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Present Bias and Quality Reduction on Daily Deal Platforms

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Abstract

We study daily deal markets, i.e. platforms where sellers, called merchants, offer coupons for their products at a heavily discounted price for a short time window. Inspired by the evidence that both merchants and customers are often unsatisfied with their experiences with daily deals, we setup a two-period model that reconciles such evidence. In the first period, the merchant sells the coupons at the (low) price imposed by the platform and can choose the quality of its product, which is unobserved by the consumers. In the second period, when the deal period has expired and the merchant is free to set the selling price, first-period customers purchase again only if they were satisfied with the quality of the product. Our crucial result is that, if the merchant has present biased preferences, the daily deal market exacerbates the risk that the merchant provides a low quality product, even though, at the beginning of the daily deal campaign, the merchant was fully aware that only a high quality product would have made the campaign profitable. We also show that it might be in the interest of the platform to set a higher price for the coupons, as this would reduce the risk of having low quality products sold on the platform, avoiding negative reputational effects.

Keywords: present bias, self control, daily deal, quality reduction

1. Introduction

Online platforms - virtual marketplaces were sellers and buyers make transactions - are nowadays extremely popular and widespread. There are online platforms to exchange any kind of products: from physical goods (e.g. Amazon) to services such as accommodation (e.g. Booking, Airbnb), transportation (e.g. Uber), and many others.

In this paper, we are mainly concerned with a particular type of online platform: daily deal sites (e.g. LivingSocial, Groupon). These are platforms where sellers (called merchants) post coupons for their products - usually services such as restaurants, gyms, spas, etc. - at a heavily discounted price for a short time window. The coupons purchased are then redeemed at a later date, when the buyers visit the merchant's shop to enjoy the service. The platform retains a share of the total revenue collected from the sale of the coupons. There are two characteristics that differentiate daily deal sites from other platforms: first, the deal is offered for a limited amount of time and, second, the coupon price is determined prior to start selling it on the website and cannot be changed throughout the period of the deal.

It is intuitive why such platforms have rapidly become popular: consumers have easy and free access to a wide range of attractive offers; for merchants, deals might represent an effective strategy to increase the firm's awareness amongst consumers, something that might be extremely valuable especially for new, yet relatively unknown, businesses. Clearly, the main goal of the merchant is not simply to sell the coupons (which are typically priced at a very low, if not negative, margin). Rather, firms would like the coupons' purchasers to become repeated customers, visiting again the firm in the future when they will pay the full price of the product. In this respect, daily deals represent an investment: the firm trade-offs a low or even negative margin in the short run for higher profits in the future (Note 1).

Despite the popularity of daily deals, users of these platforms seem to be predominantly dissatisfied with their experience. On the consumers' side, there is evidence that the online reviews posted by daily deals' purchasers tend to be worse than those left by other customers, and this is often due to a lower perceived quality of the service (see Bai et al., 2020; Byers et al., 2012a; Byers et al., 2012b). On the merchants' side, surveys report that a significant fraction of them actually lost money from the daily deal campaign, mainly because deal users fail to become repeated customers in the future (Dholakia, 2011a; Dholakia, 2011b).

It is easy to reconcile these facts: if consumers are not satisfied with the product, they are unlikely to purchase again from that merchant, and this, in turn, makes the daily deal campaign unprofitable for the seller. The question is then: why do some merchants provide a poor service to customers with coupons?

In this paper, we provide a possible answer to this question grounded on present biased preferences (or self control problems). To this end, we setup a model in which a firm (the merchant) sells her product for two periods, and, in each period, she can set the quality of her product. Offering a low quality product reduces the firm's marginal cost; however, it has a negative impact on the demand in the next period, as consumers who experience a low quality product in one period will not purchase again in the future (unless the price is unrealistically low).

Prior to start selling her product, the firm may decide to sell coupons through a daily deal platform. Joining the platform increases the visibility of the firm, expanding the size of potential customers, but the firm must sell at the (low) coupon price imposed by platform and leave a fraction of the revenues to the platform. Notice that the daily deal campaign lasts one period only: in the second period, the firm will freely set the price of her product, possibly reaping the benefits of the coupon campaign.

The crucial element of the model is that the firm may suffer of a present bias: the discount rate between the current and the next period is lower than between any future periods. As we show, this bias may lead a firm to reduce the quality of her service, even though, at the time of joining the platform, she was fully aware that the investment in the daily deals would have been profitable only conditional on making the customers satisfied with the product and, thereby, willing to purchase again in the future. In fact, at the time of choosing her selling strategy, the trade-off between low profits in the deals' period and higher profits in the next involved two future periods, and the firm foresaw the potential profitability of the investment in the daily deal campaign. However, when coupon users visit the firm to enjoy the service, the present bias bites and the firm prefers instantaneous gratification, reducing quality in order to pocket at least a small profit in that period, thereby sacrificing the future profits.

Our crucial result is that the daily deal market may exacerbate this effect: there are levels of the present bias discount rate such that a firm that decided to participate in a daily deal campaign reduces the product's quality (though she planned to provide high quality), whereas this would not have happened if the firm had decided to sell directly on the market. The intuition is simple: the trade-off between low profits in one period for high profits in the next is more unbalanced for a firm selling coupons than for a firm that sells on the market: as a consequence, the present bias has a stronger impact on the former.

We also show that quality reduction may lead a firm to ex post regretting her decision to launch the coupon campaign, as her profits in the second period -- the period in which the firm expected to reap the benefit of the daily deals -- are actually lower than what she could have earned by selling directly on the market. Finally, we show that, to avoid negative effects on its reputation, it may be in the interest of the platform to increase the price of the coupons, as this may reduce the risk that merchants reduce quality.

This paper contributes to the literature dealing with the performance of daily deal platforms from the users' point of view. Dholakia (2011a) and Dholakia (2011b) collected survey data from businesses participating in daily deals in the U.S. to assess their effectiveness: responses are multifaceted, but about one third of businesses declare that the promotion was unprofitable, and the main reason is that only a minority of customers returned to purchase at full price (similar results are also found by Wu et al., 2012, for restaurants only). Not surprisingly, some business owners regretted their decision to run daily deals. Cao et al. (2018) show that excessively low prices may reduce coupons sales, as consumers might be concerned about the actual quality of the product (Eisenbeiss et al., 2015, uncovered a similar effect but only for hedonic products). In general, the profitability of daily deals has been questioned by several authors (see, among others, Kumar & Rajan, 2012; Zhang et al., 2013; Edelman et al., 2016; Carlson & Kukar-Kinney, 2018). As far as the consumers' side of the market is concerned, Bai et al. (2020) provide formal evidence of a negative effect of daily deals on a restaurant's monthly average ratings on Yelp, and observe that the main driver of this lack of satisfaction is perceived food and service quality. These results are in line with those previously obtained by Byers et al. (2012a) and Byers et al. (2012b) (Note 2). In particular, Byers et al. (2012b) find some evidence that merchants offer a worse quality service, though their main explanation of the results is that reviews left by deal users may be worse because they are actually more genuine than the generality of reviews on Yelp, which are often biased (if not fake). In general, that the quality of the service is a crucial issue in this market has been raised by several authors. For example, Luo et al. (2014), Subramanian and Rao (2016), and Li and Wu (2018) observe that the practice of platforms to show the number of coupons sold in real time might be an effective way to increase sales, as it is taken by consumers as a signal of the quality of the merchants.

Our paper is also related to the literature on present bias preferences. Present biased preferences (also labelled hyperbolic or quasi-hyperbolic discounting) - a bias in intertemporal decision making that may lead inconsistencies between what an individual plans to do in a future date and what she actually does on that date - has been proposed as an explanation for many puzzling behaviors observed in the real world, ranging from labor market (Della Vigna & Paserman, 2005), to health club attendance (Della Vigna & Malmendier, 2006), to saving decisions (Laibson et al., 2007) (Note 3). In this respect, our paper adds another real world instance where present biased preferences may explain puzzling facts.

The rest of the paper is structured as follows. Section 2 presents the model, that is solved in Section 3. Section 4 discusses the model's implications for the daily deal market. Section 5 concludes.

2. The Model

A firm (the merchant) is the sole producer of a good or service for which she can set the quality level. The firm faces the following dynamic decision problem. At the outset (date $t = 0$), the firm decides whether to sell its product directly on the market or through an online platform that offers coupons or deals. Let $y \in \{M, C\}$ denote such decision, where M stands for "market" and C stands for "coupons".

If the firm chooses $y = M$, then, in each of the following two periods ($t = 1$ and $t = 2$), the firm is free to set the price and the quality of its product. Let (p_t, x_t) denote the price-quality pair chosen by the firm in period t ($t = 1,2$). While p_t can be any positive number, $x_t \in \{H, L\}$, where H stands for "high quality" and L for "low quality".

If, instead, the firm opts for the coupons ($y = C$), then, in period $t = 1$ (and only in that period) the firm's product is sold at the coupon price specified by the platform, denoted by p_C (Note 4). Moreover, the firm will retain only a fraction α of her revenues in $t = 1$ (the remaining fraction, $1 - \alpha$, goes to the platform provider in the form of royalties). Still, the firm is free to choose x_1 (the quality of her product in $t = 1$). In period $t = 2$, the contract with the platform is expired: the firm chooses both p_2 and x_2, and retains all the proceeds from her sales. Hence, joining the coupon platform has two downsides: the firm cannot choose the price of the product in $t = 1$ and must share that period revenues with the platform.

The technology is characterized by constant returns to scale, but depends on the quality chosen in that period: let c_H and c_L denote the (constant) unit costs associated with high and low quality, respectively, with $c_H > c_L$ (Note 5).

The demand side is characterized by a population of consumers who observe the price of the product but not its quality. We assume that, in this respect, consumers are naive in the sense that they do not form rational expectations on the quality choice by the firm. Rather,

- in $t = 1$, all consumers expect an "average" quality;
- in $t = 2$, *those consumers who purchased the product in* $t = 1$ and, thereby, learned x_1, expect that the quality of the product will be the same in $t = 2$ (i.e. $x_2 = x_1$). These consumers then adjust their demand accordingly: in particular, their willingness to pay is increased or decreased (with respect to their willingness to pay in $t = 1$) by an amount $w \in (0,1]$, depending on whether $x_1 = H$ or $x_1 = L$. It is important to remark that only those consumers who experienced the good in $t = 1$ change their willingness to pay in $t = 2$: for the remaining consumers, the demand is unchanged (Note 6).

Finally, the demand is affected by the initial decision of the firm, in that daily deals increase the visibility of the firm, allowing the firm's product to permanently (i.e. in both $t = 1$ and $t = 2$) reach a larger audience of potential customers.

Specifically, letting $q_1(p_1|y)$ denote the demand in $t = 1$ for a firm who chose y at $t = 0$ and sets the price p_1 in $t = 1$, and $q_2(p_2|y, p_1, x_1)$ denote the demand in $t = 2$ for a firm who chose y at $t = 0$, (p_1, x_1) at $t = 1$ and sets the price p_2 in $t = 2$, we assume that

$$q_1(p_1|y = M) = 1 - p_1, \tag{1}$$

$$q_2(p_2|y = M; p_1, x_1 = H) = \begin{cases} 1 - p_2 & \text{if } \ 0 \leq p_2 \leq p_1 \\ 1 - p_1 & \text{if } \ p_1 < p_2 \leq p_1 + w \\ 1 + w - p_2 & \text{if } \ p_1 + w < p_2 \leq 1 + w \\ 0 & \text{if } \ 1 + w \leq p_2 \end{cases} \tag{2}$$

$$q_2(p_2|y=M;p_1,x_1=L)=\begin{cases}1-p_2 & \text{if } \ 0\le p_2\le p_1-w\\ 1-w+p_1-2p_2 & \text{if } \ p_1-w<p_2\le \min\{1-w,p_1\}\\ \max\{1-w,p_1\}-p_2 & \text{if } \ \min\{1-w,p_1\}<p_2\le \max\{1-w,p_1\}-p_2\\ 0 & \text{if } \ \max\{1-w,p_1\}-p_2\le p_2\end{cases} \quad (3)$$

and that $q_1(p_1|y=C)=k\times q_1(p_1|y=M)$ and $q_2(p_2|y=C,p_1,x_1)=k\times q_2(p_2|y=M,p_1,x_1)$, with $k>1$ (i.e. the daily deals scale up the firm's demand by a factor k) (Note 7).

A decision by the firm is thus a vector $(y;p_1,x_1;p_2,x_2)$ of actions that are made in three different dates (it is intended that, if the firm chooses $y=C$ at date $t=0$, then p_1 is constrained to be equal to p_C). At any date, the firm's objective is to maximize the discounted flows of profits. Our crucial hypothesis is that the firm may be affected by a present bias: we model this hypothesis by assuming quasi-hyperbolic discounting as in Laibson (1997): at any date $t=0,1,2$, the firm acts to maximize

$$\Pi_t=\pi_t+\beta\left[\sum_{s=1}^{2-t}\delta^s\pi_{t+s}\right] \quad (4)$$

where π_t is the instantaneous profit in period t, $\delta\in(0,1]$ is the standard discount rate, and $\beta\in(0,1]$ captures the present bias. In words, when $\beta<1$, the discount rate between the profits in two consecutive *future* periods (δ) is larger than the discount rate between the profits in the *current* and in the *next* period (which is $\beta\delta$).

Notice that, in general, although the firm's decisions are made at different dates, to (optimally) make a decision at a date t, the firm must anticipate how she will behave in later periods. In other words, she must make a plan for the future actions. The well known implication of the present bias is that it may induce time-inconsistent choices: in choosing the current action, the decision maker makes a plan of future actions that is revised as the future unfolds.

3. Solution

In this section, we present the solution to the model. Since our goal is to provide some general intuitions on the implications of the present bias on the coupon industry, we make some simplifying restrictions on the parameters. In particular, we assume that: *(i)* $w\ge 1-c_L$, and *(ii)* $w\ge (c_H-c_L)^2/[2(1-c_H)]$. Assumption *(i)* implies that disappointed consumers will not come back: if the firm chooses low quality in $t=1$, consumers who purchased in $t=1$ will not buy again in $t=2$, because they are willing to pay no more than c_L (and the firm will certainly not choose a price below c_L). Assumption *(ii)* is useful as it allows to reduce the cases to be considered in the determination of the optimal p_1 when $y=M$. In the sequel, we will write p_t^s, with $0\le s<t$, to denote the price that, at date s, the firm *plans* to set in the future date t; accordingly, p_t^t will denote the (optimal) price of the product *chosen* by the firm at date t. We apply this same notation to the quality choice.

3.1 Date $t=2$

At date $t=2$, the firm takes $(y;p_1,x_1)$ as given, and chooses (p_2,x_2) with the goal of finding

$$\max_{[p_2,x_2]}\Pi_2(y;p_1,x_1;p_2,x_2)\ =\pi_2(y;p_1,x_1;p_2,x_2)=\left(p_2-c_{x_2}\right)q_2(p_2|y;p_1,x_1). \quad (5)$$

It is immediate to see that, since $t=2$ is the last period, the firm will certainly choose $x_2^2=L$ regardless of the previous periods' decisions (a low quality has no effect on the current demand, but is less costly to the firm). Hence, we just need to determine the optimal p_2 for each possible choice of $(y;p_1,x_1)$ (Note 8).

Suppose, first, that the firm chose $(y=M;p_1,x_1=H)$. In this case, the optimal price is

$$p_2^2(M;p_1,H)=\begin{cases}(1+c_L+w)/2 & \text{if } \ 0\le p_1\le(1+c_L-w)/2\\ p_1+w & \text{if } \ (1+c_L-w)/2<p_1\le[1+c_L-w+\sqrt{w^2+w(1-c_L)}]/2\\ (1+c_L)/2 & \text{if } \ [1+c_L-w+\sqrt{w^2+w(1-c_L)}]/2<p_2\le 1\end{cases} \quad (6)$$

with corresponding profits

$$\pi_2(M;p_1,H;p_2^2(M;p_1,H),L)=$$
$$\begin{cases}(1-c_L+w)^2/4 & \text{if } \ 0\le p_1\le(1+c_L-w)/2\\ (p_1+w-c_L)(1-p_1) & \text{if } \ (1+c_L-w)/2<p_1\le[1+c_L-w+\sqrt{w^2+w(1-c_L)}]/2\\ (1-c_L)^2/4 & \text{if } \ [1+c_L-w+\sqrt{w^2+w(1-c_L)}]/2<p_2\le 1\end{cases} \quad (7)$$

If, instead, the firm chose $(y = M; p_1, x_1 = L)$, the optimal price is

$$p_2^2(M; p_1, L) = \begin{cases} c_L & \text{if } 0 \leq p_1 \leq c_L \\ (p_1 + c_L)/2 & \text{if } c_L < p_1 \leq 1 \end{cases} \tag{8}$$

with corresponding profits

$$\pi_2(M; p_1, L; p_2^2(M; p_1, L), L) = \begin{cases} 0 & \text{if } 0 \leq p_1 \leq c_L \\ (p_1 - c_L)^2/4 & \text{if } c_L < p_1 \leq 1 \end{cases} \tag{9}$$

What if, instead, at date $t = 0$ the firm decided to sell coupons (i.e. $y = C$)? It is immediate to see that, since joining the coupon platform has only a scale effect, the optimal prices in $t = 2$ (as a function of p_1 and x_1) are the same as when $y = M$. Clearly, the corresponding profits have to be multiplied by the factor k. We thus have

$$p_2^2(C; p_1, H) = p_2^2(M; p_1, H), \qquad p_2^2(C; p_1, L) = p_2^2(M; p_1, L), \tag{10}$$

$$\pi_2(C; p_1, H; p_2^2(C; p_1, H), L) = k \times \pi_2(M; p_1, H; p_2^2(M; p_1, H), L), \tag{11}$$

$$\pi_2(C; p_1, L; p_2^2(C; p_1, L), L) = k \times \pi_2(M; p_1, L; p_2^2(M; p_1, L), L). \tag{12}$$

3.2 Date $t = 1$

At date $t = 1$, the firm takes y as given, and optimally chooses (p_1, x_1), which involves also anticipating what actions will be taken in $t = 2$. Hence, the firm solves the following problem:

$$\max_{[p_1, x_1; p_2, x_2]} \Pi_1 = \pi_1(y; p_1, x_1) + \beta \cdot \delta \cdot \pi_2(y; p_1, x_1; p_2, x_2). \tag{13}$$

It is quite immediate to see that the plan made at $t = 1$ for the future actions (p_2, x_2) will coincide with the actual decisions that will be made at $t = 2$, i.e. the plan will not be revised. To see this, let $\left(p_1^1(y), x_1^1(y); p_2^1(y), x_2^1(y)\right)$ be the solution to (13) (for given y). Then it must be that, for all (p_2, x_2),

$$\pi_1\left(y; p_1^1(y), x_1^1(y)\right) + \beta \cdot \delta \cdot \pi_2\left(y; p_1^1(y), x_1^1(y); p_2^1(y), x_2^1(y)\right) \geq \\ \pi_1\left(y; p_1^1(y), x_1^1(y)\right) + \beta \cdot \delta \cdot \pi_2(y; p_1^1(y), x_1^1(y); p_2, x_2) \tag{14}$$

or,

$$\pi_2\left(y; p_1^1(y), x_1^1(y); p_2^1(y), x_2^1(y)\right) \geq \pi_2(y; p_1^1(y), x_1^1(y); p_2, x_2), \tag{15}$$

i.e. $\left(p_2^1(y), x_2^1(y)\right)$ solves (5) for $p_1 = p_1^1(y)$ and $x_1 = x_1^1(y)$. Hence, we conclude that the optimal plan made at $t = 1$ for the actions to be taken at $t = 2$ is $x_2^1 = L$ and $p_2^1(y) = p_2^2\left(y; p_1^1(y), x_1^1(y)\right)$. Thus, the firm's optimization problem (13) reduces to:

$$\max_{[p_1, x_1]} \Pi_1 = \pi_1(y; p_1, x_1) + \beta \cdot \delta \cdot \pi_2(y; p_1, x_1; p_2^2(y; p_1, x_1), L). \tag{16}$$

Suppose, first, that the firm chose $(y = M)$. We solve for the optimal price p_1^1 conditional on the firm choosing $x_1 = H$ and $x_1 = L$, respectively (Note 9), and then we compare the value of the firm's objective function in these two cases to determine the optimal quality x_1^1. Now, conditional on $x_1 = H$, the optimal price in $t = 1$ is

$$p_1^1(M|H) = \frac{1+c_H}{2} - \frac{\beta\delta(c_H - c_L + w)}{2(1+\beta\delta)}, \tag{17}$$

with corresponding profits

$$\Pi_1(M; p_1^1(M|H), H) = \frac{[1 - c_H + \beta\delta(1 - c_L + w)]^2}{4(1-\beta\delta)}. \tag{18}$$

On the other hand, conditional on $x_1 = L$, the optimal price is

$$p_1^1(M|L) = \frac{2 + (2-\beta\delta)c_L}{4-\beta\delta}, \tag{19}$$

with corresponding profits

$$\Pi_1(M; p_1^1(M|L), L) = \frac{(1-c_L)^2}{4-\beta\delta}. \tag{20}$$

Clearly, the optimal quality choice $x_1^1(M)$ will be H or L depending on whether (18) is larger or smaller than (20). It turns out that the firm will choose H (L) when $\beta\delta$ is larger (smaller) than a threshold value $\hat{\gamma}_M$, where $\hat{\gamma}_M \in (0,1)$ is the unique value such that (18) e (20) are equal. To see this, take

$$\Pi_1(M; p_1^1(M|H), H) - \Pi_1(M; p_1^1(M|L), L) = \frac{[1-c_H+\beta\delta(1-c_L+w)]^2}{4(1-\beta\delta)} - \frac{(1-c_L)^2}{4-\beta\delta}, \tag{21}$$

and observe that that the above expression is strictly negative for $\beta\delta = 0$, strictly positive for $\beta\delta \to 1$, and strictly increasing for all $\beta\delta \in (0,1)$.

Consider now the case in which the firm chose $(y = C)$. In this case, the price is necessarily p_C, and the firm has only to choose the quality x_1. To make this decision, the firm compares

$$\begin{aligned}\Pi_1(C; p_C, H) &= \pi_1(C; p_C, H) + \beta \cdot \delta \cdot \pi_2(C; p_C, H; p_2^2(C; p_C, H), L) \\ &= (\alpha p_C - c_H)(1 - p_C) + \beta \cdot \delta \cdot \pi_2(C; p_C, H; p_2^2(C; p_C, H), L)\end{aligned} \tag{22}$$

with

$$\begin{aligned}\Pi_1(C; p_C, L) &= \pi_1(C; p_C, L) + \beta \cdot \delta \cdot \pi_2(C; p_C, L, p_2^2(C, p_C, L), L) \\ &= (\alpha p_C - c_L)(1 - p_C) + \beta \cdot \delta \cdot \pi_2(C; p_C, L, p_2^2(C, p_C, L), L)\end{aligned} \tag{23}$$

where $\pi_2(C; p_C, H; p_2^2(C; p_C, H), L)$ and $\pi_2(C; p_C, L, p_2^2(C, p_C, L), L)$ are given by (11) and (12). Again, the optimal quality choice $x_1^1(C)$ will be H or L depending on whether (22) is larger or smaller than (23). Notice that such a decision is crucially affected by the value of p_C. As before, there is a single threshold value $\hat{\gamma}_C$ (which clearly depends on p_C) such that the firm's optimal quality choice is $x_1^1(C) = H$ whenever $\beta\delta$ is greater than $\hat{\gamma}_C$, while it is $x_1^1(C) = L$ otherwise. The value of $\hat{\gamma}_C$ can easily be computed explicitly, and is given by:

$$\hat{\gamma}_C = \begin{cases} \dfrac{4(c_H - c_L)(1 - p_C)}{(1 - c_L + w)^2} & \text{if } 0 \le p_C \le \dfrac{1 + c_L - w}{2} \\ \dfrac{c_H - c_L}{p_C - c_L + w} & \text{if } \dfrac{1 + c_L - w}{2} < p_C \le c_L \\ \dfrac{4(c_H - c_L)(1 - p_C)}{4(p_C - c_L + w)(1 - p_C) - (p_C - c_L)^2} & \text{if } c_L < p_C \le \dfrac{1 + c_L - w + \sqrt{w^2 + w(1 - c_L)}}{2} \\ \dfrac{4(c_H - c_L)}{1 + p_C - 2c_L} & \text{if } \dfrac{1 + c_L - w + \sqrt{w^2 + w(1 - c_L)}}{2} \le p_C \le 1 \end{cases} \tag{24}$$

3.3 Date $t = 0$

At date $t = 0$, the firm must decide whether to sell directly on the market ($y = M$), or through the coupon platform ($y = C$). To do so, she must anticipate what actions will be taken in $t = 1$ and $t = 2$. Hence, the firm solves the following problem:

$$\max_{[y; p_1, x_1; p_2, x_2]} \Pi_0 = \beta[\delta \cdot \pi_1(y; p_1, x_1) + \delta^2 \cdot \pi_2(y; p_1, x_1; p_2, x_2)], \tag{25}$$

or, given that β and δ are strictly positive,

$$\max_{[y; p_1, x_1; p_2, x_2]} \pi_1(y; p_1, x_1) + \delta \cdot \pi_2(y; p_1, x_1; p_2, x_2). \tag{26}$$

Let $(y^0; p_1^0, x_1^0; p_2^0, x_2^0)$ denote the solution to (26). It is quite immediate to see that $x_2^0 = x_2^2 = L$, and $p_2^0 = p_2^2(y^0; p_1^0, x_1^0)$. Hence, (15) can be written as

$$\max_{[y; p_1, x_1]} \pi_1(y; p_1, x_1) + \delta \cdot \pi_2(y; p_1, x_1; p_2^2(y; p_1, x_1), L). \tag{27}$$

Notice that, for given y, this maximization problem is similar to (16), with the crucial difference that $\beta\delta$ in (16) is replaced by simply δ in (27). Hence, the results of the previous subsection can be, mutatis mutandis, applied here. We thus have that the optimal planned first period price, conditional on the firm choosing $y = M$ and $x_1 = H$ is

$$p_1^0(M|H) = \frac{1+c_H}{2} - \frac{\delta(c_H - c_L + w)}{2(1+\delta)}, \tag{28}$$

with corresponding profits

$$\Pi_0(M; p_1^0(M|H), H) = \beta\delta \frac{[1-c_H+\delta(1-c_L+w)]^2}{4(1-\delta)} \tag{29}$$

the optimal planned first period price, conditional on the firm choosing $y = M$ and $x_1 = L$ is

$$p_1^0(M|L) = \frac{2+(2-\delta)c_L}{4-\delta}, \tag{30}$$

with corresponding profits

$$\Pi_0(M; p_1^0(M|L), L) = \beta\delta\frac{(1-c_L)^2}{4-\delta}. \tag{31}$$

And (29) is larger (smaller) than (31) - the firm plans to choose high (low) quality, if she sells directly on the market - if δ is greater (smaller) than $\hat{\gamma}_M$.

When, instead, the firm chooses $y = C$, the price is necessarily p_C, and the firm's profit, conditional on $x_1 = H$, is

$$\Pi_0(C; p_C, H) = \beta\delta(\alpha p_C - c_H)(1 - p_C) + \delta \cdot \pi_2(C; p_C, H; p_2^2(C; p_C, H), L), \tag{32}$$

whereas her profit, conditional on $x_1 = L$, is

$$\Pi_0(C; p_C, L) = \beta\delta(\alpha p_C - c_L)(1 - p_C) + \delta \cdot \pi_2(C; p_C, L; p_2^2(C, p_C, L), L), \tag{33}$$

and (32) is larger (smaller) than (33) - the firm plans to choose high (low) quality, if she sells through the coupon platform - if δ is greater (smaller) than $\hat{\gamma}_C$ as defined by (24).

We thus conclude that y^0 and x_1^0 are the solution to

$$\max\{\Pi_0(M; p_1^0(M|H), H), \Pi_0(M; p_1^0(M|L), L), \Pi_0(C; p_C, H), \Pi_0(C; p_C, L)\}. \tag{34}$$

The next Proposition summarizes the solution to the model in terms of the firm's selling strategy and quality choice in $t = 1$.

PROPOSITION 1. *Let* $\hat{\beta}_M = \hat{\gamma}_M/\delta$. *If*

$$\max\{\Pi_0(M; p_1^0(M|H), H), \Pi_0(M; p_1^0(M|L), L)\} \geq \max\{\Pi_0(C; p_C, H), \Pi_0(C; p_C, L)\}, \tag{35}$$

the firm decides to sell directly on the market and

- *if* $\beta \geq \hat{\beta}_M$, *the firm plans to choose high quality and does indeed choose high quality;*
- *if* $\delta > \hat{\gamma}_M$ *and* $\beta < \hat{\beta}_M$, *the firm plans to choose high quality but then switches to low quality;*
- *if* $\delta \leq \hat{\gamma}_M$, *the firm plans to choose low quality and does indeed choose low quality.*

Let $\hat{\beta}_C = \hat{\gamma}_C/\delta$. *If*

$$\max\{\Pi_0(M; p_1^0(M|H), H), \Pi_0(M; p_1^0(M|L), L)\} < \max\{\Pi_0(C; p_C, H), \Pi_0(C; p_C, L)\}, \tag{36}$$

the firm decides to sell directly on the market and

- *if* $\beta \geq \hat{\beta}_C$, *the firm plans to choose high quality and does indeed choose high quality;*
- *if* $\delta > \hat{\gamma}_C$ *and* $\beta < \hat{\beta}_C$, *the firm plans to choose high quality but then switches to low quality;*
- *if* $\delta \leq \hat{\gamma}_C$, *the firm plans to choose low quality and does indeed choose low quality.*

We say that the firm *reduces quality* (or that the firm makes *quality reduction*) when, at date $t = 0$, the firm planned to set high quality in $t=1$ (and her initial decision was based on that plan), but then, at $t = 1$, she actually changes her plan and provides a low quality product. Notice that, for a firm that decides to sell coupons through the platform, this occurs when the standard discount rate is sufficiently high (specifically, $\delta > \hat{\gamma}_C$) and, at the same time, the present bias discount rate is sufficiently low (specifically, $\beta < \hat{\beta}_C$) (Note 10). Notice that quality reduction is not limited to the daily deal market: also a firm that decides to sell on the market may reduce quality at $t = 1$ (this occurs when $\delta > \hat{\gamma}_M$ and $\beta < \hat{\beta}_M$). However, our intuition is that quality reduction is more likely to occur on the daily deal market than outside. The reason is that, while for both selling strategy (market or coupons), a high quality at $t = 1$ involves a trade-off between current and future profits, this trade-off is more unbalanced for a firm that sells coupons, as daily deals squeeze profit margins in the first period, but potentially brings larger sales in the second. We explore this intuition and its implications in the next section.

4. Implications for the Daily Deal Industry

In this section, we discuss some qualitative implications of the model for the daily deal industry. To illustrate such implications, we fix the value of some of the model's parameters, namely:

$$c_H = 0.8, c_L = 0.5, w = 0.5, \alpha = 0.8, k = 2, \tag{37}$$

leaving the remaining parameters (δ, β and p_C) free. We remark that the choice of these parameters is not motivated by any presumption of realism, but just for their simplicity. We discuss three relevant implications for the industry.

4.1 Coupons Trigger Quality Reduction

The first implication is that the present bias may induce a firm selling coupons to provide a lower quality than she would have offered outside the platform. In other words, quality reduction is more likely to be observed on the daily deal market than on the regular market. This result is aligned with the evidence of a lower level of satisfaction of daily deal users with respect to the generality of consumers (see Section 1).

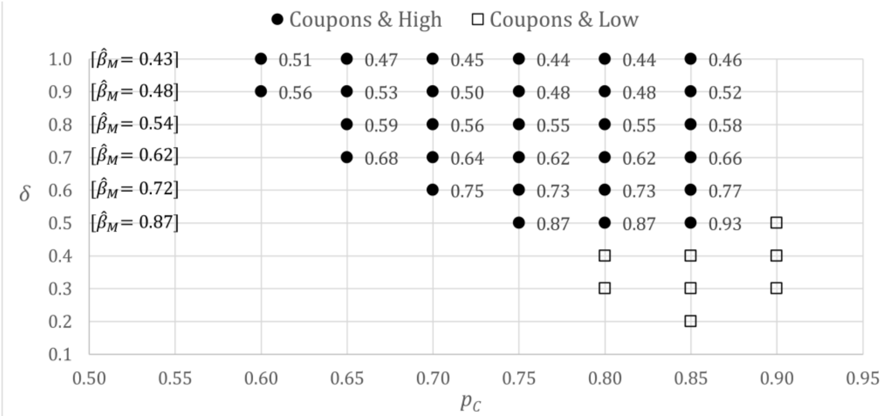

Figure 1. Optimal selling strategy and optimal quality choice

Note. Bullets indicate pairs (p_C, δ) such that $y^0 = C$ and $x_1^0 = H$. The numbers attached to each bullet is the value of $\hat{\beta}_C$ corresponding to that pair. Squares indicate pairs (p_C, δ) such that $y^0 = C$ and $x_1^0 = L$. The value of $\hat{\beta}_M$ corresponding to each value of δ is also reported.

To see this, Figure 1 shows, for a grid of values of p_C - the coupon price - and δ - the standard discount rate -, cases in which the firm optimally opts for selling coupons and plans to offer high quality in $t = 1$ (bullets), and cases in which the firm chooses to join the platform and plans to offer low quality in $t = 1$ (squares). In the remaining points, the firm prefers to sell directly on the market. Notice that, as expected, for fixed p_C, a higher δ makes daily deals more attractive for the firms to sell coupons; conversely, for given δ, a higher coupon price makes the firm more willing to sell coupons (but only up to some unrealistically high price). The figure also reports, for each bullet, the value of $\hat{\beta}_C$, i.e. the threshold value of the present bias discount rate β below which the firm would reduce quality in $t = 1$ (e.g. if $p_C = 0.7$ and $\delta = 0.8$, a firm with $\beta < 0.56$ would switch to low quality in $t = 1$, though she planned to provide high quality). Finally, the figure also reports, next to each value of δ on the vertical axis, the values of $\hat{\beta}_M$, i.e. the threshold values of the present bias discount rate β below which, had the firm decided to sell on the market, the firm would have reduced quality.

The main point is that, for any δ, the threshold values $\hat{\beta}_C$ tend always to be higher than the corresponding threshold value $\hat{\beta}_M$. In other words, there are values of the present bias discount rate β such that the firm, that optimally decided to sell coupons, reduces the quality in $t = 1$, and this quality reduction is tied to her decision of selling coupons, in the sense that, if she had decided to sell on the market (or, simply, if the daily deal platform did not exist) she would have provided high quality. As an example, suppose that the price of the coupon is $p_C = 0.6$, the firm's standard discount rate is $\delta = 1$, and her present-bias discount rate is $\beta = 0.45$. This firm decides to sell through the online platforms planning to provide high quality, but then she revises her plan and offers low quality. If, instead, this same firm had decided to sell directly on the market, she would have sticked to her initial plan of offering high quality. Broadly speaking, the coupon platform encourages quality reduction.

4.2 Firm May Regret Selling Coupons

The second consequence of the present bias is that a firm, that optimally decided to launch a coupon campaign, may ex-post regret her initial decision, in the sense that her profits in $t = 2$ - the period in which the firm expected to reap the benefits of the campaign - are lower than they would have been if the firm chose to sell on the market. Again, this result is consistent with the dissatisfaction reported by several businesses that ran daily deal campaigns (see Section 1).

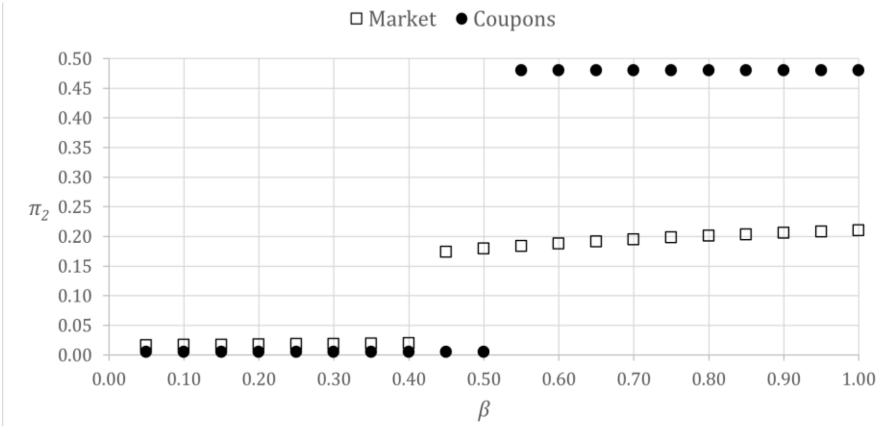

Figure 2. Actual profit in $t = 2$ (for $p_C = 0.6$ and $\delta = 1$)

This can be seen from Figure 2, which reports the actual profits of the firm in $t = 2$ as a function of her present bias discount rate β. The graph is depicted for $\delta = 1$ and $p_C = 0.6$. Notice that, with these parameters, the firm finds it optimal to sell coupons, planning to provide high quality at $t = 1$. As can be seen from the figure, if the firm's present bias discount rate is between 0.43 and 0.51, her actual profit in $t = 2$ is much lower than what the firm could have earned by selling directly on the market. Notice that these values of β are exactly those for which the firm reduces her quality in $t = 1$ - thereby undermining her future profits - while this would not have happened if the firm opted for selling on the market.

4.3 Effects on the Platform's Reputation

As we have seen before, in the presence of a present bias merchant, the online platform may end up making everybody unhappy: consumers may be disappointed by the quality of the product, which can be lower than on the market, and the firm may be dissatisfied with the low profits, which can be smaller than what she could have earned on the market.

What about the third actor of this market, the platform itself? Clearly, the platform would like the firm to join the daily deal platform, and, conditional on that, would like that the firm's revenue is maximized (because the platform gets a share of it). It is easy to see that, for any given α, the revenue of a firm that joins the platform is maximized for $p_C = 0.5$. Notice, however, that, in our example, with $p_C = 0.5$ the firm is not willing to sell coupons, regardless of δ (see Figure 1). Hence, the platform should ideally set the lowest price acceptable by the firm.

Suppose, for example, that $\delta = 1$: among those prices that make the firm willing to launch the daily deal campaign, the price that maximizes the firm's revenue (and thereby the platform's profit) is clearly $p_C = 0.6$. With such price, the firm would would plan to set high quality at the time of providing her product; however, if her present bias discount rate is below 0.51 (see again Figure 1), the firm would then reduce quality, thus making consumers and, eventually, the firm herself disappointed. This is something that the platform would like to avoid: disappointed consumers and disappointed merchants are unlikely to buy and sell again on the platform. Moreover, this could jeopardize the reputation of the platform, reducing its attractiveness to new users. Hence, it may be in the interest of the daily deal platform to increase the price of the coupon above the minimum price acceptable by the merchant: although a higher coupon price decreases the platform's revenue, it diminishes the risk of having the firm reducing quality (i.e. it decreases the threshold value of β) and, therefore, allows to avoid negative reputational effects.

5. Concluding Remarks

In this paper, we wanted to understand why daily deal market users, both consumers and merchants, seem to be predominantly dissatisfied with their experiences with the daily deals. We were able to reconcile this evidence by hypothesizing that merchants suffer from a present bias -- the discount rate between the current period's and the next period's profits is lower than between profits in two future dates. As we showed, this bias may lead a merchant that optimally decided to launch a daily deal campaign, to provide a low quality service to customers, even though, at the beginning of the campaign, the merchant was fully aware that only a high quality service would have led customers to return again in the future, making the campaign profitable. This phenomenon of quality reduction is exacerbated in daily deal markets, as the merchant's profits in the short run are squeezed (or made even negative) by the heavily discounted price of the deal, whereas the possible benefits to the merchant in terms of increased visibility materialize only in the long run.

Our paper provides another instance in which the present bias may explain some puzzling evidence taken from the real world. As it is usually observed, when an individual suffers of a self control problem and become aware of that, she may try to temper its negative effect by finding some commitment device, i.e. a means that prevents her from taking the tempting action. In the case considered in this paper, the tempting action is the reduction of quality, and a commitment device should be some means that makes it not possible for the merchant to adjust the quality of the product at the time of serving its customers. Finding such a quality constraining device may not be an easy for the kinds of product - usually not standardized services - that are typically offered through the daily deals. However, in our case, there is another subject that is likely to be interested in avoiding quality reduction, which is the daily deal platform itself. In fact, a low quality of the products sold is likely to eventually hurt the platform’s reputation. Hence, it may be in the interest of the platform to avoid quality reduction by the merchants, and this can be done by properly design the terms of the deals. In particular, we show that setting a higher price for the coupons can be a simple way to temper the potential self control problems of the merchant, thereby reducing the risk of having low quality products sold on the platform.

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Notes

Note 1. There can also be an immediate benefit in selling daily deals: in fact, this can be a way to practice price discrimination between high demand consumers, who pay the full price offline, and low demand consumers who purchase the discounted coupon. Clearly, to be effective, such strategy requires that, for some reason, high demand consumers do not have access to the daily deals; if that is not the case, there is the risk of cannibalization (customers that would pay the full price switch to the discounted coupons).

Note 2. Erdogmus and Cicek (2011) report that some deal users felt discriminated against by the merchant, with respect to other customers.

Note 3. For more on this, see the survey by Della Vigna (2009).

Note 4. We take the price of the coupon as given, saying, maybe improperly, that it is specified or imposed by the platform. In reality, it is typically the result of some bargaining between the firm and the platform, though the latter is likely to have more bargaining power. In this respect, see Zhang and Chung (2020).

Note 5. There are many ways in which a firm can easily reduce the quality of her product to save on variable costs, especially when the product is a service. For example, a restaurant owner could decide to buy cheaper ingredients, or to serve smaller portions, or to employ less waiters or a low wage unexperienced cook.

Note 6. We are implicitly assuming that the product is an experience good, i.e. a good whose quality can be fully assessed by the consumer only after consuming it. As a matter of fact, most services sold on daily deal sites belong to this category.

Note 7. The derivation of the demand function in $t = 2$ can be found in Appendix A.

Note 8. The derivation of the optimal p_2 can be found in Appendix B.

Note 9. The derivation of the optimal p_1 can be found in Appendix B.

Note 10. The firm provides low quality also when $\delta \leq \hat{\gamma}_C$; however, in this case, we do not talk about quality reduction because the low quality choice was planned since the beginning.

Appendix A

Derivation of the demand function in $t = 2$

Let's fix $y = M$ (for $y = C$, the quantity demanded for each possible price has simply to be multiplied by k). By assumptions we know that, in $t = 2$:

- if *all* consumers expect an average quality, the demand is $q_2 = 1 - p_2$;
- if *all* consumers expect a high quality, the demand is $q_2 = 1 + w - p_2$;
- if *all* consumers expect a low quality, the demand is $q_2 = 1 - w - p_2$.

In $t = 1$, the price is p_1, and thus $q_1 = 1 - p_1$ consumers purchase the product and learn the quality (which they expect to be the same in $t = 2$), whereas the remaining consumers still expect an average quality. Hence, the demand in $t = 2$ will be the sum of two segments: the q_1 consumers who purchased in $t = 1$ and thus expect either a high or a low quality, and the $(1 - q_1)$ consumers that did not buy and thus expect an average quality.

In particular, if $x_1 = H$, then the total demand in $t = 2$ is the sum of these two segments: the demand from consumers who purchased in $t = 1$, which is

$$q_2 = \begin{cases} 1 - p_1 & \text{if } 0 \le p_2 \le p_1 + w \\ 1 + w - p_2 & \text{if } p_1 + w < p_2 \le 1 + w \\ 0 & \text{if } 1 + w < p_2 \end{cases} \tag{38}$$

and the demand from consumers who did not purchase in $t = 1$, which is

$$q_2 = \begin{cases} p_1 - p_2 & \text{if } 0 \le p_2 \le p_1 \\ 0 & \text{if } p_1 < p_2 \end{cases} \tag{39}$$

Summing these two functions, we get (2). If $x_1 = L$, then the total demand in $t = 2$ is the sum of these two segments: the demand from consumers who purchased in $t = 1$, which is

$$q_2 = \begin{cases} 1 - p_1 & \text{if } 0 \le p_2 \le p_1 - w \\ 1 - w - p_2 & \text{if } p_1 - w < p_2 \le 1 - w \\ 0 & \text{if } 1 - w < p_2 \end{cases} \tag{40}$$

and the demand from consumers who did not purchase in $t = 1$, which is

$$q_2 = \begin{cases} p_1 - p_2 & \text{if } 0 \le p_2 \le p_1 \\ 0 & \text{if } p_1 < p_2 \end{cases} \tag{41}$$

Summing these two functions, we get (3).

Appendix B

Solution of the model

At $t = 2$, the firm solves the following problem:

$$\max_{[p_2, x_2]} \left(p_2 - c_{x_2}\right) q_2(p_2 | y; p_1, x_1). \tag{42}$$

It is immediate to see that it is optimal to set $x_2^2 = L$. Let's then solve for the optimal p_2 for any possible values of $(y; p_1, x_1)$. Suppose first that $y = M$ and $x_1 = H$. The profit of the firm is then

$$\pi_2 = \begin{cases} (p_2 - c_L)(1 - p_2) & \text{if } 0 \le p_2 \le p_1 \\ (p_2 - c_L)(1 - p_1) & \text{if } p_1 < p_2 \le p_1 + w \\ (p_2 - c_L)(1 + w - p_2) & \text{if } p_1 + w < p_2 \le 1 + w \end{cases} \tag{43}$$

Let's take the derivative of the profit function above. It is

$$\pi_2' = \begin{cases} -2p_2 + 1 + c_L & \text{if } 0 \le p_2 \le p_1 \\ 1 - p_1 & \text{if } p_1 < p_2 \le p_1 + w \\ -2p_2 + 1 + c_L + w & \text{if } p_1 + w < p_2 \le 1 + w \end{cases} \tag{44}$$

It can be verified that:

- if $p_1 \le (1 + c_L - w)/2$, then the derivative is:
    - strictly positive for $p_2 < (1 + c_L + w)/2$;
    - strictly negative for $p_2 > (1 + c_L + w)/2$.

  Hence, the optimal price in this case is $p_2^2 = (1 + c_L + w)/2$;
- if $(1 + c_L - w)/2 < p_1 \le (1 + c_L)/2$, then the derivative is:

- strictly positive for $p_2 < p_1 + w$;
- strictly negative for $p_2 > p_1 + w$.

Hence, the optimal price in this case is $p_2^2 = p_1 + w$;

- if $p_1 > (1 + c_L)/2$, then the derivative is:
  - strictly positive for $p_2 < (1 + c_L)/2$;
  - strictly negative for $(1 + c_L)/2 < p_2 < p_1$;
  - strictly positive for $p_1 < p_2 < p_1 + w$;
  - strictly negative for $p_1 + w < p_2$.

Hence, the profit function has two local maxima, namely $p_2 = (1 + c_L)/2$ and $p_2 = p_1 + w$. By computing the value of the profit function in these two points, it can be seen that the global maximum is $p_2^2 = p_1 + w$ if $p_1 \le \left(1 + c_L - w + \sqrt{w^2 + 2w(1 - c_L)}\right)/2$, it is $p_2^2 = (1 + c_L)/2$ if $p_1 > \left(1 + c_L - w + \sqrt{w^2 + 2w(1 - c_L)}\right)/2$.

By properly substituting p_2^2 into (43), we obtain (7).

Suppose next that $y = M$ and $x_1 = L$. Consider, first, the case $p_1 \le c_L$. This, together with assumption *(i)* made in Section 3, implies that $\max\{1 - w, p_1\} \le c_L$, and, therefore, that the firm can sell a strictly positive quantity only if $p_2 < c_L$. Hence, in this case, the firm prefers to sell nothing (i.e. she sets $p_2 \ge c_L$). Consider now the case $p_1 > c_L$. This, together with assumption *(i)*, implies that $\max\{1 - w, p_1\} = p_1$. The profit of the firm is then

$$\pi_2 = \begin{cases} (p_2 - c_L)(1 - p_2) & \text{if } 0 \le p_2 \le p_1 - w \\ (p_2 - c_L)(1 - w + p_1 - 2p_2) & \text{if } p_1 - w < p_2 \le 1 - w \\ (p_2 - c_L)(p_1 - p_2) & \text{if } 1 - w < p_2 \le p_1 \end{cases} \tag{45}$$

Let's take the derivative of the profit function above. It is

$$\pi_2' = \begin{cases} -2p_2 + 1 + c_L & \text{if } 0 \le p_2 \le p_1 - w \\ -4p_2 + 1 - \mathrm{w} + p_1 + 2c_L & \text{if } p_1 - w < p_2 \le 1 - w \\ -2p_2 + p_1 + c_L & \text{if } 1 - w < p_2 \le p_1 \end{cases} \tag{46}$$

It can be verified that the above derivative is strictly positive for $p_2 < (p_1 + c_L)/2$ and strictly negative for $p_2 > (p_1 + c_L)/2$. Hence, the optimal price is $p_2^2 = (p_1 + c_L)/2$. By properly substituting p_2^2 into (45), we obtain (9).

Finally, for $y = C$, the optimal prices in $t = 2$ with $x_1 = H$ and with $x_1 = L$, respectively, are the same as for $y = M$, because the firm's profit with $y = C$ is equal to the firm's profit with $y = M$ multiplied by k.

Consider now date $t = 1$. We show here how to derive the optimal price p_1^1 when $y = M$ (when $y = C$ the price is p_C). Suppose that $y = M$ and the firm chooses $x_1 = H$. In this case, the maximand in (16) becomes

$$\Pi_1 = \begin{cases} (p_1 - c_H)(1 - p_1) + \beta\delta(1 - c_L + w)^2/4 & \text{if } 0 \le p_1 \le (1 + c_L - w)/2 \\ (p_1 - c_H)(1 - p_1) + \beta\delta(p_1 + w - c_L)(1 - p_1) & \text{if } (1 + c_L - w)/2 < p_1 \le (1 + c_L - w + \sqrt{w^2 + w(1 - c_L)})/2 \\ (p_1 - c_H)(1 - p_1) + \beta\delta\,(1 - c_L)^2/4 & \text{if } (1 + c_L - w + \sqrt{w^2 + w(1 - c_L)})/2 < p_1 \le 1 \end{cases} \tag{47}$$

Let's take the derivative of the profit function above. It is

$$\Pi_1{}' = \begin{cases} -2p_1 + 1 + c_H & \text{if } 0 \le p_1 \le (1 + c_L - w)/2 \\ -2(1 + \beta\delta)p_1 + 1 + c_H + \beta\delta(1 + c_L - w) & \text{if } (1 + c_L - w)/2 < p_1 \le (1 + c_L - w + \sqrt{w^2 + w(1 - c_L)})/2 \\ -2p_1 + 1 + c_H & \text{if } (1 + c_L - w + \sqrt{w^2 + w(1 - c_L)})/2 < p_1 \le 1 \end{cases} \tag{48}$$

It can be verified that, under assumption *(ii)* made in Section 3, the derivative is:

- strictly positive for $p_1 < \frac{1+c_H}{2} - \frac{\beta\delta(c_H - c_L + w)}{2(1+\beta\delta)}$;
- strictly negative for $p_1 > \frac{1+c_H}{2} - \frac{\beta\delta(c_H - c_L + w)}{2(1+\beta\delta)}$.

Hence, the optimal price in this case is $p_1^1 = \frac{1+c_H}{2} - \frac{\beta\delta(c_H - c_L + w)}{2(1+\beta\delta)}$. After noticing that $\frac{1+c_L-w}{2} < p_1^1 < \frac{1+c_L-w+\sqrt{w^2+w(1-c_L)}}{2}$, then one can simply substitute p_1^1 into (47) to obtain (18).

Suppose that $y = M$ and the firm chooses $x_1 = L$. In this case, the maximand in (16) becomes

$$\Pi_1 = \begin{cases} (p_1 - c_L)(1 - p_1) & \text{if } 0 \le p_1 \le c_L \\ (p_1 - c_L)(1 - p_1) + \beta\delta(p_1 - c_L)^2/4 & \text{if } c_L < p_1 \le 1 \end{cases} \tag{49}$$

Clearly, setting $p_1 \le c_L$ cannot be optimal. It is straightforward to see that (49) is maximized at $p_1^1 = [2 + (2 - \beta\delta)c_L]/(4 - \beta\delta)$. Substituting p_1^1 into (49), we obtain (20).

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