The Dynamic Correlation of Stock Markets in the World’s Five Largest Economies—Based on DCC-GARCH Model

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Abstract

The dynamic correlation of stock markets in various countries has attracted the attention of scholars and financial investors. In this paper, the dynamic conditional correlation model and the generalized autoregressive conditional heteroskedasticity model are combined to analyze the dynamic conditional correlation coefficient matrix of the stock data of China, the United States, Britain, Germany and Japan, aiming at the five indexes of the Shanghai Securities Composite Index, the Dow Jones Index, the Financial Times Stock Exchange 100 Index, the Frankfurt DAX Index and the Nikkei Index. The results show that there is a certain correlation between the stock markets of various countries, especially the correlation coefficient of the yield of the FTSE index and the GDAXI index reaches 0.96, which a strong correlation. The conclusions of this study can provide constructive suggestions for global economic recovery.

Keywords: Dynamic conditional correlation model, Shanghai Securities Composite Index, Dow Jones Index, Financial Times Stock Exchange 100 Index, Frankfurt DAX Index, Nikkei Index

1. Introduction

Reliable estimation of the correlation between financial variables has always been a research hotspot for statistical practitioners. Derivatives pricing, portfolio optimization, risk management and hedging all need to study the correlation. The issue of stock market correlation has attracted much attention, and the links between countries and economies around the world have become increasingly close. In the context of economic globalization, if a country or region’s financial policy changes, it will quickly have an impact within its financial system and trigger global financial turmoil. Since 2019, there have been close economic and trade exchanges between countries. The five major economies of China, the United States, Britain, Germany and Japan are highly representative in the world economy. As a barometer of the financial sector, the stock market also has a very important position. This paper takes the stock indexes of five countries, China, the United States, Britain, Germany and Japan, as the research object, and focuses on estimating the changes of the dynamic correlation coefficient matrix of the five countries’ stock markets after the impact of the epidemic.

In recent years, many scholars at home and abroad have studied the correlation of stock market with different methods. Shi and Guan (2003) found that Shanghai and Shenzhen stock markets have a strong correlation at the extreme value by using the binary extreme value conditional distribution method; Lin and Chen (2010) used the multivariate Generalized autoregressive conditional heteroskedasticity model to find that the stock returns of the Shanghai Securities Composite Index, the Hong Kong Hang Seng Index and the Tokyo Nikkei Index have time-varying conditional correlation; Pan (2012) used wavelet analysis model to study the correlation between Chinese Mainland, Hong Kong and the US stock markets; Yuki Toyoshima and Shigeyuki Hamori (2013) studied the correlation between Japanese and Singapore stock markets using asymmetric dynamic conditional correlation models; Lin, Shang and Zhou (2014) used detrended fluctuation analysis to study the influence of exponential trend on the correlation of Hang Seng index; Bi (2015) used non-parametric kernel density estimation method and introduced time-varying asymmetric Copula function to study the correlation of Shanghai and Shenzhen stock markets, and found that the correlation coefficient is larger when the stock market falls; Hu Mei (2016) used Granger causality test to find that the linkage between Shanghai, Shenzhen and Hong Kong stock markets is significant in the short term; Wang, Gan et al. (2017) studied the microscopic correlation
between the Shanghai Composite Index and the Hang Seng Index after the decomposition of the binary empirical mode decomposition algorithm; Zhang Minjia (2017) found that stock market volatility is related to important events in financial markets by multifractal detrended cross-correlation analysis; Daniel Pyeong Kang Kim et al. (2021) used Logistic regression and support vector machine to study the relationship between Twitter and Tesla stock value.

The correlation fluctuation of the stock market is usually related to the current background, and the correlation fluctuation of the stock market under different economic environments will be different. Ioana MOLDOVAN (2011) used multiple regression analysis to study the correlation of stock markets between the US, the UK and Japan before and during the financial crisis; Juha Kotkatvuori-Ornberg, Jussi Nikkinen et al. (2013) used the augmented DCC model to study the stock market correlation of 50 stocks during the financial crisis; Paramati, Zakari et al. (2018) discussed the stock market correlation between Australia and China under bilateral trade links using least squares, dynamic least squares and fully modified least squares methods; Sonali Das, Riza Demirer et al. (2019) used functional data analysis to study the impact of global crisis on stock market correlation. In the past three years, the COVID-19 has had a huge impact on us. Many factories have been forced to close down, a large number of employees have lost their jobs, and individual enterprises cannot afford to pay wages. Whether these have an impact on the stock market is a question worth considering.

Because the Pearson correlation coefficient is used to measure the linear correlation between variables, and there are many assumptions that need to be met, it is not suitable for time-varying data such as stock market. Therefore, Engle (2002) proposed a dynamic conditional correlation model (DCC) based on multivariate Generalized autoregressive conditional heteroskedasticity model to solve the estimation of dynamic correlation between financial variables; Since then, many scholars have also widely used this model. Liu Liping, Ma Dan and Bai Wanping (2015) improved the DCC model proposed by Engle; They decomposed the conditional covariance matrix of the standardized residual in the DCC model by spectral decomposition, and then applied the threshold function to the orthogonal complement of the covariance matrix composed of the first K principal components, which improved the estimation efficiency. Wang (2019) used the DCC model to study the correlation of rates of return between real estate and financial industries; Li (2022) used DCC model to analyze the time-varying correlation between stock index futures and spot; Xiong Jingbo and Zhang Xiaolei (2022) used the DCC model to study the dynamic correlation between the stock markets of China and the United States under the impact of the COVID-19; Qing Xin (2022) used DCC model to study digital currency hedging and risk aversion in the context of COVID-19; Ringim Salim Hamza et al. (2022) used the DCC model to study the correlation between Russian energy prices and economic policy uncertainty. This paper also uses the DCC model to study the correlation fluctuation of the stock market.

This paper is divided into five parts. The first part is the introduction, the second part is the model introduction, the third part is the data selection and processing, the fourth part is the empirical analysis, and the last part is the conclusion.

2. DCC-GARCH Model

For the study of time series data, the common autoregressive moving average (ARMA) model is mainly suitable for linear models in univariate and homoscedastic situations. In the case of heteroscedasticity, the American statistician Robert F. Engle proposed the autoregressive conditional heteroscedasticity (ARCH) model in 1982. On this basis, Bollerslov proposed the generalized autoregressive conditional heteroscedasticity (GARCH) model in 1985, which is more widely used than the autoregressive conditional heteroscedasticity model. The autoregressive conditional heteroscedasticity model is only applicable to the short-term autocorrelation process of the heteroscedasticity function. If the long-term autocorrelation process is encountered in practice, the autoregressive conditional heteroscedasticity model will produce a high moving average order, which will not only increase the difficulty, but also affect the final estimation effect. The GARCH model just makes up for this shortcoming. Subsequently, Engle proposed a dynamic conditional correlation (DCC) model based on the GARCH model. This model has the flexibility of univariate GARCH, but does not have the complexity of multivariate GARCH. In addition, it also considers the influence of past market information on the correlation coefficient matrix or covariance matrix, and contributes to the estimation and modeling of large-dimensional data covariance matrix.

The DCC-GARCH model is widely used in the process of multivariate volatility modeling. The application of this model mainly focuses on two aspects. First, when the number of variables is large, the multivariate time-varying covariance matrix can be estimated, commonly used in portfolio; secondly, the dynamic correlation coefficient matrix between variables can be estimated, which is often used to analyze the linkage between
variables or the change of correlation coefficient between variables over time.

The structure of GARCH (p,q) model is as follows:

\[
\begin{align*}
x_t &= f(t,x_{t-1},x_{t-2},\ldots) + \epsilon_t \\
\epsilon_t &= \sqrt{h_t}e_t \\
h_t &= \lambda_0 + \sum_{j=1}^{p} \eta_j h_{t-j} + \sum_{i=1}^{q} \lambda_i \epsilon_{t-i}^2
\end{align*}
\]

(1)

The parameters satisfy the following constraints:

\[
\begin{align*}
0 &\leq \lambda_i < 1, \ i = 1,2,\ldots, q \\
0 &\leq \eta_j < 1, \ j = 1,2,\ldots, p \\
0 &\leq \sum_{j=1}^{p} \eta_j + \sum_{i=1}^{q} \lambda_i < 1
\end{align*}
\]

(2)

In the formula, \( f(t,x_{t-1},x_{t-2},\ldots) \) mainly obtains the mean estimation of the sequence, and realizes the zero mean of the residual sequence, that is, \( E(\epsilon_t) = 0 \). The residual sequence \( \epsilon_t \) has the characteristic of conditional heteroscedasticity. By constructing the autoregressive moving average model of residual square, the conditional heteroscedasticity \( h_t \) of the sequence is obtained. \( \epsilon_t \) is the residual fluctuation after the original sequence extracts the deterministic information and the conditional heteroscedasticity information. Because the relevant information in the mean and variance of the sequence is extracted cleanly, it is a real white noise sequence.

The DCC-GARCH(1,1) model assumes that the residuals \( \epsilon_t \) of the five time series are normally distributed with a mean of 0 and a covariance matrix of \( H_t \). The modeling process is as follows:

\[
\begin{align*}
\epsilon_t | \Phi_{t-1} &\sim N(0,H_t) \\
\tau_t &= \mu_t + \epsilon_t \\
\epsilon_t &= D_t e_t \\
D_t^2 &= diag(\omega_i + diag(\alpha_i) \epsilon_{t-1}^2) + diag(\beta_i) D_{t-1}^2 \\
H_t &= D_t R_t D_t \\
D_t &= diag(\sqrt{h_{t,tt}}) \\
Q_t &= (1-m-n) \tilde{Q} + m \epsilon_{t-1} e_{t-1}^2 + n Q_{t-1} \\
R_t &= diag(Q_t^{-1} \tilde{Q} \diag(Q_t^{-1})
\end{align*}
\]

(3)

Where \( D_t \) is the conditional standard deviation matrix obtained by establishing the univariate GARCH(1,1) model for the five variables at time \( t \), \( e_t \) is the standard residual sequence at time \( t \), \( Q_t \) is the conditional covariance matrix of the standard residual sequence \( \epsilon_t \), \( \tilde{Q} \) is the unconditional variance matrix of the standard residual sequence, \( m \) and \( n \) are the coefficient matrices respectively, and \( R_t \) is the finally obtained dynamic conditional correlation coefficient matrix. Estimated parameters using two-step estimation method.

3. Data Selection and Description

In the latest ranking of the top 20 economies for 2022, the top five are the US, China, Japan, Germany and the UK. Therefore, this paper selects the Shanghai Securities Composite Index (SSEC) as the proxy variable of the Chinese stock market, selects the Dow Jones Index (DJI) as the proxy variable of the American stock market, selects the Financial Times Stock Exchange 100 Index (FTSE) as the proxy variable of the British stock market, selects the Frankfurt DAX Index (GDAXI) as the proxy variable of the German stock market, and selects the Nikkei Index (N225) as the proxy variable of the Japanese stock market. Different countries have different legal holidays, stock market trading time is different, and there is time difference. In order to ensure that the data of different countries are comparable at the same time, these asynchronous data need to be deleted, and finally 456 groups of samples are obtained.

The descriptive statistical characteristics of the closing prices of the five indexes are shown in Table 1. The results show that the skewness and kurtosis of the five groups of indexes are less than 0, indicating that the probability distribution of the sample data is skewed to the left and the steepness is less than the normal distribution. Combined with the time series diagram of five groups of sample data (Figure 1), it is found that the index of the SSEC began to decline in mid-January 2020, while the other four indexes all fell sharply at the end of February, which is due to the COVID-19 epidemic first appeared in China, and then spread to all countries in the world. Until the end of March, the five indexes fell to the lowest value. From April 2020, reasonable prevention and control measures began to take effect, and each index showed a slow upward trend.

Through the time series diagram, it is found that the five groups of sample data are obviously not stable, and the
ADFe,
Next, the residual sequence white noise test (Ljung-Box test) and parameter significance test were performed on the five fitting models respectively. The original hypothesis of the residual white noise test is that the residual sequence is a pure white noise sequence, and the original hypothesis of the parameter significance test is that the parameters are not significantly different from 0. The results show that the p value of the LB test is much larger than the significance level of 0.05, which indicates that the residual sequence has realized white noise, that is, there is no autocorrelation in the residual term, and all parameters are significantly non-zero, which indicates that the five fitting models are effective fitting models.

4.2 Establishment of DCC-GARCH Mode

The premise of GARCH model is that the residual sequence has heteroscedasticity effect, so it is necessary to test the heteroscedasticity of the residual sequence (LM test). The null hypothesis is that there is no heteroscedasticity effect in the residual sequence. The test results of the sample data in this paper show that the p values are all less than the significance level of 0.05, rejecting the original hypothesis, indicating that the five groups of sequence residuals have heteroscedastic effects and meet the GARCH modeling conditions.

Previous studies have found that GARCH(1,1) model can better fit the fluctuation of time series, so GARCH(1,1) is selected to model, and the model fitting results are shown in Table 3. The table shows that the values for $\alpha$ and $\beta$ are positive, and the values for $\alpha + \beta$ are less than 1, indicating that the modeling results are statistically significant at the 0.05 significance level. On the basis of these good models, the DCC-GARCH(1,1) model is established for the five index returns, and the results are shown in Table 4. The results show that the values of $m$ and $n$ are greater than 0, and the value of $m + n$ is smaller than 1, which indicates that the model is reasonable and effective, and that there must be dynamic correlation between every two index return series.

### Table 2. ARIMA model fitting results

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter estimation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSEC</td>
<td>ARIMA(0,1,0)</td>
</tr>
<tr>
<td>DJI</td>
<td>ARIMA(0,1,3)</td>
</tr>
<tr>
<td>FTSE</td>
<td>ARIMA(1,1,1)</td>
</tr>
<tr>
<td>GDAXI</td>
<td>ARIMA(0,1,0)</td>
</tr>
<tr>
<td>N225</td>
<td>ARIMA(2,1,1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>$m$</th>
<th>$n$</th>
<th>$m+n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSEC&amp;DJI</td>
<td>0.038</td>
<td>0.785</td>
<td>0.823</td>
</tr>
<tr>
<td>SSEC&amp;FTSE</td>
<td>0.072</td>
<td>0.756</td>
<td>0.828</td>
</tr>
<tr>
<td>SSEC&amp;GDAXI</td>
<td>0.077</td>
<td>0.804</td>
<td>0.881</td>
</tr>
<tr>
<td>SSEC&amp;N225</td>
<td>0.087</td>
<td>0.685</td>
<td>0.772</td>
</tr>
<tr>
<td>DJI&amp;GDAXI</td>
<td>0.016</td>
<td>0.949</td>
<td>0.965</td>
</tr>
<tr>
<td>DJI&amp;FTSE</td>
<td>0.016</td>
<td>0.949</td>
<td>0.965</td>
</tr>
</tbody>
</table>

4.3 Result Analysis

From the parameters of the fitted DCC-GARCH model, it is preliminarily judged that there is a dynamic correlation between the indexes. Therefore, the conditional correlation coefficient values are further extracted and the conditional correlation coefficient diagram is drawn (Figure 2). This is the focus of this study and the direct manifestation of the correlation between the stock markets. It can be seen that the conditional correlation coefficients between the return series of each index are mostly positive, and only 18 values are less than 0, which indicates that there is a positive correlation between the indexes. It is worth noting that the correlation coefficient...
between the yield of the FTSE and the GDAXI is up to 0.96, very close to 1, which proves that the stock markets of the two countries have a strong correlation. The correlation coefficient between the DJI, the FTSE and the GDAXI also fluctuates slightly between 0.4 and 0.8, and there is also a high correlation. The correlation coefficient between the remaining countries is roughly between 0.2 and 0.6, although relatively small, but it is also ideal.

In December 2019, a small amount of pneumonia of unknown origin was detected in Wuhan, China, after which it was found to be infectious. It then broke out on a large scale in Wuhan, China, in January 2020 and later spread across the country. At the end of February, China adopted a key measure to decisively block the spread of the epidemic, made major decisions on coordinating epidemic prevention and control, economic and social development, and orderly resumption of work and production, and made unremitting efforts to achieve effective prevention and control by the end of April. However, the other four countries began to flood in large areas in mid-March. From the conditional correlation coefficient diagram, it is found that the correlation between the stock index returns of China and the other four countries decreased sharply after reaching the maximum on February 4, 2020, which is the most serious period of the epidemic in China, followed by a brief upward trend, while the correlation coefficient between the remaining countries also showed a downward trend after reaching the highest point in March 2020, which is precisely because the epidemics in different countries are not synchronized and the economic environment is different. Further observation shows that the correlation between stock returns in China and the other four countries fluctuates sharply, and remains low from the end of 2020 to the end of 2021. Therefore, countries around the world should work together to eliminate the economic downturn caused by the epidemic, restore and enhance the positive correlation between stock markets and economies of various countries, and promote global economic recovery.

5. Conclusion

China, the United States, Britain, Germany and Japan are the world’s five largest economies in 2022, with strong economic representation. Studying the market relations of these countries is conducive to predicting global economic changes and taking corresponding measures. This paper uses the daily stock index data from December 3, 2019 to December 30, 2021 to study the dynamic correlation of the stock markets of the five countries under the COVID-19 epidemic through the DCC-GARCH model. The study found that the correlation of stock markets in various countries fluctuated sharply under the impact of the epidemic, and the correlation generally decreased. This is because the stock markets are no longer affected by the new coronavirus pneumonia epidemic, and there is asynchrony.

The conclusions of this paper provide new case support for the theoretical research on the dynamic correlation of financial markets. First, the co-movement of stock markets in various countries is obviously declining under the impact of the epidemic. In the process of research and investment, we need to pay attention to the consistent changes in global financial markets and be alert to the impact of global events on financial markets. In the context of the continued impact of the epidemic, investors must think carefully about the prevention and control of systemic risks to prevent huge losses. Second, the linkage of the stock market affected by the global epidemic has gradually decreased. In the later analysis and investment process of the stock market, investors should pay more attention to the domestic economic recovery, focus on the actual domestic investment, and make reasonable judgments. In the face of the downward pressure on the economy, countries should balance the relationship between epidemic prevention and economic recovery, so as to achieve orderly supervision and stabilize corporate confidence. Due to the complexity of the epidemic situation and the differences in national conditions, epidemic prevention measures should be as far as possible in line with national conditions, keep pace with the times, adjust and optimize the situation of each country in the century, and truly achieve “epidemic situation should be prevented, economy should be stabilized, development should be safe”, and the differences of each market should be considered when making investment decisions. Investors should not be too anxious about foreign stock markets, but should pay more attention to the domestic stock market environment changes, based on their own national conditions to make reasonable investment decisions. Third, countries should trust each other, and investors should also trust the financial market. A smaller trust distance among nations is related to a higher stock market correlation (Yuna Liu, 2020). It is the future direction of all countries in the world to work together to formulate a more effective and beneficial epidemic prevention policy for restoring economic development. Countries should consciously eliminate trade barriers, put aside the idea of zero-sum game, work together, share experience, and seek win-win cooperation. We should actively explore cooperation in digital, green, and sustainable fields, and work together to get out of the haze of the COVID-19 epidemic as soon as possible and strive to promote normal economic development.
Conflict of interest
The author declares no conflicts of interest in this paper.

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References


Xuan, W., Zhihong, G., Junling, C., & Kaijian, H. (2017). Research on the correlation between Shanghai and Hong Kong stock markets based on BEMD and Copula. *Journal of Beijing University of Chemical Technology (Natural Science Edition),* 44(01), 118-123.


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