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Smart Money in the NCAA Men's Basketball Tournament

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Abstract

Although bracket pools in the NCAA Men's Basketball Tournament are a game, in practice most players do not compete strategically. Instead, they are more likely to choose brackets as though they are playing a lottery. When faced with such unsophisticated opponents, the game simplifies to a finance problem where you must choose an optimal portfolio of brackets. The brackets that pay the highest return are modal brackets in which higher seeds are always picked to beat lower seeds until the Final Four. When playing multiple brackets, the optimal strategy is to diversify across possible winners in the first round of the Final Four. We have found both theoretically and empirically that enormous returns can be earned with this approach.

Keywords: portfolio choice, lotteries, metric games, diversification, overconfidence

1. Introduction

Sports betting has become a major industry worldwide. In the United States alone, it is believed that hundreds of billions of dollars are gambled each year. While most of these games of chance simply involve picking a winner, some of the most popular actually require players to allocate a portfolio. For example, in fantasy football, playing Patrick Mahomes, Travis Kelce, and Clyde Edwards-Helaire together on your fantasy team can generate extraordinarily high scores when the Saints have a good game, but can be a bust if Brees has a bad day so his receivers make few catches and also have bad days. Playing Mahomes, Stefan Diggs, and Derrick Henry, who are all on separate teams in real life, is a much safer hedging strategy-except during a week where their respective teams play each other, in which case their performances will be interdependent.

Billions of dollars are also bet on the NCAA Men's Basketball Tournament with Warren Buffett offering a billion dollar prize to anyone who can pick a perfect bracket. The ease with which one can enter a bracket into a pool also confers aspects of portfolio choice to this game as well. If you enter two or more brackets, should you choose nearly identical brackets? Completely different brackets?

In addressing these questions, we see a major difference between bracket pools and fantasy football. Choosing a fantasy football team is like choosing a traditional financial portfolio in that you must make a tradeoff between maximizing your expected score and minimizing its variance. In contrast, bracket pools belong to another class of games, which we will call metric games, where the winner is whichever player comes closest to choosing or hitting a target in some metric space. The canonical example of a such a metric game is horse- shoes, in which each player tries to throw a horseshoe closest to a fixed pole. Bracket pools work the opposite way, as if everyone lays their horseshoe on the ground before the pole gets thrown onto the field. First, everyone chooses their bracket, and then the true bracket generated by the NCAA Tournament is slowly revealed as games are played.

Optimal strategies for picking brackets will depend on the sophistication of your opponents. If most players recognize that you are playing a metric game, then game theory will be needed to determine your optimal strategy. Today, however, most players treat bracket pools like a lottery. It is, in fact, akin to a lottery, but a very special kind of lottery called a roll-down lottery. Suppose the lottery has six numbers. In most lotteries, if no one buys a ticket with all six numbers, then the current pot is added to the pot of the next lottery, and so on until there is a draw where somebody picks all six winning numbers. In contrast, for a roll-down lottery, if no one matches six numbers, then the winnings are spread among the closest matches with five or, if necessary, four numbers. Recently, it has come to light that a retired couple in Michigan, Jerry and Marge Selbee, won 26 million dollars by recognizing that a roll-down lottery is a metric game and choosing combinations of tickets accordingly (Note 1). That same approach can be generalized to choosing an NCAA tournament bracket, albeit with the constraint that it would be difficult to scale up the winning strategy to win millions of dollars. Whereas the Selbees could buy ten thousand tickets for one lottery, most pools set a limit on how many brackets you can enter, and it is hard to imagine that anyone could get away with joining tens of thousands of simultaneous office pools.

Nevertheless, the Selbees had the right idea that the optimal approach to a metric game where you have to guess at a stochastic target is to maximize your win probability. In a typical state-run lottery, each combination of numbers is supposed to be equally likely. In contrast, some of the possible brackets in the NCAA tournament are much, much more likely to be realized than others. The 64 teams (68 if you include the preliminary rounds) in the tournament are not equally skilled. While the NCAA selection committee does not have a perfect record of ranking teams, their seedings have correlated fairly well with the quality of teams. Based on historical results, a #1 seed does have a better than even chance of beating any lower seeded team, and this greatly simplifies the task of picking an optimal bracket as compared to what the Selbees had to do.

One rarely sees people choose brackets where they always pick the higher seed to win, perhaps because this seems like too simple-minded a strategy (Note 2). There are eight such modal brackets, distinguished only by what you predict will happen to the #1 seeds when they get to the Final Four. We find under fairly typical circumstances that they earn a higher mean return than other brackets. If other players view the game as a lottery, an optimal portfolio will consist of a prudently chosen combination of modal brackets.

To authenticate this finance-based approach to picking brackets, two data generating processes need to be estimated: the process that determines who wins the actual basketball games and the process that determines how opposing players pick their brackets. In order to quantify the conditions under which it is optimal to play modal brackets, we estimate parsimonious models of both processes. We employ a logistic regression to estimate the probability that the top-seeded team in a game will win as a function of the two seeds. To create opposing brackets, we assume noise players pick the higher-seeded team to win with a constant probability for all the games in the bracket (Note 3). We can then characterize pools based on the number of brackets in the pool, the scoring scheme for different rounds of the tournament, the prize structure, and the sophistication of the opposing players as measured by this probability of picking higher seeds. Under all of the standard forms of the tournament, enormous returns of 100% or more can be earned by playing a modal bracket against players of typical sophistication as evidenced by brackets posted on the internet.

To validate our recommendation that you always pick a modal bracket, we also do a bootstrapping exercise that does not depend on any assumptions about the data generating processes that determine the winners of the actual games and the opposing brackets. Using actual brackets entered into a large professionally run pool over several years, we can compute the ex post return for each modal bracket over the several years when matched against brackets randomly chosen from the professional pool. This confirms that modal brackets would often have earned large returns in actual play.

If a pool does not put a limit on how many brackets you can enter, a diversified portfolio would include all eight modal brackets. However, the larger the pool, the more the pool will behave like a lottery. Although the win probability for a modal bracket will always be higher than the win probability for other brackets, our estimated win probability for a modal bracket is only 50% higher than the estimated win probability for a close to modal bracket in which the top seed always wins except for one #2 seed that upsets three #1 seeds on the way to win the tournament (Note 4). In a pool of a thousand or more brackets, if your bracket has twice the probability of winning the tournament as most of your competitors (Note 5), playing a modal bracket will yield a measurable improvement in your expected return, yet this may be economically insignificant. The gains from playing a modal bracket would be much larger in a small pool of only ten to twenty brackets, though the size of the pot will be in proportion to the size of the pool.

Some gains can be obtained by diversifying intratemporally across pools. If you are limited to playing in two pools and entering one bracket into each, you will maximize your Sharpe ratio, i.e. the reward to risk ratio (Note 6), by playing a com- bination in which different teams win each of the two first-round games. So for example, consider 2008, the only occasion when all four #1 seeds made it to the Final Four. North Carolina played Kansas, and Memphis played UCLA in the penultimate round. Suppose you chose one bracket where Kansas beat North Carolina and Memphis beat UCLA with Kansas winning the whole thing, as actually happened. If you entered another bracket in another pool, that should optimally have had North Carolina beat Kansas and UCLA beat Memphis. The expected return for the portfolio would have been the same whether you chose North Carolina or UCLA to win the championship in this second bracket.

To highlight the advantages of this approach, we should note that the authors have each individually applied this strategy in bracket pools run in every tournament since we started writing this paper. While theoretically there ought to be years when we do not win a prize, that has not happened yet (Note 7). Even in 2019 when only a single #1 seed made it to the Final Four, the four modal brackets that picked Virginia, the ultimate winner, to play in the championship all did well even if they did not pick Virginia to win. Because a #3 seed and a #5 seed both did make it into the Final Four, a result that hardly anybody predicted, most everyone's brackets busted, not just brackets that picked Duke, Gonzaga, or North Carolina to win.

As far as we are aware, this is the first paper to recognize the metric-game aspect of bracket pools. Kvam and Sokol (2006) focused on estimating the actual probability distribution and beating the projections of well known sports forecasters like Sagarin. Using a more high-tech computational approach than us, Clair and Letscher (2007) focus on how to compute the optimal bracket given data on the actual probability distribution and the probability distribution for opponents' brackets without providing much economic insight that can be used by laypeople in practice.

The paper is organized as follows. In Section 2 we describe how bracket pools typically work. In Section 3 we calibrate both the data generating process that determines the outcome of games in the tournament and the parameters of the noise traders. In Section 4 we discuss the advantages of choosing one of the eight modal brackets and the circumstances under which this is optimal. In Section 5 we report our results from diversifying brackets across two separate but contemporaneous pools. We conclude in Section 6.

2. The Mechanics of Bracket Pools

Under the current format of the NCAA Men's Basketball tournament, each year 68 teams are chosen to participate in a single-elimination tournament. Unlike the "national championship" for football, which is the result of a competition between four subjectively chosen teams, usually from the so-called Power Five conferences, the basketball tournament is a meritocracy in that any Division I team can win its way into the tournament by winning its conference championship. The selection committee then subjectively chooses the remaining 36 at-large teams.

Eight of the 68 teams play in an early qualification round. This is a fairly recent innovation that has not been incorporated into most bracket pools. Instead, the four pairs of teams in these play-in games are treated indifferently, so effectively there are 64 teams. Not counting the play-in games, each team will potentially play two games per week over the course of three weeks with a maximum of six possible games, though, of course, only half the teams survive at each stage.

The 64 teams are divided into four subbrackets labeled East (E), Midwest (M), South (S), and West (W) based on the location where the four remaining teams in each subbracket will play in the second week. The 16 teams in each subbracket are ranked from 1 to 16. In the first round, the top seed plays the sixteenth seed, the second seed plays the fifteenth seed, the third seed plays the fourteenth seed, and so on. For the three remaining rounds in the subbracket, matchups continue under the assumption that the higher seed will win. Thus, for example, the winner between the first and sixteenth seeds will play the winner of the game between the eighth and ninth seeds regardless of which teams win the first-round games. The four surviving teams from each of the four subbrackets are collectively known as the Sweet Sixteen. The victors in the first round of the second week are called the Elite Eight. These then compete to determine the Final Four who gather in one location in the third week to determine the ultimate champion. Which regional champion plays which varies from year to year (Note 8). To fix notation, we will assume the Eastern regional champion plays the Midwest regional champion while the Southern regional champion plays the Western regional champion. Then the winners of these two matchups play for the league championship.

Most gambling on the tournament is an assessment of a bettor's ability to predict the entire bracket. In a typical pool, players will submit one or more brackets, paying a fee for each bracket. In our modeling, we will assume the pools are not for profit and the total of all the fees makes up the pot, which will be divided up among the winners. In for-profit pools, the manager will take a commission and the remainder will be dispersed to the winners. The majority of office and family pools are not for profit, though some very large pools are managed by professional bookmakers. While large pools offer larger prizes, they also curtail the advantage of playing an optimally chosen bracket, so they are of less interest to us here.

Suppose there are m brackets in the pool, and we normalize the size of the pot to one. Then the cost of entering a bracket will be 1/m. If you earn the prize w 2: 0, the ex post return will be

$$r = \frac{w}{\frac{1}{m}} - 1 = mw - 1$$

Likewise, if E[w] is the expected prize, the expected return will be

$$E[r] = mE[w] - 1 \tag{2}$$

Prizes are determined by how well your bracket compares to the actual bracket generated by the wins and losses of the 64 teams. Brackets score points based on how many winners are chosen correctly. The scoring scheme will depend on the pool, though usually more points are awarded for correct predictions in later rounds. Two common schemes are a linear scheme and a power-of-two scheme. Under the linear scheme, a predicted win in the first round scores 1 point, a win in the second round scores 2 points, up to a predicted tournament champion that scores 6 points. For the power-of-two scheme, a predicted win in the first and second rounds again score 1 and 2 points respectively, but a win in the third round scores 4 points, in the fourth round scores 8 points, up to a predicted champion that scores 32 points. The power-of-two scheme puts more weight on predicting the correct champion while the linear scheme puts more relative weight on predicting what happens in the early rounds. Let us denote the scoring weights by $s_1, ..., s_6$. Then a linear scoring scheme will have $s_i = i$ and a power-of-two scoring scheme will have $s_i = 2^{i-1}$.

Note that you cannot predict a team will lose in an early round and then win in a later round. Viewed in terms of the team, when you pick a bracket you are predicting which round you think each team will be eliminated at. You must predict that thirty-two teams will be eliminated in the first round, sixteen teams in the second round, eight in the third round, four in the fourth round, two in the fifth found, and finally one in the last round, leaving one champion that is never eliminated. If you predict a team will survive to a late round and it gets eliminated early on, any future points that could have been earned by that team will be gone. Your bracket is said to "bust" if one or more teams that you predict to go to the Final Four or to win the tournament lose beforehand. In a winner-takes-all tournament, the whole pot will go to the owner of the bracket with the highest score. However, most pools will offer small prizes to the second- or third-place brackets while giving the majority of the pot to the first-place bracket (Note 9). If there are p prizes, we denote the prize vector by (1, ...,), where i 2: 0 is the prize for the ith place winner, and

$$\sum_{i=1}^{p} \pi_i = 1 \tag{3}$$

The most common prize vectors that we have observed are π = (0.75, 0.25) and = (0.6, 0.3, 0.1), which we call 3-1 and 6-3-1 payoffs respectively since the prizes are in proportion to those numbers. Ties in scoring are possible. Various schemes are used to break ties. We assume that if there is a tie for the $l_1 < l_2 < \ldots < l_w$ places, then a ranking is imposed on those players by randomly choosing a permutation of these players.

3. Calibration

To calibrate the data generating process that determines the outcome of a bracket pool, we start with the tabulated data on the probability of the top seed beating the bottom seed as a function of the seeds in tournament games played between 1985 and 2018. This matrix is presented in Table 1. Note that there are several examples of matchups that did not occur during this period. For example, a #1 seed has never played a #14 or #15 seed.

Table 1. Frequency at which the top seed beats the lower seed when matched up for games in tournaments between 1985 and 2018

t/b	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	35/64	19/32	44/61	38/45	8/11	6/7	55/70	62/68	5/6	4/7	19/19	4/4	0	0	135/136
2		31/51	3/7	0/4	24/31	56/81	2/8	0/1	29/47	14/16	1/1	0	0	127/136	0
3			4/7	2/3	40/69	9/15	1/2	2/2	9/13	31/48	0	0	114/136	1/1	0
4				43/73	2/4	3/5	4/9	2/3	2/2	0	24/36	107/36	0	0	0
5					1/1	0	1/2	1/3	1/1	0	90/136	14/17	0	0	0
6						5/7	0/1	0	4/6	87/136	0	0	14/16	0	0
7							1/2	0	85/136	0/4	0	0	1/1	2/3	0
8								68/136	0	1/1	0/1	1/1	0	0	0
9									0	0/1	0	1/1	0	0	1/1
10										1/3	0	0	1/1	5/5	0
11											0	0	5/5	0	0
12												8/11	0	0	0
13													0	0	0
14														0	0
15															0

Note. That there has never been a matchup between several seeds.

We estimate a logistic probability model that depends on the two seeds, t and b, so

$$p = \frac{1}{1+exp(-L(t,b))}$$

where L(t, b) is a cubic polynomial of the top seed t and bottom seed b, where t < b. Coefficients are listed in Table 2.

Table 2. Coefficients of logit model

Variable	Coefficient	Standard Error	T-stat	p
constant	-0.86960	0.60098	-1.4469	0.1479
t	-0.21661	0.33525	-0.6461	0.5182
b	0.89339	0.29631	3.0150	0.0026
t2	0.16950	0.06562	2.5828	0.0098
tb	-0.19706	0.09105	-2.1642	0.0304
b2	-0.08202	0.04202	-1.9520	0.0509
t3	-0.016316	0.00445	-3.6690	0.0002
t2b	0.011839	0.00584	2.0280	0.0426
tb2	-0.00568	0.00499	1.1379	0.2551
b3	0.00334	0.00167	1.9978	0.0457

The probabilities for the outcome of games predicted by this model are reported in Table 3. The top seed is always favored according to this probability model, although there are a few anomalous matchups in Table 1 for which the bottom seed has been more likely to win than the top seed. For example, #11 seeds have won all four of their matchups against #7 seeds, and likewise #5 seeds have won their four matchups against #2 seeds.

Table 3. Probability at which the top seed beats the lower seed when matched up using the cubic logistic regression estimates

t/b	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	55%	64%	71%	75%	78%	81%	83%	84%	86%	89%	91%	94%	96%	98%	99%
2		58%	63%	65%	67%	68%	69%	71%	73%	76%	80%	85%	90%	94%	97%
3			59%	60%	60%	60%	60%	60%	62%	66%	71%	78%	85%	91%	96%
4				58%	57%	56%	55%	55%	57%	61%	67%	75%	83%	91%	96%
5					56%	55%	54%	54%	57%	61%	68%	76%	85%	93%	97%
6						54%	54%	55%	58%	64%	72%	81%	89%	95%	98%
7							52%	54%	59%	67%	76%	85%	93%	97%	99.1%
8								51%	58%	68%	78%	88%	95%	98%	99.6%
9									51%	64%	78%	90%	96%	99.2%	99.8%
10										54%	74%	89%	96%	99.2%	99.9%
11											60%	84%	96%	99.2%	99.9%
12												68%	92%	99%	99.9%
13													79%	97%	99.8%
14														89%	99.2%
15															96%

Goodness of fit measures for logistic regressions are difficult to interpret, but Figure 2 shows the sample frequency of games for which the difference between the actual win probability and the predicted win probability is above a given error. For example, we see from the x coordinate of where the graph crosses the y = 0.1 line that only ten percent of games in the sample have a difference in probabilities bigger than 9 percentage points. Similarly, only 1 percent of games have a difference bigger than 43 percentage points.

As a further check on our model probability distribution, Table 4 compares the frequency at which a team of a given seed wins the tournament to the probabilities generated by our distribution. Our probability model overpredicts the probability that a #2 seed will win the tournament. Whereas #1 seeds

have won more than lost in matchups against all other seeds, #2 seeds have fared quite poorly when matched against #4 seeds, #5 seeds, #8 seeds, and #9 seeds, and our model fails to capture this phenomenon. However, since our recommended strategy is to always choose a #1 seed over a lower seed, underpredicting the chance that a #1 seed will win the tournament actually biases the model against our findings.

To calibrate the distribution of opposing brackets, we assume that opposing players pick the top seed to win an individual game with a constant probability pn [0, 1] (Note 10). Based on a sampling of past brackets that people have posted on the internet, we found that typical players pick the top seed to win in 70% of the games in a tournament, although we will also consider what happens as pn varies. In particular, we will be interested in how the optimal strategy varies opposing players become more sophisticated and pn approaches 1.

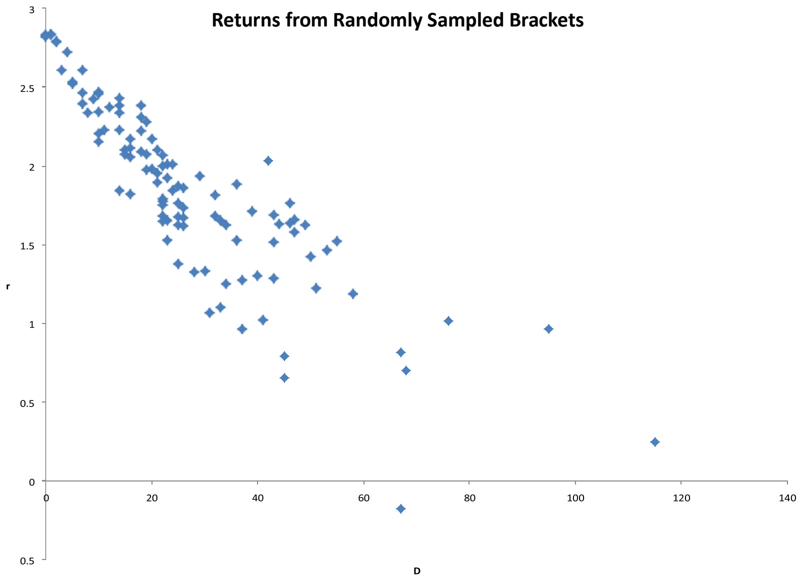

Figure 1. Scatter plot of the mean return and the D score of 110 randomly chosen brackets in pools with 20 brackets, power of two scoring, 3-1 payoffs, and opposing players who choose the top seed with probability pn = 0.7

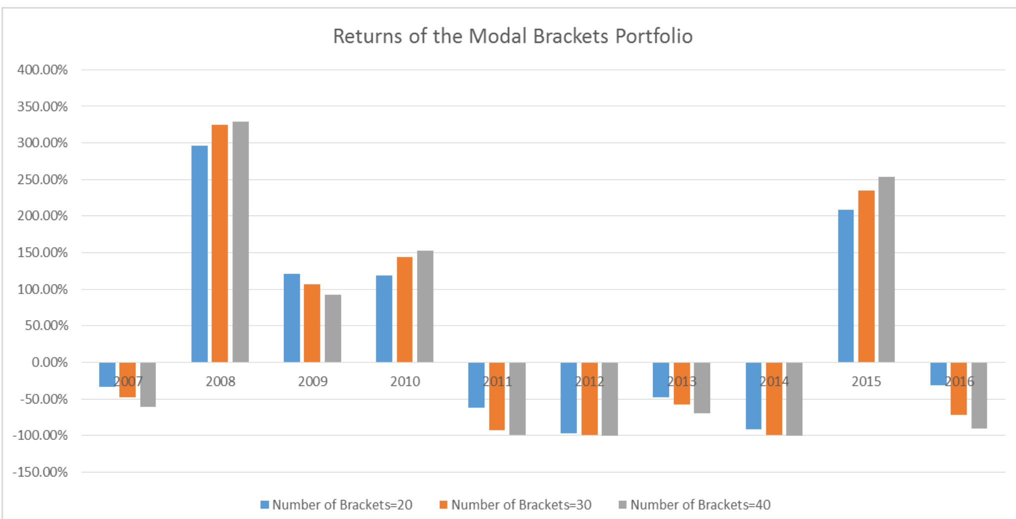

Figure 2. Return computed by bootstrapping from playing a portfolio of all eight modal brackets in separate pools of size 20, 30, and 40 from 2007 to 2016

4. Modal Brackets

According to the estimated probabilities of a higher seed winning in Table 3, the higher seed always has a higher probability of winning. Thus a bracket with the highest probability of being the actual result of the tournament and winning Warren Buffet's prize will be obtained by always picking the higher seed. Since the Final Four consists only of the #1 seeds, and there are 8 possible outcomes in a two-stage tournament of these four teams, there are eight such brackets. We call them modal brackets since they correspond to the modes of the probability distribution for the actual tournament bracket. A modal bracket can be represented by the triple (i, j, k). Note, however, that since we have ignored the ranking of the four regions in a tournament, our model predicts that the eight modal brackets should earn the same expected return.

4.1 Simulating Bracket Pools with Probability Models

First, let us consider the performance of modal brackets in simulations using our estimated probability models as calibrated in Section 2. Table 5 reports the win fraction, or probability of winning a prize (Note 11), and the expected return from playing a single modal bracket as we vary the characteristics of the pool along several dimensions: the prize vector , the scoring scheme s, the size of the pool m, and the sophistication of the other players in the pool as measured by the probability p_n (Note 12).

Several trends become apparent as we examine Table 5. Intuitively, one would expect that the win fraction and return would decrease with the sophistication of the other agents. This is borne out by their dependence on pn except when pn approaches 1. In that limit when most of the players either pick modal or close to modal brackets, the game-theoretic aspects of a bracket pool become more pronounced. In particular, if everyone else is playing a modal bracket, it will no longer be optimal to play a modal bracket because the bracket pool will then in fact revert to a lottery so the expected return will be zero (Note 13). It is instead better to pick a bracket that commands a region of the bracket space with a probability larger than 1/m where your bracket is the closest bracket in play. Calculating such brackets is beyond the scope of this paper. How- ever, picking the top seed to win each game with a high probability strictly less than one mimics that calculation more closely than picking modal brackets. Thus the return and win fraction have a hump-shaped dependence on pn with a maximum at some value of pn slightly less than 1.

Table 4. Actual frequencies (between 1985 and 2018) and predicted probabilities for a team of each seed to win the tournament

Seed	Actual Championship Frequency	Predicted Championship Frequency
1	0.647	0.56656
2	0.118	0.24298
3	0.118	0.08588
4	0.029	0.03862
5	0.000	0.02046
6	0.029	0.01676
7	0.029	0.01083
8	0.029	0.00706
9	0.000	0.00460
10	0.000	0.00361
11	0.000	0.00217
12	0.000	0.00045
13	0.000	0.00002
14	0.000	0.00000
15	0.000	0.00000
16	0.000	0.00000

Note. Probabilities were computed based on 100000 simulations.

Not surprisingly, the win fraction decreases with the size of the pool since you are less likely to have one of the best brackets if you are competing against more brackets. However, the dependence of the expected return on the size of the pool is more complicated. As we have already discussed, when $p_n = 1$, the expected return is zero independent of the size of the pool. However, for pn that are low relative to 1 but still greater than equal to 0.5, the expected return actually increases with the size of the pool despite the falling win fraction. This is because the win fraction falls more slowly than 1/m, the cost of entering the pool. For example, in Panel A, as we increase m tenfold from 10 to 100 for $p_n = 0.5$, the win probability only decreases by 44%. Thus the expected prize only falls by half yet the cost of entering the pool is ten times less, so the return increases over fivefold from four hundred to twenty-one hundred percent. In contrast, for pn close to 1 the game-theoretic aspects of the bracket pool become more pronounced as the size of the pool increases, and the expected return of playing a modal bracket will go negative for large m.

In comparing Panels A and C to Panels B and D of Table 5, we can see how the win fraction and mean return change as we vary the prizes. As one would expect, increasing the number of prizes increases the chance of winning so the win fractions are higher with 6-3-1 payoffs. Mean returns are not as sensitive to the prize structure. Mirroring the dependence of the return on the size of the pool, the returns are slightly higher with 3-1 payoffs for low pn, but this trend is reversed for pn close to 1.

Similarly, comparing Panels A and B to Panels C and D, we can see how the scoring scheme affects results. This has a much bigger impact than the prize vector, especially for $p_n = 0.5$. For 6-3-1 payoffs and m = 100 (Panel A), the win fraction with power of two scoring is 56% and the mean return is twenty-one hundred percent. Switching to linear scoring (Panel C) increases the win fraction to 90% and the mean return to forty-six hundred percent! However, for $p_n = 0.9$, the disparity in results between the scoring schemes diminishes considerably and may no longer be statistically significant. Linear scoring presumably favors the modal brackets because it punishes brackets that do poorly in the early rounds, and a modal bracket is more likely on average to get these early games right. In contrast, the performance of a bracket with power-of-two scoring depends almost entirely on how prescient it is regarding the final rounds. The early rounds usually only matter for breaking ties between players who make the same predictions regarding the champions.

Table 5A. Power-of-two scoring with 6-3-1 Payoff

Noise Probability	0.5	0.6	0.7	0.8	0.9
Number of Brackets=10					
Win Fraction	0.9906	0.9529	0.8333	0.6282	0.4212
Return	423.40%	336.39%	218.26%	109.91%	31.94%
Number of Brackets=30					
Win Fraction	0.9011	0.6530	0.3716	0.2079	0.1196
Return	1114.11%	637.39%	297.69%	115.23%	8.70%
Number of Brackets=100					
Win Fraction	0.5580	0.2624	0.1379	0.0692	0.0241
Return	2108.99%	939.03%	420.83%	125.82%	-33.80%

Note. All returns are statistically different from zero.

Table 5B. Power-of-two scoring with 3-1 Payoff

Noise Probability	0.5	0.6	0.7	0.8	0.9
Number of Brackets=10					
Win Fraction	0.9539	0.8430	0.6396	0.4229	0.2643
Return	529.32%	395.03%	238.13%	111.33%	27.34%
Number of Brackets=30					
Win Fraction	0.7776	0.4818	0.2587	0.1428	0.0762
Return	1276.21%	679.63%	312.75%	117.04%	5.11%
Number of Brackets=100					
Win Fraction	0.4246	0.1994	0.1036	0.0446	0.0141
Return	2312.60%	1039.95%	455.75%	117.73%	-40.65%

Note. All returns are statistically different from zero.

Table 5C. Linear scoring with 6-3-1 Payoff

Noise Probability	0.5	0.6	0.7	0.8	0.9
Number of Brackets=10					
Win Fraction	0.9970	0.9873	0.9533	0.8379	0.5741
Return	478.35%	438.35%	359.25%	230.70%	81.23%
Number of Brackets=30					
Win Fraction	0.9762	0.9088	0.7277	0.4170	0.1591
Return	1525.78%	1247.75%	801.95%	324.56%	39.30%
Number of Brackets=100					
Win Fraction	0.9015	0.6960	0.3814	0.1272	0.0281
Return	4603.34%	2997.26%	1320.68%	302.26%	-25.40%

Note. All returns are statistically different from zero.

Table 5D. Linear scoring with 3-1 Payoff

Noise Probability	0.5	0.6	0.7	0.8	0.9
Number of Brackets=10					
Win Fraction	0.9885	0.9548	0.8650	0.6638	0.3734
Return	615.66%	551.14%	433.29%	256.49%	77.30%
Number of Brackets=30					
Win Fraction	0.9480	0.8317	0.5844	0.2829	0.0938
Return	1887.22%	1488.32%	886.43%	322.01%	27.82%
Number of Brackets=100					
Win Fraction	0.8420	0.5927	0.2794	0.0814	0.0142
Return	5507.92%	3445.62%	1379.73%	291.85%	-40.35%

Note. All returns are statistically different from zero.

While we have established-in simulations-that exorbitant returns can be earned playing a modal bracket, albeit with unsophisticated opponents, we have not yet shown that modal brackets actually earn the highest mean return. In other settings, we could demonstrate this by examining the properties of the gradient and Hessian of the mean return, but that does not work here because the bracket space is discrete and not continuous (Note 14). We can, however, compute the return at a random sample of points in the bracket space and show that brackets generally have a lower return the more they deviate from modality.

One way to measure this deviation is what we call the D score of the bracket. This is calculated as if we were calculating the score of a bracket except points are earned when a bracket picks an outcome that differs from a modal bracket. So, for example, in the introduction we discussed a bracket that is modal except one #2 seed beats all comers. This #2 seed would win three additional games, in the fourth, fifth, and sixth rounds. Thus the D score for this bracket will be s4 + s5 + s6. The D score of a modal bracket will, by construction, be zero.

Fig. 1 shows a scatter plot of the mean return and D score for 110 randomly chosen brackets in a pool of 20 brackets with 3-1 payoffs and a power of two scoring scheme (Note 15). The opposing brackets are chosen with pn = 0.7. One hundred of the brackets that our player is considering were generated randomly by having the top seed be picked with probability 0.9. The remaining ten were chosen with the top seed being picked with probability 0.99. One hundred thousand simulations were used to compute the mean for the first 100 brackets. Since we wanted to verify that modal brackets are superior than close to modal brackets, we used one million simulations for the ten remaining brackets in order to compute the mean return more precisely. The graph clearly demonstrates a general tendency of the mean return to decrease with D. The brackets in the sample with the highest return had D = 1. These were both cases where a #9 seed was picked to beat a #8 seed. Picking #9 seeds over #8 seeds might actually be a wise deviation from modality in practice since, if we include data from 2019 in our sample, they actually have a slight advantage. Our probability model, however, estimates that a #8 seed will beat a #9 seed with a 51% probability, so theoretically modal brackets should earn a higher return. The higher return for the two brackets with D = 1 is not, however, statistically significant. Thus we conclude that modal brackets do, indeed, earn the highest return.

4.2 Bootstrapping Results

A fair criticism of the previous results is that they depend on the parametric assumptions we made in Section 2 and our calculations of the mean return are purely theoretical. In fact we can see that modal brackets would often have earned extremely high returns in actual practice if played during actual tournaments in recent years. While the sample of tournament results is very small-we used data from 34 tournaments to estimate our probability model- data on the sorts of brackets that people play are much more plentiful. Table 6 presents the results of a bootstrapping exercise in which we compute the mean return for each of the eight modal brackets in each year of the tournament between 2007 and 2016. Drawing from a sample of realized scores from actual brackets played in each year (Note 16), we match each of the modal brackets against a random resample of 20, 30, or 40 brackets, computing the mean return for each bracket over 10000 resamples. The eight returns from each modal bracket in each year are ranked from best to worst.

There are some years like 2012 in which all of the modal brackets basically have a win fraction of zero and a mean return of 100%. That year, the Final Four teams were Kentucky, a #1 seed; Ohio State and Kansas, both #2 seeds; and Louisville, a #4 seed. Though Kentucky went on to win the tournament, Kansas was projected by many to do well. Thus there were likely many brackets that picked Kentucky and Kansas to make the final, as happened, and these would have beaten any of the modal brackets. However, the downside of playing a bracket pool is limited to the entrance fee. In two of the ten years in the sample, a modal bracket could have earned a return of over one thousand percent.

The variation in performance across modal brackets can be averaged out by playing each bracket in a separate pool. The return that would be obtained by doing so is plotted in Fig. 2. In four of the ten years, this return would have been positive and usually over 100%.

Averaging across the columns in Fig. 2, the mean return that could have been earned during the sample period is only slightly positive (Note 17). We are not advocating that playing bracket pools is an opportunity to make money. But for the millions of people who have already decided to play a bracket pool and paid their fee, the outside option is to play a bad bracket and earn a return of-100%. Once you have accounted for this, the relative gain from playing a modal bracket in 2008 was a return of roughly 400%, not 300%. On average, the benefits of playing a modal return were actually enormous during the sample period.

5. Diversification of Brackets

Suppose that you are limited to playing in two pools with 30 brackets each. Both pools have participants that choose the top seed to win in each game with pn = 0.7. We fix one bracket to be (E, W, E) and then consider what happens as we vary the bracket in the other pool. Relevant statistics regarding the win fraction and return are reported in Table 7 (Note 18). The Sharpe ratio is the ratio of the mean return to the standard deviation of the return. Rational investors will generally choose a portfolio with the highest Sharpe ratio.

Qualitative results do not depend on the prize vector or scoring scheme. Note that the mean return is essentially the same regardless of the portfolio choice. However, the win fraction and the standard deviation of the return do vary with the portfolio. Naturally, the win fraction is smallest and the standard deviation is highest if you play (E, W, E) in both pools since then you are not diversifying. Playing different brackets in the two pools will increase your win fraction and lower your standard deviation. The best combination, which maximizes both the win fraction and the Sharpe ratio, is to pick the Midwest and South #1 seeds to win their respective first-round games. Whether you then choose the Midwest or South #1 seeds to win the championship has no effect on the mean return. Thus if you are playing a portfolio of two brackets, you want to diversify across the outcomes of the first round of the Final Four.

Table 6. Returns of Modal Brackets from Bootstrapping Played Brackets from 2007 to 2016

	1 (Best)	2	3	4	5	6	7	8
Year 2016								
Number of Brackets=20	37.25%	37.19%	37.17%	37.17%	-99.93%	-99.93%	-99.93%	-99.93%
Number of Brackets=30	-43.34%	-43.37%	-43.40%	-43.42%	-100.00%	-100.00%	-100.00%	-100.00%
Number of Brackets=40	-81.01%	-81.03%	-81.06%	-81.07%	-100.00%	-100.00%	-100.00%	-100.00%
Year 2015								
Number of Brackets=20	897.86%	695.18%	286.89%	-5.41%	-5.42%	-5.54%	-99.06%	-99.06%
Number of Brackets=30	1237.86%	848.73%	205.95%	-69.64%	-69.69%	-69.70%	-99.98%	-99.99%
Number of Brackets=40	1494.04%	902.66%	107.74%	-92.13%	-92.14%	-92.19%	-100.00%	-100.00%
Year 2014								
Number of Brackets=20	-91.63%	-91.63%	-91.64%	-91.67%	-91.69%	-91.69%	-91.69%	-91.70%
Number of Brackets=30	-99.39%	-99.39%	-99.39%	-99.40%	-99.40%	-99.41%	-99.41%	-99.41%
Number of Brackets=40	-99.96%	-99.96%	-99.97%	-99.97%	-99.97%	-99.97%	-99.97%	-99.97%
Year 2013								
Number of Brackets=20	313.77%	-98.62%	-98.63%	-98.63%	-99.98%	-99.98%	-99.98%	-99.98%
Number of Brackets=30	240.91%	-99.97%	-99.97%	-99.97%	-100.00%	-100.00%	-100.00%	-100.00%
Number of Brackets=40	140.14%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%
Year 2012								
Number of Brackets=20	-72.06%	-99.81%	-99.98%	-99.98%	-100.00%	-100.00%	-100.00%	-100.00%
Number of Brackets=30	-95.88%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%
Number of Brackets=40	-99.55%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%
Year 2011								
Number of Brackets=20	-61.47%	-61.48%	-61.49%	-61.51%	-61.52%	-61.53%	-61.53%	-61.53%
Number of Brackets=30	-92.98%	-92.98%	-92.99%	-92.99%	-93.00%	-93.00%	-93.00%	-93.03%
Number of Brackets=40	-98.99%	-98.99%	-98.99%	-98.99%	-98.99%	-99.00%	-99.01%	-99.01%

Year 2010								
Number of Brackets=20	696.97%	696.97%	-22.46%	-22.46%	-99.76%	-99.76%	-99.76%	-99.77%
Number of Brackets=30	852.12%	852.12%	-78.18%	-78.18%	-100.00%	-100.00%	-100.00%	-100.00%
Number of Brackets=40	908.92%	908.92%	-95.07%	-95.07%	-100.00%	-100.00%	-100.00%	-100.00%
Year 2009								
Number of Brackets=20	593.54%	593.53%	14.78%	14.78%	-62.19%	-62.19%	-62.20%	-62.24%
Number of Brackets=30	673.01%	672.41%	-57.91%	-57.91%	-93.17%	-93.17%	-93.18%	-93.19%
Number of Brackets=40	656.43%	656.13%	-87.69%	-87.69%	-99.04%	-99.06%	-99.06%	-99.06%
Year 2008								
Number of Brackets=20	812.26%	678.28%	678.28%	129.47%	129.47%	129.47%	-91.60%	-91.60%
Number of Brackets=30	1069.42%	818.01%	818.01%	31.05%	31.05%	31.05%	-99.40%	-99.41%
Number of Brackets=40	1229.92%	858.16%	858.16%	-38.66%	-38.66%	-38.66%	-99.97%	-99.97%
Year 2007								
Number of Brackets=20	362.94%	-46.26%	-79.91%	-99.96%	-99.97%	-99.97%	-100.00%	-100.00%
Number of Brackets=30	308.37%	-87.84%	-97.50%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%
Number of Brackets=40	213.29%	-97.86%	-99.77%	-100.00%	-100.00%	-100.00%	-100.00%	-100.00%

Table 7. Portfolio characteristics if you play E, W, E in one pool and the other bracket in the second pool.

(A) Scoring: Power of Two. Payout: 3-1. Number of brackets =30. Number of simulations = 100,000

Sec. Brac.	E, W, E	E, W, W	E, S, E	E, S, S	M, W, M	M, W, W	M, S, M	M, S, S
Win Frac.	0.334	0.408	0.355	0.439	0.455	0.439	0.472	0.472
Mean Ret.	3.069	3.094	3.085	3.085	3.053	3.088	3.071	3.080
SD Ret.	7.144	6.023	6.967	5.600	5.340	5.604	5.158	5.186
Sharpe	0.430	0.514	0.443	0.551	0.572	0.551	0.595	0.594

(B) Scoring: Power of Two. Payout: 6-3-1. Number of brackets =30. Number of simulations = 100,000

Sec. Brac.	E, W, E	E, W, W	E, S, E	E, S, S	M, W, M	M, W, W	M, S, M	M, S, S
Win Frac.	0.471	0.528	0.500	0.575	0.600	0.580	0.626	0.627
Mean Ret.	2.918	2.957	2.930	2.937	2.935	2.956	2.957	2.938
SD Ret.	5.865	5.015	5.706	4.613	4.353	4.604	4.183	4.164
Sharpe	0.497	0.590	0.514	0.637	0.674	0.642	0.707	0.705

(C) Scoring: Linear. Payout: 3-1. Number of brackets =30. Number of simulations = 100,000

Sec. Brac.	E, W, E	E, W, W	E, S, E	E, S, S	M, W, M	M, W, W	M, S, M	M, S, S
Win Frac.	0.708	0.717	0.726	0.746	0.753	0.745	0.770	0.770
Mean Ret.	8.879	8.870	8.869	8.883	8.861	8.891	8.883	8.852
SD Ret.	8.721	8.567	8.501	8.179	8.066	8.202	7.858	7.821
Sharpe	1.108	1.035	1.043	1.086	1.099	1.084	1.131	1.132

(D) Scoring: Linear. Payout: 6-3-1. Number of brackets =30. Number of simulations = 100,000

Sec. Brac.	E, W, E	E, W, W	E, S, E	E, S, S	M, W, M	M, W, W	M, S, M	M, S, S
Win Frac.	0.825	0.828	0.840	0.853	0.858	0.852	0.872	0.872
Mean Ret.	7.987	7.941	7.982	7.988	7.996	7.974	7.999	7.967
SD Ret.	6.667	6.582	6.505	6.289	6.184	6.278	6.000	5.991
Sharpe	1.978	1.206	1.227	1.270	1.293	1.270	1.333	1.330

6. Concluding Remarks

Given the sophistication of most participants in NCAA Men's Basketball Tournament bracket pools, we find that the best brackets are modal brackets in which the top seed is always predicted to beat a lower seed until the Final Four. If you enter multiple brackets, the optimal portfolio will diversify across the possible outcomes of the first round of the Final Four. At this point though, it is important to remember the stock financial advice that past returns are no guarantee of future success. Modal brackets are optimal when playing against opponents who exhibit overconfidence (Thaler, 2000) and believe that their subjective ranking of teams in the tournament is more accurate than the selection committee's. If the conclusions of this paper are disseminated through the population of sports fans and the adoption of modal brackets becomes wide- spread in future bracket pools, the game-theoretic aspects of bracket pools will become more significant, and the advice of this paper will no longer be valid.

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Notes

Note 1. 60 Minutes January 27, 2019: https://www.cbsnews.com/news/jerry-and-marge-selbee-how-a-retired-couple-won-millions-using-a-lottery-loophole-60-minutes/

Note 2. One article that does advise players not to overthink picking #1 seeds is https://www.bloomberg.com/news/articles/2019-03-17/how-to-win-your-office-ncaa-pool-use-game-theory-pick-duke, which however focuses on a game theory approach to analyzing the game rather than the finance approach that we employ here. .

Note 3. In the first round we assume everyone picks #1 seeds over #16 seeds since, up to and including the 2019 tournament, there has only been one instance of a #16 team upsetting a #1 team.

Note 4. We need to simulate the model to calculate probabilities for winning a pool. However, these should scale with the probability of winning Warren Buffett's prize. We estimate the latter probability to be $1.82x10^{-11}$ for a modal bracket. A bracket where a single #2 team upsets all of the #1 teams it plays will have a corresponding win probability of $1.19x10^{-11}$. Note that there are $9.22x10^{18}$ brackets in total.

Note 5. The estimated probability for an antimodal bracket, in which the lower seed always wins, to get Warren Buffett's prize is $8.71x10^{-73}$. Thus a modal bracket will have an astronomically larger win probability than a very poorly chosen bracket.

Note 6. To be more precise, the Sharpe ratio is the ratio of the expected return to the standard deviation of the return.

Note 7. The 2020 tournament is still months away as of this writing.

Note 8. The selection committee typically tries to assign the #1 seeds to regions so the second- week venues will be geographically convenient, though this can be difficult if multiple #1 seeds come from the same part of the country. The committee also ranks the #1 seeds. Which regional champion plays which is determined by this seeding. If the #1 seeds all win their regions-though this has only happened once-the first round of the Final Four would have the best #1 seed play the fourth best #1 seed and the second best play the third best.

Note 9. We have found rare examples where early prizes are awarded to the best scoring bracket after the first or second week. Sometimes a booby prize is given to the last-place bracket.

Note 10. Acknowledging the common wisdom that you should never pick a #16 seed to upset a #1 seed since there has only ever been one such upset, we do assume they pick the #1 seeds to win in the first round.

Note 11. For all of the cases that we consider, the size of the pool is large enough that any prize.

would be larger than the cost of entering the pool. Thus the win fraction is the probability of realizing a positive return.

Note 12. We use 100000 simulations to compute the mean return and win fraction for each type of

bracket pool.

Note 13. Or of it is a for-profit bracket pool, the expected return will be negative.

Note 14. Choosing a portfolio of brackets is an example of combinatorial finance.

Note 15. We get qualitatively similar results with other prize vectors and scoring schemes.

Note 16. We only need the opposing scores to do this exercise. Our sample came from a large online pool, www.gscotthenderson.com, that typically hosts over a hundred brackets each year and posts everybody's score.

Note that their scoring scheme is s (1, 2, 5, 8, 10, 15). To match the payoff structure used in the pool, we also used the prize vector 7r (.625, .25, .125). They also give prizes for performance in the earlier rounds, but we did not account for this.

Note 17. It might actually be negative since we did not account for the profit taken by the pool in computing the prizes.

Note 18. 100000 simulations were used for each case.