192%	14.196	0.006%	-2.701%	-5.365%	5.575%
60	Nashira	19	7	0.112	False	21.59	13.406%	1.115	-0.0232343	DFL	-2.215%	81.771%	79.492%	71.987%	3.489%	4.017%	3.855	7.973	-2.687	0.070%	0.000%	0.809%	35.886	0.000%	5.949%	8.089%	-0.741%
61	Nekkar	30	5	0.121	False	156347.13	-2.464%	0.816	0.0000043	EBIT	3.130%	57.306%	54.144%	41.702%	0.323%	12.118%	-0.723	5.163	2.932	23.785%	0.001%	0.339%	18.121	0.001%	10.431%	23.864%	-1.747%
62	Nembus	19	2	0.192	False	6.22	2.692%	1.465	0.1886049	EBIT Growth	-51.454%	68.419%	64.472%	60.365%	5.772%	-1.665%	0.469	5.714	1.722	32.275%	0.002%	5.218%	17.332	0.010%	1.905%	3.044%	18.353%
63	Okab	30	2	0.241	False	6.41	-0.243%	1.050	0.8539157	Net Profit Growth	24.514%	59.566%	56.571%	37.165%	-13.056%	32.463%	-0.039	4.067	3.676	48.461%	0.019%	0.052%	19.888	0.000%	17.240%	43.831%	-18.136%
64	Polaris	19	2	0.077	False	9.10	2.450%	0.637	-0.3717114	Revenue Growth	31.464%	74.414%	71.216%	63.419%	13.586%	-5.789%	1.280	6.818	-2.367	10.942%	0.000%	1.543%	23.267	0.002%	5.764%	8.807%	12.266%
65	Pollux	13	9	0.157	False	272022.61	-11.776%	1.341	0.0000053	Total Revenue	37.242%	74.813%	69.775%	58.376%	-22.408%	33.808%	-1.650	3.754	2.269	6.497%	0.188%	2.332%	14.851	0.101%	7.931%	12.824%	-18.376%
66	Procyon	8	7	0.086	False	3571.31	35.716%	1.456	0.0010009	EBIT	24.692%	89.863%	85.809%	21.048%	-7.984%	72.745%	4.583	2.554	5.327	0.297%	2.550%	0.156%	22.163	0.327%	53.482%	165.442%	-33.994%
67	Proxima Centauri	20	8	0.424	False	9.21	-24.752%	2.992	1.3528689	Revenue Growth	-48.683%	49.337%	43.377%	16.271%	26.261%	0.845%	-1.327	3.811	3.101	10.106%	0.070%	0.324%	8.278	0.309%	22.699%	109.773%	65.613%
68	Ran	30	11	0.133	False	10.29	1.328%	0.803	0.5546561	Revenue Growth	-10.014%	49.012%	45.236%	37.339%	5.305%	2.592%	0.419	4.775	2.244	33.914%	0.003%	1.660%	12.977	0.011%	5.736%	14.522%	4.939%
69	Regulus	30	1	0.236	False	8.38	-10.304%	1.223	1.5676866	Revenue Growth	26.248%	67.123%	64.688%	32.053%	-15.411%	48.046%	-1.481	3.768	5.184	7.513%	0.041%	0.001%	27.563	0.000%	30.292%	88.069%	-26.533%
70	Rigel	30	5	0.461	False	8.47	8.230%	1.162	0.3507840	EBIT Growth	34.560%	50.020%	46.318%	19.779%	-14.455%	40.994%	0.836	1.943	3.853	20.529%	3.123%	0.033%	13.511	0.009%	23.773%	105.448%	-40.657%
71	Rukbat	30	1	0.151	False	6.20	1.198%	0.763	0.1368025	EBIT Growth	14.787%	53.421%	49.970%	41.137%	-5.334%	14.167%	0.373	4.587	2.438	35.605%	0.005%	1.082%	15.483	0.003%	6.803%	15.760%	-7.287%
72	Sabik	30	2	0.700	False	7.34	-8.007%	3.252	1.0431281	Revenue Growth	24.367%	51.787%	48.216%	33.295%	-11.224%	26.145%	-0.506	3.596	3.011	30.841%	0.064%	0.279%	14.501	0.005%	12.621%	35.456%	-16.945%
73	Salm	30	2	0.377	False	7.08	4.743%	1.269	0.8022490	Revenue Growth	46.753%	58.550%	55.480%	36.282%	-25.978%	45.175%	0.590	2.735	3.616	28.010%	0.544%	0.061%	19.070	0.001%	17.001%	44.181%	-38.198%
74	Sceptrum	30	4	0.258	False	38.06	-3.890%	1.615	0.0210173	Net Profit Growth	20.687%	58.888%	55.842%	48.163%	-8.682%	16.362%	-0.693	5.102	2.423	24.715%	0.001%	1.119%	19.337	0.001%	5.892%	11.796%	-8.944%
75	Segin	23	6	0.674	False	8.81	-66.316%	0.573	2.9495085	Revenue Growth	41.062%	90.322%	89.354%	14.083%	-14.443%	89.714%	-3.726	0.539	12.226	0.067%	29.790%	0.000%	93.325	0.000%	71.365%	396.723%	-90.304%
76	Sheratan	7	11	0.090	False	385.32	-12.717%	3.493	-0.0056385	DOL	9.536%	93.343%	90.014%	62.456%	26.894%	0.664%	-1.953	6.763	-3.847	6.130%	0.125%	0.918%	28.043	0.443%	21.301%	31.000%	5.735%
77	Sirius	30	4	0.224	False	142.88	6.379%	0.968	-0.0067416	DTL	-20.507%	52.700%	49.197%	30.129%	-9.431%	28.499%	1.353	3.522	-3.393	9.368%	0.077%	0.108%	15.041	0.004%	16.658%	51.196%	-16.493%
78	Spica	30	9	0.208	False	6.99	3.401%	0.456	1.1736798	EBIT Growth	20.679%	78.921%	77.359%	7.919%	-6.221%	75.661%	0.737	1.761	9.321	23.382%	4.481%	0.000%	50.544	0.000%	66.265%	597.304%	-52.261%
79	Subra	7	1	0.119	False	56431.53	-31.220%	1.455	0.0000794	Total Revenue	31.438%	80.284%	70.426%	67.971%	-9.978%	12.434%	-0.870	3.287	1.190	21.680%	1.515%	14.999%	8.144	3.887%	-2.883%	-3.932%	-10.215%
80	Syrma	30	2	0.458	False	6.92	-6.785%	1.359	1.7553717	Revenue Growth	1.514%	55.970%	52.709%	6.810%	1.218%	44.681%	-0.609	2.399	5.308	27.383%	1.182%	0.001%	17.161	0.002%	42.686%	425.879%	-3.242%
81	Tabit	30	12	0.248	False	167.61	11.998%	0.189	0.0050453	DOL	11.602%	25.253%	19.717%	-0.563%	-0.948%	21.228%	2.197	0.688	2.841	1.838%	24.874%	0.422%	4.561	1.966%	16.812%	578.806%	-32.402%
82	Talitha	30	4	0.243	False	284035.24	26.449%	1.439	-0.0000058	Total Revenue	-14.152%	56.041%	52.785%	43.689%	-5.171%	14.267%	2.558	4.883	-2.529	0.823%	0.002%	0.880%	17.211	0.002%	7.155%	15.679%	-6.829%
83	Tarazed	30	5	0.648	False	7.60	2.316%	0.072	2.7830618	Revenue Growth	24.211%	47.295%	43.391%	-0.300%	-1.708%	45.399%	0.162	0.083	4.755	43.642%	46.723%	0.003%	12.114	0.018%	40.232%	1273.611%	-93.275%
84	Tarf	30	5	0.250	False	7.45	3.043%	0.044	0.7077304	Revenue Growth	18.191%	48.347%	44.521%	-1.408%	-0.539%	46.468%	0.538	0.133	4.917	29.755%	44.757%	0.002%	12.636	0.013%	42.432%	2031.153%	-87.051%
85	Tejat	30	8	0.405	False	10.80	17.909%	1.069	0.0998807	Net Profit Growth	-23.612%	18.943%	12.939%	4.517%	5.086%	3.337%	1.963	2.022	1.926	3.000%	2.660%	3.236%	3.155	5.870%	5.130%	65.695%	29.015%
86	Thuban	30	1	0.188	False	7.68	4.024%	0.538	1.5421632	Revenue Growth	39.682%	75.182%	73.343%	27.017%	-22.456%	68.782%	1.016	2.406	7.047	15.921%	1.161%	0.000%	40.896	0.000%	43.810%	148.339%	-53.747%
87	Tiaki	30	11	0.112	False	20.42	2.391%	0.807	1.0461645	Revenue Growth	-0.622%	59.169%	56.144%	47.734%	2.285%	6.125%	0.826	5.739	2.524	20.801%	0.000%	0.890%	19.563	0.001%	6.607%	13.339%	0.274%
88	Timir	24	8	0.237	False	48.88	-47.971%	1.422	0.0621072	Risk Rate	17.463%	70.173%	67.332%	54.274%	-8.348%	21.406%	-2.627	5.651	3.130	0.787%	0.001%	0.253%	24.703	0.000%	11.070%	19.676%	-8.819%
89	Titawin	30	7	0.228	False	18.80	2.295%	0.962	2.8039710	Revenue Growth	35.449%	55.838%	52.567%	31.705%	-18.421%	39.283%	0.464	2.973	3.649	32.330%	0.307%	0.056%	17.069	0.002%	18.507%	54.337%	-30.315%
90	Toliman	10	11	0.669	False	125.57	-34.310%	7.962	-0.2215783	EBIT Growth	-17.101%	94.084%	92.393%	7.391%	-0.452%	85.455%	-0.875	2.880	-9.508	20.539%	1.182%	0.001%	55.660	0.005%	74.713%	422.569%	-36.081%
91	Tonatiuh	30	6	0.391	False	198452.11	48.894%	2.203	-0.0000134	Total Revenue	-5.849%	60.182%	57.232%	40.613%	-1.352%	17.971%	3.599	5.168	-3.447	0.063%	0.001%	0.094%	20.404	0.000%	14.572%	34.157%	-3.755%
92	Tureis	27	9	0.277	False	114715.63	14.354%	0.867	-0.0000105	DOL	-36.970%	49.835%	45.655%	27.662%	-17.105%	35.098%	2.376	2.420	-3.046	1.292%	1.173%	0.278%	11.921	0.025%	15.211%	49.964%	-31.755%
93	Uruk	30	11	0.135	False	26.74	7.157%	1.465	-0.0116294	Adj EPS	3.158%	73.576%	71.619%	71.276%	3.121%	-2.777%	1.746	8.625	-1.157	4.608%	0.000%	12.871%	37.591	0.000%	-0.647%	-0.896%	0.425%
94	Vega	18	5	0.630	False	16.51	20.939%	1.689	0.2950689	Adj EPS Growth	39.729%	55.519%	49.589%	20.874%	-17.589%	46.304%	1.172	1.429	3.180	12.971%	8.673%	0.311%	9.361	0.230%	24.061%	94.251%	-46.924%
95	Veritate	30	2	0.245	False	862.53	5.907%	0.813	0.0006327	DTL	-14.171%	32.619%	27.628%	19.248%	5.187%	3.193%	1.120	3.233	2.060	13.638%	0.161%	2.460%	6.535	0.484%	5.595%	25.395%	9.923%
96	Wezen	30	7	0.147	False	7.73	7.353%	0.723	0.0449841	Adj EPS Growth	-0.219%	36.391%	31.679%	29.743%	1.184%	0.751%	2.260	3.698	1.339	1.603%	0.049%	9.584%	7.723	0.223%	-0.487%	-1.514%	0.080%
97	Xihe	30	4	0.183	False	713625.65	-6.127%	0.941	0.0000013	Total Revenue	-11.346%	47.121%	43.204%	28.394%	6.579%	8.231%	-1.184	4.271	2.881	12.343%	0.011%	0.384%	12.030	0.018%	12.341%	39.986%	8.288%
98	Yildun	29	12	0.702	False	12.43	38.694%	-0.008	-0.1556543	DFL	-0.684%	16.615%	10.201%	-3.704%	0.378%	13.527%	2.476	-0.010	-2.276	1.007%	49.612%	1.566%	2.590	9.422%	10.201%	9631199.734%	-270.905%
99	Zaurak	30	3	0.130	False	69143.73	19.896%	0.836	-0.0000060	Total Revenue	-6.648%	62.195%	59.394%	55.851%	-0.404%	3.947%	4.726	6.264	-1.856	0.003%	0.000%	3.723%	22.209	0.000%	2.021%	3.522%	-1.931%
100	Zibal	30	7	0.419	False	6.40	-4.561%	0.324	1.7404087	Revenue Growth	39.367%	75.421%	73.601%	13.165%	-13.650%	74.086%	-0.508	0.697	8.069	30.787%	24.600%	0.000%	41.426	0.000%	57.442%	355.478%	-82.013%

Such a result mainly depends on the correlation between the stock market and corporate performance: in the overlap case, the mean correlation reaches 17.4%, while in the bridge case, the correlation is -1.7%. There are 36 negative correlations between R_M and the corporate performance ψ_i selected as the second regressor with a mean of -14.7%, while the mean value for the remaining 64 correlations is 23.2%. The mean value of the correlation for all 100 stocks is 9.5%.

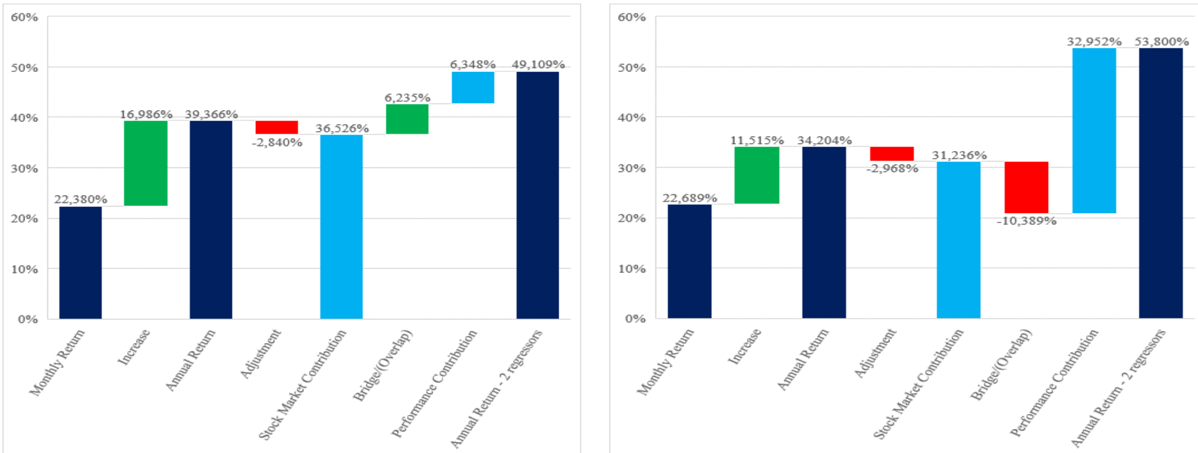

Figure 8. Commonality Analysis: Bridge and Overlap Split

There are no significant correlations between the weights assumed by the various stocks in the optimised portfolios, with or without short sales, and the $\overline{R^2}$ value assumed in the various analyses carried out so far. Bridge and overlap cannot explain short sales or zero weights in the two different kinds of optimised portfolios, and we cannot deduce any correlation between weights on one side and the joint performance of the stock market and the corporate business on the other.

Moving from one to two regressors, $\overline{R^2}$ increases by 15.5%, but the impact of the second regressor results in a mean 16% decrease in the β due to stock market performance. Moreover, this reduction is entirely clustered in the overlap cases, resulting in a reduction of the corresponding β by 27.5%; in the bridge case, there is a slight increase in the stock market β of 0.7%. In essence, the traditional β could overestimate the impact of stock market performance on the stock return in the overlap case with a second variable affected by the corporate performance. This effect would happen when the two performances are positively correlated; if the correlation is negative, this effect is less observable.

Table 16. Summary of the MLR regressors and commonality analysis

2nd Regressor Name	Bridge / Overlap	#	30Y Monthly Returns			30Y Annual Returns			30Y Annual Returns (MLR)							R² Adjusted % Change	$_2\beta_M$ % Change over Monthly Returns	$_2\beta_M$ % Change over Annual Returns	Stock Market / Corp. Performance Correlation
			Intercept	β_M	R^2	Intercept	β_M	R^2	Intercept	$_2\beta_M$	$_2\beta_\pi$	R^2 Adjusted	Stock Market Contribution	Bridge / (Overlap)	Corporate Performance Contribution				
Revenue Growth	Bridge	9	0.013	0.923	17.522%	0.131	1.326	33.716%	-0.047	1.493	0.8947716	52.863%	30.412%	7.767%	14.683%	19.147%	61.801%	12.588%	-4.657%
	Overlap	21	0.010	1.297	26.281%	0.117	1.570	27.592%	-0.007	0.874	1.3699230	55.010%	24.905%	-12.292%	42.398%	27.418%	-32.623%	-44.325%	24.780%
	Total	*30*	*0.011*	*1.185*	*23.653%*	*0.121*	*1.497*	*29.430%*	*-0.019*	*1.060*	*1.2273776*	*54.366%*	*26.557%*	*-6.274%*	*34.083%*	*24.937%*	*-10.563%*	*-29.198%*	*15.949%*
EBIT Growth	Bridge	3	0.010	0.801	19.811%	0.095	1.090	49.211%	0.062	1.187	0.2157426	51.712%	47.063%	4.944%	-0.294%	2.501%	48.121%	8.860%	-22.850%
	Overlap	11	0.013	1.068	19.900%	0.084	2.260	24.741%	0.037	1.556	0.4994303	49.919%	20.708%	-8.648%	37.859%	25.178%	45.631%	-31.158%	14.016%
	Total	*14*	*0.013*	*1.011*	*19.881%*	*0.086*	*2.009*	*29.985%*	*0.042*	*1.477*	*0.4386401*	*50.303%*	*26.355%*	*-5.736%*	*29.683%*	*20.318%*	*46.054%*	*-26.505%*	*6.116%*
Total Revenue	Bridge	5	0.010	0.878	20.761%	0.102	1.239	43.445%	0.141	1.291	-0.0000056	47.151%	40.925%	6.448%	-0.222%	3.705%	47.077%	4.189%	1.691%
	Overlap	6	0.009	0.790	21.124%	0.093	1.274	48.037%	0.136	1.164	0.0000083	53.103%	45.099%	-6.644%	14.649%	5.067%	47.243%	-8.608%	7.927%
	Total	*11*	*0.010*	*0.830*	*20.959%*	*0.097*	*1.258*	*45.950%*	*0.138*	*1.222*	*0.0000020*	*50.398%*	*43.202%*	*-0.693%*	*7.889%*	*4.448%*	*47.163%*	*-2.877%*	*5.092%*
DFL	Bridge	6	0.008	0.851	24.286%	0.103	0.726	37.842%	0.125	0.756	-0.0168809	47.095%	35.498%	3.677%	7.920%	9.253%	-11.153%	4.133%	6.043%
	Overlap	2	0.014	0.859	9.900%	0.094	1.566	35.729%	0.198	1.459	-0.0764959	36.787%	33.375%	-6.314%	9.725%	1.058%	69.734%	-6.876%	-17.897%
	Total	*8*	*0.010*	*0.853*	*20.690%*	*0.101*	*0.936*	*37.313%*	*0.143*	*0.932*	*-0.0317847*	*44.518%*	*34.967%*	*1.180%*	*8.371%*	*7.205%*	*9.212%*	*-0.471%*	*0.058%*
Adj Diluted EPS Growth	Bridge	4	0.008	0.670	18.716%	0.094	0.619	23.241%	0.085	0.636	0.0957734	28.043%	20.500%	2.108%	5.435%	4.801%	-5.047%	2.774%	-2.502%
	Overlap	3	0.014	1.088	31.909%	0.151	1.863	54.391%	0.099	1.347	0.1260876	62.634%	52.097%	-8.041%	18.578%	8.243%	23.827%	-27.707%	23.779%
	Total	*7*	*0.011*	*0.849*	*24.370%*	*0.118*	*1.152*	*36.591%*	*0.091*	*0.941*	*0.1087652*	*42.868%*	*34.042%*	*-2.241%*	*11.067%*	*6.276%*	*10.807%*	*-18.350%*	*8.761%*
Net Income Growth	Bridge	3	0.016	1.060	19.913%	0.165	2.462	26.645%	0.088	3.098	-0.0989484	29.892%	22.569%	8.935%	-1.611%	3.247%	192.337%	25.826%	10.068%
	Overlap	4	0.007	1.029	23.027%	0.056	1.449	39.687%	0.027	1.138	0.2719808	59.819%	37.533%	-10.601%	32.887%	20.132%	10.586%	-21.459%	24.239%
	Total	*7*	*0.011*	*1.042*	*21.693%*	*0.103*	*1.883*	*34.097%*	*0.053*	*1.978*	*0.1130111*	*46.993%*	*31.119%*	*-2.229%*	*18.102%*	*12.896%*	*89.800%*	*5.039%*	*18.166%*
Risk Rate	Bridge	1	-0.002	1.246	42.180%	-0.015	1.291	58.595%	-0.088	1.332	0.0190421	82.166%	57.116%	6.645%	18.406%	23.571%	6.866%	3.131%	-4.806%
	Overlap	4	0.008	0.868	20.928%	0.067	1.363	53.270%	-0.149	1.204	0.0354121	60.013%	51.291%	-11.915%	20.637%	6.743%	38.629%	-11.683%	31.972%
	Total	*5*	*0.006*	*0.944*	*25.179%*	*0.051*	*1.349*	*54.335%*	*-0.137*	*1.229*	*0.0321381*	*64.444%*	*52.456%*	*-8.203%*	*20.191%*	*10.109%*	*30.244%*	*-8.847%*	*24.616%*
DOL	Bridge	2	0.005	1.325	24.270%	-0.078	2.418	48.450%	-0.053	2.524	-0.0033341	58.056%	43.760%	14.440%	-0.144%	9.606%	90.491%	4.399%	6.761%
	Overlap	3	0.010	0.890	19.076%	0.127	0.950	30.127%	0.113	0.665	0.0017848	43.590%	27.361%	-17.332%	33.560%	13.463%	-25.271%	-29.943%	9.011%
	Total	*5*	*0.008*	*1.064*	*21.153%*	*0.045*	*1.537*	*37.456%*	*0.046*	*1.409*	*-0.0002628*	*49.376%*	*33.921%*	*-4.623%*	*20.079%*	*11.920%*	*32.376%*	*-8.335%*	*8.111%*
DTL	Bridge	4	0.008	0.931	27.374%	0.070	0.974	45.369%	0.064	1.043	0.0028378	50.567%	43.139%	6.853%	0.576%	5.199%	11.947%	7.087%	-9.534%
	Overlap	1	0.007	0.993	24.789%	0.065	1.159	32.538%	0.064	0.968	-0.0067416	49.197%	30.129%	-9.431%	28.499%	16.658%	-2.509%	-16.493%	-20.507%
	Total	*5*	*0.007*	*0.944*	*26.857%*	*0.069*	*1.011*	*42.803%*	*0.064*	*1.028*	*0.0009219*	*50.293%*	*40.537%*	*3.596%*	*6.160%*	*7.491%*	*8.906%*	*1.680%*	*-11.728%*
EBIT	Bridge	2	0.009	0.968	24.098%	0.046	1.658	57.353%	0.128	1.640	-0.0000050	68.601%	53.930%	6.323%	8.348%	11.248%	69.326%	-1.116%	0.638%
	Overlap	2	0.015	0.977	23.009%	0.134	1.727	35.439%	0.302	1.321	0.0004720	67.913%	28.703%	-5.171%	44.381%	32.473%	35.135%	-23.529%	8.320%
	Total	*4*	*0.012*	*0.973*	*23.554%*	*0.090*	*1.693*	*46.396%*	*0.215*	*1.480*	*0.0002335*	*68.257%*	*41.316%*	*0.576%*	*26.365%*	*21.861%*	*52.153%*	*-12.550%*	*4.479%*
Adj Diluted EPS	Bridge	2	0.005	1.144	33.949%	0.041	1.222	47.039%	0.011	1.234	0.0092546	62.333%	45.147%	2.895%	14.291%	15.294%	7.837%	0.943%	0.899%
	Overlap	1	0.006	0.332	5.954%	0.058	0.610	36.362%	-0.093	0.426	0.0432832	55.778%	34.089%	-19.636%	41.325%	19.416%	28.444%	-30.125%	35.787%
	Total	*3*	*0.005*	*0.873*	*24.617%*	*0.047*	*1.018*	*43.480%*	*-0.024*	*0.964*	*0.0205974*	*60.148%*	*41.461%*	*-4.615%*	*23.302%*	*16.668%*	*10.447%*	*-5.260%*	*12.528%*
EBIT* (Bloomberg data)	Overlap	1	0.005	0.666	15.746%	0.034	1.118	41.594%	0.134	1.089	-0.0000156	38.862%	39.508%	-1.597%	0.951%	-2.732%	63.613%	-2.610%	-13.687%
	Total	*1*	*0.005*	*0.666*	*15.746%*	*0.034*	*1.118*	*41.594%*	*0.134*	*1.089*	*-0.0000156*	*38.862%*	*39.508%*	*-1.597%*	*0.951%*	*-2.732%*	*63.613%*	*-2.610%*	*-13.687%*
		100	**0.010**	**1.008**	**22.562%**	**0.096**	**1.460**	**36.320%**	**0.045**	**1.220**	**0.4448716**	**51.877%**	**33.405%**	**-3.573%**	**22.045%**	**15.556%**	**21.017%**	**-16.439%**	**9.559%**

4.7.7 MLR Details and Comparison with OLS

There is no multicollinearity in the chosen 200 explanatory variables of each MLR when VIF measures the test. Instead, when we observe the Condition Number, many regressors are affected by high values, much higher than 30, and this occurs when we use a regressor with significantly different values from R_i and R_M, for example, when using data in dollars or when the percentages of corporate performance are several orders of magnitude, even 1000 times higher in absolute value. Although there is a matrix ill-conditioning problem, it does not appear to be ascribable to multicollinearity.

Table 16 summarises the results of the OLS regressions on monthly and annual returns and MLR on yearly returns, all over 30 years or the shorter available timespan. The second most used regressor is Revenue Growth in 30 cases, followed by EBIT Growth in 14 instances and Total Revenue in 11 cases. These three corporate performance variables stand for 55% of the second regressors. Also, other regressors assume a moderate relevance while the Adjusted Dividends, the Adjusted Basic EPS and the Net Income are absent. From this analysis, it seems that the stock market appraises corporate performance variables linked to real markets and operating profitability rather than variables directly related to net profitability. Corporate business expectancies outclass the ability to generate immediate profit, which is entirely unexpected but an utterly logical behaviour from a perspicuous market.

We point out that in the overlap case with Revenue Growth and EBIT Growth, the $\overline{R^2}$ share ascribable to the stock market is smaller than the corporate performance. This effect also occurs in the overlap case where DOL, EBIT, and the Adjusted Diluted EPS appear.

In all cases but one, $\overline{R^2}$ grows consistently from one regressor to two regressors with annual returns, especially if the second regressor is Revenue Growth (+24.9%), EBIT Growth (+20.3%) and EBIT (+21.8%). The only case this does not occur is when the EBIT regressor is concerned about a firm that does not communicate such data, which is available only through Bloomberg.

We have already seen that passing from one to two regressors with annual returns entails a β reduction to 1.220 from 1.460 (Table 16) due to the stock market performance by about 16%; if we compare the annual return ${}_2\beta_M$ (1.220) with the monthly returns β (1.008), we note that it is 21% underestimated. In both cases, the evaluation varies according to the presence of bridges or overlaps, which largely depend on the correlation between the stock market and corporate performance. In general, a high correlation between the stock market and corporate performance implies the presence of an overlap, while a low or negative correlation implies the presence of a bridge.

In essence, the empirical evidence suggests that the stock market performance alone cannot explain stock return; it is necessary to introduce a variable representing the corporate performance to improve $\overline{R^2}$ remarkably. This result is not totally in contrast with CAPM, which partly assumes stock prices and returns as exogenous data, without providing a convincing explanation of the market adjustment process facing an essential deviation from its theoretical assumptions.

4.8 Conclusions from Empirical Evidence

Having completed the analysis of the empirical evidence topics, which means asset allocation, security market lines, and integration of the stock market’s and corporate business’s joint performance, the time has come to draw a unitary conclusion.

The asset allocation test has highlighted that portfolio optimisation leads to the physiological presence of short sales, reaching almost 50% of the stock positions as risky assets increase. Such a requirement makes the optimised portfolio inconsistent with the market portfolio. At the same time, comparing the optimised portfolio *sine* and market portfolio raises further concerns about stock market efficiency and optimisation since it appears sub-optimised compared to a 100-stock portfolio *sine*, with inactive stocks reaching over 70% of the positions.

Considering that β and R^2 of a common stock belonging to an optimised portfolio are parameters which, as a result of the optimisation, assume respectively increasing and decreasing trends as the number of stocks increases, we must ask about the β eligibility to express the risk of a stock not belonging to an optimised portfolio or belonging to a non-optimised portfolio like the market one or even worse when the stock is compared but not belonging to a non-optimised portfolio.

Furthermore, we must reflect on the efficient portfolio frontier: is it an optimal and achievable theoretical framework or unachievable but valuable as a benchmark? We cannot answer that question, but the doubt is legitimate.

The security market line is affected by the problems that emerged with asset allocation, and the possibility that it changes rapidly in a few weeks exacerbates it. On one side, the instability of the β_M, due to the stock market performance, is not unexpected but makes the relation between the stock market and the stock excess returns less explicative. Furthermore, finding more space in the financial communication for the data relating to the rolling β would be appropriate.

The integration into an MLR relation of the stock market and corporate performance to explain the annual returns of risky assets in a long-term perspective has highlighted a greater explanatory power of the relation (28) compared to (21), measured by $\overline{R^2}$, higher than 51% on average. The transition from the monthly return to the annual return determines a greater explanatory power of equation (21). The S&P 500 Index return has the most significant explanatory power. However, this role finds valid competitors in some explanatory variables of the corporate performance, such as Revenue Growth, EBIT Growth and EBIT, which in specific cases play a role even more critical than stock market performance.

Analysing in detail the 100 MLR integrations, we have seen that in the case of a negative correlation between the stock market and corporate performance, a bridge arises, and the transition to the annual return determines a clear improvement of the first variable in explaining the stock return while the second variable remains marginal.

In the presence of a positive correlation, the opposite occurs: first, an overlap arises, and the corporate performance acquires a crucial explanatory power, often superior to the stock market performance. In that case, R^2 and β_M are respectively significantly under and overestimated in the framework of the OLS with annual returns. If the correlation is strongly negative, this outcome is less important.

The hypothesis that relation (21), the classic CAPM equation, is not the best possible explanation of the behaviour of the risky asset return appears not entirely implausible considering the empirical evidence presented. The MLR integration of the stock market and corporate performance with annual returns has greater explanatory power than the CAPM equation, at least in temporal local conditions.

5. Conclusions

This paper aims to identify a relation between systematic risk and corporate performance represented by DOL and DFL, based on the essay by Mandelker et al. (1984) or through further theoretical analyses that will need empirical evidence.

In Section 2, we examined the Mandelker et al. (1984) contribution, concluding that if such a link exists, it is not in the form presented by the two economists, as DOL and DFL are not static parameters. The study by Miller et al. (1961), also referred to by Fama et al. (2015), permits linking the return of common stock to corporate performance without being able to specify a link with systematic risk. If such a link exists, it would be invisible. Furthermore, we must consider the impact of dilution, which needs more focus.

Section 3 briefly reviewed CAPM, highlighting a divergence in short positions, entirely physiological in a 100-stock optimised portfolio but completely absent in a market portfolio. Another topic concerns the inconsistency of R^2 in the CAPM empirical evidence, a significant share of the risky asset return variability is unexplained. Starting from this perspective, we hypothesized that CAPM might be an incomplete theory due to the total absence of corporate performance variables, given that the stock returns are, by assumption, exogenous data. Furthermore, we have observed that neglecting the correlation between the stock market and corporate performance, in addition to determining low R^2 values, around 30%, according to Roll (1988), masks the presence of omitted variables since critical overlapping phenomena may be present. Finally, we hypothesized that stock returns derive from the joint performance of the stock market and the corporate business, which could lead to the feedback effect of the former on the latter, both at the level of single stock and portfolio return. From this perspective, the portfolio risk could be due to the joint corporate business portfolio, filtered by the stock market feedback effect. It is an exciting hypothesis, worthy of further study, but not easy to verify empirically due to the presence of an excessive number of unknowns compared to the known variables.

In Section 4, we first analysed two CAPM topics, the asset allocation and the security market line, through the empirical evidence of 100 common stocks included in the S&P 500 Index in the 1991-2020 timespan.

Asset allocation shows that short sales create a notable detachment from the market portfolio, represented by its proxy; unfortunately, this gap persists by using the KKT condition to expunge short sales. The stock market does not appear as efficient and optimised as imagined, even without short sales. Alternatively, Merton’s efficient analytical frontier could be a splendid theoretical framework unachievable by the market. In any case, the portfolio optimisation process with short sales, as the number of common stocks increases, leads to increasingly higher β values, which in turn determine an increasingly insignificant R^2 value; with short sales constraints, optimisation leads to the deactivation of an overwhelming number of stocks compared to the active ones.

Furthermore, the security market line, or more simply β, seems to be affected by considerable instability, even in a few weeks, proving to be influenced by both the corporate performance and the sector in which the listed company operates.

From the empirical evidence, we could conclude that CAPM is not the correct theory to explain the behaviour of stock returns, and this puts at risk many of its practical applications, such as the calculation of the cost of equity via β: Fama et al. (2003) had already pointed out this almost twenty years ago.

The integration of the stock market and corporate performance into a long-term MLR relation highlighted that the transition from monthly to annual returns leads to a significant improvement in R^2 in explaining the stock return variability. The introduction of corporate performance shows the presence of a correlation with the stock market performance, entirely ignored by CAPM, and this allows for achieving better $\overline{R^2}$, both with negative and positive correlations. In the latter case, by separating the contribution of the stock market, we highlighted an overlap between the two variables, casting light on the corporate performance contribution to improve $\overline{R^2}$ significantly. In this context, we emphasize that neglecting corporate performance can lead to significant errors in the β estimate within CAPM.

In general, it is impossible to separate the impact of the stock market and corporate performance on systematic risk inside CAPM due to its incompleteness. At a theoretical level, it seems likely to build an alternative general theory that integrates the two performances in a single system of linear equations, allowing to separate the share of the systematic risk due to the stock market from the corporate business and highlighting the feedback effect of the former on the latter. In such a theory, some variables linked to DOL and DFL can play an important role, as the empirical evidence has highlighted, providing encouraging results in keeping this direction. Unfortunately, we cannot identify corporate performance with a specific variable: those linked to DOL seem to play a significant role, especially Revenue Growth, EBIT Growth, Total Revenue and EBIT.

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