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An Examination of Calculations: From Anaerobic Mono-digestion to Co-digestion

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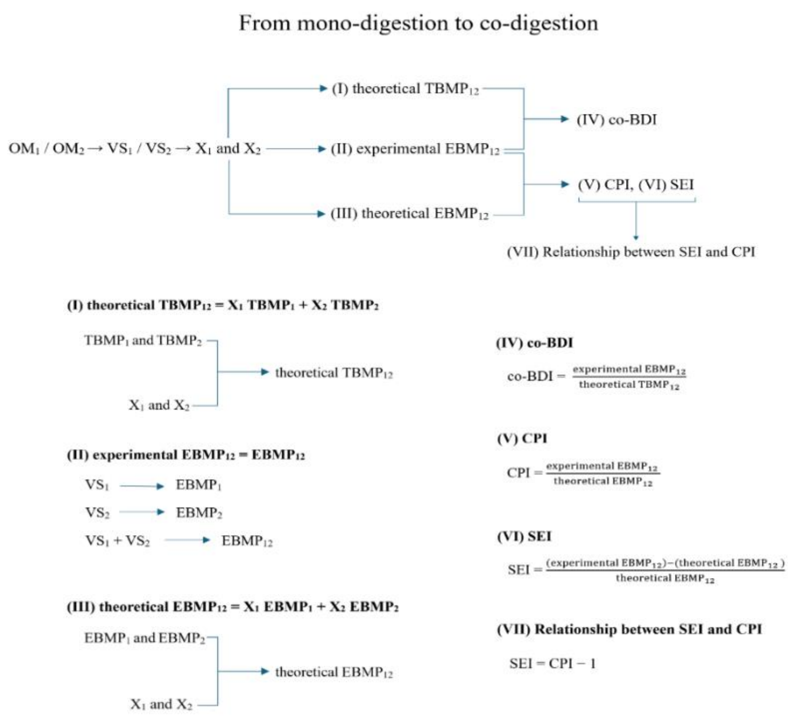
Abstract

Anaerobic digestion is a biochemical process and form of sustainable technology. In comparison to mono-digestion, co-digestion is more effective in the utilization of diverse resources, supply of nutrient balance, and overall digestibility and stability. The co-digestion biodegradability index, co-digestion performance index, and synergistic effect index are generally used as anaerobic metrics to evaluate the co-digestion performance. Although they are numerical parameters, little information on their working procedures has been revealed. There is also a lack of supporting calculations for some published data hence their values cannot be verified. The aim of this article is to reconsider and examine the mathematical calculations from anaerobic mono-digestion to co-digestion. The method of study is a pen-and-paper analysis that processes both experimental and theoretical data from past studies. The article serves four purposes. First, it identifies processing parameters. Second, it shows step-by-step procedures of a series of calculations. Third, it identifies the core parameter in the calculation of anaerobic co-digestion. Fourth, it establishes the relationships between co-digestion biodegradability index, co-digestion performance index, and synergistic effect index. Critically, the mass fraction of volatile solid is identified as the core parameter that determines the valid calculations of co-digestion.

Graphical Abstract



Keywords: co-digestion biodegradability index (co-BDI), co-digestion performance index (CPI), synergistic effect index (SEI), mass fraction of volatile solid (X_i)

1. Introduction

Anaerobic digestion (AD) is a biochemical process and form of sustainable technology for treating wastes, producing biofuel, and reducing environmental impact. The biomethane potential (BMP) test (Holliger et al., 2016; Holliger et al., 2021; Koch et al., 2020; Raposo et al., 2011) is a measure of biodegradability of an organic matter (OM), in which experimental biomethane potential (EBMP) (Owen et al., 1979) and theoretical biomethane potential (TBMP) (Angelidaki & Sanders, 2004) are two critical parameters for counting mono-digestion biomethane production. The biodegradability index (BDI, %) (Nielfa et al., 2015) refers to the ratio percentage of EBMP to TBMP.

Mono-digestion is a type of AD process, which contains one single type of OM, such as chicken manure, corn stover, rice straw, and biological sludge. When an empirical formula of an OM is found, the quantitative amount of biomethane (nCH_4) can be calculated by Buswell's equation (BEq) (Yuen & Lau, 2023; 2024a). Consequently, the TBMP can be determined by nCH_4 and its corresponding empirical formula (Yuen & Lau, 2023; Yuen et al., 2024b). BEq (Buswell & Mueller, 1952; Boyle, 1977) is represented as:

$$C_xH_yO_zN_vS_u+\frac{4x-y-2z+3v+2u}{4}H_2O\rightarrow\frac{4x+y-2z-3v-2u}{8}CH_4+\frac{4x-y+2z+3v+2u}{8}CO_2+vNH_3+uH_2S$$

Co-digestion occurs when more than one type of OM is digested simultaneously. It utilizes diverse resources, supplies nutrient balance, and improves overall digestibility and stability (Angelidali & Ahring, 1997; United States of Environmental Protection Agency, 2014). The co-digestion performance can be evaluated by three quantitative metrics: co-digestion biodegradability index (co-BDI), co-digestion performance index (CPI), and synergistic effect index (SEI).

The co-digestion biodegradability index (co-BDI, %) (Li et al., 2018) of two or more OM ($n \geq 2$) is demonstrated in the following mathematical equations:

$$\text{co-BDI (\%)} = \frac{EBMP_{CO}}{\text{sum of weighted } TBMP_{mono}} \times 100\%$$

$$\text{co-BDI (\%)} = \frac{EBMP_{CO}}{X_1\ TBMP_1+X_2\ TBMP_2+\cdots+X_n\ TBMP_n} \times 100\%$$

$$\text{co-BDI (\%)} = \text{co-BDI} \times 100\%$$

The co-digestion performance index (CPI) (Ebner et al., 2016; Labatut et al., 2011) is established to evaluate the interaction effect of two or more OM. The mathematical equation of CPI is shown as:

$$CPI = \frac{EBMP_{CO}}{\text{sum of weighted } EBMP_{mono}} \text{ or } CPI_{i,n} = \frac{B_{i,n}}{B_{oi,n}} = \frac{B_{i,n}}{\sum_i^n \%VS_i B_{oi,n}}$$

The CPI values are described and quantified by three types of co-digestion interactions, and they are shown in the following:

$CPI > 1$, the synergistic effect, with positive interaction

$CPI = 1$, the additive effect, without mutual interaction

$CPI < 1$, the antagonistic effect, with negative interaction

The synergistic effect index (SEI, %) (Li et al., 2018; Li et al., 2013) for two OM are represented by the following mathematical equations:

$$\text{SEI (\%)} = \frac{(EBMP_{CO})-(\text{ sum of weighted } EBMP_{mono})}{\text{sum of weighted } EBMP_{mono}} \times 100\%$$

$$\text{SEI (\%)} = \frac{(EBMP_{CO})-(X_1\ EBMP_1+X_2\ EBMP_2)}{X_1\ EBMP_1+X_2\ EBMP_2} \times 100\%$$

$$\text{SEI (\%)} = \frac{\text{SEI}-\text{difference}}{X_1\ EBMP_1+X_2\ EBMP_2} \times 100\%$$

$$\text{SEI-difference} = (EBMP_{CO}) - (X_1\ EBMP_1 + X_2\ EBMP_2)$$

X_1 and X_2 = mass fraction

$$\text{SEI (\%)} = \text{SEI} \times 100\%$$

The SEI-difference value is used to evaluate and quantify co-digestion interactions that are shown in the following:

SEI-difference > 0, the synergistic effect, with positive interaction

SEI-difference = 0, the additive effect, without mutual interaction

SEI-difference < 0, the antagonistic effect, with negative interaction

Measurements and calculations are two major components that make up the BMP test. Although the co-BDI, CPI, and SEI can be used to evaluate the co-digestion biodegradability, some articles do not provide supporting calculations and data that can be verified.

The aim of this article is to reconsider and examine the mathematical calculations from anaerobic mono-digestion to co-digestion. The method of study is a pen-and-paper analysis that explores both experimental and theoretical data from past studies. The article serves four purposes. First, it identifies processing parameters. Second, it shows step-by-step procedures of a series of calculations. Third, it identifies the core parameter in the calculation of anaerobic co-digestion. Fourth, it quantifies the key parameters to establish the relationships between co-BDI, CPI and SEI.

Examples are provided to demonstrate the processing parameters of the working procedures. Among them, the mass fraction of volatile solid (X_i) plays a dominant role in the co-digestion calculations. In addition, the co-digestion experimental biomethane potential (experimental $EBMP_{CO}$), co-digestion theoretical-experimental biomethane potential (theoretical $EBMP_{CO}$) and co-digestion theoretical biomethane potential (theoretical $TBMP_{CO}$) are shown to be correlated parameters among co-BDI, CPI and SEI for the co-digestion mathematical equations.

2. Methodology, Methods, and Materials

2.1 Methodology

Quantitative analysis is the core of data processing, in which the collection and manipulation of numerical data produces meaningful information. In this article, the pen-and-paper analysis is adapted for the parameter-based co-digestion calculations. Calculations are conducted according to experimental data and theoretical data from past studies.

The method is designed to recalculate and examine the quantitative parameters of anaerobic co-digestion. Figure 1 illustrates the sequences of calculations from mono-digestion to co-digestion.

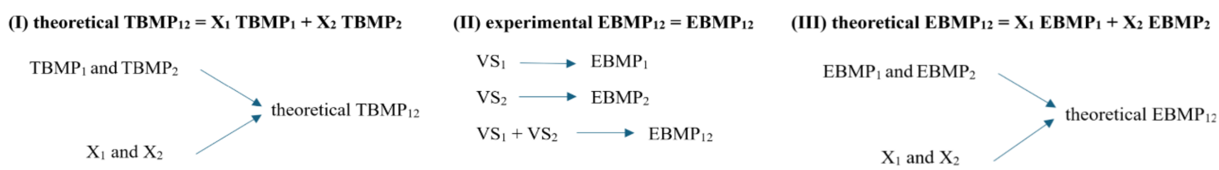


Figure 1. The flowchart: calculations from mono-digestion to co-digestion

2.2 Proximate Analysis (PA)

The physical composition of OM is measured by the proximate analysis (PA) (American Public Health Association, 1999). Volatile solid of OM (VS) refers to the degradable amount in total solid of OM (TS) (Hamilton & Zhang, 2016). The mass relationships are shown in the following:

For wet OM: mass of OM = mass of moisture + mass of TS

For aqueous OM: mass of OM = mass of water + mass of TS

For dry OM: mass of OM = mass of TS

For dry OM: mass of TS = mass of VS + mass of ash

2.3 Experimental Biomethane Potential (EBMP)

The EBMP is defined as the ratio of maximum amount of produced volumetric methane (VCH_4) to mass of added volatile solid (Owen et al., 1979). The equation of EBMP is shown as:

$$\text{EBMP (at STP, mL/g)} = \frac{\text{maximum amount of produced volumetric methane}}{\text{mass of added volatile solid}}$$

$$= \frac{V_{CH4}}{VS_{added}}$$

2.4 Theoretical Biomethane Potential (TBMP)

Based on the empirical formula of $C_xH_yO_zN_vS_u$, the TBMP (Yuen & Lau, 2023; 2024c) can be counted by the following mathematical equation:

$$\text{TBMP (at STP, mL/g)} = \frac{22400\ (\text{mole of biomethane})}{\text{empirical formula mass of volatile solid}}$$

$$= \frac{22400\ (\text{n}CH_4)}{\mu C_xH_yO_zN_vS_u}$$

2.5 Biodegradability Index (BDI)

The BDI evaluates the biodegradability of OM. The BDI (%) (Nielfa et al., 2015) is defined by the ratio percentage of EBMP to TBMP.

$$\text{BDI (\%)} = \frac{\text{EBMP}}{\text{TBMP}} \times 100\%$$

$$\text{BDI (\%)} = \text{BDI} \times 100\%$$

2.6 Collecting Data from Literature

The PA, mono- and co-digestion experimental EBMP data are collected from literature. After recalculations of selected published mono-digestion experimental EBMP, the co-digestion theoretical EBMP can be found. The co-digestion theoretical TBMP can be counted by the mono-digestion theoretical TBMP.

3. Introducing Mono-digestion and Co-digestion Biodegradability Indexes

3.1 Mono-digestion

The mathematical relationships of mono-digestion are shown in the following:

$$\text{BDI (\%)} = \frac{\text{EBMP}}{\text{TBMP}} \times 100\%$$

$$\text{BDI} = \frac{\text{EBMP}}{\text{TBMP}}$$

For mono-digestion (n = 1)

$$VS_1\text{: } BDI_1 = \frac{EBMP_1}{TBMP_1}$$

$$VS_2\text{: } BDI_2 = \frac{EBMP_2}{TBMP_2}$$

$$VS_i\text{: } BDI_i = \frac{EBMP_i}{TBMP_i}$$

mass of VS in type one $OM_1 = VS_1$

mass of VS in type two $OM_2 = VS_2$

mass of VS in type i $OM_i = VS_i$

3.2 Co-digestion

The co-BDI is defined as the ratio of experimental $EBMP_{CO}$ to theoretical $TBMP_{CO}$, and is shown in the following:

$$\text{co-BDI} = \frac{\text{experimental } EBMP_{CO}}{\text{theoretical } TBMP_{CO}}$$

For co-digestion (n = 2, two types of OM)

Based on the mono-digestion parameters: $EBMP_1$ and $EBMP_2$, co-digestion mathematical equations of the experimental $EBMP_{12}$ and theoretical $TBMP_{12}$ are shown in the following:

$$VS_1, VS_2, \text{ and } (VS_1+VS_2)\text{: co-BDI} = \frac{\text{experimental } EBMP_{12}}{\text{theoretical } TBMP_{12}}$$

$$\text{experimental co-}VCH_4 = (VS_1+VS_2)\ EBMP_{12}$$

$$\text{theoretical co-VCH}_4 = \text{VS}_1\ \text{TBMP}_1 + \text{VS}_2\ \text{TBMP}_2$$

$$\text{experimental EBMP}_{12} = \text{EBMP}_{12} = \frac{\text{experimental co}-\text{VCH}_4}{(\text{VS}_1+\text{VS}_2)}$$

$$\text{theoretical TBMP}_{12} = \text{TBMP}_{12} = \frac{\text{theoretical co}-\text{VCH}_4}{(\text{VS}_1+\text{VS}_2)}$$

$$= \frac{\text{VS}_1\ \text{TBMP}_1 + \text{VS}_2\ \text{TBMP}_2}{(\text{VS}_1+\text{VS}_2)}$$

$$= \frac{\text{VS}_1}{(\text{VS}_1+\text{VS}_2)}\ \text{TBMP}_1 + \frac{\text{VS}_2}{(\text{VS}_1+\text{VS}_2)}\ \text{TBMP}_2$$

$$= \text{X}_1\ \text{TBMP}_1 + \text{X}_2\ \text{TBMP}_2$$

$$\text{X}_1 = \frac{\text{VS}_1}{\text{VS}_1+\text{VS}_2}$$

$$\text{X}_2 = \frac{\text{VS}_2}{\text{VS}_1+\text{VS}_2}$$

sum of VS_1 and VS_2 = $(VS_1+VS_2) = VS_{total}$

$$\text{X}_1 = \frac{\text{VS}_1}{\text{VS}_{\text{total}}}$$

$$\text{X}_2 = \frac{\text{VS}_2}{\text{VS}_{\text{total}}}$$

$$\text{co-BDI}_{12} = \frac{\text{experimental EBMP}_{12}}{\text{theoretical TBMP}_{12}}$$

$$\text{co-BDI}_{12} = \frac{\text{EBMP}_{12}}{\frac{\text{VS}_1}{\text{VS}_1+\text{VS}_2}\text{TBMP}_1 + \frac{\text{VS}_2}{\text{VS}_1+\text{VS}_2}\text{TBMP}_2}$$

$$\text{co-BDI}_{12} = \frac{\text{EBMP}_{12}}{\frac{\text{VS}_1}{\text{VS}_{\text{total}}}\text{TBMP}_1 + \frac{\text{VS}_2}{\text{VS}_{\text{total}}}\text{TBMP}_2}$$

$$\text{co-BDI}_{12} = \frac{\text{EBMP}_{12}}{\text{X}_1\ \text{TBMP}_1 + \text{X}_2\ \text{TBMP}_2}$$

For co-digestion ($n \geq 2$)

$$\text{co-BDI} = \frac{\text{experimental EBMP}_{12....n}}{\text{theoretical TBMP}_{12....n}}$$

$$\text{co-BDI} = \frac{\text{EBMP}_{12....n}}{\text{X}_1\ \text{TBMP}_1 + \text{X}_2\ \text{TBMP}_2 + \cdots + \text{X}_n\ \text{TBMP}_n}$$

$$\text{co-BDI} = \frac{\text{EBMP}_{\text{CO}}}{\text{X}_1\ \text{TBMP}_1 + \text{X}_2\ \text{TBMP}_2 + \cdots + \text{X}_n\ \text{TBMP}_n}$$

$$\text{co-BDI} = \frac{\text{EBMP}_{\text{CO}}}{\Sigma \text{X}_\text{i}\ \text{TBMP}_\text{i}}$$

$$\text{co-BDI}\ (\%) = \frac{\text{EBMP}_{\text{CO}}}{\Sigma \text{X}_\text{i}\ \text{TBMP}_\text{i}} \times 100\%$$

sum of $VS_i = \Sigma VS_i = VS_{total}$

$$\text{mass fraction of volatile solid } (\text{X}_\text{i}) = \frac{\text{VS}_\text{i}}{\Sigma \text{VS}_\text{i}} = \frac{\text{VS}_\text{i}}{\text{VS}_{\text{total}}}$$

4. Counting Mass Fraction of Volatile Solid (X_i)

4.1 Unit of OM_i

OM is a complex organic mixture. Some examples of OM include biomass, biowaste, and wastewater. The unit of mass of OM (OM_i) is either kg or L. When the unit is L, OM_i is in liquid form and when the unit is kg, it is either in solid or liquid form.

4.2 Mixing Ratio: OM_1/OM_2

OM_1 and OM_2 must have the same unit for a mixing ratio OM_1/OM_2. A mixing ratio OM_1/OM_2 can either be a volume ratio ($volume_1/volume_2$) or mass ratio ($mass_1/mass_2$).

4.3 Mass Ratio: VS_1/VS_2

The unit of volatile solid (VS_i) can be g/kg or g/L. When a datum of VS_i = 393.5 g/kg is provided, 393.5 g of VS can be found in 1 kg of OM. A mass ratio VS_1/VS_2 must have the same unit.

4.4 Unit of $VS_{i'}$

The unit of resulted mass of volatile solid ($VS_{i'}$) is gram (g). The unit of OM_i must match the unit of VS_i. When the unit of OM_i is L, the corresponding unit of VS_i is g/L. When the unit of OM_i is kg, the corresponding unit of VS_i is g/kg.

4.5 Resulted Mass Ratio: $VS_{1'}/VS_{2'}$

The relationships between mixing ratio OM_1/OM_2, mass ratio VS_1/VS_2, and resulted mass ratio $VS_{1'}/VS_{2'}$ are established below:

$$VS_{1'} = OM_1 \times VS_1$$

$$VS_{2'} = OM_2 \times VS_2$$

$$\frac{VS_{1'}}{VS_{2'}} = \frac{OM_1 \times VS_1}{OM_2 \times VS_2}$$

$$VS_{total} = VS_{1'} + VS_{2'}$$

$$VS_i = \frac{VS_{total}}{OM_1 + OM_2}$$

4.6 Mass Fractions of Two Volatile Solids: X_1 and X_2

The X_1 and X_2 can be counted as follows:

$$X_1 = \frac{VS_{1'}}{VS_{total}} = \frac{VS_{1'}}{VS_{1'} + VS_{2'}}$$

$$X_2 = \frac{VS_{2'}}{VS_{total}} = \frac{VS_{2'}}{VS_{1'} + VS_{2'}}$$

Example 1. A list of data is provided.

OM	$\frac{OM1}{OM_2}$ (mixing ratio)	VS_i (g/kg)	X_1	X_2
Organic Matter-1	100/0	393.5	1	0
Co-digestion	80/20	?	?	?
Organic Matter-2	0/100	56.9	0	1

When a mixing ratio (OM_1/OM_2 = 80/20) is chosen, the resulting mass ratio ($VS_{1'}/VS_{2'}$), total mass of volatile solid (VS_{total}), VS_i in a mixing ratio (OM_1/OM_2 = 80/20), and mass fractions of volatile solids (X_1 and X_2) can be determined.

Step 1. Find $\frac{VS_{1'}}{VS_{2'}}$

$$\frac{OM_1}{OM_2} = \frac{80}{20}$$

$$\frac{VS_1}{VS_2} = \frac{393.5}{56.9}$$

$$\frac{VS_1{\prime}}{VS_2{\prime}} = \frac{OM_1 \times VS_1}{OM_2 \times VS_2}$$

$$\frac{VS_1{\prime}}{VS_2{\prime}} = \frac{80 \times 393.5}{20 \times 56.9} = \frac{31480}{1138}$$

Step 2. Find VS_{total} and VS_i

$$VS_{total} = VS_{1'} + VS_{2'} = 31480 + 1138 = 32618$$

VS_i in a mixing ratio $\frac{OM_1}{OM_2}$ (80/20) = $\frac{VS_{total}}{OM_1+OM_2} = \frac{32168}{80+20} = \frac{32168}{100} = 326.18$

The VS_i in the mixing $\frac{OM_1}{OM_2}$ (80/20) sample equals 326.18 (g/kg).

Step 3. Find X_1 and X_2

$$X_1 = \frac{VS_{1'}}{VS_{1'}+VS_2{\prime}} = \frac{VS_1{\prime}}{VS_{total}} = \frac{31480}{32618} = 0.965$$

$$X_2 = \frac{VS_{2'}}{VS_{1'}+VS_2{\prime}} = \frac{VS_2{\prime}}{VS_{total}} = \frac{1138}{32618} = 0.035$$

5. Counting Co-digestion Biodegradability Index

5.1 Working Flowchart of Co-digestion Biodegradability Index (co-BDI)

The working flowchart for determination of co-BDI is shown in Figure 2.

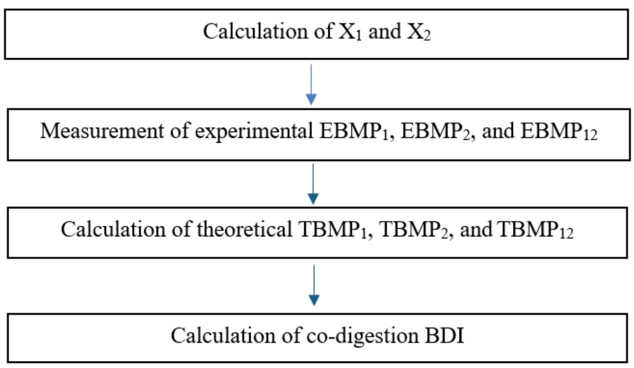


Figure 2. Co-BDI: working flowchart for two OM

5.2 Co-digestion Working Procedures

The working procedures, processing parameters, and mathematical relationships for counting co-BDI are shown in Table 1.

Table 1. Co-digestion BDI: working procedures, processing parameters, and mathematical relations

Working procedure	Collect and/or measured parameters	Measured and/or calculated parameters
Collect OM_i and count mixing ratio	OM_1, OM_2	$\frac{OM_1}{OM_2}$
Collect VS_i and count mass ratio	VS_1, VS_2	$\frac{VS_1}{VS_2}$
Count resulted mass ratio	$VS_{1'}$, $VS_{2'}$	$VS_{1'} = OM_1 \times VS_1$ $VS_{2'} = OM_2 \times VS_2$ $\frac{VS_{1'}}{VS_{2'}} = \frac{OM_1 \times VS_1}{OM_2 \times VS_2}$
Count VS_{total} and VS_i	VS_{total}, VS_i	$VS_{total} = VS_{1'} + VS_{2'}$ $VS_i = \frac{VS_{total}}{OM_1+OM_2}$
Calculate mass fraction of volatile solid (X_i)	X_1 and X_2	$X_1 = \frac{VS_{1'}}{VS_{total}} = \frac{VS_{1'}}{VS_{1'}+VS_{2'}}$ $X_2 = \frac{VS_{2'}}{VS_{total}} = \frac{VS_{2'}}{VS_{1'}+VS_{2'}}$
Collect experimental mono-digestion and co-digestion EBMP	$EBMP_1$, $EBMP_2$, and $EBMP_{12}$	Experimental $EBMP_{12} = EBMP_{12}$
Collect or count mono-digestion theoretical $TBMP_i$, and count co-digestion theoretical $TBMP_{12}$	$TBMP_1$, $TBMP_2$, and $TBMP_{12}$	Theoretical $TBMP_{12} = TBMP_{12}$ $TBMP_{12} = X_1\,TBMP_1 + X_2\,TBMP_2$
Calculate mono-BDI and co-BDI	BDI_1, BDI_2, and BDI_{12}	$BDI_1 = \frac{EBMP_1}{TBMP_1}$ $BDI_2 = \frac{EBMP_2}{TBMP_2}$ $BDI_{12} = \frac{\text{experimental } EBMP_{12}}{\text{theoretical } TBMP_{12}}$
Calculate co-BDI (%)	BDI_{12} (%)	$BDI_{12}\ (\%) = \frac{\text{experimental } EBMP_{12}}{\text{theoretical } TBMP_{12}} \times 100\%$

5.3 Step-by-step Procedures

Based on the collected data of OM_i, VS_i, EBMP, and TBMP (Nielfa et al., 2015), the working procedures for calculations of co-BDI are demonstrated in Example 2. The selected data of OM_i, $\frac{OM_{OFMSW}}{OM_{BS}}$, VS_i, mono- and co-digestion experimental EBMP, and mono-digestion theoretical TBMP (through calculation from BEq) are summarized in Table 2.

Table 2. Collected data of OM, $\frac{OM_{OFMSW}}{OM_{BS}}$, VS_i, EBMP, and TBMP

OM	$\frac{OM_{OFMSW}}{OM_{BS}}$ (mixing ratio)	VS_i (g/kg)	EBMP	TBMP
OFMSW	100/0	393.5	201.5	494.3
Co-digestion1	80/20	-	220.6	-
Co-digestion2	60/40	-	217.5	-
Co-digestion3	40/60	-	212.3	-
Co-digestion4	20/80	-	200.2	-
BS	0/100	56.9	164.5	333.9

Example 2. The mixing ratio of $\frac{OM_{OFMSW}}{OM_{BS}}$ that equals $\frac{60}{40}$ is selected as an example. The collected data of VS_i, EBMP, and TBMP are used to recalculate the co-BDI.

Step 1. Collect data of $\frac{OM_{OFMSW}}{OM_{BS}}$, VS_i, EBMP, and TBMP.

OM	$\frac{OM_{OFMSW}}{OM_{BS}}$	VS_i	EBMP	TBMP
OFMSW	100/0	393.5	201.5	494.3
Co-digestion2	60/40	?	217.5	?
BS	0/100	56.9	164.5	333.9

Step 2. Identify mass ratio $\frac{VS_{OFMSW'}}{VS_{BS'}}$ and count VS_{total} and VS_i.

$$\frac{OM_{OFMSW}}{OM_{BS}} = \frac{60}{40}$$

$$\frac{VS_{OFMSW}}{VS_{BS}} = \frac{393.5}{56.9}$$

$$\frac{VS_{OFMSW'}}{VS_{BS'}} = \frac{OM_{OFMSW} \times VS_{OFMSW}}{OM_{BS} \times VS_{BS}}$$

$$\frac{VS_{OFMSW'}}{VS_{BS'}} = \frac{60 \times 393.5}{40 \times 56.9} = \frac{23610}{2276}$$

$$VS_{total} = VS_{OFMSW'} + VS_{BS'}$$

$$VS_{total} = 23610 + 2276 = 25886$$

$$VS_i = \frac{VS_{total}}{OM_1 + OM_2} = \frac{25886}{60+40} = \frac{25886}{100} = 258.86$$

Step 3. Calculate mass fractions X_{BS} and X_{OFMSW}.

$$X_{BS} = \frac{VS_{BS'}}{VS_{total}} = \frac{VS_{BS'}}{VS_{OFMSW'} + VS_{BS'}} \text{ and } X_{OFMSW} = \frac{VS_{OFMSW'}}{VS_{total}} = \frac{VS_{OFMSW'}}{VS_{OFMSW'} + VS_{BS'}}$$

$$X_{OFMSW} + X_{BS} = 1$$

$$\because \frac{VS_{OFMSW'}}{VS_{BS'}} = \frac{60 \times 393.5}{40 \times 56.9} = \frac{23610}{2276}$$

$$\therefore X_{OFMSW} = \frac{VS_{OFMSW'}}{VS_{OFMSW'}+VS_{BS'}} = \frac{23610}{23610+2276} = \frac{23610}{25886} = 0.912$$

$$X_{BS} = \frac{VS_{BS'}}{VS_{OFMSW'}+VS_{BS'}} = \frac{2276}{23610+2276} = \frac{2276}{25886} = 0.088$$

Step 4. Collect $TBMP_{OFMSW}$ ($\frac{OM_{OFMSW}}{OM_{BS}} = \frac{100}{0}$, OM_{OFMSW} = 100%) and $TBMP_{BS}$ ($\frac{OM_{OFMSW}}{OM_{BS}} = \frac{0}{100}$, OM_{BS} = 100%)

$TBMP_{OFMSW}$ = 494.3

$TBMP_{BS}$ = 333.9

Step 5. Collect experimental EBMP $_{OFMSW\text{-}BS}$ ($\frac{OM_{OFMSW}}{OM_{BS}} = \frac{60}{40}$)

$EBMP_{OFMSW\text{-}BS}$ = 217.5

Step 6. Count theoretical TBMP $_{OFMSW\text{-}BS}$ ($\frac{OM_{OFMSW}}{OM_{BS}} = \frac{60}{40}$)

$$TBMP_{OFMSW\text{-}BS} = X_{OFMSW}\ TBMP_{OFMSW} + X_{BS}\ TBMP_{BS}$$

$$= (0.912)(494.3)+(0.088)(333.9)$$

$$= 450.802+29.383$$

$$= 480.185$$

Step 7. Count co-BDI and co-BDI (%)

$$\text{co-BDI} = \frac{\text{experimental } EBMP_{OFMSW-BS}}{\text{theoretical } TBMP_{OFMSW-BS}}$$

$$\text{co-BDI} = \frac{EBMP_{OFMSW-BS}}{X_{OFMSW}\ TBMP_{OFMSW} + X_{BS}\ TBMP_{BS}}$$

$$= \frac{217.5}{480.185}$$

$$= 0.453$$

$$\text{co-BDI (\%)} = \frac{EBMP_{OFMSW-BS}}{X_{OFMSW}\ TBMP_{OFMSW} + X_{BS}\ TBMP_{BS}} \times 100\%$$

$$= 0.453 \times 100\%$$

$$= 45.3\%$$

Using the same working procedures, all collected and calculated data of $\frac{OM_{OFMSW}}{OM_{BS}}$, $\frac{VS_{OFMSW'}}{VS_{BS'}}$, VS_{total}, X_{OFMSW}, X_{BS}, experimental mono- and co-EBMP, theoretical mono- and co-TBMP, co-BDI, and co-BDI (%) are summarized in Table 3.

Table 3. Calculated co-digestion data and resulted co-BDI

OM	$\frac{OM_{OFMSW}}{OM_{BS}}$	$\frac{VS_{OFMSW'}}{VS_{BS'}}$	VS_{total}	X_{OFMSW}	X_{BS}	Experimental EBMP	Theoretical TBMP	Co-BDI	Co-BDI (%)
OFMSW	100/0	$= \frac{0 \times 393.5}{100 \times 56.9}$ $= \frac{0}{5690}$	= 0+5690 = 5690	$= \frac{5690}{5690}$ = 1	$= \frac{0}{5690}$ = 0	201.5	494.3	0.408	40.8%
Co-digestion1	80/20	$= \frac{80 \times 393.5}{20 \times 56.9}$ $= \frac{31480}{1138}$	= 31480+1138 = 32618	$= \frac{31480}{32618}$ = 0.965	$= \frac{1138}{32618}$ = 0.035	220.6	488.687	0.451	45.1%
Co-digestion2	60/40	$= \frac{60 \times 393.5}{40 \times 56.9}$ $= \frac{23610}{2276}$	= 23610+2276 = 25886	$= \frac{23610}{25886}$ = 0.912	$= \frac{2276}{25886}$ = 0.088	217.5	480.185	0.453	45.3%
Co-digestion3	40/60	$= \frac{40 \times 393.5}{60 \times 56.9}$ $= \frac{15740}{3414}$	= 15740+3414 = 19154	$= \frac{15740}{19154}$ = 0.822	$= \frac{3414}{19154}$ = 0.178	212.3	465.749	0.456	45.6%
Co-digestion4	20/80	$= \frac{20 \times 393.5}{80 \times 56.9}$ $= \frac{7870}{4552}$	= 7870+4552 = 12422	$= \frac{7870}{12422}$ = 0.634	$= \frac{4552}{12422}$ = 0.366	200.2	435.593	0.460	46.0%
BS	0/100	$= \frac{100 \times 393.5}{0 \times 56.9}$ $= \frac{39350}{0}$	= 39500+0 = 39500	$= \frac{0}{39500}$ = 0	$= \frac{39500}{39500}$ = 1	164.5	333.9	0.493	49.3%

In Example 2, two OM (n = 2) are mixed for the co-digestion calculations. When the experimental $EBMP_{12}$ and theoretical $TBMP_{12}$ are determined, the co-BDI can be quantified accordingly. In cases where three or more OM (n > 2) are mixed, the general mathematical equations for co-digestion are derived, and are shown in the following:

$$\text{co-BDI} = \frac{EBMP_{CO}}{X_1\ TBMP_1 + X_2\ TBMP_2 + \cdots + X_n\ TBMP_n}$$

$$\text{co-BDI} = \frac{EBMP_{CO}}{\Sigma X_i\ TBMP_i}$$

6. Introducing Co-digestion Performance Index and Synergistic Effect Index

6.1 Co-digestion Performance Index (CPI)

The CPI is established to evaluate the interaction effect of two or more OM. The modified mathematical equation of CPI is shown as:

$$CPI = \frac{\text{experimental } EBMP_{CO}}{\text{theoretical } EBMP_{CO}}$$

$$CPI = \frac{\text{experimental } EBMP_{CO}}{X_1\ EBMP_1 + X_2\ EBMP_2 + \cdots + X_n\ EBMP_n}$$

$$CPI\ (\%) = \frac{\text{experimental } EBMP_{CO}}{X_1\ EBMP_1 + X_2\ EBMP_2 + \cdots + X_n\ EBMP_n} \times 100\%$$

$$\text{theoretical } EBMP_{CO} = X_1\ EBMP_1 + X_2\ EBMP_2 + \cdots + X_n\ EBMP_n$$

6.2 Co-digestion Synergistic Effect Index (SEI)

The SEI is modified as the following mathematical equations:

$$SEI = \frac{(\text{experimental } EBMP_{CO}) - (\text{theoretical } EBMP_{CO})}{\text{theoretical } EBMP_{CO}}$$

$$SEI\ (\%) = \frac{(\text{experimental } EBMP_{CO}) - (X_1\ EBMP_1 + X_2\ EBMP_2 + \cdots + X_n\ EBMP_n)}{X_1\ EBMP_1 + X_2\ EBMP_2 + \cdots + X_n\ EBMP_n} \times 100\%$$

6.3 Relationship between CPI and SEI

The mathematical equations of CPI and SEI are shown as:

$$CPI = \frac{\text{experimental } EBMP_{CO}}{\text{theoretical } EBMP_{CO}}$$

$$\text{SEI} = \frac{(\text{experimental EBMP}_{CO}) - (\text{theoretical EBMP}_{CO})}{\text{theoretical EBMP}_{CO}}$$

The difference between CPI and SEI is shown below:

$$\text{CPI} - \text{SEI} = \frac{\text{experimental EBMP}_{CO}}{\text{theoretical EBMP}_{CO}} - \frac{(\text{experimental EBMP}_{CO}) - (\text{theoretical EBMP}_{CO})}{\text{theoretical EBMP}_{CO}}$$

$$\text{SEI} = \text{CPI} - 1$$

SEI-difference is defined as the difference between experimental EBMP_{CO} and theoretical EBMP_{CO}, and is shown in the following:

$$\text{SEI-difference} = \text{experimental EBMP}_{CO} - \text{theoretical EBMP}_{CO}$$

The mathematical equations of SEI and SEI (%) are modified:

$$\text{SEI} = \frac{(\text{experimental EBMP}_{CO}) - (\text{theoretical EBMP}_{CO})}{\text{theoretical EBMP}_{CO}}$$

$$\text{SEI} = \frac{\text{SEI} - \text{difference}}{\text{theoretical EBMP}_{CO}}$$

$$\text{SEI (\%)} = \frac{\text{SEI} - \text{difference}}{\text{theoretical EBMP}_{CO}} \times 100\%$$

$$\text{SEI (\%)} = \text{SEI} \times 100\%$$

7. Calculation of Co-digestion Performance Index and Synergistic Effect Index

7.1 Working Flowchart for Co-digestion CPI and SEI

The working flowchart for determination of CPI is shown in Figure 3.

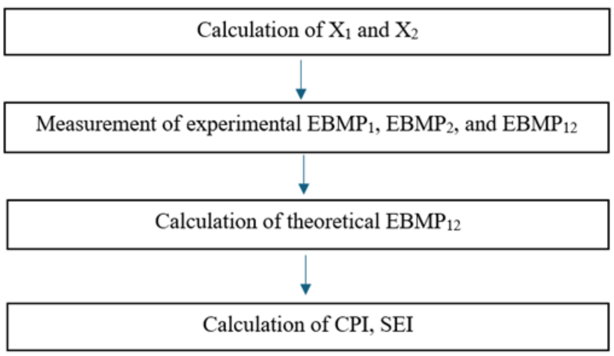


Figure 3. Co-digestion CPI and SEI: working flowchart

7.2 Step-by-step Procedures for Counting CPI and SEI

The data are chosen from literature to calculate CPI and SEI. With reference to Table 2, the counting procedures are demonstrated in Example 3.

Example 3. The mixing ratio of $\frac{OM_{OFMSW}}{OM_{BS}}$ that equals $\frac{20}{80}$ is selected as an example. The collected data of VS_i and EBMP are used to calculate the corresponding CPI.

Step 1. Choose the data of $\frac{OM_{OFMSW}}{OM_{BS}}$ that equals $\frac{20}{80}$, VS_i, and EBMP.

OM	$\frac{OM_{OFMSW}}{OM_{BS}}$	VS_i	Experimental EBMP	Theoretical EBMP
OFMSW	100/0	393.5	201.5	201.5
Co-digestion4	20/80	?	200.2	?
BS	0/100	56.9	164.5	164.5

Step 2. Identify mass ratio $\frac{VS_{OFMSW'}}{VS_{BS'}}$, count VS_{total} and VS_i.

$$\frac{OM_{OFMSW}}{OM_{BS}} = \frac{20}{80}$$

$$\frac{VS_{OFMSW}}{VS_{BS}} = \frac{393.5}{56.9}$$

$$\frac{VS_{OFMSW'}}{VS_{BS'}} = \frac{OM_{OFMSW} \times VS_{OFMSW}}{OM_{BS} \times VS_{BS}}$$

$$\frac{VS_{OFMSW'}}{VS_{BS'}} = \frac{20 \times 393.5}{80 \times 56.9} = \frac{7870}{4552}$$

$VS_{total} = VS_{OFMSW'} + VS_{BS'}$

$VS_{total} = 7870 + 4552 = 12422$

$$VS_i = \frac{VS_{total}}{OM_{OFMSW}+OM_{BS}} = \frac{12422}{20+80} = \frac{12422}{100} = 124.22$$

Step 3. Calculate mass fractions X_{BS} and X_{OFMSW}.

$$X_{BS} = \frac{VS_{BS'}}{VS_{total}} = \frac{VS_{BS'}}{VS_{OFMSW'}+VS_{BS'}} \text{ and } X_{OFMSW} = \frac{VS_{OFMSW'}}{VS_{total}} = \frac{VS_{OFMSW'}}{VS_{OFMSW'}+VS_{BS'}}$$

$X_{OFMSW} + X_{BS} = 1$

$$\because \frac{VS_{OFMSW'}}{VS_{BS'}} = \frac{20 \times 393.5}{80 \times 56.9} = \frac{7870}{4552}$$

$$\therefore X_{OFMSW} = \frac{VS_{OFMSW'}}{VS_{OFMSW'}+VS_{BS'}} = \frac{7870}{7870+4552} = \frac{7870}{12422} = 0.634$$

$$X_{BS} = \frac{VS_{BS'}}{VS_{OFMSW'}+VS_{BS'}} = \frac{4552}{7870+4552} = \frac{4552}{12422} = 0.366$$

Step 4. Collect $EBMP_{OFMSW}$ ($\frac{OM_{OFMSW}}{OM_{BS}} = \frac{100}{0}$, OM_{OFMSW} = 100%) and $EBMP_{BS}$ ($\frac{OM_{OFMSW}}{OM_{BS}} = \frac{0}{100}$, OM_{BS} = 100%)

$EBMP_{OFMSW} = 201.5$

$EBMP_{BS} = 164.5$

Step 5. Collect $EBMP_{OFMSW\text{-}BS}$ ($\frac{OM_{OFMSW}}{OM_{BS}} = \frac{20}{80}$)

Experimental $EBMP_{OFMSW\text{-}BS} = 200.2$

Step 6. Calculate theoretical $EBMP_{OFMSW\text{-}BS}$ ($\frac{OM_{OFMSW}}{OM_{BS}} = \frac{20}{80}$)

Theoretical $EBMP_{OFMSW\text{-}BS} = X_{OFMSW}\, EBMP_{OFMSW} + X_{BS}\, EBMP_{BS}$

$= (0.634)(201.5)+(0.366)(164.5)$

$= 127.751+60.207$

$= 187.958$

Step 7. Count CPI

$$CPI = \frac{\text{experimental } EBMP_{OFMSW-BS}}{\text{theoretical } EBMP_{OFMSW-BS}}$$

$$CPI = \frac{EBMP_{OFMSW-BS}}{X_{OFMSW}\ EBMP_{OFMSW} + X_{BS}\ EBMP_{BS}}$$

$$= \frac{200.2}{187.958}$$

$$= 1.065$$

Step 8. Count SEI-difference

$$\text{SEI-difference} = \text{experimental } EBMP_{OFMSW\text{-}BS} - \text{theoretical } EBMP_{OFMSW\text{-}BS}$$

$$= 200.2 - 187.958$$

$$= +12.242$$

Step 9. Count SEI and SEI (%)

$$SEI = \frac{\text{experimental } EBMP_{OFMSW-BS} - \text{theoretical } EBMP_{OFMSW-BS}}{\text{theoretical } EBMP_{OFMSW-BS}}$$

$$SEI = \frac{\text{SEI}-\text{difference}}{\text{theoretical } EBMP_{OFMSW-BS}}$$

$$= \frac{+12.242}{187.958}$$

$$= +0.065$$

$$SEI\ \% = \frac{\text{SEI}-\text{difference}}{\text{theoretical } EBMP_{OFMSW-BS}} \times 100\%$$

$$= +0.065 \times 100\%$$

$$= 6.5\ \%$$

Using the same working procedures, collected data, and all calculated data of $\frac{OM_{OFMSW}}{OM_{BS}}$, $\frac{VS_{OFMSW'}}{VS_{BS'}}$, VS_{total}, X_{OFMSW}, X_{BS}, theoretical TBMP, co-BDI are summarized in Table 4.

Table 4. Summarize calculated data and resulting CPI, SEI-difference, SER, SER (%)

OM	$\frac{OM_{OFMSW}}{OM_{BS}}$	X_{OFMSW}	X_{BS}	Experimental $EBMP_{OFMSW\text{-}BS}$	Theoretical $EBMP_{OFMSW\text{-}BS}$	CPI	SEI-difference	SER	SER (%)	Effect
OFMSW	100/0	$= \frac{39350}{39350}$ = 1	$= \frac{0}{39350}$ = 0	201.5	201.5	1	0	-	-	-
Co-digestion1	80/20	$= \frac{31480}{32618}$ = 0.965	$= \frac{1138}{32618}$ = 0.035	220.6	200.206	1.102	+20.394	0.102	10.2%	Synergistic
Co-digestion2	60/40	$= \frac{23610}{25886}$ = 0.912	$= \frac{2276}{25886}$ = 0.088	217.5	198.244	1.097	+19.256	0.097	9.7%	Synergistic
Co-digestion3	40/60	$= \frac{15740}{19154}$ = 0.822	$= \frac{3414}{19154}$ = 0.178	212.3	194.914	1.089	+17.386	0.089	8.9%	Synergistic
Co-digestion4	20/80	$= \frac{7870}{12422}$ = 0.634	$= \frac{4552}{12422}$ = 0.366	200.2	187.958	1.065	+12.242	0.065	6.5%	Synergistic
BS	0/100	$= \frac{0}{5690}$ = 0	$= \frac{5690}{5690}$ = 1	164.5	164.5	1	0	-	-	-

Through the designated procedures, the resulting theoretical $EBMP_{OFMSW\text{-}BS}$ and CPI values in Table 4 are shown to conform with the published theoretical $EBMP_{OFMSW\text{-}BS}$ and CPI data (Nielfa et al., 2015). In this study, all co-digestion parameters CPI, SEI-diffetence, SER, and SER (%) can be determined using the mathematical equations in Examples 2 and 3. The developed working procedures are suitable for co-digestion (mixing of two or more OM) calculations. The mathematical equations for the co-digestion metrics are summarized in Table 5.

Table 5. Determination of co-digestion metrics: processing parameters

co-BDI	CPI	SEI
$= \frac{\text{experimental } EBMP_{CO}}{\text{theoretical } TBMP_{CO}}$ $= \frac{\text{experimental } EBMP_{CO}}{X_1\, TBMP_1 + X_2\, TBMP_2 + \cdots + X_n\, TBMP_n}$ $= \frac{\text{experimental } EBMP_{CO}}{\Sigma X_i\, TBMP_i}$	$= \frac{\text{experimental } EBMP_{CO}}{\text{theoretical } EBMP_{CO}}$ $= \frac{\text{experimental } EBMP_{CO}}{X_1\, EBMP_1 + X_2\, EBMP_2 + \cdots + X_n\, EBMP_n}$ $= \frac{\text{experimental } EBMP_{CO}}{\Sigma X_i\, EBMP_i}$	$= \frac{(\text{experimental } EBMP_{CO}) - (\text{theoretical } EBMP_{CO})}{\text{theoretical } EBMP_{CO}}$ $= \frac{\text{experimental } EBMP_{CO} - \Sigma X_i\, EBMP_i}{\Sigma X_i\, EBMP_i}$

8. Importance of Mass Fraction of Volatile Solid (X_i)

The X_i is the core parameter in the calculations of anaerobic co-digestion. With reference to Table 1 and Example 1, the working procedure for determining X_i is developed. Regarding two or more OM ($n \geq 2$), the mathematical representation of X_i is established through a series of processing parameters as shown in Table 6.

Table 6. Determination mass fractions of volatile solids: processing parameters

Type of Parameter	Mathematical Representation (n = 2)	Mathematical Representation (n > 2)
Experimental parameter: OM_i	Mixing ratio: OM_1/OM_2	Mixing ratio: $OM_1/OM_2/\ldots OM_n$
Experimental parameter: VS_i	Mass ratio: VS_1/VS_2	Mass ratio: $VS_1/VS_2/\ldots VS_n$
Calculated parameter	Resulted mass: $VS_{1'} = OM_1 \times VS_1$ Resulted mass: $VS_{2'} = OM_2 \times VS_2$	Resulted mass: $VS_{1'} = OM_1 \times VS_1$ Resulted mass: $VS_{2'} = OM_2 \times VS_2$ Resulted mass: $VS_{n'} = OM_n \times VS_n$
	Resulted mass ratio: $VS_{1'}/VS_{2'}$	Resulted mass ratio: $VS_{1'}/VS_{2'}/\ldots VS_{n'}$
	$VS_{total} = VS_{1'} + VS_{2'}$	$VS_{total} = VS_{1'} + VS_{2'} + \ldots + VS_{n'}$
	$VS_i = \frac{VS_{total}}{OM_1 + OM_2}$	$VS_i = \frac{VS_{total}}{OM_1 + OM_2 + \cdots + OM_n}$
Calculated parameter	$X_1 = \frac{VS_1'}{VS_{total}} = \frac{VS_1'}{VS_1' + VS_2'}$ $X_2 = \frac{VS_2'}{VS_{total}} = \frac{VS_2'}{VS_1' + VS_2'}$	$X_1 = \frac{VS_1'}{VS_{total}} = \frac{VS_1'}{VS_1' + VS_2' + \ldots + VS_n'}$ $X_2 = \frac{VS_2'}{VS_{total}} = \frac{VS_2'}{VS_1' + VS_2' + \ldots + VS_n'}$ $X_n = \frac{VS_n'}{VS_{total}} = \frac{VS_n'}{VS_1' + VS_2' + \ldots + VS_n'}$
	$X_1 = \frac{(OM_1 \times VS_1)}{(OM_1 \times VS_1) + (OM_2 \times VS_2)}$ $X_2 = \frac{(OM_2 \times VS_2)}{(OM_1 \times VS_1) + (OM_2 \times VS_2)}$	$X_1 = \frac{(OM_1 \times VS_1)}{(OM_1 \times VS_1) + (OM_2 \times VS_2) + \cdots + (OM_n \times VS_n)}$ $X_2 = \frac{(OM_2 \times VS_2)}{(OM_1 \times VS_1) + (OM_2 \times VS_2) + \cdots + (OM_n \times VS_n)}$ $X_n = \frac{(OM_n \times VS_n)}{(OM_1 \times VS_1) + (OM_2 \times VS_2) + \cdots + (OM_n \times VS_n)}$

Based on the mixing ratio OM_1/OM_2 /…/OM_n (OM_i unit = kg) and mass ratio VS_1/VS_2/…/VS_n (VS_i unit = g/kg), the general mathematical equations for calculating X_i are established as follows:

$$X_1 = \frac{(OM_1 \times VS_1)}{(OM_1 \times VS_1)+(OM_2 \times VS_2)+\cdots+(OM_n \times VS_n)}$$

$$X_2 = \frac{(OM_2 \times VS_2)}{(OM_1 \times VS_1)+(OM_2 \times VS_2)+\cdots+(OM_n \times VS_n)}$$

$$X_n = \frac{(OM_n \times VS_n)}{(OM_1 \times VS_1)+(OM_2 \times VS_2)+\cdots+(OM_n \times VS_n)}$$

$$X_n = \frac{OM_n \times VS_n}{\sum OM_i \times VS_i}$$

$$\sum X_i = 1$$

The accuracy of theoretical $TBMP_{12}$ and theoretical $EBMP_{12}$ as well as co-BDI, CPI, and SRI rely on the validity of X_i.

8-1 Relationships between X_i, theoretical $TBMP_{CO}$, and theoretical $EBMP_{CO}$

When the co-digestion is mixed with two or more OM ($n \geq 2$), the mathematical representations of the theoretical $TBMP_{12}$ and theoretical $EBMP_{12}$ are shown in the following:

$$\text{Theoretical } TBMP_{CO} = \sum X_i \, TBMP_i$$

$$\text{Theoretical } EBMP_{CO} = \sum X_i \, EBMP_i$$

8-2 Relationships between X_i, Co-BDI, CPI, and SRI

The co-BDI metric is relevant to the experimental $EBMP_{CO}$ and theoretical $TBMP_{CO}$. Both the CPI and SEI metrics are correlated to the experimental $EBMP_{CO}$ and theoretical $EBMP_{CO}$. The mathematical equations of three co-digestion metrics are shown:

$$\text{co-BDI} = \frac{\text{experimental } EBMP_{CO}}{\text{theoretical } TBMP_{CO}} = \frac{\text{experimental } EBMP_{CO}}{\Sigma X_i \, TBMP_i}$$

$$\text{CPI} = \frac{\text{experimental } EBMP_{CO}}{\text{theoretical } EBMP_{CO}} = \frac{\text{experimental } EBMP_{CO}}{\Sigma X_i \, EBMP_i}$$

$$\text{SEI} = \frac{(\text{experimental } EBMP_{CO})-(\text{theoretical } EBMP_{CO})}{\text{theoretical } EBMP_{CO}} = \frac{\text{experimental } EBMP_{CO} - \Sigma X_i \, EBMP_i}{\Sigma X_i \, EBMP_i}$$

$$\text{SEI} = \text{CPI} - 1$$

9. Conclusion

Although the co-BDI, CPI, and SEI are important anaerobic co-digestion metrics, some articles do not provide supporting calculations and data that can be verified. This article demonstrates the working procedures for anaerobic calculations from mono-digestion to co-digestion. By identifying and recalculating all processing parameters in the co-digestion series, this article reaches the following conclusions:

9-1 Processing Parameters

Theoretical parameters come from calculations. The recalculations of co-digestion reveals that four sets of processing parameters are needed:

(i) Mono-digestion parameters: individual experimental $EBMP_i$ and theoretical $TBMP_i$

(ii) Mixing ratio, mass ratio, and mass fractions of volatile solids: $OM_1/OM_2/OM_i$, $VS_1/VS_2/VS_i$, and X_i

(iii) Co-digestion experimental parameter: experimental $EBMP_{CO}$

(iv) Co-digestion theoretical parameters: theoretical $EBMP_{CO}$ and theoretical $TBMP_{CO}$

9-2 Mass Fraction of Volatile Solid as the Core Parameter

(i) The working procedure for calculating X_i is developed and the mathematical equation is derived and shown as

“$X_n = \frac{(OM_n \times VS_n)}{(OM_1 \times VS_1)+(OM_2 \times VS_2)+\cdots+(OM_n \times VS_n)} = \frac{OM_n \times VS_n}{\sum OM_i \times VS_i}$”.

(ii) The X_i is identified as the core parameter in co-digestion calculations. The accuracy of theoretical $TBMP_{CO}$ and theoretical $EBMP_{CO}$ as well as co-BDI, CPI, and SRI rely on the validity of X_i. If X_i is invalid, miscalculations of co-digestion parameters and metrics will occur.

9-3 Co-digestion Metrics

Co-digestion experimental $EBMP_{CO}$, theoretical $EBMP_{CO}$, and theoretical $TBMP_{CO}$ are correlated parameters to establish the mathematical equations of co-BDI, CPI, and SEI.

(i) The mathematical relationships between theoretical $TBMP_{CO}$, theoretical $EBMP_{CO}$, and X_i are established as "theoretical $TBMP_{CO} = \sum X_i\ TBMP_i$" and "theoretical $EBMP_{CO} = \sum X_i\ EBMP_i$".

(ii) The mathematical equation of co-BDI is related to the experimental $EBMP_{CO}$ and theoretical $TBMP_{CO}$. It is demonstrated as "co-BDI = $\frac{\text{experimental } EBMP_{CO}}{\text{theoretical } TBMP_{CO}}$".

(iii) The co-digestion metrics CPI and SEI are relevant to the experimental $EBMP_{CO}$ and theoretical $EBMP_{CO}$. Their mathematical equations are established as: "CPI = $\frac{\text{experimental } EBMP_{CO}}{\text{theoretical } EBMP_{CO}}$" and "SEI = $\frac{(\text{experimental } EBMP_{CO})-(\text{theoretical } EBMP_{CO})}{\text{theoretical } EBMP_{CO}}$". In addition, the mathematical relationship between CPI and SEI is derived as "SEI = CPI – 1".

List of Abbreviations

AD	anaerobic digestion
BDI (%)	mono-digestion biodegradability index in percentage ratio
BDI	mono-digestion biodegradability index in numerical ratio
BEq	Buswell's equation
BMP	biomethane potential
co-BDI (%)	co-digestion biodegradability index in percentage ratio
co-BDI	co-digestion biodegradability index in numerical ratio
CPI	co-digestion performance index
experimental co-VCH_4	co-digestion experimental maximum amount of produced volumetric methane
experimental $EBMP_{CO}$	co-digestion experimental EBMP
EBMP	experimental biomethane potential
$EMBP_{co}$	co-digestion experimental biomethane potential
$EBMP_i$	mono-digestion experimental biomethane potential
$EBMP_{ij}$	co-digestion experimental biomethane potential
EPA	Environmental Protection Agency
kg	kilogram
L	liter
OM	organic matter
OM_i	mass of organic matter
PA	proximate analysis
SEI (%)	synergistic effect index in percentage ratio
SEI	synergistic effect index in numerical ratio
TBMP	theoretical biomethane potential
$TBMP_{co}$	co-digestion theoretical biomethane potential

$TBMP_i$	mono-digestion theoretical biomethane potential
$TBMP_{ij}$	co-digestion theoretical biomethane potential
theoretical co-VCH_4	co-digestion theoretical maximum amount of produced volumetric methane
theoretical $EBMP_{CO}$	co-digestion theoretical EBMP
theoretical $TBMP_{CO}$	co-digestion theoretical TBMP
TS	total solid of organic matter
VCH_4	mono-digestion experimental maximum amount of produced volumetric methane
VS	volatile solid of organic matter
VS_{added}	mass of added volatile solid
VS_i	concentration of volatile solid in gram per kilogram
$VS_{i'}$	resulted mass of volatile solid in gram
X_i	mass fraction of volatile solid

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Authors contributions

Dr. Pong Kau Yuen is responsible for designing the study, drafting, and revising the manuscript. Dr. Cheng Man Diana Lau is responsible for revising the manuscript. Kuok In Gabriel Yuen is responsible for figures drawing and data processing. All authors read and approved the final manuscript.

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There are no conflicts to declare.

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Data sharing statement

No additional data are available.

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