Optimizing Batch Size in a Flow-Oriented Synchronized Production

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Abstract

This study was prepared for a leading company, Miele GmbH, the global premium brand of domestic appliances and commercial machines in the field of laundry care, dishwashing and disinfection in Germany. The production line of Miele GmbH in Bielefeld, Germany was analyzed to develop a model that can be used for all the firms in the group.

Dynamic programming models are widely used by companies to efficiently meet the demand for a variety of products. In a flow shop, each product has to be processed by a number of machines in synchronized lines. The production smoothing problems under the presence of setup and processing times vary among the products. The master production-inventory problem of Miele GmbH was divided into two sub-problems which were concerned with determining the batch sizes and production sequences of products, respectively. A dynamic programming procedure was developed to solve the batching problem for the current problem. A dynamic computational study for the first case was conducted so that the solution method is effective in meeting the goals of the firm and efficient in its computational requirements.

Scientific problem –the firm has a problem of high logistics. Moreover, they want to decrease the cost of production in order to compete with their competitors. Their competitors start to make some of their production in low labour countries such as China. Miele is a special brand for upper level. However, they now have a more strict competition with new global players and one way to stay competitive in the market is to decrease the costs and find new market segments.

The aim of the research –The dynamic programming algorithm is suggested to them to decrease the costs. The numbers of products are decreased to explain the algorithm. An example with calculations of this algorithm was explained in this study. The number of variables and constraints can be increased. After the logic of algorithm is understood, it can be applied many similar problems. The algorithm can be developed by using different software such as Java. Then, the variables and values of algorithm can be input for the algorithm and the results can be gotten in a short time.

The object of the research – Inventory and Production mix projects.

The methods of the research –Dynamic Programming

Keywords: Dynamic programming, Batch size, JIT, Inventory, Production, Operation research

1. Introduction

With concept of globalization, trade barriers have been decreased, however the cost of different businesses increases. As a result, firms try to decrease the costs of material storage, warehouses, production and product distribution (Rimienė, 2007) Manufacturing industry is in transforming from the traditional high volume low-mix production to low volume high-mix production. The equipment used in the processes are worth millions of dollars, thus dedicating separate production lines to different products is not profitable. Therefore, the lines are operated as mixed-model production lines. The production runs are short enough to require several changeovers in a period of time. Different machines have different setup and processing times for different products.

Rau et al, (2008) recently, numerous valuable studies have been devoted to the integrated production–inventory problem for supply chains. Most of them have considered the situation of constant demand; few have studied other demand patterns. However, the demand rate with a time-varying pattern reflects the actual environment. Over the past two and a half decades, due to the phenomenon of product life cycle or boom-and-bust seasons, the demand pattern has been plausibly assumed to be a linear in production or inventory problems. In addition, a number of studies for a linear trend in demand have focused on replenishment or production policies for manufacturing systems; but, few have considered for supply chain application.

The optimization programming models contain several hundreds of variables and constraints. After deciding which product and how many of them will be produced. The sensitivity analysis can be carried out and the variables range can be found or changed. (Kurlavicius, 2009) Optimization means flexibility and this flexibility can be provided with linear or dynamic optimization programming. We have used dynamic programming logic to find a solution for the problem.

Finished and semi-finished products are stored in warehouse and it has no direct impact on production, but long time storing and not providing semi-finished products on time result in extra costs. (Rimienė, 2007) Storing semi-finished products in huge among machines and lines causes waiting and not using space well during production. The best way is to produce and send the semi-finished products to assembly lines in the firm. However, the firm has assembly lines in different firms for different products. They buy some parts from different suppliers in Germany and other countries. Moreover, they provide support for their suppliers. As a result, they need to control all steps of Supply Chain in a proper way. Dynamic Programming allows adding many constraints and this cannot be provided with existing model that they had used. They have used Economic Order Quantity with some changes to find batch-size in their production for each machine.

Moreover, Miele GmbH wants to sustain its high quality brand image in the market, but cost is the main source of competition now. Consumers mainly prefer products with lower costs. In the battle field of cost reduction, the firm is to decrease its all costs.

2. Theoretical Background

A literature review regarding the integrated production–inventory policy and linear trend in demand is as follows:

Ghare and Schrader (1963) were the first to consider ongoing deterioration of inventory with constant demand. Banerjee (1986) developed a joint economic lot size model for a single-buyer, single-vendor system. Goyal (1988) generalized Banerjee's model by relaxing the assumption of the lot-for-lot policy of the vendor. Hill (1989) derived a central-warehouse, multi-retailer model with shortage by using simulation. Lu and Posner (1994) introduced two heuristic procedures for the one-warehouse multi-retailer system. Ha and Kim (1997) made the analysis of integration between a buyer and a supplier by setting up the mathematical model in which the inventory cost of vendor is derived through a discontinuous saw-tooth inventory-level function. Wee and Jong (1998) studied the producer's integration between multipart and finished product with multi-lot-size for deteriorating items. Rau et al. (2008) provided an integrated inventory model for a deteriorating item in a three-echelon supply chain environment. Rosenblatt and Lee (1986) and Porteus (1986) developed EMQ (Economic Manufacturing Quantity) models. They assumed that the process is an in-control state at the beginning of the production run and it may shift to an out-of-control state after a certain time. Only good-quality products are manufactured in the in-control state and defective items are produced at the out-of-control state. These defective items are reworked at a cost. Khouja and Mehrez (1994) extended a classical inventory model to the cases where production rate is a variable under managerial control and production cost becomes a function of production rate. They showed that the optimal production rate is smaller than the production rate which minimized unit production cost. In their model, the process might shift to an out-of-control state after a certain time that follows exponential distribution. In out-of-control state, a certain percentage of total products are defective items which are reworked at a cost. Recently, Sana et al. (2010) developed the EMQ model in an imperfect production system where the defective items are sold at a reduced price. As a result, the demand function of defective items without reworking is a non-linear function of reduction rate. Wagner and Whitin (1958) discussed the discrete version of this problem, applying the dynamic programming approach. Khanra and Chaudhuri (2003) extended an EOQ (Economic Order Quantity) model of perishable items over a finite-time horizon, considering quadratic time-varying demand Adkins argued that the ‘‘True’’ value of the holding cost is not necessarily the best one to use in the EOQ equation. Instead, he suggested that the proper question to ask when seeking the parameters for the EOQ equation is: ‘‘what values will provide the proper lot size to meet aggregate inventory investment objectives?’’ Brown et al. showed that, under certain conditions, even relatively small lot-size errors could be extremely costly to a firm. They claimed that their result is important to theoreticians and practitioners due to the importance of lot sizing in computer-based production and inventory control systems. Jones cautioned that most manufacturers, who use the EOQ formula to minimize relevant annual costs, end up over estimating their lot sizes. This was attributed to the fact that most accountants fail to identify which costs are relevant.

According to Stivalla et al. (2010), Dynamic Programming (DP) is a powerful technique for solving any optimization problem for which an optimal solution can be efficiently computed from optimal solutions to its sub problems. The idea is to avoid re-computing the optimal solution to these sub problems by reusing previously computed values. Thus, for dynamic programming to be useful, the same sub problems must be encountered often enough while solving the original problem. Dynamic programming can be easily implemented by using either a ``bottom-up'' or ``top-down'' approach. In the ``bottom-up'' approach, the solution to every single sub problem is computed and stored in the dynamic programming matrix, starting from the smallest sub problems until the solution to the entire problem is finally computed.

Dynamic Programming (DP) represents a united framework for solving stochastic, multistage control problems found in process industries and other application areas. (Wong, 2009) Central to DP is the cost-to-go function (which scores the desirability of any arbitrary state) which can be theoretically obtained by solving (usually off-line) for the fixed-point of Bellman's optimality equations. Optimal control is achieved through the on-line solution of a single stage problem which reflects the trade-off between immediate costs (manifested through a single-stage term) and future costs reflected by the value function of a candidate next state). Systems with large state and action spaces suffer from a `curse of dimensionality'; where representing and obtaining the cost-to-go compactly and efficiently becomes highly nontrivial. To circumvent this, the authors proposed an Approximate Dynamic Programming (ADP) method for solving process control problems, which suffer all the more from the said curse due to the presence of continuous state and action spaces.

The basic idea is to use carefully-designed simulations to uncover a control-relevant part of the state space (which is a finite-sized subset of the original state space) and employ an appropriate function approximated generalization. The focus was mainly on the control of deterministic, nonlinear systems. For stochastic systems, however, the off-line and on-line computations involving Bellman's equations require a minimization over the sum of a single-stage cost and the expected value function of a candidate next state. Since an analytical expression for the expectation is usually unknown, solving such a problem may be cumbersome.

Höfferl (2009), the type of the dynamic problem considered is even more challenging in terms of its complexity. We do not have only one fixed person, who has to run through exactly one state of the stages from the starting point to the final destination. We have more optional entities at the starting point, which change their personality through the stages and all the decisions are interrelated. We do not even know which entities are used at what time at the starting state in the literature. It was first studied by Kosten (1967) & (Kosten, 1973), who described a custodian who dispatches trucks whenever the number of waiting passengers exceeds a certain threshold. Deb and Serfozo (1973) were the first to prove that in steady state the optimal decision rule of this system was monotone and therefore had a control limit structure, thereby proving the optimality of Kosten's custodian. They assume that the waiting cost per customer is an increasing function of the number of customers waiting in the queue.

Once the structure of an optimal policy is known, the primary problem is one of determining the expected costs for a given control strategy, and then using this function to find the optimal control strategy.

Powell (1985) was the first to introduce general holding and cancellation strategies, where a vehicle departure might be cancelled for a fixed period of time if a dispatch rule has not been satisfied within a period of time (reflecting, for example, the inability to keep a driver sitting and waiting). Powell and Humblet (1986) present a general, unified framework for the analysis of a broad class of (Markovian) dispatch policies, assuming stationary, stochastic demands. Reviews of this literature are contained in the Medhi (1984) study of the multiproduct single link problem by determining frequencies at which several products have to be shipped to minimize transportation and inventory costs. In Bertazzi et al. (2000) techniques of neuro-dynamic programming are implemented to approximate a stochastic, multiproduct version of the problem. Stivala et al (2010) implemented a method for parallelizing top-down dynamic programs in a straightforward way by a careful choice of a lock-free shared hash table implementation and randomization of the order in which the dynamic program computes its sub problems.

3. The Existing Production-Inventory Model of Miele GmbH

To solve the production-inventory cost problem of Miele GmbH, the production and inventory of firm of the group in Bielefeld was analyzed. Based on this analysis, a model was prepared. To explain the model, five different machines (M1-M5) of the firm were chosen as shown in Figure 1. There are small boxes between machines to store the parts if they cannot be sent to the next machine or they can be stored in the big warehouse as shown in Figure 1 of the firm in order to be reprocessed. Freight carriers are used between storage boxes and warehouse. The customers of the firm are either from the firm itself or from other firms. To determine the batch of each product and the order of production over a time period, this method was developed. Four different products (A, B, C and D) are selected to explain the system and calculations.

Parameters of Algorithm:

<u>Machines:</u>

- Number of different machines (M1-M5)

<u>Others:</u>

- Setup time for each machine is different
- Cycle time for each A, B, C and D parts
- Capacity of each machine
- Waiting times between machines (can be assumed zero)
- Storage between machines is limitless
- Batch size for each product
- Machine cost (€/h) (can be added to the whole cost)
- Stock cost
- Stock out cost resulted in penalty cost
- Staff cost (can be considered in production cost)
- Average setup cost for each machine related to each product

Decisions can be static. In a static model, it is assumed that all decisions are made at a single point of time. In our case, it will be shown how to use dynamic models to determine optimal decisions in multi-periods. Dynamic models arise when the decision maker makes decisions at more than one point in time. In a dynamic model, decisions made during the current period influence decisions made during future periods. For example, the firm wants to learn how many parts for each product to produce during each month. If it produces a large number of units during the current month, this would reduce the number of units that should be produced during future months. Two situations were developed to find an optimal inventory-production solution.

3.1 Alternative Production Methods

Two alternative production methods were suggested for the system. Both have some advantages and disadvantages. The second one is more complex but after the program is prepared, the calculations will be easier. A sample for the first situation of dynamic backward programming for a product was calculated and calculations are shown at the end of article.

<u>In the first situation:</u>

In the first situation, firm can produce the whole amounts of parts for each product demanded during the planning period in one setup. In this case, it is expected that the inventory costs will increase while the setup costs will decrease as shown in Table 1.

<u>In the second situation:</u>

In this case as shown in Table 2, four products are produced daily, but as seen, there are so many setups of machines leading to high setup costs. However, it is expected that the inventory cost will be less as compared with the first situation. The batch size for each case will be found and costs for each case will be compared. The differences between the first situation and the second situation are production costs, time spent for setup of machines, inventory costs, amount produced daily, machine working hours etc. The amount of demand plays an important role for each time period. In this case, fewer products can be produced while setups cause losing time. Changing the dies of a machine can take even hours. As seen from the table, four times the dies each day or period. The strategy of the firm plays an important role for the selection of production method. The strategy can change from time to time and the demand will be the main source of criteria. When there is a high demand for different products in a short time period. Then, the second situation can be used. The period of production can be even a week or month and in this case the second situation will be more advantageous.

How many parts for each case will be produced?

Which case should be selected for production?

Minimum cost found by each case will be the base for decisions.

The firm must determine which products (A, B, C and D) should be produced during each of the next four quarters (Periods or times). At the beginning of each period, the firm must decide how many parts should be produced during that period. For simplicity, it is assumed that parts manufactured during a period can be used to meet demand for that period.

4. An application of Inventory-Production Dynamic model at Miele GmbH

A solution of a production-inventory problem of the dynamic programming model is constructed. After that the application of the dynamic programming model is realized on the production-inventory problem at Miele GmbH. Following this, the results taken by application of the model are explained. In this research, the deterministic dynamic programming model is used. Because the deterministic dynamic programming model solves problems directly, not by estimating, it is selected for the model to find the batch size. On the other hand, while dynamic programming is an optimization technique a technique that parts the problems into series of such problems. This model will be constructed on a production-inventory problem. And then, application of this model will be realized using data belonging to four periods. Following these, the results determined by the application of model will be exalted. Then, an algorithm for four products will be given.

Production planning is a study that is used to take a series of decisions belonging to future parts of time in factories. One of the problems which are necessary for production planning is to balance the inventories. Developed Decision Models (DDM) related to production planning can be solved by special techniques of variables and functions. A DP approach is an effective method. In this situation, four different products during periods will be produced. Basic parameters Miele GmbH production-inventory model:

n : 1,2,3, and 4 planning period of time,

k: A, B, C, and D products

D_{kn} : Demand values of every n period of time,

L_{kn} : Inventory level of beginning of n^{th} period,

X_{kn} : Production values in n^{th} period,

h_k : Holding cost of a unit property,

$L(X_{kn})$: $h.L_{kn}$ holding cost,

$f(X_{kn})$: Production cost

C(n) : Total cost.

Steps of Model:

1. Decision step: Every period is considered as a decision step (n=1, 2, 3, and 4) and k =A, B, C, and D

2. State variable: Inventory value at the beginning (L_{kn}).

The inventory-value in a period of time is related to inventory, production and demand value in other periods.

$$L_{k(n+1)=} L_{kn} + X_{kn} - D_{kn}$$

3. *Decision variable:* Production value related to every period of time (X_{kn}).

4. *Optimum Decision Rule:* Optimal production which has the expression of $X_{kn}*(L_{kn})$ will be determined as the function beginning inventory value for every period of time.

5. *Total cost function related to every period of time is equal to sum of production and holding costs:*

$$C(n)= f(X_{kn}) + L(X_{kn}) = a + b X_{kn} + c X^2{}_{kn} + h.L_{kn}$$

6. *Objective function related to every period of time f (n):*

$$f_{kn}*(L_{kn})= \min(C_{kn} ((X_{kn},L_{kn}) + f_{k(n+1)*} L_{kn} +X_{kn} - D_{kn})),$$

$$0 \leq X_{kn} \leq K$$

$$X_{kn} + L_{kn} \geq D_{kn}$$

Limitations of the model taken from the firm:

- Production capacity of Miele is 15.000 units for a period of time for product A.
- Function of production cost is $f(X_n)= a + b X_n + c X^2{}_n$
- Maximum inventory unit should not pass more than 4.000 units in every period.
- Production amount $X_n* (L_n)$ which minimizes the total cost for every value of L_n named condition variable is determined in every decision step. An optimal decision rule that minimizes the total cost was developed for every beginning inventory starting from last to first periods of time. In application, data belongs to four periods of time. Amounts of demand, in four periods of time for product A by Miele GmbH are estimated as given below in Table 3:

Function of production cost:

$$f(X_n)=325 + 9 X_n + (1/500.000) X_n{}^2 \quad (€)$$

For simplification, $(1/500.000) X_n{}^2$ (€) part will not be added to the function. The aim is to show that the cost function can be parabolic or other kinds.

$$f(Xn)=325 + 9 X_n$$

a=325 is setup cost of production

Cost of keeping a unit in inventory is h = 1 €

Decision model of problem under these situations is:

$L_{n+1}= L_n + X_n - D_n$ (n= 1, 2, 3, 4)

$L_n + X_n \geq D_n$

$L_n \leq 4.000$

$X_n \leq 15.000$

Sn, X_n and $D_n \geq 0$ and Integer

Objective:

$$\text{Min } C_n = C_1 + C_2 + C_3 + C_4$$

Or

$$\text{Min } C_n = f(X_1) + h.L_1 + f(x_2) + h.L_2 + f(x_3) + h.L_3 + f(x_4)$$

It was started to find a solution the problem from fourth period of time (Backward solution of DP). See tables 5-8 for each period of time.

Due to high inventory costs, the firm should not keep inventory during 2, 3 and 4 periods as shown in Table 4. But if we decrease inventory cost and use our real production cost functions, what will happen?

If function of production cost:

$f(X_n)=325 + 9 X_n + (1/500.000) X_n^2$ (€) and Cost of keeping a unit in inventory is h = 0,6 €. It is suggested that the inventory should be carried out during the first three periods as shown in Table 4. These calculations are done for just one product. The same calculations should be carried out for each product. The length of period is defined according to time that demand is to be met.

The capacity of each machine for a product should be also considered. In this model, to avoid complexity, calculations for four products for the second situation are not calculated. For that purpose, dynamic programs such as java-based ones or solvers can be developed by considering firm specific requests. In this model for the first period, 11.000 A products will be produced with one setup. An assembly line was chosen for that product. The number of lines for different products can be increased. Now high setup costs prevent multi-product production for a period. A simulation model of the problem can be employed to test the sensitivity of the production. The range of resources consumed can be determined. Having applied a set of production rules to the given facts and modelling results within the module of decision analysis and inference, conclusions and suggestions are made.

5. Discussion and Conclusion

In today's global competition, none company has a secure place at the market. Everyone can fall down from the market segments. China, India and other low labour countries are searching for new markets and costumers. Especially, China has a huge amount of production potential and they make really cheap production. Miele GmbH has noticed that threat and wants to decrease the costs to gain competition position against it in its internal and external markets with high quality products. Optimization is a beneficial way to decrease costs. Thus, the dynamic programming was suggested and it was found that costs can be decreased in a flexible manner. A linear algorithm for the same problem was also developed.

The deterministic dynamic programming that applied in Miele GmbH is more appropriate model than dynamic linear programming. 4 periods deterministic dynamic programming calculations for A product was prepared to show how the model works. As seen from the model, the program searches for the optimal solution or the best alternative. When we consider 4 time periods, the production of A seems beneficial according to the model in first situation. In the same line, 4 different products can be produced but it is not seen as profitable. If the number of machines decreases, then in one period, four different products can be profitable with low setup costs. There is a high demand for each product when compared with the full capacity. But when the amount of demand for each product decreases, the multi-product production can be more profitable.

The firm decided to develop a program based on that model by using Java Programming. When the huge production amount of the firm is considered, this model provides very beneficial results. The second situation is a more complex model. The number of parameters and calculations increases but with suitable software, the calculations can be carried out in short times. A simulation program could be a helpful tool to illustrate the model. Following this, in order to approach zero inventories in every period of time, the firm should decrease the costs of production and holding. But, the quality of product should stay the same, when Miele GmbH reaches its purposes, Just-In-Time (JIT) production can be adopted by the firm. The firm has very long experience in production of high quality products and high value markets. It should enter in middle value markets.

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Table 1. One product production per day and one setup case

Time / Period	Period1	Period2	Period3	Period4
Time1	Setup for A and Production of A parts			
Time2	Setup for B and Production of B parts			
Time3	Setup for C and Production of C parts			
Time4	Setup for D and Production of D parts			

Table 2. Many products manufactured per time with many setups

Time / Period	Period1			Period2			Period3			Period4	
Time1	Setup for A	A parts	A parts	Setup for B	B parts	B parts	Setup for C	C parts	C parts	Setup for D	D parts
Time2	Setup for A	A parts	A parts	Setup for B	B parts	B parts	Setup for C	C parts	C parts	Setup for D	D parts
Time3	Setup for A	A parts	A parts	Setup for B	B parts	B parts	Setup for C	C parts	C parts	Setup for D	D parts
Time4	Setup for A	A parts	A parts	Setup for B	B parts	B parts	Setup for C	C parts	C parts	Setup for D	D parts

Table 3. Amounts of demand according to periods

Periods (n)	**Amounts of Demand (D_n)**
1	12.000
2	13.500
3	15.000
4	10.000

Table 4. Results of Model for a Product

			End of period	**COST**		
Period	**Li**	**xi**	**inventory**	**Production**	**Holding**	**Total**
1	4000	8000	0	72.325,00 €	4.000,00 €	76.325,00 €
2	0	12000	0	121.825,00 €	0,00 €	121.825,00 €
3	0	15000	0	135.325,00 €	0,00 €	135.325,00 €
4	0	10000	0	90.325,00 €	0,00 €	90.325,00 €
Total Cost				**419.800,00 €**	**4.000,00 €**	**423.800,00 €**

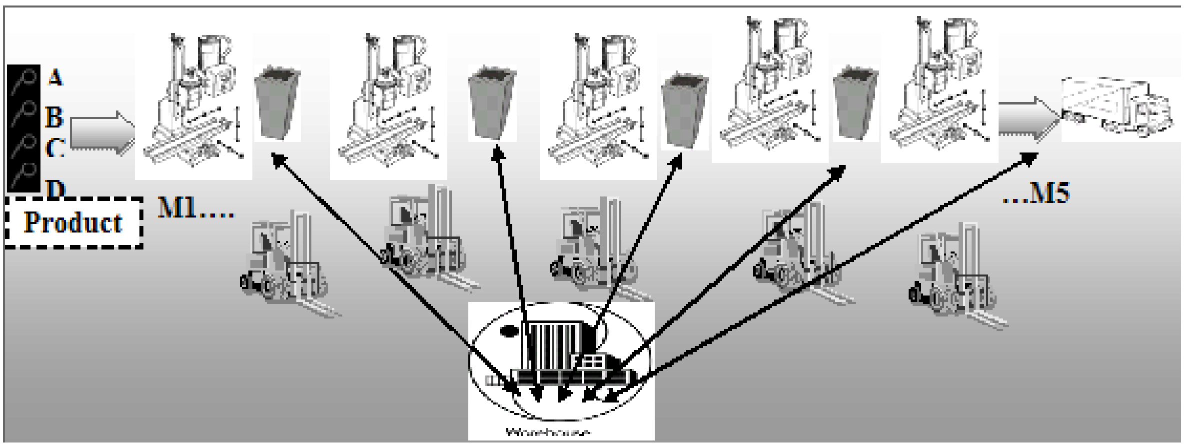

Figure 1. The design of system

Appendix

Solution:

Table 5. Fourth Period

		Fourth Period			
		COST			
L4	**X4**	**Production**	**Holding c.**	**Total**	**End of Period Inventory(L5)**
	10000	**90.325,00 €**	**0,00 €**	**90.325,00 €**	**0**
	11000	99.325,00 €	0,00 €	99.325,00 €	1.000
	12000	108.325,00 €	0,00 €	108.325,00 €	2.000
	13000	117.325,00 €	0,00 €	117.325,00 €	3.000
0	14000	126.325,00 €	0,00 €	126.325,00 €	4.000
	9500	**85.825,00 €**	**500,00 €**	**86.325,00 €**	**0**
	10500	94.825,00 €	500,00 €	95.325,00 €	1000
	11500	103.825,00 €	500,00 €	104.325,00 €	2000
	12500	112.825,00 €	500,00 €	113.325,00 €	3000
500	13500	121.825,00 €	500,00 €	122.325,00 €	4000
	9000	**81.325,00 €**	**1.000,00 €**	**82.325,00 €**	**0**
	10000	90.325,00 €	1.000,00 €	91.325,00 €	1000
	11000	99.325,00 €	1.000,00 €	100.325,00 €	2000
	12000	108.325,00 €	1.000,00 €	109.325,00 €	3000
1000	13000	117.325,00 €	1.000,00 €	118.325,00 €	4000
	8500	**76.825,00 €**	**1.500,00 €**	**78.325,00 €**	**0**
	9500	85.825,00 €	1.500,00 €	87.325,00 €	1000
	10500	94.825,00 €	1.500,00 €	96.325,00 €	2000
	11500	103.825,00 €	1.500,00 €	105.325,00 €	3000
1500	12500	112.825,00 €	1.500,00 €	114.325,00 €	4000
	8000	**72.325,00 €**	**2.000,00 €**	**74.325,00 €**	**0**
	9000	81.325,00 €	2.000,00 €	83.325,00 €	1000
	10000	90.325,00 €	2.000,00 €	92.325,00 €	2000
	11000	99.325,00 €	2.000,00 €	101.325,00 €	3000
2000	12000	108.325,00 €	2.000,00 €	110.325,00 €	4000
	7500	**67.825,00 €**	**2.500,00 €**	**70.325,00 €**	**0**
	8500	76.825,00 €	2.500,00 €	79.325,00 €	1000
	9500	85.825,00 €	2.500,00 €	88.325,00 €	2000
	10500	94.825,00 €	2.500,00 €	97.325,00 €	3000
2500	11500	103.825,00 €	2.500,00 €	106.325,00 €	4000
	7000	**63.325,00 €**	**3.000,00 €**	**66.325,00 €**	**0**
	8000	72.325,00 €	3.000,00 €	75.325,00 €	1000
	9000	81.325,00 €	3.000,00 €	84.325,00 €	2000
	10000	90.325,00 €	3.000,00 €	93.325,00 €	3000
3000	11000	99.325,00 €	3.000,00 €	102.325,00 €	4000
	6500	**58.825,00 €**	**3.500,00 €**	**62.325,00 €**	**0**
	7500	67.825,00 €	3.500,00 €	71.325,00 €	1000
	8500	76.825,00 €	3.500,00 €	80.325,00 €	2000
	9500	85.825,00 €	3.500,00 €	89.325,00 €	3000
3500	10500	94.825,00 €	3.500,00 €	98.325,00 €	4000
	6000	**54.325,00 €**	**4.000,00 €**	**58.325,00 €**	**0**
	7000	63.325,00 €	4.000,00 €	67.325,00 €	1000
	8000	72.325,00 €	4.000,00 €	76.325,00 €	2000
	9000	81.325,00 €	4.000,00 €	85.325,00 €	3000
4000	10000	90.325,00 €	4.000,00 €	94.325,00 €	4000

Table 6. Third Period

		Third Period					
		COST			**Future Cost**		
L3	**X4**	**Production**	**Holding c.**	**Total**	**End of Period Invent(L5)**	**F4(S4)**	**Total Cost**
0	**15000**	**135.325,00 €**	**0,00 €**	**135.325,00 €**	**0**	**90.325,00 €**	**225.650,00 €**
500	**14500**	**130.825,00 €**	**500,00 €**	**131.325,00 €**	**0**	**90.325,00 €**	**221.650,00 €**
	15000	135.325,00 €	500,00 €	135.825,00 €	500	85.825,00 €	221.650,00 €
1000	**14000**	**126.325,00 €**	**1.000,00 €**	**127.325,00 €**	**0**	**90.325,00 €**	**217.650,00 €**
	15000	135.325,00 €	1.000,00 €	136.325,00 €	**1000**	81.325,00 €	217.650,00 €
1500	**13500**	**121.825,00 €**	**1.500,00 €**	**123.325,00 €**	**0**	**90.325,00 €**	**213.650,00 €**
	14500	130.825,00 €	1.500,00 €	132.325,00 €	1000	81.325,00 €	213.650,00 €
	15000	135.325,00 €	1.500,00 €	136.825,00 €	1500	78.325,00 €	215.150,00 €
2000	**13000**	**117.325,00 €**	**2.000,00 €**	**119.325,00 €**	**0**	**90.325,00 €**	**209.650,00 €**
	14000	126.325,00 €	2.000,00 €	128.325,00 €	1000	81.325,00 €	209.650,00 €
	15000	135.325,00 €	2.000,00 €	137.325,00 €	2000	74.325,00 €	211.650,00 €
2500	**12500**	**112.825,00 €**	**2.500,00 €**	**115.325,00 €**	**0**	**90.325,00 €**	**205.650,00 €**
	13500	121.825,00 €	2.500,00 €	124.325,00 €	1000	81.325,00 €	205.650,00 €
	14500	130.825,00 €	2.500,00 €	133.325,00 €	2000	74.325,00 €	207.650,00 €
	15000	135.325,00 €	2.500,00 €	137.825,00 €	2500	70.325,00 €	208.150,00 €
3000	**12000**	**108.325,00 €**	**3.000,00 €**	**111.325,00 €**	**0**	**90.325,00 €**	**201.650,00 €**
	13000	117.325,00 €	3.000,00 €	120.325,00 €	1000	81.325,00 €	201.650,00 €
	14000	126.325,00 €	3.000,00 €	129.325,00 €	2000	74.325,00 €	203.650,00 €
	15000	135.325,00 €	3.000,00 €	138.325,00 €	3000	66.325,00 €	204.650,00 €
3500	**11500**	**103.825,00 €**	**3.500,00 €**	**107.325,00 €**	**0**	**90.325,00 €**	**197.650,00 €**
	12500	112.825,00 €	3.500,00 €	116.325,00 €	1000	81.325,00 €	197.650,00 €
	13500	121.825,00 €	3.500,00 €	125.325,00 €	2000	74.325,00 €	199.650,00 €
	14500	130.825,00 €	3.500,00 €	134.325,00 €	3000	66.325,00 €	200.650,00 €
	15000	135.325,00 €	3.500,00 €	138.825,00 €	3500	62.325,00 €	201.150,00 €
4000	**11000**	**99.325,00 €**	**4.000,00 €**	**103.325,00 €**	**0**	**90.325,00 €**	**193.650,00 €**
	12000	108.325,00 €	4.000,00 €	112.325,00 €	1000	81.325,00 €	193.650,00 €
	13000	117.325,00 €	4.000,00 €	121.325,00 €	2000	74.325,00 €	195.650,00 €
	14000	126.325,00 €	4.000,00 €	130.325,00 €	3000	66.325,00 €	196.650,00 €
	15000	135.325,00 €	4.000,00 €	139.325,00 €	4000	62.325,00 €	201.650,00 €

Table 7. Second Period

		Second Period					
		COST			**Future Cost**		
L2	**X2**	**Production**	**Holding c.**	**Total**	**End of Period Invent(L3)**	**F3(S3)**	**Total Cost**
	13500	**121.825,00 €**	**0,00 €**	**121.825,00 €**	**0**	**225.650,00 €**	**347.475,00 €**
	14500	130.825,00 €	0,00 €	130.825,00 €	1.000	217.650,00 €	348.475,00 €
0	15000	135.325,00 €	0,00 €	135.325,00 €	1.500	213.650,00 €	348.975,00 €
	13000	**117.325,00 €**	**500,00 €**	**117.825,00 €**	**0**	**225.650,00 €**	**343.475,00 €**
	14000	126.325,00 €	500,00 €	126.825,00 €	1000	217.650,00 €	344.475,00 €
500	15000	135.325,00 €	500,00 €	135.825,00 €	2000	209.650,00 €	345.475,00 €
	12500	**112.825,00 €**	**1.000,00 €**	**113.825,00 €**	**0**	**225.650,00 €**	**339.475,00 €**
	13500	121.825,00 €	1.000,00 €	122.825,00 €	1000	217.650,00 €	340.475,00 €
	14500	130.825,00 €	1.000,00 €	131.825,00 €	2000	209.650,00 €	341.475,00 €
1000	15000	135.325,00 €	1.000,00 €	136.325,00 €	2500	205.650,00 €	341.975,00 €
	12000	**108.325,00 €**	**1.500,00 €**	**109.825,00 €**	**0**	**225.650,00 €**	**335.475,00 €**
	13000	117.325,00 €	1.500,00 €	118.825,00 €	1000	217.650,00 €	336.475,00 €
	14000	126.325,00 €	1.500,00 €	127.825,00 €	2000	209.650,00 €	337.475,00 €
1500	15000	135.325,00 €	1.500,00 €	136.825,00 €	3000	201.650,00 €	338.475,00 €
	11500	**103.825,00 €**	**2.000,00 €**	**105.825,00 €**	**0**	**225.650,00 €**	**331.475,00 €**
	12500	112.825,00 €	2.000,00 €	114.825,00 €	1000	217.650,00 €	332.475,00 €
	13500	121.825,00 €	2.000,00 €	123.825,00 €	2000	209.650,00 €	333.475,00 €
	14500	130.825,00 €	2.000,00 €	132.825,00 €	3000	201.650,00 €	334.475,00 €
2000	15000	135.325,00 €	2.000,00 €	137.325,00 €	3500	197.650,00 €	334.975,00 €
	11000	**99.325,00 €**	**2.500,00 €**	**101.825,00 €**	**0**	**225.650,00 €**	**327.475,00 €**
	12000	108.325,00 €	2.500,00 €	110.825,00 €	1000	217.650,00 €	328.475,00 €
	13000	117.325,00 €	2.500,00 €	119.825,00 €	2000	209.650,00 €	329.475,00 €
	14000	126.325,00 €	2.500,00 €	128.825,00 €	3000	201.650,00 €	330.475,00 €
2500	15000	135.325,00 €	2.500,00 €	137.825,00 €	3500	197.650,00 €	335.475,00 €
	10500	**94.825,00 €**	**3.000,00 €**	**97.825,00 €**	**0**	**225.650,00 €**	**323.475,00 €**
	11500	103.825,00 €	3.000,00 €	106.825,00 €	1000	217.650,00 €	324.475,00 €
	12500	112.825,00 €	3.000,00 €	115.825,00 €	2000	209.650,00 €	325.475,00 €
	13500	121.825,00 €	3.000,00 €	124.825,00 €	3000	201.650,00 €	326.475,00 €
3000	14500	130.825,00 €	3.000,00 €	133.825,00 €	4000	193.650,00 €	327.475,00 €
	10000	**90.325,00 €**	**3.500,00 €**	**93.825,00 €**	**0**	**225.650,00 €**	**319.475,00 €**
	11000	99.325,00 €	3.500,00 €	102.825,00 €	1000	217.650,00 €	320.475,00 €
	12000	108.325,00 €	3.500,00 €	111.825,00 €	2000	209.650,00 €	321.475,00 €
	13000	117.325,00 €	3.500,00 €	120.825,00 €	3000	201.650,00 €	322.475,00 €
3500	14000	126.325,00 €	3.500,00 €	129.825,00 €	4000	193.650,00 €	323.475,00 €
	9500	**85.825,00 €**	**4.000,00 €**	**89.825,00 €**	**0**	**225.650,00 €**	**315.475,00 €**
	10500	94.825,00 €	4.000,00 €	98.825,00 €	1000	217.650,00 €	316.475,00 €
	11500	103.825,00 €	4.000,00 €	107.825,00 €	2000	209.650,00 €	317.475,00 €
	12500	112.825,00 €	4.000,00 €	116.825,00 €	3000	201.650,00 €	318.475,00 €
4000	13500	121.825,00 €	4.000,00 €	125.825,00 €	4000	193.650,00 €	319.475,00 €

Table 8. First Period

		First Period					
		COST			**Future Cost**		
L1	**X1**	**Production**	**Holding c.**	**Total**	**End of Period Invent (L2)**	**F2(S2)**	**Total Cost**
	12000	**108.325,00 €**	**0,00 €**	**108.325,00 €**	**0**	**347.475,00 €**	**455.800,00 €**
	13000	117.325,00 €	0,00 €	117.325,00 €	1.000	339.475,00 €	456.800,00 €
	14000	126.325,00 €	0,00 €	126.325,00 €	2.000	331.475,00 €	457.800,00 €
0	15000	135.325,00 €	0,00 €	135.325,00 €	3.000	323.475,00 €	458.800,00 €
	11500	**103.825,00 €**	**500,00 €**	**104.325,00 €**	**0**	**347.475,00 €**	**451.800,00 €**
	12500	112.825,00 €	500,00 €	113.325,00 €	1000	339.475,00 €	452.800,00 €
	13500	121.825,00 €	500,00 €	122.325,00 €	2000	331.475,00 €	453.800,00 €
	14500	130.825,00 €	500,00 €	131.325,00 €	3000	323.475,00 €	454.800,00 €
500	15500	139.825,00 €	500,00 €	140.325,00 €	4000	315.475,00 €	455.800,00 €
	11000	**99.325,00 €**	**1.000,00 €**	**100.325,00 €**	**0**	**347.475,00 €**	**447.800,00 €**
	12000	108.325,00 €	1.000,00 €	109.325,00 €	1000	339.475,00 €	448.800,00 €
	13000	117.325,00 €	1.000,00 €	118.325,00 €	2000	331.475,00 €	449.800,00 €
	14000	126.325,00 €	1.000,00 €	127.325,00 €	3000	323.475,00 €	450.800,00 €
1000	15000	135.325,00 €	1.000,00 €	136.325,00 €	4000	315.475,00 €	451.800,00 €
	10500	**94.825,00 €**	**1.500,00 €**	**96.325,00 €**	**0**	**347.475,00 €**	**443.800,00 €**
	11500	103.825,00 €	1.500,00 €	105.325,00 €	1000	339.475,00 €	444.800,00 €
	12500	112.825,00 €	1.500,00 €	114.325,00 €	2000	331.475,00 €	445.800,00 €
	13500	121.825,00 €	1.500,00 €	123.325,00 €	3000	323.475,00 €	446.800,00 €
1500	14500	130.825,00 €	1.500,00 €	132.325,00 €	4000	315.475,00 €	447.800,00 €
	11000	**99.325,00 €**	**2.000,00 €**	**101.325,00 €**	**0**	**347.475,00 €**	**448.800,00 €**
	12000	108.325,00 €	2.000,00 €	110.325,00 €	1000	339.475,00 €	449.800,00 €
	13000	117.325,00 €	2.000,00 €	119.325,00 €	2000	331.475,00 €	450.800,00 €
	14000	126.325,00 €	2.000,00 €	128.325,00 €	3000	323.475,00 €	451.800,00 €
2000	15000	135.325,00 €	2.000,00 €	137.325,00 €	4000	315.475,00 €	452.800,00 €
	9500	**85.825,00 €**	**2.500,00 €**	**88.325,00 €**	**0**	**347.475,00 €**	**435.800,00 €**
	10500	94.825,00 €	2.500,00 €	97.325,00 €	1000	339.475,00 €	436.800,00 €
	11500	103.825,00 €	2.500,00 €	106.325,00 €	2000	331.475,00 €	437.800,00 €
	12500	112.825,00 €	2.500,00 €	115.325,00 €	3000	323.475,00 €	438.800,00 €
2500	13500	121.825,00 €	2.500,00 €	124.325,00 €	4000	315.475,00 €	439.800,00 €
	9000	**81.325,00 €**	**3.000,00 €**	**84.325,00 €**	**0**	**347.475,00 €**	**431.800,00 €**
	10000	90.325,00 €	3.000,00 €	93.325,00 €	1000	339.475,00 €	432.800,00 €
	11000	99.325,00 €	3.000,00 €	102.325,00 €	2000	331.475,00 €	433.800,00 €
	12000	108.325,00 €	3.000,00 €	111.325,00 €	3000	323.475,00 €	434.800,00 €
3000	13000	117.325,00 €	3.000,00 €	120.325,00 €	4000	315.475,00 €	435.800,00 €
	8500	**76.825,00 €**	**3.500,00 €**	**80.325,00 €**	**0**	**347.475,00 €**	**427.800,00 €**
	9500	85.825,00 €	3.500,00 €	89.325,00 €	1000	339.475,00 €	428.800,00 €
	10500	94.825,00 €	3.500,00 €	98.325,00 €	2000	331.475,00 €	429.800,00 €
	11500	103.825,00 €	3.500,00 €	107.325,00 €	3000	323.475,00 €	430.800,00 €
3500	12500	112.825,00 €	3.500,00 €	116.325,00 €	4000	315.475,00 €	431.800,00 €
	8000	**72.325,00 €**	**4.000,00 €**	**76.325,00 €**	**0**	**347.475,00 €**	**423.800,00 €**
	9000	81.325,00 €	4.000,00 €	85.325,00 €	1000	339.475,00 €	424.800,00 €
	10000	90.325,00 €	4.000,00 €	94.325,00 €	2000	331.475,00 €	425.800,00 €
	11000	99.325,00 €	4.000,00 €	103.325,00 €	3000	323.475,00 €	426.800,00 €
4000	12000	108.325,00 €	4.000,00 €	112.325,00 €	4000	315.475,00 €	427.800,00 €