International Journal of Business and Management
www.ccsenet.org/journal.html
Vol. 3, No. 11
November 2008

Optimization of Security Investment Portfolio Based on Improved Simulated Annealing Algorithm

Lin Xiao & Zhongyuan Wu
School of Management, Tianjin Polytechnic University, Tianjin 300384, China

Abstract

Considered of multi-local optimal solution in investment portfolio, the paper proposed an improved Simulated Annealing Algorithm. This algorithm applied multiplicator function which transform security investment model into nonrestraint optimization problem, optimized key process and parameter of basal Simulated Annealing Algorithm. It also solved initial temperature and how to get the result, balanced speed and precision, then improved efficiency of algorithm. At last, this paper used date to validate the feasibility of algorithm.

Keywords: Simulated Annealing Algorithm, Security investment portfolio, Parameter optimization

1. Foreword

The aim of security investment is achieve return while the investor should take on risk. In order to increase return and avoid or decrease risk, security investment portfolio which use mathematics to seek balance between expected return and investment risk. Markowitz-an America economist found a new model, which means Mean-Variance model. This model applied mean and variance of investment portfolio return distributing to represent expected return and risk, which is the basic of modern investment portfolio. Classical M-V model:

$$
\begin{aligned}
& Min \quad f(X) = X^{\mathrm{T}}VX \\
& s.t \quad R^{\mathrm{T}}X = \delta \\
& \sum_{i=1}^{n} x_i = 1 \\
& x_i \geq 0 \qquad i = 1, 2, \mathrm{L}\ , n \qquad (1)
\end{aligned}
$$

Where n the number of security investment, the expected return of security is i is r_i, X_i means funds proportion of security i in total funds, $V = (\sigma_{ij})$ and δ stand for Covariance matrix of r_i and Expected profit.

Solve a More restrictive conditions is the crux of model, so the difficult issue is solving universal optimization. Most non-linear programming model has much solution with a lot of portion optimization. Therefore, different initial solution has different optimization. Until now, many scholars research the issue. An iterative algorithm base on gradient slope is the most fundamental algorithm. This paper proposes an optimization of security investment portfolio based on improved Simulated Annealing Algorithm.

2. Simulated Annealing Algorithm

Simulated Annealing Algorithm base on the annealing solid principles, which is thermodynamic process: solid will be heated to melt, and then slowly cooled until solidified into a regular crystal. When heating, with the internal temperature rises, solid particles gradually turns into a state of disorder, then internal energy increase; while slowly and gradually cooling at the orderly, then each temperature achieves a balanced, finally reaches ground state at normal atmospheric temperature so internal energy reduces into minimum. In 1982, Kirkpateick first realizes the similarity between solid annealing process and combinatorial optimization problems, then introduce into Metropolis criteria, which using Monte Carlo to simulate the process of solid achieving thermal equilibrium in a normal atmospheric temperature and cooling schedule with the control algorithms. With each temperature T of decreasing process, an iterative process-"new judge to accept or discard" is continuing. Simulated Annealing Algorithm is a heuristic, which not only can iterate to the direction of objective function reducing, but also accept deterioration in a certain range , therefore simulated annealing algorithm can jump out the "trap" of local optimization to have overall optimal solution of portfolio optimization problem. In theory, as long as the simulation process is fully, simulated annealing algorithm can convergence into achieve global optimum with a probability.

The steps of simulated annealing algorithm can be expressed as follows:

Step1: Initialization. Given the initial temperature $T = T_0$, randomly generated initial state $X = X^0$, determine the temperature of each iteration times L;

Step2: In temperature T, $k = 1, 2, \ldots, L$ repeat step3 and step6

Step3: Produce a neighboring state X';

Step4: Calculate the incremental evaluation function $\Delta f = f(X) - f(X')$;

Step5: If $\Delta f < 0$, accept new solution; while, if $\exp(-\Delta f / T)$ more than a random number

Between 0 and 1 accepted.

Step6: If the conditions met termination then output the current optimal solution, ending calculation.

Step7: Annealing, turn into step2.

3. Algorithm and choice of key parameters

3.1 Portfolio optimization model

This problem to solve is model (1); we apply multiplication function which can transform model M-V into unconstrained optimization problem of penalty coefficients to solve:

$$\min \quad L(x) = f(X) + M(\max(0, (\delta - R^{\mathrm{T}} X)) + \left|\sum_{i=1}^{n} x_i - 1\right|)$$

$$s.t. \quad X^c \le X \le X^f \tag{2}$$

Where X^c and X^f is ceiling and threshold, in this paper X^c and X^f is 0 and 1, M is penalty function.

When using simulated annealing to solve these problems, the algorithm relies on a set of parameters to control the process, the selection of parameters directly related to the algorithm's merits. Then the following discusses the key parameters.

3.2 The choice of initial temperature T

One of the important factors, which affect research possibility of simulated annealing, is the setting of initial temperature T_0. The higher initial temperature also with the possibility of a global optimum, so the number of iterative will be substantially increased and reducing the possibility of the algorithm; instead, the global search capabilities will be affected. Therefore, the initial temperature is set at a reasonable time, which can maximize the quality of the final solution. After several experiments, we use the following formula to determined initial temperature.

$$T_0 = \frac{\overline{\Delta f^{+}}}{\ln \dfrac{m_2}{m_2 \chi - m_1 (1 - \chi)}} \tag{3}$$

In the formula, m_1 and m_2 stands for reduce of objective function and increase of number respectively, Δf^{+} is average incremental, χ is the initial acceptance rate of new solution-must approach 1.

3.3 Terminal temperature and length of Markove Chain

Along with the slowly reduced, simulated annealing algorithm gradually converges into global optimal solution. Only the terminal temperature is fully small, algorithm can get high quality of the optimal solution; however, in order to reduce the amount of computation, terminal temperature should not be too high. Usually the length of Markove Chain is $100n$, n is dimension of solution.

3.4 Formation of adjacent state

Adjacent state is that from current to achieve with one moving, just new solution. The formation of new state generally with a random number generator, then select an initial state and zoom it. Assuming at one state, X^k is $(X_1, X_2, \ldots X_n)$, format function at adjacent state is

$$x_i^{k+1} = x_i^k \pm \alpha(b - a) \tag{4}$$

Where, α is a random number at $(0,1)$, the range of X_i is $[a, b]$.

3.5 Attenuation function of temperature

We use attenuation function of temperature to simulate the process of cooling down, only in small decrement can simulate solid that has minimal energy at lowest temperature. So, the process of attenuation is naturally lower cooling, can reach balance at each state and finally achieve ground state. This paper approaches heuristic criteria for cooling:

$$T_k = \frac{T_0}{k^m}, k = 1, 2, \ldots \quad (5)$$

Where T_0 is initial temperature; m is constant above or equal 1, generally 3; T_k is temperature after k .

3.6 Acceptance criteria of adjacent state

When X turned from X^k to X^{k+1}, it should have a criterion that judges which state can accept. This paper adopts Metropolis criteria, with the corresponding transfer probability:

$$p = \exp(\Delta f / T) \quad (6)$$

Where Δf is the change in the value of objective function. Form this formula, we can see that simulated Annealing Algorithm not only accept optimum solution but also accept deterioration in certain range.

4. Examples

We assume security investment portfolio $X = (X_1, X_2, X_3)$ which formed by the three securities, where X_1, X_2, X_3 stands for the proportion of funds. According to the historic data between 1981 and 2000, we calculate expect rate of profit $E(X) = (11.30, 18.50, 7.55)$,risk $\delta^2(X) = (2.74, 11.02, 0.08)$, covariance matrix

$$Cov(X) = \begin{bmatrix} 1 & -0.1959 & -0.0289 \\ -0.1959 & 1 & -0.0134 \\ -0.0289 & -0.0134 & 1 \end{bmatrix}$$

Assume expect rate of profit is 13%, the globally optimal solution is $X = (0.51, 0.32, 0.17)$, optimal objective function is 0.0151.

We use simulated annealing algorithm to solve the problem and then according to formula 3 determine the initial temperature after 50 tests, accept probability χ is 0.95; terminal temperature is 0.001; length of Markove Chain is 300; attenuation function of temperature is 3.

Table 1. the solution of simulated annealing algorithm

Initial temperature	Value of objective function	Relative error (%)
(0.3,0.3,0.4)	0.01510	0
(0, 0, 1)	0.01498	0.794701987
(0.5,0.5,0)	0.01510	0
(0.1,0.5,0.4)	0.01510	0
(0,1,0)	0.01511	0.033225166

From table 1, we can see that the algorithm has not sensitive to initial temperature. Global optimal solution could achieve at any initial value with small error.

5. Conclusions

In this study, we proposed simulated annealing to solve the model which based on investment portfolio model of Markowitz, and its key processes and parameters were optimized to achieve a balance between speed and accuracy, so that has strong robustness. According to the results of research, we can see that the simulated annealing algorithm has the advantages to solve the problem.

References

Michael W.Trosset. (2001). *What is simulated annealing, Optimization and Engineering*. 2,201-213.

S.Kirkpatrick, C.D.Gelatt. (1983). *Science, Optimization by Simulated Annealing*, 220:617-680.

E.H.L. (1989).Aarts, Simulated Annealing and Boltzmann machines, John Wiley and Sons.

Zhang,Wei, Zhou, Qun & Sun, Denbao. (2001). Genetic Algorithm for the optimal security investment portfolio. *Quantitative and Technical Economics*. 10:114-116.