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Liquidation Risk: Evidence on Non-Linearities in Uncovered Interest Parity

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Abstract

Prospective shocks that force the immediate liquidation of securities to raise liquidity determine the *ex-ante* excess returns on currencies – a liquidation premium to compensate the investor for their liquidation risk even if they have forward cover. This liquidation premium behaves non-linearly, as postulated by the liquidity-risk augmented uncovered interest rate parity theory. The success of uncovered interest parity is, thus, conditional on the severity of the shock and the levels of interest rates. We examine the empirical validity of these non-linearity propositions using data on five major currencies, and document that the failure of uncovered interest parity is more pronounced when liquidity shocks are more severe and interest rate levels are higher.

Keywords: uncovered interest rate parity, forward premium puzzle, liquidity risk-augmented UIP, liquidation risk, liquidation premium, non-linearity in uncovered interest parity

1. Introduction

The uncovered interest parity (UIP) theory characterizing the flow of international capital holds that with free capital mobility, the interest rate differential between domestic currency and foreign currency bonds – the forward premium -- equals the expected appreciation of the foreign currency. The same-currency returns on domestic and foreign bonds are equal because the high interest rate currency is expected to depreciate, depriving investors of systemic opportunities for earning excess profits. In practice high interest rate currencies do not depreciate sufficiently and often appreciate. Therefore, *ex-post* returns on high interest rate currencies exceed those on low interest rate currencies, resulting in persistent excess returns. The forward premium consistently underpredicts foreign currency appreciation in the data, and this empirical failure of UIP has been labelled the forward premium puzzle.

The search for plausible resolutions of the puzzle has developed in several directions, and among these is the role of liquidity constraints. One such is the liquidity risk-augmented UIP (ℓ-UIP) theory, which focuses on the effects of sudden declines in asset prices when the supply of liquidity is not infinitely elastic, and investors are forced to liquidate their bonds in the face of adverse liquidity events (Kumar, 2020). It points to particular non-linearities that affect how well UIP performs against the data: the failure of UIP is more pronounced when liquidity shocks are more severe, and when the levels of interest rates are higher. The purpose of this paper is to examine the empirical validity of these non-linearity propositions.

Liquidity shocks compel rapid-fire divestment of securities by investors in a crisis state and lead to capital losses in a liquidity constrained world. The ℓ-UIP theory states that losses in the liquidity crisis state are directly proportional to interest rates in the normal or no-crisis state, embodying a ‘higher they are, harder they fall’ analogy. The prospect of loss resulting from sudden declines in asset prices and forced liquidation of default free bonds alters the incentives for international arbitrageurs since the *expected* return on bonds also incorporates investors’ perceptions of liquidity shocks; the expected return is less than the corresponding nominal interest rate. In this environment, zero-arbitrage prevails when the *expected* return on domestic bonds equals the *expected* return on foreign bonds plus the expected appreciation of the foreign currency. The excess return on currencies found in the data is thus interpreted as the compensation for the investors’ liquidation risk-bearing: it is the liquidation risk premium.

Our results show that the failure of UIP is more pronounced when the liquidation risk is high relative to when it is low. Thus, we add to earlier empirical findings in the literature on non-linearities in the behavior of UIP such as Bansal (1997), who uses a term structure framework to show that the *sign* of the forward premium matters: that UIP is more clearly rejected when the interest rate differential is positive (in favor of US Dollar assets) relative to when it is negative. In a similar vein, an important prediction of the ℓ-UIP theory is that the failure of UIP is more prominent when interest rate *levels* are high relative to when they are low. This empirical regularity on the asymmetry in the performance of UIP complements studies on non-linearities in the behavior of UIP such as Baillie and Kiliç (2006), who show that the failure or success of UIP depends on whether or not a risk-adjusted forward premium and other macro transition variables cross certain thresholds.

Broadly, explanations of these findings run along two threads: one assumes a time-varying risk premium in the presence of risk aversion and rational expectations, whereas the other assumes systematic expectations errors that allow excess returns to persist. Support for time-varying risk premia is found in Dahlquist and Pénasse (2022) and Bak and Park (2022) who attribute roles to proxies of latent components in explaining the slope parameter, Lustig and Verdelhan (2019) who examine the effect of incomplete spanning on the currency risk premium, Verdelhan (2010) who examines the role of habit and varying risk-aversion in the presence of consumption growth risk, Lustig and Verdelhan (2007) who show that excess returns compensate for the cost of consumption growth risk, Backus et al. (2001) who incorporate the role of the term structure of interest rates in the failure of UIP, and the celebrated work of Fama (1984).

Schmitt-Grohé and Uribe (2022), on the other hand, find in a model with portfolio frictions that deviations from UIP are conditioned on whether monetary shocks are permanent or temporary. The role of systematic forecast errors is explored in Bacchetta and van Wincoop (2010) who show that infrequent portfolio adjustments cause predictable excess returns to currencies, and Evans and Lewis (1995) who investigate the effects of regime-switching in explaining the Peso problem. Kumar (2020) emphasizes liquidity constraints, arguing that expected rather than notional returns drive international financial market equilibrium even with forward cover and shows that UIP fails without invoking either systematic errors or risk aversion, though it will be negative only under risk-aversion. Engel et al. (2019) show that excess returns on foreign bonds vary directly with the US and foreign interest rate differentials due to the role of ‘liquidity returns’ on the dollar, and Nagayasu (2014) finds that “the risk premium seems to be closely associated with liquidity constraints.”

Our findings are also related to recent literature on the role of uncertainty in the failure of UIP. For instance, Islam et al. (2023) conclude that UIP failed across developed and emerging economies during the Covid period which was characterized by heightened foreign exchange rate volatility, and Jamali et al. (2023) show that excess returns on currencies vary systematically and negatively with investment risk factors. Ismailov and Rossi (2018) find that UIP is less likely to hold when uncertainty is high, since gains from arbitrage become more uncertain in higher uncertainty environments relative to low uncertainty environments, Husted et al. (2018) show that financial and macroeconomic uncertainty increases currency excess returns, and Farhi and Gabaix (2016) argue that the tail risk of rare disasters is an important determinant of the risk premium.

Evidence on the failure of UIP has covered a lot of ground since the early works of Hansen and Hodrick (1980) and Fama (1984) among others, but these findings are not uncontested or unqualified. For instance, Burnside et al. (2011) show that traditional risk factors are not related to currency excess returns, and Froot and Frankel (1989) could not confirm the existence of a risk premium in survey data. Likewise, Cover and Mallick (2012) find that UIP performs well in UK data, Lee (2011) fails to confirm failure of UIP in cross-sectional data, and Chinn (2006) and Boudoukh et al. (2005) find that UIP performs well for longer maturities.

Yet, the documented failure of UIP is an enduring puzzle in international economics, and this paper provides new empirical insights from the liquidity risk-augmented UIP framework into the implications of limited internal capital markets stemming from the arbitragers’ limited capital during times of crisis. In section 2 we fix certain notations and concepts pertaining to the forward premium puzzle and ℓ-UIP, in section 3 we discuss data and estimation issues, in section 4 we present our empirical findings and related commentary on non-linearities in the effects of liquidity shocks on UIP. Concluding remarks are in section 5.

2. Forward Premium Puzzle and ℓ – UIP

According to the canonical UIP theory that characterizes the international financial market equilibrium, the interest rate on domestic bonds equals the interest rate on foreign bonds plus the expected appreciation of the foreign currency. Since the high interest rate currency is expected to depreciate, the same-currency returns on domestic and foreign bonds are equal, precluding systematic opportunities for earning excess profits. The canonical UIP relationship is:

$$\Delta s_{t+1}^{e} = i_t - i_t^{*} \tag{1}$$

In equation (1), $\Delta s_{t+1}^{e} = \ln S_{t+1}^{e} - \ln S_t$ is the approximate one-period ahead expected depreciation rate of the domestic currency, where S_t is domestic currency price of a unit of foreign currency at time *t*. The domestic and foreign interest rates are i_t and i_t^* (asterisks denote foreign variables), respectively. Thus, the UIP predicts that on average, interest rate differentials are offset by expected currency depreciations, and currencies yielding higher interest are expected to depreciate.

Now, if the one-period ahead forward rate is F_t, then the forward premium is given by $f_t \equiv \ln F_t - \ln S_t$. Further, if covered interest parity holds and errors are white noise, then $f_t = i_t - i_t^*$, and equation (1) can be written as (Note 1):

$$\Delta s_{t+1}^{e} = f_t \tag{2}$$

In this form, the canonical UIP predicts that the forward premium is an unbiased predictor of future currency depreciation, or equally that on average the risk premium is zero, i.e., $\Delta s_{t+1}^{e} - f_t = 0$. Empirical tests of the canonical UIP utilize *ex post* one-period-ahead currency depreciation rates in estimating equations of the form:

$$\Delta s_{t+1} = \alpha + \beta f_t + \varepsilon_{t+1} \tag{3}$$

Equation (3) is tested assuming the joint null $H_0 : (\alpha = 0, \beta = 1)$. Under this null, if ε_{t+1} is white noise and investors are risk neutral then the forward premium is a rational forecast of the future currency depreciation.

However, a large body of evidence rejects $\beta = 1$, finding that the estimated slope coefficient $\beta < 1$, that often $\beta < 0$. The latter implies that currencies with higher interest rates are expected to appreciate rather than depreciate. Therefore, given (2) and (3), the one-period-ahead forecast error, $\Delta s_{t+1} - \Delta s_{t+1}^{e} = \alpha + (\beta - 1) f_t + \varepsilon_{t+1}$, also interpreted as excess return or the risk premium, is negatively correlated with the forward premium. Indeed, the forward premium anomaly has endured over different currency samples and sample periods, evident in the ranges of estimated slope coefficients over the decades: -0.21 to -1.15 (Fama, 1984), -2.02 to -3.50 (Evans & Lewis, 1995), -0.74 to -1.84 (Backus et al., 2001), -0.55 to -3.06 (Bacchetta & van Wincoop, 2010), and 0.61 to -0.97 (Kumar, 2020).

2.1 Liquidity Shock and UIP Failure: An Example

As an illustration of how liquidity shocks explain the forward premium puzzle, let the interest rates on zero-coupon default-free US dollar (USD) and yen (JPY) denominated assets be $i = 0.05$ and $i^* = 0.02$, respectively. On average the UIP theory predicts a one-period ahead USD depreciation of 0.03 = 0.05 – 0.02. But conditional on a shock forcing rapid-fire asset sales at low prices, the returns will be lower, possibly negative. The risk-neutral investor will base their decision on *expected returns* from these bonds which are *less* than their notional returns. Assuming that shocks are symmetric and propagate worldwide, let us say that the expected returns (R^e) are both 20% lower: $R^e = 0.04$ and $R^{*e} = 0.016$ on USD and JPY assets, respectively. Then the *logic* of the canonical UIP should, in the data, show an expected one-period-ahead USD depreciation of $\Delta s_{+1}^{e} = 0.04 - 0.016 = 0.024$, though the canonical UIP itself predicts $\Delta s_{+1}^{e} = 0.03$. If covered interest parity holds, then the implied forward premium of $f = i - i^* = 0.03$ correlates with an anticipated USD depreciation $\Delta s_{+1}^{e} = 0.024$, and on average an *ex post* USD depreciation $\Delta s_{+1} = 0.024$. So the UIP regression slope coefficient in (3) ends up being 0.8 rather than 1.0, which manifests itself as the forward premium puzzle.

In the ℓ-UIP theory the downward bias of the slope coefficient, the -0.2 deviation from unity, reflects a *liquidation premium* required to compensate this investor for the larger potential loss of holding the high interest asset in the crisis. In canonical UIP the data will show an excess return on USD assets equal to 0.05 – (0.02 + 0.024) = +0.006: this is 0.2 of the interest rate differential of 0.03. The ℓ-UIP theory thus argues that prospective liquidation shocks alter the terms of financial arbitrage and credit constraints exacerbate the risk to investors: *ceteris paribus*, the risk premium will increase with the severity of the financial shock and will be increase with the level of the interest rates, both of which result in larger declines the expected returns R^e and R^{*e}.

2.2 ℓ-UIP Condition and Testable Hypotheses

In contrast to (2) it has been shown in Kumar (2020) that the liquidity risk-augmented uncovered interest parity condition can be expressed as:

$$\Delta s_{t+1}^{e} = [1 - \mathrm{B}_t(\cdot)] f_t \tag{4}$$

In (4), $\mathrm{B}_t(\cdot)$ is a measure of the forward premium bias or the liquidation premium noted above, and is a function of financial market conditions and investor risk preference. To fix certain notations and concepts relating to ℓ-UIP and summarize key elements in that study which inform us of the channels by liquidity shocks affect the performance of UIP relevant for setting up our testing framework, we note that:

$$\mathrm{B}_t = \frac{\pi}{(1-\pi)}\ell\bar{\theta}(i_t, i_t^*) \quad (5)$$

In (5) $\pi > 0$ is the probability of a liquidity crisis state obtaining, and $\ell > 0$ is an indicator of the severity of the crisis. θ is the marginal utility of returns conditional on a liquidity crisis relative to the marginal utility of returns conditional on no-crisis and is a measure of risk tolerance or the curvature of the investor utility function. The term $\bar{\theta}_t = 0.5[\theta(i_t) + \theta(i_t^*)]$. If the marginal utility of interest income is u_i, then:

$$\theta(i) = \frac{u_i(-\ell\, i)}{u_i(i)} \geq 1 \quad (6)$$

Risk-neutrality implies $\theta = 1$ and risk-aversion implies $\theta > 1$. θ reflects the investors' subjective valuation of loss due to a shock relative to the gain under normal conditions.

The theoretical slope coefficient of f under ℓ-UIP in (5) is $1 - \mathrm{B}(\cdot)$; the deviation from unity by $\mathrm{B}(\cdot)$ is due to liquidation risk, which is the compensation required by the investor for bearing the risk of loss due to a potential liquidation of assets in a crisis. The slope coefficient depends on indicators of the likelihood and severity of the liquidity shock, and interest rates levels via the curvature of the utility function. Hence, following a discussion of data and some estimation issues in the following section, in formulating the estimating equations we utilize the property of (6) that its partial derivatives are positive: $\theta_\ell > 0$ and $\theta_i > 0$. These yield the testable hypotheses that the slope coefficient decreases as the severity of the liquidity shock ℓ and interest rate levels increase.

3. Data & Estimation Issues

We test equation (4) using monthly and quarterly data on five major currencies: the Canadian Dollar (CAD), the Swiss Franc (CHF), the German Mark (DEM) obtained by converting the Euro/US Dollar rate at the legacy rate 1.95583 DEM/Euro, the British Pound (GBP) and the Japanese Yen (JPY). Spot rates are US Dollar (USD) prices of foreign currencies. Sample periods are 1978m08-2021m05 for monthly data, and 1978q4-2021q1 for quarterly data. Assuming covered interest parity holds, the forward premium on currency j ($f^j = i^{USD} - i^j$) is calculated from 1-month and 3-month Eurocurrency rates. Credit conditions as a correlate for the severity of liquidity shocks are measured by the credit index of the Chicago Fed's National Financial Conditions Index database (Note 2). We estimate several models using the *ex-post* risk premium $\Delta s_{t+1} - f_t$ as the regressand.

Tests of UIP using the standard estimating equation in (3) can be critiqued along several lines in light of the issues raised by the $\ell - \mathrm{UIP}$ framework. Firstly, in view of (4), canonical tests of UIP regressing Δs_{+1} on f are invalid because omitting the term $-\mathrm{B}(\cdot)$ from the estimating equation causes a specification bias. Secondly, this omitted term, which is then included in the error term, is a function of f so the errors are not orthogonal to the regressor and result in invalid parameter estimates. Regressing $\Delta s_{+1} - f$ on f circumvents the non-orthogonality issue and it is, therefore, the desired regressor on theoretical grounds. Yet, thirdly, a specification error persists due to the exclusion of the liquidity shock ℓ, as well as the preference curvature term $\bar{\theta}$ that in turn depends on interest rate levels: both introduce non-linearities in the UIP relationship via the liquidity shock mechanism, and they are both strongly correlated.

With these issues in mind we examine whether the data support the testable hypotheses of (4) that in the regression of $\Delta s_{+1} - f$ on f, the coefficient of f is negative, and that there are non-linearities in the UIP condition. The mechanisms from which the non-linearities arise are firstly, changes in liquidity conditions captured by larger values of ℓ and, secondly, changes in interest rate levels. The slope coefficient decreases as (a) the severity of liquidity events increases, and (b) as interest rate levels increase because $\bar{\theta}(i, i^*)$ in (5) is increasing in the levels of interest rates.

4. Results

We present in the following section the estimating equations and results of these non-linearity propositions. In brief, Models 1 & 2 are tests of the canonical UIP relationship in equation (3). Models 3-5 address the non-linearity postulates of ℓ-UIP: Model 3 examines the effects of the severity of liquidity shocks, proxied by tightening credit conditions, and the pair of Models 4 & 5 estimate the impact of changing interest rate levels on the slope coefficient in equation (4).

4.1 Canonical UIP Tests

Tests of the canonical UIP relationship are provided below despite the econometric issues discussed above to contextualize the salient effects of liquidity shocks, as well as for comparison with the wider literature on UIP. Model 1 uses the estimating equation for the canonical UIP:

$$\Delta s_{t+1}^{j} - f_t^{j} = \alpha^{j} + (\beta^{j} - 1) f_t^{j} + \varepsilon_{t+1}^{j} \tag{7}$$

Equation (7) is estimated using OLS for each currency *j*, and diagnostics for these single-equation estimations are based on Newey-West HAC standard errors under the null $H_0:(\alpha^j = 0, \beta^j - 1 = 0)$ for all *j*.

Model 1 also includes a pooled sample of the five currencies using Seemingly Unrelated Regression (SUR) with country fixed effects and β restricted to be equal across currencies:

$$\Delta s_{t+1}^{j} - f_t^{j} = \alpha + \gamma^{j} + (\beta - 1) f_t^{j} + \varepsilon_{t+1}^{j} \tag{8}$$

The coefficient β is restricted for reasons discussed in Bilson (1981); the ratios of standard deviations of $\Delta s_{t+1}^{j} - f_t^{j}$ to those of f_t^{j} are 13.9-18.3 in 1-month, and 8.6-10.5 in 3-month data (see Appendix B). The SUR is estimated using GLS. The CSSUR estimates correct for cross-sectional heteroskedasticity and contemporaneous covariance between currencies: $E(\varepsilon_{it}\varepsilon_{jt}) = \sigma_{ij}$ and $E(\varepsilon_{is}\varepsilon_{jt}) = 0$. Following Lee (2011), TSSUR estimates correct for arbitrary period heteroskedasticity and serial correlation of residuals within each cross-section but residuals across cross-sections are uncorrelated: $E(\varepsilon_{it}\varepsilon_{jt}) = \sigma_t^2$ and $E(\varepsilon_{is}\varepsilon_{jt}) = 0,\ i \neq j$. Though TSSUR inferences are based on this scheme, given the limited number of cross-sections we use the same parameter restrictions as above and estimate it using unweighted least squares. Both variants are panel corrected standard errors of Beck and Katz (1995) and are adjusted for degrees of freedom. Single series and panel unit root tests (not presented) confirmed stationary using ADF, PP and KPSS tests, and LM tests confirmed the absence of serial correlation in residual series at conventional levels.

Since exchange rates are known to embed variance memory in their distributions, Model 2 fits either a GARCH (1,1) or a GARCH (2,1) specification with the following mean and conditional variance equations:

$$\begin{aligned} &\Delta s_{t+1}^{j} - f_t^{j} = \alpha^{j} + (\beta^{j} - 1) f_t^{j} + \varepsilon_{t+1}^{j} \\ &\sigma_{t+1}^{j2} = \omega^{j} + \sum_{k=1,2} \delta_k \varepsilon_{t+1-k}^{j2} + \eta \sigma_t^{j2} \quad (k = 1 \text{ or } 2) \end{aligned} \tag{9}$$

Models 1 and 2 are estimated for both 1-month and 3-month data, and the corresponding results are presented in Tables 1 and 2, respectively. We suppress the presentation of the constant terms in the results tables and focus on the slope coefficient β, noting that the constants impound Jensen's inequality terms (see, for instance, Engel, 1996; Baillie, 2011; and Olmo & Pilbeam, 2011) and panel heterogeneity terms. The constants were not statistically significantly different from zero with exceptions for JPY fixed effects.

Table 1. Canonical UIP

		Single Equation Estimates					Panel	
Maturity		CAD	CHF	DEM	GBP	JPY	CSSUR	TSSUR
1-Month	β	0.04	**-1.02**	-0.42	**-1.45**	**-1.54**	**-0.93**	**-1.00**
		(1.65)	(-2.95)	(-1.88)	(-2.12)	(-3.79)	(-6.07)	(-5.79)
	Adj. R^2	0.001	0.015	0.024	0.019	0.026	0.013	0.015
	AIC	-4.93	-3.96	-4.10	-4.23	-4.11		
	F	1.4	11.1	4.5	10.9	14.6	7.65	9.06
	D.W.	2.12	2.03	1.98	1.91	1.90	2.00	1.98
	T (NT for SUR)	513	513	513	513	513	2565	2565
3-Month	β	0.12	**-1.10**	**-0.43**	**-1.33**	**-1.74**	**-0.84**	**-1.03**
		(1.52)	(-3.77)	(-2.02)	(-2.29)	(-3.86)	(-5.61)	(-5.50)
	Adj. R^2	0.002	0.054	0.017	0.049	0.063	0.032	0.041
	AIC	-3.95	-2.84	-2.91	-3.12	-2.82		
	F	1.36	10.6	3.99	9.66	12.4	6.55	8.23
	D.W.	1.82	2.08	1.92	1.79	1.98	1.90	1.95
	T (NT for SUR)	170	170	170	170	170	850	850

Note. Boldface denotes statistical significance at 5% or less: *t*-statistics in parentheses for $H_0: \beta - 1 = 0$. Sample periods: 1-Month 1978m08-2021m05, 3-Month 1978q3-2021q1. Constants not presented. Single equation estimates: Standard errors are HAC robust. T=observations after exclusions. No constant is statistically significant at conventional levels except for JPY. SUR estimates: β restricted across currencies. Robust panel corrected standard errors are used. CSSUR robust to contemporaneous covariance and cross-section heteroskedasticity. TSSUR robust to arbitrary period heteroskedasticity and serial correlation of residuals within each cross-section. N=5 cross-sections.

Table 1 presents Model 1 estimates of canonical UIP and holds little surprise: $\hat{\beta} < 1$ in all cases for both 1-month and 3-month data except CAD. They are not significantly different from unity (though the point estimates are close to zero) for CAD, but in all other instances, they are statistically significant at conventional levels and negative, ranging between 0.04 for CAD to -1.54 for JPY (1-month), and 0.12 for CAD to – 1.74 for JPY (3-month). There is little difference in the monthly and quarterly estimates, or between the single-equation and SUR estimates. Residuals were free of serial autocorrelation.

Table 2. Canonical UIP GARCH Estimates

Maturity		CAD	CHF	DEM	GBP	JPY
1-Month	β	**-0.47**	**-1.12**	**-0.65**	**-1.21**	**-1.02**
		(-2.89)	(-2.60)	(-1.95)	(-1.80)	(-2.39)
	ARCH	**0.04**	**0.08**	**0.05**	0.06	0.05
		(2.47)	(1.90)	(2.50)	(1.22)	(1.34)
	GARCH	**0.95**	**0.88**	**0.90**	**0.89**	**0.91**
		(43.1)	(15.48)	(22.4)	(9.07)	(13.49)
	Adj. R^2	0.00	0.019	0.006	0.018	0.025
	AIC	-5.11	-4.03	-4.14	-4.28	-4.14
	D.W.	2.12	2.03	1.98	1.91	1.89
	T	513	513	513	513	513
3-Month	β	-0.04	-0.82	-0.39	**-1.51**	**-2.41**
		(-1.76)	(-0.71)	(-1.41)	(-2.28)	(-4.28)
	LR Variance	0.0002	0000	0.0004	**0009**	**0.0006**
		(1.03)	(0.01)	(0.79)	(2.03)	(2.98)
	ARCH (-1)	0.07	0.04	**-0.02**	0.26	**-0.1**
		(1.42)	(0.59)	(-0.47)	(1.54)	(-3.43)
	ARCH (-2)		0.01	0.12		
			(0.14)	(1.70)		
	GARCH	**0.92**	**0.94**	**0.75**	0.38	**0.93**
		(15.3)	(11.0)	(3.40)	(1.59)	(19.8)
	Adj. R^2	0.001	0.053	0.017	0.040	0.058
	AIC	-4.07	-2.83	-2.87	-3.17	-2.82
	D.W.	1.82	2.07	1.92	1.78	1.98
	T (NT for SUR)	170	170	170	170	170

Note. Boldface denotes statistical significance at 5% or less: *z*-statistics in parentheses for $H_0: \beta - 1 = 0$. Standard errors are HAC robust. Sample periods: 1-Month 1978m08-2021m05, 3-Month 1978q3-2021q1. Correlograms for all currencies indicate absence of serial correlation in errors; ARCH LM tests confirm that errors are homoscedastic. T=observations after adjustments. Constants are not presented. No constant is significant at conventional levels except for JPY.

Estimates in Table 2 show significant GARCH effects in all monthly estimates and all but GBP 3-month estimates. Estimated ARCH and GARCH terms sum to less than unity in all cases. Except for DEM (3-month), the point estimates of the slope coefficients closely track the OLS estimates in Table 1, 1-month estimates ranging between -0.47 for CAD and -1.21 for GBP, and 3-month estimates ranging between -0.04 for CAD and – 2.41 for JPY.

These estimates are similar to findings in Chinn (2006) and Frankel and Poonawala (2010) who use different sample periods and currencies and conform with prior studies that have endured over various currency samples and sample periods, evident in the estimated ranges of β in the various studies noted in section 2. Yet we know from the discussion at the top of this section that under $\ell-\text{UIP}$ the canonical estimating equations of UIP can be misleading due to misspecification, because the forward premium bias as measured by the liquidation risk premium $-\text{B}(\cdot)f$ depends not only on the forward premium f but also on the severity of the liquidity shock ℓ and risk preference measure $\bar{\theta}$ that varies directly with interest rate levels.

4.2 Tests of $\ell-\text{UIP}$

A direct test of the $\ell-\text{UIP}$ equation (4) is precluded since neither the severity of the liquidity shock ℓ, nor the utility curvature term $\bar{\theta}(i_t, i_t^*)$ are observable. But we know that β varies inversely with ℓ, and since $\theta_i > 0$ and $\theta_{i*} > 0$, β also varies inversely with interest rate levels i and i^*. Thus, $\ell-\text{UIP}$ identifies liquidity constraints as a separate source of non-linearity in the UIP relationship that adds to those documented by Bansal (1997), Baillie and Bollerslev (2000) and Baillie and Kiliç(2006), for instance (Note 3).

In the following sections we present evidence of non-linearity in UIP by conditioning the slope coefficient on an indicator of ℓ, and i and i^*, to test the hypotheses that, firstly, the liquidation risk premium is larger the larger is the severity of the liquidity shock, and secondly, that the liquidity risk premium is larger the higher are the levels of interest rates. Both factors make the slope coefficient smaller and increase the forward premium bias.

4.2.1 Severity of the Liquidity Shock

We utilize tightness of credit conditions as an indicator of the severity of liquidity shocks as proxied by the Credit Index (CI) of the Chicago Fed's National Financial Conditions Index database. Some three dozen indicators are included in the Credit Index including the Markit Investment Grade 5-yr Senior CDS Index, various FRB Senior Loan Officer Surveys, Moody's Baa corporate bond/10-yr Treasury yield spread and MBA Serious Delinquencies (Note 4). Positive values of CI imply tighter-than-average credit conditions and negative values indicate looser-than-average credit conditions.

Model 3 estimates the effect of liquidity shocks by employing a dummy variable D_t^{CI} such that:

$$D_t^{CI} = \begin{cases} 1 \text{ if } CI \geq 0 \\ 0 \text{ otherwise} \end{cases} \tag{10}$$

We separate the forward premium by the CI dummy and implement a modified UIP equation to examine if the slope coefficients are different in periods when the credit conditions are looser relative to when they are tighter than average. Including intercept and slope CI dummy variables, the estimating equation of Model 3 is:

$$\Delta s_{t+1}^j - f_t^j = \alpha_1^j + D_t^{CI}\alpha_2^j + [(1-D_t^{CI})(\beta_1 - 1)]f_t^j + [D_t^{CI}(\beta_2 - 1)]f_t^j + \varepsilon_{t+1}^j \tag{11}$$

The null is $H_0: \alpha_1^j = \alpha_2^j = (\beta_1 - 1) = (\beta_2 - 1) = 0$. In light of the $\ell-$UIP theory we expect that $\beta_2 < \beta_1 < 1$.

Model 3 is estimated as an SUR system with restrictions on the slope parameters β_1 and β_2 for reasons discussed in Model 1. We use the CSSUR and TSSUR error structures there described, for both monthly and quarterly data. Estimations include the full sample as well as two equal sub-samples to assess parameter stability. Sub-sample 1 covers the periods 1978m08-1999m12 and 1978q3-1999q3; sub-sample 2 covers the periods 2000m01-2021m05 and 1999q4-2021q1. Robust panel corrected standard errors are used in inferences.

Table 3. ℓ-UIP: Severity of Liquidity Crisis

	Full Sample		Sub-sample 1		Sub-sample 2	
1-Month	CSSUR	TSSUR	CSSUR	TSSUR	CSSUR	TSSUR
β_1	**-0.14**	**-0.62**	**-0.80**	**-1.02**	0.32	**-0.32**
	(-2.18)	(-2.97)	(-2.96)	(-2.74)	(0.66)	(-2.66)
β_2	**-1.54**	**-1.24**	**-1.51**	**-1.28**	0.11	**-1.70**
	(-6.17)	(-4.97)	(-5.33)	(-4.31)	(0.72)	(-3.53)
Wald $p\beta_1 = \beta_2$	**0.036**	**0.382**	**0.355**	**0.774**	**0.896**	**0.134**
Adj. R^2	0.015	0.015	0.026	0.022	0.003	0.031
F	4.45	4.44	4.08	3.63	1.40	4.71
D.W.	2.01	1.98	1.93	1.97	2.11	2.03
NT	2565	2565	1280	1280	1280	1280
$\sum D_t^{CI}$	221	221	120	120	101	101
3-Month						
β_1	0.19	**-0.21**	**-0.67**	**-0.82**	1.00	0.89
	(1.54)	(-2.00)	(-2.75)	(-2.09)	(0.00)	(0.23)
β_2	**-1.60**	**-1.58**	**-1.17**	**-1.45**	**-2.37**	**-1.47**
	(-6.12)	(-5.39)	(-4.41)	(-4.20)	(-2.54)	(-2.64)
Wald $p\beta_1 = \beta_2$	**0.008**	**0.077**	**0.530**	0.55	**0.046**	**0.026**
Adj. R^2	0.038	0.041	0.051	0.059	0.023	0.066
F	4.09	4.27	3.04	3.37	1.92	3.73
D.W.	1.93	1.94	1.87	1.94	2.09	2.05
NT	850	850	420	420	425	425
$\sum D_t^{CI}$	73	73	40	40	33	33

Note. Boldface denotes statistical significance at 5% or less: *t*-statistics in parentheses for $H_0: \beta - 1 = 0$. Wald shows *p*-values. Sample dates are 1978m08-2021m05 for 1-month & 1978q3-2021q1 for 3-month data. CSSUR uses cross-section as the panel unit and is robust to contemporaneous covariance and cross-section heteroskedasticity of residuals. TSSUR assumes that the errors for a cross-section are heteroskedastic and serially correlated (cross-section clustered). Both procedures utilize panel corrected standard errors methodology of Beck and Katz (1995). NT=number of observations after exclusions where N=5 cross-sections.

The results are presented in Table 3. For the full sample and both sub-samples, for monthly as well as quarterly data, the point estimates of the slope coefficients $\beta_2 < \beta_1$ which is in line with the $\ell-\text{UIP}$ theory: the required liquidation risk premium is larger when credit conditions are tighter and liquidity shocks would be more severe. For instance, CSSUR estimates for the full sample monthly data show that estimated $\beta_2 = 11\beta_1$ with Wald test rejecting the equality of β_1 and β_2. Overall, nineteen of the twenty-four estimated parameters presented are statistically significant at *p*-values < 0.05. Wald tests reject the equality of the two slope coefficients in three of the four estimated parameter pairs for the full sample, none for the first sub-sample and two of the four pairs for the second sub-sample. Evidence on the effect of tighter credit conditions via $\ell-\text{UIP}$ mechanism is stronger for quarterly data, though qualitatively there do not seem to be pattern differences between monthly and quarterly liquidation risk premia. Using only pairs of statistically significant point estimates, as measured by the estimated values $(\beta_2 - 1)/(\beta_1 - 1)$, we see that the liquidation risk premium on average was 91% larger for the full sample, and 29% and 104% larger for the first and second sub-samples, respectively. Thus, investors systematically sought a higher liquidation premium when faced with more severe prospective liquidity shocks, and such premium was higher in the second sub-sample relative to the first.

4.2.2 Interest Rate Levels

In this section we examine whether and to what extent the levels of interest rates impact the liquidation risk premium and show up as downward biased slope coefficients. We saw in (4) and (5) that the channel by which interest rate levels affect the liquidation risk premium is the risk tolerance term θ. With diminishing marginal utility $\overline{\theta} > 1$, reflecting the subjective valuation of utility loss at the margin should a liquidity shock occur relative to utility gains at the margin should it not. Since $\theta_i > 0$, this relative utility value of the loss *increases with the interest rate level* and motivates the investor to seek an offsetting liquidation risk premium. The $\ell-\text{UIP}$ theory predicts that the liquidity discount rises with interest rates, or that the slope coefficient declines as interest rates rise.

In Models 4 & 5, we separate periods of 'low' and 'high' interest rates by defining two dummy variables $D_t^{>}$ and $D_t^{j\square}$ such that:

$$D_t^{>} = \begin{cases} 1 \ if \ (i_t^{USD} \geq \overline{i}^{USD}) \\ 0 \quad otherwise \end{cases} ; \ D_t^{j\square} = \begin{cases} 1 \ if \ (i_t^{USD} \geq \overline{i}^{USD} \ and \ i_t^j \geq \overline{i}^{\,j}) \\ 0 \qquad otherwise \end{cases} \tag{12}$$

In (12) $\overline{i}^{USD}$ & $\overline{i}^{\,j}$ are the sample mean or median interest rates on USD and foreign currency-*j* denominated assets (Note 5). Model 4 includes $D_t^{>}$ to capture high versus low interest rates on USD assets. Likewise, Model 5 utilizes $D_t^{j\square}$ to control for periods when *both* USD and the corresponding currency-*j* assets have high or low interest rates. Estimates for Models 4 and 5 report results using both the sample mean and the sample median as the separating parameters between high and low interest rate periods.

The relevant estimating equations for both Models are given in equation (13), written without dummy superscripts:

$$\Delta s_{t+1}^j - f_t^{\,j} = \alpha_1^j + D_t \alpha_2^j + [(1-D_t)(\beta_1 - 1)] f_t^{\,j} + [D_t(\beta_2 - 1)] f_t^{\,j} + \varepsilon_{t+1}^j \tag{13}$$

Under the joint null $H_0 : \alpha_1^j = \alpha_2^j = (\beta_1 - 1) = (\beta_2 - 1) = 0,$ equation (13) is estimated as an SUR system, and as before, based on the $\ell-\text{UIP}$ we expect that $\beta_2 < \beta_1 < 1$: the prediction is that the coefficient of f_t^j is less than unity and it is smaller in high interest rate periods when $D_t^{>} = 1$ or $D_t^{j\square} = 1,$ than in low interest rate periods. Inferences for the SURs are based on the error covariance structures described under equation (8) as CSSUR and TSSUR.

Full Sample Results: Tables 4 & 5 provide the CSSUR and TSSUR estimates, respectively, for $\ell-\text{UIP}$ Models 4 & 5 for both 1-month and 3-month maturities. Between these tables, the sixteen point estimates of β_1 $(D_t = 0)$ range from 1.06 to -0.51. Of these, nine are not statistically different from unity; all estimates using only the mean or median US interest rate ($D_t = D_t^{>}$) as the separating parameter for low and high rates are in this category. Seven β_1 estimates are negative, all statistically significant. These finding suggest that the conventional UIP performs better when interest rates are low. Indeed, this result echoes the finding in Baillie and Kiliç (2006) that UIP holds within certain ranges of the transition variables considered in that study, but which did not include the levels of interest rates as done here.

Table 4. ℓ-UIP : Effects of Interest Rate Levels (Full Sample. CSSUR Estimates)

Maturity	$\bar{i} = Mean(i_t)$		$\bar{i} = Median(i_t)$	
1-Month	Model 4	Model 5	Model 4	Model 5
β_1	1.06	**-0.44**	1.01	**-0.25**
	(0.08)	(-3.09)	(0.015)	(-2.55)
β_2	**-1.68**	**-2.02**	**-1.70**	**-1.94**
	(-6.51)	(-6.32)	(-6.54)	(-6.64)
Wald $p\beta_1 = \beta_2$	**0.002**	**0.012**	**0.002**	0.061
Adj. R^2	0.016	0.016	0.016	0.016
F	4.74	4.84	4.75	4.83
D.W.	2.00	2.01	2.00	2.01
NT	2565	2565	2565	2565
$\sum D_t$	259		257	
3-Month				
β_1	0.92	**-0.14**	0.95	**-0.23**
	(0.100)	(-2.36)	(0.06)	(-2.40)
β_2	**-1.54**	**-2.05**	**-1.53**	**-1.72**
	(-6.11)	(-6.28)	(-6.09)	(-6.25)
Wald $p\beta_1 = \beta_2$	**0.006**	**0.004**	**0.006**	**0.018**
Adj. R^2	0.037	0.041	0.037	0.038
F	3.95	4.29	3.96	4.04
D.W.	1.92	1.93	1.92	1.91
NT	850	850	850	850
$\sum D_t$	85		86	

Note. Boldface denotes statistical significance at 5% or less: *t*-statistics in parentheses for $H_0: \beta - 1 = 0$. *Wald* shows *p*-values. CSSUR uses cross-section as the panel unit. Panel corrected standard errors of Beck and Katz (1995) robust to contemporaneous covariance and cross-section heteroskedasticity of residuals. NT = number of observations after exclusions where N = 5 cross-sections. Model 4 uses $D = D^{>}$ & Model 5 uses $D = D^{j\square}$ for currency *j* as described in equation (12). Full sample dates are 1978m08-2021m05 for 1-month & 1978q3-2021q1 for 3-month data.

Table 5. ℓ-UIP : Effects of Interest Rate Levels (Full Sample, TSSUR Estimates)

Maturity	$\bar{i} = Mean(i_t)$		$\bar{i} = Median(i_t)$	
1-Month	Model 4	Model 5	Model 4	Model 5
β_1	0.58	**-0.51**	0.50	**-0.20**
	(0.60)	(-2.98)	(0.73)	(-2.24)
β_2	**-1.81**	**-2.01**	**-1.82**	**-2.12**
	(-5.83)	(-5.63)	(-5.84)	(-6.04)
Wald $p\beta_1 = \beta_2$	**0.005**	**0.044**	**0.006**	**0.010**
Adj. R^2	0.017	0.018	0.017	0.019
F	5.14	5.33	5.10	5.43
D.W.	1.99	1.99	1.99	1.99
NT	2565	2565	2565	2565
$\sum D_t$	259		257	
3-Month				
β_1	0.76	**-0.33**	0.88	0.07
	(0.31)	(-2.68)	(-0.15)	(1.66)
β_2	**-2.30**	**-2.58**	**-2.32**	**-2.59**
	(-6.64)	(-5.96)	(-6.63)	(-6.56)
Wald $p\beta_1 = \beta_2$	**0.001**	**0.004**	**0.001**	**0.001**
Adj. R^2	0.055	0.056	0.055	0.058
F	5.45	5.62	5.49	5.75
D.W.	1.98	1.99	1.98	1.99
NT	850	850	850	850
$\sum D_t$	85		86	

Note. Boldface denotes statistical significance at 5% or less: *t*-statistics in parentheses for $H_0: \beta - 1 = 0$. *Wald* shows *p*-values. TSSUR assumes that the errors for a cross-section are heteroskedastic and serially correlated (cross-section clustered) and utilize panel corrected standard errors methodology of Beck and Katz (1995). NT = number of observations after exclusions where N = 5 cross-sections. Model 4 uses $D = D^{>}$ & Model 5 uses $D = D^{j\square}$ for currency *j* as described in equation (12). Full sample dates are 1978m08-2021m05 for 1-month & 1978q3-2021q1 for 3-month data.

Yet this optimistic outlook for the conventional UIP is fades when the interest rates are high ($D_t = 1$). All sixteen estimates of the slope parameter conditioned on high interest rates are negative and several standard deviations from unity, ranging between -1.53 and -2.59. And in all sixteen cases, the estimated $\beta_2 < \beta_1$ with Wald tests rejecting the null $\beta_1 = \beta_2$ with *p*-values < 0.05 in fifteen of the sixteen cases, and *p* < 0.1 in all cases. Taken together, these results are striking across models, maturities, and covariance structures, offering traction in the data for the liquidity-risk-augmented formulation of UIP. They point to liquidity shocks as a source of non-linearity in UIP that depends on the *levels* of interest rates.

Sub-sample Results: Regression results for the first sub-sample are provided in Table 6. All point estimates of the slope coefficients are negative, and twenty-eight of thirty-two estimates of β_1 and β_2 are statistically significant with *p*-values < 0.05. In all sixteen pairs across models and maturities, these point estimates support the $\ell-\mathrm{UIP}$ hypothesis that $\beta_2 < \beta_1$ Wald tests reject the null of $\beta_1 = \beta_2$ for five (β_2, β_1) pair estimates with *p*-values < 0.05, and another four pair estimates with *p*-values < 0.1. For the second sub-sample in Table 7, all point estimates of the slope coefficients are less than one, but none of the CSSUR estimates for either β_1 or β_2 differ statistically from one and Wald tests do not reject their equality. TSSUR estimates provide some rehabilitation for $\ell-\mathrm{UIP}$: in four of the eight pairs, those which use the mean US interest rate as the separating variable between high and low interest rates $(D_t = D_t^{>})$, we find the estimated values of $\beta_2 < \beta_1 < 0$ at conventional levels of significance with Wald tests decisively rejecting the equality of these two coefficients.

Table 6. ℓ-UIP: Effects of Interest Rate Levels (Sub-sample 1)

	CSSUR				TSSUR			
Maturity	$\bar{i} = Mean(i_t)$		$\bar{i} = Median(i_t)$		$\bar{i} = Mean(i_t)$		$\bar{i} = Median(i_t)$	
1-Month	Model 4	Model 5	Model 4	Model 5	Model 4	Model 5	Model 4	Model 5
β_1	**-0.68**	**-0.68**	**-0.52**	**-0.63**	**-0.29**	**-0.47**	**-0.27**	**-0.43**
	(-2.89)	(-2.16)	(-2.38)	(-3.24)	(-1.95)	(-2.78)	(-1.85)	(-2.53)
β_2	**-1.93**	**-2.13**	**-1.93**	**-1.80**	**-2.52**	**-2.89**	**-2.56**	**-2.62**
	(-5.81)	(-5.34)	(-6.09)	(-5.33)	(-5.96)	(-5.88)	(-6.25)	(-5.74)
Wald $p\beta_1 = \beta_2$	0.107	0.144	**0.079**	**0.088**	**0.012**	**0.005**	**0.011**	**0.010**
Adj. R^2	0.029	0.028	0.029	0.027	0.028	0.028	0.028	0.027
F	4.48	4.35	4.42	4.19	4.29	4.26	4.40	4.24
D.W.	1.94	1.93	1.94	1.93	1.98	1.97	1.99	1.98
NT	1280	1280	1280	1280	1280	1280	1280	1280
$\sum D_t$	113		129		113		129	
3-Month								
β_1	**-0.42**	**-0.69**	**-0.51**	**-0.77**	-0.24	**-0.78**	**-0.36**	**-0.85**
	(-2.44)	(-3.42)	(-2.32)	(-3.30)	(1.64)	(-3.18)	(-1.66)	(-3.10)
β_2	**-1.66**	**-1.58**	**-1.40**	**-1.29**	**-2.57**	**-2.46**	**-2.19**	**-2.04**
	(-5.03)	(-4.55)	(-4.84)	(-4.34)	(-5.50)	(-4.30)	(-4.84)	(-3.95)
Wald $p\beta_1 = \beta_2$	0.116	0.226	0.274	0.480	**0.020**	**0.088**	**0.083**	**0.224**
Adj. R^2	0.060	0.056	0.054	0.051	0.069	0.059	0.062	0.056
F	3.42	3.26	3.19	3.06	3.80	3.41	3.52	3.28
D.W.	1.89	1.88	1.88	1.87	2.00	1.98	1.97	1.96
NT	420	420	420	420	420	420	420	420
$\sum D_t$	37		43		37		43	

Note. Boldface denotes statistical significance at 5% or less: *t*-statistics in parentheses for $H_0: \beta - 1 = 0$. Wald shows *p*-values. Sub-sample 1 covers 1978m08-1999m12 for 1-month & 1978q3-1999q3 for 3-month data. CSSUR uses cross-section as the panel unit and is robust to contemporaneous covariance and cross-section heteroskedasticity of residuals. TSSUR assumes that the errors for a cross-section are heteroskedastic and serially correlated (cross-section clustered). Both procedures utilize panel corrected standard errors methodology of Beck and Katz (1995). NT=number of observations after exclusions where N=5 cross-sections. Model 4 uses $D = D^{>}$ & Model 5 uses $D = D^{j}$ for currency *j* as described in equation (12).

Table 7. ℓ-UIP : Effects of Interest Rate Levels (Sub-sample 2)

	CSSUR				TSSUR			
Maturity	$\bar{i} = Mean(i_t)$		$\bar{i} = Median(i_t)$		$\bar{i} = Mean(i_t)$		$\bar{i} = Median(i_t)$	
1 Month	Model 4	Model 5	Model 4	Model 5	Model 4	Model 5	Model 4	Model 5
β_1	0.55	0.32	0.78	0.47	**-1.03**	**-0.73**	**-0.85**	-0.17
	(0.43)	(0.81)	(0.21)	(0.62)	(-3.98)	(-3.90)	(-3.52)	(-1.50)
β_2	-4.42	-5.21	-1.08	-2.98	**-7.58**	**-9.14**	-0.76	-2.10
	(-1.54)	(-1.49)	(-0.77)	(-1.35)	(-4.21)	(-3.97)	(-1.11)	(-1.21)
Wald $p\beta_1 = \beta_2$	0.172	0.192	0.523	0.259	**0.002**	**0.001**	0.954	0.473
Adj. R^2	-0.002	-0.003	-0.004	-0.003	0.025	0.024	0.013	0.034
F	0.68	0.69	0.57	0.63	4.03	3.91	2.58	3.79
D.W.	2.10	2.11	2.10	2.11	2.02	2.02	2.02	1.92
NT	1280	880	1280	1280	1280	1280	1280	880
$\sum D_t$	48		57		48		57	
3 Month								
β_1	-0.34	-0.41	-0.03	-0.39	**-0.03**	**0.05**	0.32	**0.06**
	(-1.29)	(-1.61)	(-0.94)	(-1.55)	(-1.95)	(-2.22)	(-1.25)	(-2.03)
β_2	-3.28	-2.50	-0.02	-0.61	**-6.61**	**-11.39**	-0.68	**-3.35**
	(-1.06)	(-0.73)	(-0.36)	(-0.53)	(-2.70)	(-3.34)	(-0.82)	(-1.79)
Wald $p\beta_1 = \beta_2$	0.482	0.664	0.997	0.943	0.**022**	**0.002**	0.64	0.17
Adj. R^2	-0.008	-0.008	-0.007	-0.005	0.031	0.040	0.008	0.016
F	0.71	0.70	0.74	0.79	2.22	2.62	1.32	1.64
D.W.	2.04	2.04	2.05	2.04	2.05	2.07	2.04	2.04
NT	425	425	425	425	425	425	425	425
$\sum D_t$	15		19		15		19	

Note. Boldface denotes statistical significance at 5% or less: *t*-statistics in parentheses for $H_0: \beta - 1 = 0$. Wald shows *p*-values. Sub-sample 1 covers 2000m01-2021m05 for 1-month & 1999q4-2021q1 for 3-month data. CSSUR uses cross-section as the panel unit and is robust to contemporaneous covariance and cross-section heteroskedasticity of residuals. TSSUR assumes that the errors for a cross-section are heteroskedastic and serially correlated (cross-section clustered). Both procedures utilize panel corrected standard errors methodology of Beck and Katz (1995). NT=number of observations after exclusions where N=5 cross-sections. Model 4 uses $D = D^{>}$ & Model 5 uses $D = D^{j}$ for currency *j* as described in equation (12).

The variability in sub-sample results and non-linearity by interest rate levels are broadly consistent with Engel et al. (2022) and the quasi-meta-study by Cheung and Wang (2022) that the slope parameter is not stable over time. The full sample results are clearly more supportive of $\ell -$ UIP than either sub-sample, of which the first is more supportive than the second. The reason for this may lie in the pattern of interest rates and, therefore, of the risk tolerance term $\bar{\theta}(i,i^*)$ during these periods. The variance of US interest rates in the full period and the 1st and 2nd sub-periods, respectively, were 1.1×10^{-5}, 0.8×10^{-6} and 2.4×10^{-6} for 1-month rates; the corresponding variances for 3-month rates were 1.0×10^{-4}, 7.3×10^{-5} and 2.2×10^{-5}. The very small variation in interest rates in the 2nd sub-period would likely have erased any meaningful distinction between high and low interest rates.

Therefore, for the second subsample, the separating parameter for high and low interest rates was modified in Model 4 such that $D_t^{>} = 1\ if\ i_t^{USD} \geq [\bar{i}^{USD} + 1.0\sigma(i_r^{USD})]$ and likewise for both interest rates in the dummy variable D_t^{j} for all *j*, where s denotes the second sub-sample standard deviations. Despite changing the threshold for these separating parameters, the low variation of interest rates may still have attenuated the variation of $\bar{\theta}(i,i^*)$ and diffused the distinction between 'low' and 'high' interest rate periods within the sub-periods, especially the second sub-period, thus leading to the inability of Wald tests to reject the null of $\beta_1 = \beta_2$ more frequently than in the full sample period.

4.2.3 Implied Relative Liquidation Premia

Finally, though it is not possible to recover the implied values of the risk tolerance term $\bar{q}$ assuming the model is correctly specified the point estimates $(\hat{\beta}_1 - 1)$ and $(\beta_2 - 1)$ yield the implied $\bar{\theta}$-ratio: the value of $\bar{\theta}$ in the high interest period relative to the low interest period. Conditional on the forward premium *f*, based on (5):

$$\text{Implied } \bar{\theta}\text{-ratio} \equiv \frac{\hat{\beta}_2 - 1}{\hat{\beta}_1 - 1} = \frac{\bar{\theta}\,|_{D_t=1}}{\bar{\theta}\,|_{D_t=0}} \quad (14)$$

The implied $\bar{\theta}$-ratio measures the ratio of the liquidation premium during high interest rate period *relative* to the low interest rate period within a given sample.

Table 8. Implied $\bar{\theta}$-ratios

		1-Month		3-Month	
Sample	Model	$\bar{i} = Mean(i_t)$	$\bar{i} = Median(i_t)$	$\bar{i} = Mean(i_t)$	$\bar{i} = Median(i_t)$
Full Sample	Model 4	--	--	--	--
	Model 5	1.99	2.60	2.69	3.86
Sub-sample 1	Model 4	2.73	2.80	--	2.35
	Model 5	2.65	2.53	1.94	1.64
Sub-sample 1	Model 4	4.23	--	7.39	--
	Model 5	5.86	--	13.04	4.63

Note. Implied $\bar{\theta}$-ratio $= (\hat{\beta}_2 - 1) \div (\beta_1 - 1)$ obtained from TSSUR estimates in Tables 5-7, not reported if $(\hat{\beta}_1 - 1)$ is not statistically different from zero. $\bar{\theta}$-ratio $= 1$ when $H_0 : \beta_1 = \beta_2$ cannot be rejected at conventional levels. Model 4 uses $D = D^{>}$; Model 5 uses $D = D^{j}$ for currency *j*. Full sample dates are 1978m08-2021m05 for 1-month & 1978q3-2021q1 for 3-month data. Corresponding Sub-sample 1 data are for 1978m08-1999m12 & 1978q3-1999q3; Sub-sample 2 data are for 2001m01-2021m05 & 1999q4-2021q1.

Table 8 presents the implied $\bar{\theta}$-ratios for Models 4 & 5 for different maturities and sample periods based on TSSUR estimates in Tables 5-7. Considering only the statistically significant pairs of estimated coefficients for which the estimated $\beta_1 - 1 \neq 0$ for the full sample the range of implied $\bar{\theta}$-ratios is 1.99-3.86 with an average implied $\bar{\theta}$-ratio $= 2.79$ across cells. This finding suggests that during high interest rate periods investors sought liquidation premia that were 99%-286% larger compared to low interest rate periods: investors anticipated confronting larger losses during periods of high interest rates relative to periods of low interest rates and therefore required commensurately relatively larger liquidation premia in high interest rate periods as compensation. In the first sub-period the $\bar{\theta}$-ratios range between 1.64-2.80 with the average across models and maturities of 2.38. In the second sub-period the range is 4.23-13.04 with an average across cells of 7.03, substantially higher than the first sub-period. It should be noted, though, that these results do not enlighten the absolute size of liquidation premia, only premia during high interest rate periods relative to low interest rate periods *within* each sample period.

5. Concluding Remarks

We add to the existing literature by inquiring into the empirical validity of liquidity constraints as a potential resolution to the forward premium puzzle. By examining data on five major currencies, we find broad traction for the non-linearity propositions of the liquidity risk-augmented UIP framework that considers the effect on UIP of sudden declines in asset prices when investors are forced to liquidate their bond holdings in the face of adverse liquidity events when the supply of liquidity is not infinitely elastic. We find the evidence to be broadly consistent with the propositions that the failure of UIP is more pronounced when prospective liquidity shocks are more severe, and when interest rate levels are high.

When markets are liquidity-constrained then sudden asset liquidations cause asset price reductions, subjecting the investor to potential losses. Consequently, the incentives for international interest arbitrage transactions are affected by the potential liquidity shock. Moreover, investors who nominally hedge their foreign currency exposure by taking positions in forward markets are not free from liquidation risk due to potential black swan events that prevent delivery of foreign currency on the due date. They will thus seek compensation for the liquidation risk, a liquidation premium. In this liquidity-risk-augmented formulation of UIP, investors' required liquidation premium can place a wedge between the expected currency depreciation and the interest rate differential predicted by canonical UIP theory.

The empirical support for non-linearities in UIP in accordance with tighter or looser credit conditions, and high and low interest rates, also offers an explanation of observed instabilities in the slope parameter of the canonical UIP equation across time and currencies. While this paper adds insights into the forward premium puzzle by validating the potential role of liquidity factors in arbitrage decisions, other areas of inquiry suggest themselves. Chinn (2006) has found that the forward premium bias is less noticeable in longer relative to shorter horizons though Lee (2011) suggests the opposite. It bears consideration whether the non-linearities in uncovered interest parity are more or less pronounced at longer horizons, and if so, why. We also know that the performance of uncovered interest parity is very different for emerging markets and developed economies (Bansal & Dahlquist, 2000). Examining the extent to which non-linearities in the uncovered interest parity relationship are affected by capital controls, or market segmentation, or other financial-structural differences can lend meaningful insights. These are promising lines of future research.

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Notes

Note 1. The thrust of recent papers by Cerutti et al. (2021), Engel and Wu (2022) and Jiang et al. (2013) point to deviations from covered interest parity in the post global financial crisis period due to factors such as financial regulation, exchange market liquidity, unorthodox monetary policy, and risk tolerance.

Note 2. Details on data sources are in Appendix A and summary statistics are in Appendix B.

Note 3. The size of *f* in absolute terms, its sign, and regime transitions predicated on monetary growth rate and income differentials are some of the contributing factors to non-linearities in UIP in these studies.

Note 4. The National Financial Conditions Index includes risk and leverage indexes also; the results broadly are qualitatively like CI, but the latter is statistically most compelling.

Note 5. The limitation of the binary specification of interest rate regimes is offset in an important way because by allowing pooled estimation, information in contemporaneous covariances, crucial in financial markets can be utilized, unlike single-equation estimations for modelling regime change such as Logistic Smooth Transition Dynamic regressions as in Baillie and Kiliç (2006).

Appendix A.

Data Sources

Monthly spot rates are from IMF International Financial Statistics. Legacy DEM/EUR exchange rate starting 1999:01 obtained from European Central Bank: http://www.ecb.int/press/pr/date/1998/html/pr981231_2.en.html. Interest rates are monthly Eurocurrency rates from Datastream. The original interest rates were annualized per cent (i_A). They were converted into monthly rates (i_M) using $i_M = (1+0.01i_A)^{1/12} - 1$, and into quarterly rates (i_Q) using $i_Q = (1+0.01i_A)^{1/4} - 1$. The Credit Index (CI) is from the Federal Reserve Bank of Chicago's National Financial Conditions Index database. CI has a mean of zero and standard deviation of 1. CI > 0 imply tighter than normal credit conditions, and CI < 0 imply looser than normal credit conditions.

Appendix B. Summary Statistics

1978m08 – 2021m05		CAD	CHF	DEM	GBP	JPY	USD
$\Delta s_{+1} - f$	Mean	0.000337	-0.00083	-0.00053	0.000411	-0.00102	
	Median	0.000441	-0.00246	-0.00093	0.000216	-0.00302	
	Max	0.088801	0.128477	0.105328	0.135142	0.145866	
	Min	-0.12338	-0.16122	-0.11985	-0.12234	-0.12096	
	St. Dev.	0.020557	0.033615	0.031173	0.029382	0.031293	
	T	513	513	513	513	513	
							CI
$f = i^{USD} - i^{j}$	Mean	-0.00044	0.002003	0.000944	-0.00103	0.002102	0.223968
	Median	-0.00036	0.001788	0.001107	-0.00055	0.001765	-0.07792
	Max	0.00318	0.011425	0.008168	0.005666	0.009522	3.407684
	Min	-0.004	-0.0043	-0.00514	-0.00569	-0.0028	-0.5898
	St. Dev.	0.001125	0.002427	0.002042	0.001734	0.002054	0.817168
	T	514	514	514	514	514	514
i	Mean	0.004409	0.001959	0.003021	0.005007	0.00186	0.003971
	Median	0.003593	0.001415	0.00282	0.004605	0.000359	0.003993
	Max	0.016926	0.00864	0.01084	0.014744	0.010794	0.015661
	Min	7.50E-05	-0.00117	-0.00046	7.50E-05	-0.00054	0.000104
	St. Dev.	0.003565	0.002277	0.002542	0.003823	0.002473	0.003341
	T	514	514	514	514	514	514
1978q3 – 2021q1							
$\Delta s_{+1} - f$	Mean	0.000988	-0.00304	-0.00195	0.00092	-0.00321	
	Median	3.76E-05	-0.00698	-0.0032	0.001629	-0.00591	
	Max	0.089875	0.170281	0.142334	0.14495	0.16216	
	Min	-0.13377	-0.16908	-0.13595	-0.20382	-0.16135	
	St. Dev.	0.033369	0.05988	0.056611	0.051785	0.060635	
	T	170	170	170	170	170	
							CI
$f = i^{USD} - i^{j}$	Mean	-0.00133	0.005921	0.002828	-0.00302	0.006325	0.226266
	Median	-0.00099	0.005489	0.003276	-0.00157	0.00546	-0.071689
	Max	0.009043	0.027457	0.019283	0.012283	0.024633	3.201001
	Min	-0.01185	-0.01261	-0.01556	-0.01656	-0.00549	-0.565251
	St. Dev.	0.003408	0.006925	0.005986	0.005164	0.005801	0.814081
	T	171	171	171	171	171	171
i	Mean	0.01361	0.006252	0.009375	0.015326	0.00585	0.012253
	Median	0.010914	0.004809	0.008637	0.014195	0.001248	0.012224
	Max	0.050026	0.025275	0.032874	0.04527	0.029454	0.043624
	Min	0.000137	-0.00226	-0.00125	0.000225	-0.00101	0.00045
	St. Dev.	0.010817	0.006953	0.007769	0.011466	0.007579	0.010004
	T	171	171	171	171	171	171